Exam for the lecture

"Econometrics I"

for students in the M.Sc. programmes winter term 2017/18

Please fill in us	sing block letters	::				
Name: Surname				Vorname: Name		
Studiengang Course of study:				Geburtsort: Place of birth		
Matrikelnummer: Student ID				Bachelor University:		
Declaration:						
			PLEASE S	SIGN!!!		
I hereby decla	re that I am able	to be examined.				
Signature:						
Preliminary re	emarks:					
examiner.		enrolment/matri		on all paper sheets provided for the examiner.	answers by the	
Result: (TO B	E FILLED IN (ONLY BY THE	EXAMINER!)			
Problem	1	2	3	Home Assignment	Σ	
Points earned						
Grade						
Kiel,				Professor Dr. Kai C	 Carstensen	

Examination in Econometrics I

(Winter Term 2017/18)

Examination regulation

February 15, 2018, 12:30 - 13.30

Preliminary remarks:

- 1. Please read these instructions carefully!
- 2. At the beginning of the exam, fill in the cover sheet and hand in after the exam is finished!
- 3. You are permitted to use the following auxiliary tools:
 - (a) a non-programable pocket calculator,
 - (b) the formulary for Econometrics I without notes!
- 4. Write your name and enrollment (matriculation) number on every sheet of paper!
- 5. Don't use a pencil!
- 6. The exam problems are printed on 3 pages plus 2 double sheets for answers. Check your exam for completeness!
- 7. Round your solutions to 4 decimal places.
- 8. You have 60 minutes in total to answer the exam questions.

Good luck!

Problem 1 (20 points)

Consider the following economic model

$$y = \beta_1 x_1 + \beta_2 x_2 + u.$$

All variables are demeaned. The **second** regressor x_2 is thought to be endogenous and an instrument z is available for it. From a sample of 100 observations the following sample moments are obtained:

$$\sum_{t=1}^{T} x_{1,i}^2 = 3, \quad \sum_{i=1}^{N} x_{2,i}^2 = 1.5 \quad \sum_{i=1}^{N} x_{1,i} x_{2,i} = 2$$

$$\sum_{t=1}^{T} x_{1,i} y_i = 1.8, \quad \sum_{i=1}^{N} x_{2,i} y_i = 0.9$$

$$\sum_{t=1}^{T} x_{1,i} z_i = 2, \quad \sum_{i=1}^{N} x_{2,i} z_i = 0.8 \quad \text{and} \quad \sum_{i=1}^{N} y_i z_i = 0.8.$$

- 1. (3P) What properties are necessary for z to be a good instrument? Explain shortly!
- 2. **(6P)** Estimate $\beta = (\beta_1, \beta_2)'$ by IV estimation!
- 3. (8P) Furthermore you obtain

$$\widehat{Avar}(\hat{\beta}_{IV}) = \begin{pmatrix} 0.5 & 0.25 \\ 0.25 & 1 \end{pmatrix}.$$

Suppose $\gamma = (\beta_1 - \beta_2)^3$ can be given structural interpretation. Test whether $\gamma = -0.5$. If you have not solved task 2, take $\hat{\beta}_1 = 1.25$ and $\hat{\beta}_2 = 1.8$.

4. (3P) Assume now that more than one suitable instrument is available for x_2 . State a procedure to estimate the parameters consistently taking all available instruments into account. Describe its mechanism in 2 or 3 sentences.

Problem 2 (20 points)

Prof. Wild Guess would like to examine the relationship between vote share and expenditure of the Republican party in the US. Therefore he sets up the following model:

$$\ln(RV) = \beta_{10} + \gamma_{12} \ln(RE) + \beta_{11}(DI - RI) + \beta_{12}RPS + u_1$$

$$\ln(RE) = \beta_{20} + \gamma_{21} \ln(RV) + \beta_{22} \ln(I) + \beta_{23} \ln(G) + \beta_{24} \ln(ED) + u_2$$

where RV is the Republican vote share in an electoral district in percent, RI (DI) is a dummy whether a Republican (Democrat) is incumbent, RE is the Republican expenditure in 1000\$, RPS is an index for the republican vote share of previous elections, I is median family income in 1000\$, G the Gini coefficient and ED the median years of schooling completed. Unfortunately he can't remember the content of the Econometrics I lecture (since he is old and never cared about it), thus he seeks for aid. Can you help him, answering the following questions?

- 1. (2P) What type of model is he faced with? Explain in one sentence!
- 2. **(4P)** Is the model identified?

After you have answered these questions he opens his econometrics software package and produces the following output:

$$\ln(RV) = 27.6 + 1.24 \ln(RE) - 17.72(DI - RI) + 4.8RPS$$

$$\ln(RE) = -31.7 - 0.9 \ln(RV) + 0.8 \ln(I) + 4.29 \ln(G) - 3.76 \ln(ED)$$

He is interested in correctly interpreting the effect of schooling on Republican votes:

3. **(9P)** What are the **effects** of schooling on Republic votes? Calculate and interpret! [Hints: Consider your answer for part 1. State all necessary calculations.]

He seems very pleased with your answers as you leave his office, but a few month later he calls you on the phone: "Do you remember my estimation problem you helped me with? Well, I've handed in a paper to the famous journal American Bulls Hit Bingo, but got a rejection, since the supervisor – a colleague of mine – believes that the Republican vote share is influenced by schooling, which is not accounted for in my model!"

4. (3P)How can be convince his colleague that his model is adequate? Explain in detail!

Although he finds your explanation reasonable, he doesn't believe that he will be able to convince his colleague.

5. (2P) Is there another way to incorporate the suggestions made by his colleague which might lead to an acceptance of his paper? Explain!

Problem 3 (20 points)

A.(9P) Consider the linear model

$$E[y|x] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 x_2.$$

Assume that $E[x_2|x_1] = 0$ and x_1 is demeaned. Derive the linear projection $L(y|1, x_1, x_2)$.

B.(5P) Show that the feasible generalized least squares (FGLS) estimator

$$\hat{\beta}_{FGLS} = (X'\hat{\Omega}^{-1}X)^{-1}X'\hat{\Omega}^{-1}Y$$

for the linear model $Y = X\beta + U$ is unbiased if the assumptions for GLS are met. Additionally you can assume that $\hat{\Omega}$ is a function of the regressors only. State in each step the used assumptions!

C.(6P) Prove that the Wald statistic under H_0 : $\mathbf{R}\boldsymbol{\theta} = \mathbf{r}$ is asymptotically χ_Q^2 distributed for consistent, asymptotically normal estimators $\hat{\boldsymbol{\theta}}$, i.e., for estimator with the property $\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \stackrel{d}{\to} \operatorname{Normal}(\mathbf{0}, \mathbf{V})$. Note that Q denotes the number of restrictions under H_0 and $Q \leq K$ where K is the number of parameters.