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Examination in Econometrics I

(Winter Term 2018/19)

Examination regulation

March 25, 2019 , 8.30 - 9.30

Preliminary remarks:

1. Please read these instructions carefully!
2. At the beginning of the exam, fill in the cover sheet and hand in after the exam is finished!
3. You are permitted to use the following auxiliary tools:
   (a) a non–programable pocket calculator,
   (b) **the formulary for Econometrics I without notes!**
4. Write your name and enrollment (matriculation) number on every sheet of paper!
5. Don't use a pencil!
6. The exam problems are printed on 3 pages. Check your exam for completeness!
7. **Round your solutions to 4 decimal places.**
8. For all tests use a significance level of 5%, if nothing else is specified.
9. You have 60 minutes in total to answer the exam questions.

Good luck!

<u>Problem 1</u> (13 points)

1. **(4P)** Explain briefly which assumptions might be violated when OLS is inconsistent. Furthemore, give 3 reasons why this might be the case.

2. **(3P)** Discuss the problem of an omitted variable bias (OVB) using a diagram. How would you deal with it (3-4 sentences in total)?

Consider the following model:

$$\begin{aligned} \log(wage) = \beta_0 + \beta_1 exper + \beta_2 tenure + \beta_3 married + \beta_4 south \\ + \beta_5 urban + \beta_6 black + \beta_7 educ + \gamma abil + \nu \end{aligned} \tag{1}$$

where *exper* is the labor market experience, *tenure* denotes the number of years with current employer, *married* is a dummy variable equal to unity if married, *south* is a dummy variable for the southern region, *urban* is a dummy variable for living in an SMSA (Standard Metropolitan Statistical Area), black is a race indicator, and *educ* denotes years of schooling. *abil* refers to the unobserved ability for which you have no data available. ν has zero mean and is uncorrelated with all the variables in the model (including *abil*). The unobservable *abil* is thought to be correlated with *educ*.

3. **(4P)** You have a single proxy variable *iq* for unobserved ability. Suppose the linear projection of *iq* on all regressors given in (1) is

$$iq = \delta_0 + \delta_1 abil + u, \tag{2}$$

   where $\delta_1 \neq 0$. By construction, u has zero mean and is uncorrelated with all regressors of (1). Assume u is also uncorrelated with ν. Show that you cannot consistently estimate (1) by OLS when you use *iq* as a proxy for ability.

4. **(2P)** Which estimator would you apply instead to consistently estimate (1) when you use *iq* as a proxy for ability? Explain briefly.

Problem 2 (19 points)

You are given the model

$$y = \beta_1 x_1 + u$$

where the scalar variable x_1 is correlated with u. Therefore, two instruments $\boldsymbol{z} = (z_1, z_2)$ are used for which $E(u^2|\boldsymbol{z}) = \sigma^2$ holds. Furthermore, the instruments are uncorrelated to u and are not perfectly correlated with each other. The following sample statistics are available based on a sample of 100 observations:

$$\begin{pmatrix} \sum_{i=1}^N z_{1i}^2 & \sum_{i=1}^N z_{1i}z_{2i} \\ \sum_{i=1}^N z_{2i}z_{1i} & \sum_{i=1}^N z_{2i}^2 \end{pmatrix} = \begin{pmatrix} 3.8 & 0.5 \\ 0.5 & 5.2 \end{pmatrix}$$

$$\begin{pmatrix} \sum_{i=1}^N z_{1i}^2 & \sum_{i=1}^N z_{1i}z_{2i} \\ \sum_{i=1}^N z_{2i}z_{1i} & \sum_{i=1}^N z_{2i}^2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.27 & -0.03 \\ -0.03 & 0.19 \end{pmatrix}$$

$$\begin{pmatrix} \sum_{i=1}^N z_{1i}x_{1i} \\ \sum_{i=1}^N z_{2i}x_{1i} \end{pmatrix} = \begin{pmatrix} 3.2 \\ 4.7 \end{pmatrix}$$

$$\begin{pmatrix} \sum_{i=1}^N z_{1i}y_i \\ \sum_{i=1}^N z_{2i}y_i \end{pmatrix} = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix}$$

The parameters of the regression of x_1 on $\boldsymbol{z}$ are estimated as $\hat{\gamma}_1 = 0.73$ and $\hat{\gamma}_2 = 0.83$ with $\widehat{Avar}(\hat{\boldsymbol{\gamma}}) = \begin{pmatrix} 0.12 & 0.05 \\ 0.05 & 0.11 \end{pmatrix}$.

1. **(4P)** Which estimation technique is able to incorporate both instruments z_1 and z_2 to estimate β_1? Describe its mechanism in 2 or 3 sentences!
2. **(7P)** Test for joint relevance of $\boldsymbol{z} = (z_1, z_2)$ at the 5% level and state the conclusion!
3. **(3P)** Estimate β_1 using both instruments!
4. **(5P)** Calculate $\widehat{Avar}(\hat{\beta}_1)$ of your previous estimation using $\hat{\sigma}^2 = 1.2$.

Problem 3 (20 points)

The following model jointly determines the number of policemen ($npol$, in thousands) and the crime rate ($crate$, annual number of crimes per thousand citizens) in a city:

$$\begin{aligned} crate &= \delta_{10} + \gamma_{12} npol + \delta_{11} csize + \delta_{12} youngmen + u_1 \\ npol &= \delta_{20} + \gamma_{21} crate + \delta_{21} csize + \delta_{22} nfire + \delta_{23} taxinc + u_2, \end{aligned}$$

where $csize$ denotes the size of the city (in thousands of citizens), $youngmen$ is the proportion of young men in the population of the city (in percent), $nfire$ denotes the number of firefighters in the city and $taxinc$ the tax income of the city (in millions of euros). Assume that $csize$, $youngmen$, $nfire$ and $taxinc$ are each uncorrelated to the error term of their own equation. Based on a random sample of N cities, you obtain the following estimated model, where all effects are statistically significant:

$$\hat{\mathbf{\Delta}} = \begin{pmatrix} \hat{\delta}_{10} & \hat{\delta}_{20} \\ \hat{\delta}_{11} & \hat{\delta}_{21} \\ \hat{\delta}_{12} & 0 \\ 0 & \hat{\delta}_{22} \\ 0 & \hat{\delta}_{23} \end{pmatrix} = \begin{pmatrix} -10.25 & -8.53 \\ 2.26 & 0.0192 \\ 2.89 & 0 \\ 0 & 0.0060 \\ 0 & 0.016 \end{pmatrix}$$

$$\hat{\mathbf{\Gamma}} = \begin{pmatrix} -1 & \hat{\gamma}_{21} \\ \hat{\gamma}_{12} & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0.0112 \\ -21.54 & -1 \end{pmatrix}$$

1. **(4P)** Discuss identification of each equation.
2. **(2P)** What is the direct (structural) effect of tax income on the number of policemen in a city? Interpret!
3. **(5P)** What is the total effect of the number of firefighters in a city on the crime rate? Interpret!
4. **(5P)** A newspaper claims based on your regression results: "The presence of firefighters on the street discourages potential criminals." Comment on this statement and sketch a diagram for explanation!
5. **(4P)** Now suppose that γ_{12} and γ_{21} are equal to zero. What kind of model is this? Explain briefly! Do you obtain consistent estimates by using equationswise OLS? Why or why not?

Problem 4 (8 points)

Consider the estimator $\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X} + \boldsymbol{C})^{-1}\boldsymbol{X}'y$ for the regression model $y = \boldsymbol{X}\boldsymbol{\beta} + u$, where $\boldsymbol{C}$ is a matrix of fixed numbers. Assume the standard conditions $E(u|\boldsymbol{x}) = 0$ and $plim\left(\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{x}_i'\boldsymbol{x}_i\right) = \boldsymbol{A}$ hold, with $\boldsymbol{A}$ being an invertible matrix of fixed numbers. Furthermore, $\boldsymbol{X}'\boldsymbol{X} + \boldsymbol{C}$ is invertible.

1. **(4P)** Show that the estimator is biased if $\boldsymbol{C} \neq \boldsymbol{0}$.
2. **(4P)** Show that the estimator is consistent if $\boldsymbol{C}$ is a matrix of fixed numbers independent of the sample size.