

Problem Set 1: Recap

Exercises

1. Car makers want to estimate a demand model of the form

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + u_i, \quad (i = 1, \dots, N)$$
$$Y = X\beta + U, \quad U \sim \mathcal{N}(0, \sigma^2 I_N),$$

where

y_i = Sales in thousand cars in country i (log)

x_{i1} = disposable income in billion Euro in country i (log)

x_{i2} = Gas price in Euro/l in country i (log).

They collect the sample statistics

$$X'X = \begin{bmatrix} 10 & 10 & 20 \\ 10 & 25 & 10 \\ 20 & 10 & 80 \end{bmatrix}, \quad (X'X)^{-1} = \begin{bmatrix} 0.38 & -0.12 & -0.08 \\ -0.12 & 0.08 & 0.02 \\ -0.08 & 0.02 & 0.03 \end{bmatrix}, \quad X'Y = \begin{bmatrix} 20 \\ 36 \\ 16 \end{bmatrix}$$

and $Y'Y = 63.52$.

- (a) How many observations do they have?
- (b) Estimate β . How to interpret $\hat{\beta}_2$?
- (c) Assume the gas price increases by 7.5 % due to a reduction in oil production by OPEC. What is the expected change in car sales?
- (d) Estimate the error variance.
- (e) Calculate the standard errors of $\hat{\beta}$.
- (f) Calculate and interpret the coefficient of determination R^2 .
- (g) Test $H_0 : \beta_1 = 0.6$ with a two-sided test and $\alpha = 0.1$.
- (h) What is the economic meaning of $H_0 : \beta_2 = 0$ vs. $H_1 : \beta_2 \neq 0$? Can you formulate a related pair of hypotheses that makes more sense economically?
- (i) Test the joint hypothesis $H_0 : \beta_2 = -0.1$ and $\beta_1 = 0.6$ with $\alpha = 0.05$.

2. Consider the following model:

$$\begin{aligned}y_i &= \beta_0 + \beta_1 x_{i1} + e_i, & (i = 1, \dots, N) \\ \mathbf{y} &= X\boldsymbol{\beta} + \mathbf{e},\end{aligned}$$

where y_i is tax revenue in hundreds of € per capita in 2019 in country i and x_{i1} is the population share in percent of people with an academic degree in country i . The following sample statistics are obtained

$$X'X = \begin{bmatrix} 50 & 1960 \\ 1960 & 83800 \end{bmatrix}, \quad X'y = \begin{bmatrix} 1680 \\ 73250 \end{bmatrix}.$$

- (a) How many observations do we have?
- (b) Estimate $\boldsymbol{\beta}$. How to interpret $\hat{\beta}_0$?
- (c) Estimate the error variance, use $y'y = 67000$.
- (d) Assume in the following that $Var(\mathbf{e}) = \sigma^2 I_N$ (What does that mean?). Calculate the standard errors of $\hat{\boldsymbol{\beta}}$.
- (e) A researcher claims that an increase in academics in the population share by 1 percentage point is associated with an increase of tax revenue by more than 100 € per capita (in 2019) on average. Test it on a 10 % level! Which critical value do you suggest and why?
- (f) Shortly discuss how ability and measurement issues can bias our estimate $\hat{\boldsymbol{\beta}}$!