Advanced Statistics I

Probability calculus

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It all starts with data



Data have been around for some time in one form or another;

... one would like to make use of them.

This takes us to quantitative methods:

- reduce the amount of data to a presentable form
- construct models
- draw conclusions
- make predictions

Today's outline

Kick-off

- Probability and statistics
- 2 The course
- 3 Q & A
- 4 Up next

Outline

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Quantitative methods?

We focus here on building and analyzing models with uncertainty. 1

The basic premise is the presence of some apparent or intrinsic randomness:

- Sampling variability
- Variables not under control
- ... the works.

¹Related, not irrelevant topics: measuring and communicating uncertainty, making decisions under uncertainty, design of experiments, etc.

Randomness and statistics

Say you want to draw conclusions from a data sample.

Will your conclusions hold for more than the given sample?

Typically yes, if the sample is representative.

- Random samples are as representative as it gets without prior knowledge.
- But then we need to understand how randomness affects our analyses.

More generally, we aim to construct and analyze models including random components.

No magic!

In particular, we are concerned with the distribution of possible outcomes of nondeterministic phenomena, and

... rely heavily on probability theory to do the job.

Spelled out:

- We can't forecast single outcomes!
- But we may be able to tell how likely they are.

Why probability theory?

Actually, it is not that obvious that we should resort to probability theory.²

But:

- Distributions arise naturally in a sampling framework
- Probability calculus also delivers the tools to manipulate distributions.
- There are several interpretations for the notion of probability, but there is only one probability calculus.
- This allows for a transparent/comprehensible use of probability theory, which is essential for the acceptance of statistical analyses.

²Fuzzy logic also allows for uncertainty.

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Aim of the course

This course is, together with AdvStat II, essential for deploying quantitative methods.

After successfully participating, you will become ...

- ... quite familiar with probability calculus,
- and, in particular,
 - ... able to work with models having (fully characterized) stochastic components.

Outline I

- Elements of Probability Theory:
 - Sample Space and Events
 - Probability
 - Operation Properties of the Probability Function
 - Onditional Probability
 - Independence
 - Total Probability Rule and Bayes' Law
- Random Variables and their Probability Distributions:
 - Univariate Random Variables
 - Univariate Cumulative Distribution Functions
 - Multivariate Random Variables
 - Marginal Distributions
 - 6 Conditional Distributions
 - 6 Independence of Random Variables

Outline II

- Moments of Random Variables:
 - Expectation of a Random Variable
 - Expectation of a Function of Random Variables
 - Conditional Expectation
 - Moments of a Random Variable
 - Moment-Generating Functions
 - Joint Moments and Moments of Linear Combinations
 - Means and Variances of Linear Combinations of Random Variables
- Parametric Families of Density Functions:
 - Discrete Density Functions
 - 2 Continuous Density Functions
 - Normal Family of Densities
 - Exponential Class of Distributions
 - 6 (Multivariate) Extensions

Outline III

- Basic Asymptotics:
 - Convergence of Number and Function Sequences
 - Onvergence Concepts for Sequences of Random Variables
 - Weak Laws of Large Numbers
 - Central Limit Theorems
 - Asymptotic Distributions of Functions for Asymptotically Normally Distributed Random Variables

Textbooks

Main ones

- Mood, A. M., Graybill, F. A. and D.C. Boes (1974, 3rd ed.).
 Introduction to the Theory of Statistics. McGraw-Hill.
- Mittelhammer, R. C. (1996). Mathematical Statistics for Economics and Business. Springer.

Other useful ones

- Wassermann, L. (2004). All of Statistics: A Concise Course in Statistical Inference. Springer.
- Casella, G. and R. Berger (2002, 2nd ed.). Statistical Inference. Duxbury.
- Rohatgi, V. K. und A. K. Saleh (2001, 2nd ed.). An Introduction to Probability Theory and Mathematical Statistics. John Wiley & Sons.
- Linton, O. (2017). Probability, Statistics and Econometrics. Academic Press.

Materials & workflow

- Detailed lecture notes via OLAT ... but use wisely: http://pss.sagepub.com/content/25/6/1159
- Weekly slides & videos will be available in due time (OLAT)
- Stipped classroom:
 - Prepare for class using screencasts
 - In class we discuss details and have Q&As (on site and via BigBlueButton/OLAT)

Tutorial

- Mariia Okuneva, Uliana Zaspa
- Problem sets will be available in due time via OLAT
- 3 You have the choice between three different time slots
- non-compulsory PC tutorial (R): every second week (tutored by Mariia Okuneva and Uliana Zaspa, online)
- May earn bonus points

Why R

- Open source and free check out https://cran.r-project.org
- Packages for almost anything
- High interest
- ٥

Exam & grades

- Written exam (pass with at least 50 out of 100 pts)
 problems similar to, or even the same as the pen & paper tutorial
- probably online
- either way you're allowed to use a formulary (OLAT in due time)
- don't forget the bonus points (max 15 on top of written exam)
- Extra: collection of problems and old exams (OLAT, around December)

Office hours

When you have questions outside class:

- Try first the OLAT forum
- Then the weekly Q&A sessions
- Also ask directly, mdeme@stat-econ.uni-kiel.de, mokuneva@stat-econ.uni-kiel.de or uzaspa@stat-econ.uni-kiel.de
- otherwise by appointment

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Have I forgotten anything?

Any specific questions?

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Coming up

Events and Probability