T02 : Tomke Splettstößer , Robert Hennig

$$1.1: \quad q = \frac{e^{r\Delta t} - d}{u - d}$$

$$1.2: \quad \exp\left(\left(2r+\sigma^2\right)\Delta t\right) = q u^2 + (1-q) d^2$$

$1.3: \quad q = 0{,}5$

Plug 1.3 into 1.1: $\quad 0{,}5 = \dfrac{e^{r\Delta t} - d}{u - d}$

$$\Leftrightarrow 0{,}5u - 0{,}5d = e^{r\Delta t} - d$$

$$\Leftrightarrow \quad u = 2e^{r\Delta t} - d$$

$\Rightarrow$ substitute u in 1.2: $\quad \exp\left(\left(2r+\sigma^2\right)\Delta t\right) = 0{,}5 \cdot \left(2e^{r\Delta t} - d\right)^2 + 0{,}5 d^2 \quad \big| \cdot 2$

$$2 \cdot \exp\left(\left(2r+\sigma^2\right)\Delta t\right) = 4e^{2r\Delta t} + d^2 - 4de^{r\Delta t} + d^2 \quad \big| - 4e^{2r\Delta t}$$

$$2 \cdot e^{2r\Delta t} \cdot e^{\sigma^2\Delta t} - 4e^{2r\Delta t} = 2d^2 - 4de^{r\Delta t} \quad \big| :2 \quad + e^{2r\Delta t} - e^{2r\Delta t}$$

$$e^{2r\Delta t} \cdot e^{\sigma^2\Delta t} - e^{2r\Delta t} = \left(d - e^{r\Delta t}\right)^2$$

$$e^{2r\Delta t} \cdot \left(e^{\sigma^2\Delta t} - 1\right) = \left(d - e^{r\Delta t}\right)^2 \quad \big| \sqrt{\ }$$

square root does not have to be positive!
−0,5

$$\pm e^{r\Delta t} \cdot \sqrt{\left(e^{\sigma^2\Delta t} - 1\right)} = d - e^{r\Delta t}$$

$$d = e^{r\Delta t} \cdot \left(\pm\sqrt{\left(e^{\sigma^2\Delta t} - 1\right)} + 1\right)$$

$$u = 2e^{r\Delta t} - d$$

In fact,
$d = e^{r\Delta t}\left(1 - \sqrt{e^{\sigma^2\Delta t} - 1}\right)$
$u = \dots$

$$= 2e^{r\Delta t} - e^{r\Delta t} \cdot \left(\sqrt{\left(e^{\sigma^2\Delta t} - 1\right)} + 1\right)$$

$$u = e^{r\Delta t} \cdot \left(\mp\sqrt{\left(e^{\sigma^2\Delta t} - 1\right)} + 1\right)$$

Interpretation of $q = 1/2$ missing −1

2,5/4