Problem Set 5: Inference

Review the Concepts and Proofs

1. Define type I and type II error.
2. What are size and power of a test? What is the significance level?
3. Explain how to set up null and alternative hypotheses of a test. Is there any relationship to the significance level?
4. Show that the squared t-statistic is exactly the same as the Wald statistic for one parameter restriction, i.e., show that the Wald test is a generalization of the t-test.
5. Would you recommend to use a heteroscedasticity robust estimator (e.g., White estimator) of the covariance matrix of $\hat{\boldsymbol{\beta}}$? Why or why not?

Exercises

1. Consider the following linear model

$$y_i = \beta_0 + x_i\beta_1 + e_i, \quad e_i \sim (0, \sigma^2), \quad i = 1, \ldots, N,$$

where e_i is iid. To estimate this model the following data is given:

$$X'X = \begin{pmatrix} 100 & -50 \\ -50 & 150 \end{pmatrix} \quad X'y = \begin{pmatrix} -60 \\ 200 \end{pmatrix}$$

$$y'y = 300$$

   (a) Perform a Wald test for the null hypothesis $\beta_0 = 0$ and $\beta_1 = 1$.

   (b) Now suppose your sample only contains 12 observations. Is the Wald test still suitable? If not, suggest a different test and state additional necessary assumptions for it.

2. Consider the following model for the relationship between savings (S) and income (I) (for which all OLS assumptions are assumed to hold):

$$log(S_i) = \beta_0 + \beta_1 log(I_i) + u_i.$$

The following sample moments and estimates are given:

$$\sum_{i=1}^{N} x_i' x_i = \begin{pmatrix} 100 & 950 \\ 950 & 9970 \end{pmatrix} \quad \sum_{i=1}^{N} x_i' y_i = \begin{pmatrix} 150 \\ 2000 \end{pmatrix}$$

$$\left(\sum_{i=1}^{N} x_i' x_i \right)^{-1} = \begin{pmatrix} 0.1055 & -0.0101 \\ -0.0101 & 0.0011 \end{pmatrix} \quad \sum_{i=1}^{N} y_i' y_i = 600$$

$$\hat{\beta} = \begin{pmatrix} -4.28 \\ 0.61 \end{pmatrix} \quad \hat{\sigma}^2 = 0.2565$$

$$\sum_{i=1}^{N} \hat{u_i}^2 x_i' x_i = \begin{pmatrix} 3100 & 32100 \\ 32100 & 333000 \end{pmatrix}$$

$$\left(\sum_{i=1}^{N} x_i' x_i \right)^{-1} \left(\sum_{i=1}^{N} \hat{u_i}^2 x_i' x_i \right) \left(\sum_{i=1}^{N} x_i' x_i \right)^{-1} = \begin{pmatrix} 0.0677 & -0.0025 \\ -0.0025 & 0.0032 \end{pmatrix}$$

(a) Estimate the asymptotic standard error of $\hat{\beta}_1$ under homoscedasticity and heteroscedasticity!

(b) A researcher claims that an increase in income by 1% typically is associated with an increase in savings on average by 0.55%. Test this hypothesis on a 5% level under homoscedasticity and heteroscedasticity! What can you conclude?

(c) Which of the two variance structures above would you suggest for the relationship between savings and income? Explain!