

Exam for the lecture

"Econometrics I"
for students in the M.Sc. programmes
winter term 2017/18

03.04.2018

Please fill in using block letters:

| | | |
|--|--|--------------------------------------|
| Name: <i>Surname</i> | | Vorname: <i>Name</i> |
| Studiengang <i>Course of study:</i> | | Geburtsort: <i>Place of birth</i> |
| Matrikelnummer: <i>Student ID</i> | | Bachelor University: |

Declaration:

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|---|
| PLEASE SIGN!!! |
| <p>I hereby declare that I am able to be examined.</p> <p style="text-align: center;">_____</p> <p style="text-align: center;">Signature:</p> |

Preliminary remarks:

- Write down your name and enrolment/matriculation number on all paper sheets provided for answers by the examiner.
- To write down your answers, use only the paper provided by the examiner.

Result: (TO BE FILLED IN ONLY BY THE EXAMINER!)

| Problem | 1 | 2 | 3 | 4 | Home Assignment | Σ |
|---------------|---|---|---|---|-----------------|----------|
| Points earned | | | | | | |
| Grade | | | | | | |

Kiel,

Professor Dr. Kai Carstensen

Examination in Econometrics I
(Winter Term 2017/18)

Examination regulation

April 03, 2018 , 14:30 - 15.30

Preliminary remarks:

1. Please read these instructions carefully!
2. At the beginning of the exam, fill in the cover sheet and hand in after the exam is finished!
3. You are permitted to use the following auxiliary tools:
 - (a) a non-programable pocket calculator,
 - (b) **the formulary for Econometrics I without notes!**
4. Write your name and enrollment (matriculation) number on every sheet of paper!
5. Don't use a pencil!
6. The exam problems are printed on 4 pages plus 5 pages for answers. Check your exam for completeness!
7. **Round your solutions to 4 decimal places.**
8. For all tests use a significance level of 5%, if nothing else is specified.
9. You have 60 minutes in total to answer the exam questions.

Good luck!

Problem 1 (15 points)

Consider the following regression for US college students:

$$colGPA = \beta_1 hsGPA + \beta_2 PL + u \quad (1)$$

where $colGPA$ and $hsGPA$ denote the average exam grade in college and high school (higher grade is better). PL is a binary variable indicating whether a student takes extra private lessons during college time. Interest lies on estimating the causal effect of private lessons on $colGPA$.

1. **(2P)** Do you think it makes sense to include $hsGPA$ in the regression? Briefly explain your answer.
2. **(4P)** Suppose some college students have a part-time job to support themselves while others can rely on scholarships or their parents. Explain precisely how and why the time spent on the job may affect your estimate of β_2 !
3. **(7P)** Suppose now (1) is a valid model and OLS yields $\hat{\beta}_1 = 1.5$ and $\hat{\beta}_2 = 50$ based on a sample of 100 students. Additionally, you obtain

$$\widehat{Avar}(\hat{\beta}) = \begin{pmatrix} 5 & 10 \\ 10 & 120 \end{pmatrix}.$$

A colleague of yours is questioning whether your causal effect of PL on $colGPA$ is significant. Additionally he does not believe that success in high school is relevant for success in college. Test on a 5%-level for joint significance of β_1 and β_2 !

4. **(2P)** Explain in two or three sentences the difference between a proxy and an instrument!

Name and Enrollment Number:

Problem 2 (12 points)

Consider the regression $y = \exp(\mathbf{x}\boldsymbol{\theta}) + u$ with objective function $q(x, \theta) = (y - \exp(\mathbf{x}\boldsymbol{\theta}))^2$. Let $\mathbf{x} = (x_1 \ x_2 \ \cdots \ x_k)$ and $\boldsymbol{\theta} = (\theta_1 \ \theta_2 \ \cdots \ \theta_k)'$. Additionally you can assume that the all regressors are strictly exogenous.

1. **(3P)** Derive the score.
2. **(3P)** Derive the Hessian.
3. **(4P)** Calculate the partial effect of x_k on $E[y|\mathbf{x}]$. What is the difference to a linear model? Explain briefly.
4. **(2P)** Suppose \mathbf{x} contains different weather variables and y denotes the temperature in degree Celsius. Is the model above suitable for this case? Why or why not?

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Problem 3 (15 points)

Suppose the following production function of firms $Y = AK^\alpha L^\beta$, where Y is firm's output (in \$), K is firm's capital (in \$), and L is the total wage bill (in \$). You want to infer the population parameters A , α , and β .

1. **(7P)** Based on this economic model derive an econometric model. Explain in detail which assumptions you have to make if you would like to estimate this model by OLS consistently (use mathematical, economic and econometric arguments).
2. **(3P)** The following OLS estimates are obtained: $\hat{\alpha} = 0.8$ and $\hat{\beta} = 0.6$. Interpret $\hat{\alpha}$. What does α mean economically?
3. **(5P)** How would you test the null of constant returns to scale? State the null explicitly, if applicable in matrix form. Which test do you recommend? Discuss.

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Problem 4 (18 points)

- A. (9P) Consider the linear model

$$E[y|x] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2.$$

Assume for simplicity that $E[x_2|x_1] = 0$ and $E[x_1] = \mu$ and all other moments exist. Derive the linear projection $L(y|1, x_1)$.

- B. (3P) Show that the OLS estimator

$$\hat{\beta}_{OLS} = (X'X)^{-1}X'Y$$

for the linear model $Y = X\beta + U$ is consistent if the assumptions for OLS are met.

- C. (6P) Consider the simple linear model
- $\mathbf{Y} = \mathbf{X}\beta + \mathbf{U}$
- , where some regressors are thought to be endogenous. Assume that a set of valid instruments
- \mathbf{Z}
- is available. Show that for the case of exact identification of the given model the IV estimator
- $\hat{\beta}_{IV}$
- is exactly the same as the 2SLS estimator
- $\hat{\beta}_{2SLS}$
- . Precisely explain each step of your proof.

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