

### Problem Set V: GMM Estimation

1. A standard version of the consumption-based CAPM says that

$$\mathbb{E} \left[ \beta \left( \frac{C_t}{C_{t+1}} \right)^\gamma (1 + r_{t+1}) - 1 \middle| \mathcal{I}_t \right] = 0,$$

where  $r_t$  is the real rate of return,  $C_t$  is consumption,  $\mathcal{I}_t$  is the information set available in period  $t$ ,  $0 < \beta < 1$  is a time discount factor, and  $\gamma$  measures the curvature of the utility function (and  $1/\gamma$  is the elasticity of intertemporal substitution). Your data file provides quarterly US time series data from 1960Q1 to 2016Q4. It includes real personal consumption expenditures of nondurable goods,  $C$  (chain index, seasonally adjusted, per capita), the associated price index,  $P$  (seasonally adjusted), and the nominal 3-month Treasury Bill rate,  $r$ .

- (a) Load the data into Stata. Prepare them to obtain  $C_t/C_{t+1}$  and  $r_t$ .
- (b) Estimate  $\beta$  and  $\gamma$  by means of GMM with identity weighting matrix, using the above orthogonality condition. Start with a baseline instrument set that includes a vector of ones, the first four lags of consumption growth  $\Delta \log(C_t)$ , inflation  $\Delta \log(P_t)$ , and the real rate  $r_t$ . Discuss your findings.
- (c) Check how sensitive your results are to changes in the instrument set.
- (d) Show that a log-linearization under certainty equivalence yields the approximate condition

$$\mathbb{E} [\log(\beta) - \gamma \Delta \log(C_{t+1}) + r_{t+1} | \mathcal{I}_t] = 0.$$

Estimate this equation by GMM using an identity weighting matrix and the baseline instrument set. Compare your results.