Econometric Methods
PC-tutorial: Generalized Method of Moments (GMM)

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Exe

A standard version of the consumption-based CAPM says that

$$E\left[\beta\left(\frac{C_t}{C_{t+1}}\right)^{\gamma}(1+r_{t+1})-1|I_t\right]=0,$$

- where r_t is the real rate of return
- C_t is consumption
- I_t is the information set available in period t
- $0<\beta<1$ is a time discount factor, and
- γ measures the curvature of the utility function (and $1/\gamma$ is the elasticity of intertemporal substitution).

GMM estimator

The generalized method of moments (GMM) estimator uses moment conditions of population

$$E(g(w_i, \theta_0)) = 0$$

Let L is the number of moment conditions and P is the number of the parameter
If L = P: GMM estimator, $\hat{\theta}$, is the solution of sample counterpart:

$$N^{-1} \sum_{i=1}^{N} g(w_i, \hat{\theta}) = 0$$

If L > P: GMM estimator, $\hat{\theta}$, minimizes a quadratic form in $\sum_{i=1}^{N} g(w_i, \theta)$

$$min = \left[\sum_{i=1}^{N} g(w_i, \theta)' \right]' \hat{\Xi} \left[\sum_{i=1}^{N} g(w_i, \theta)' \right]$$

where $\hat{\Xi}$ is an L x L symmetric, positive semidefinite weighting matrix.
Efficient GMM estimator: is the one generated from optimal weight matrix.

Denote $r(w_i, \theta_0)$ be the forecast error, e.g the deviation from 1.

$$r(w_i, \theta_0) \equiv \beta \left(\frac{C_t}{C_{t+1}} \right)^{\gamma} (1 + r_{t+1}) - 1,$$

where $w_i = (C_t, C_{t+1}, r_{t+1})'$ and $\theta_0 = (\beta_0, \gamma_0)'$
Z_i is the instruments set including vector of ones, the first four lags of consumption growth, inflation, and the real rate.

The conditional expectation implies that

$$E[Z_i' r(w_i, \theta_0)] = 0$$

In this case, $g(w_i, \theta_0) = Z_i' r(w_i, \theta_0)$

a) Load the data into Stata. Load the data into Stata. Prepare them to obtain C_t/C_{t+1} and r_t

b) Estimate β and γ by means of GMM with identity weighting matrix, using the above orthogonality condition. Start with a baseline instrument set that includes a vector of ones, the first four lags of consumption growth $\Delta log(C_t)$, inflation $\Delta log(P_t)$, and the real rate r_t. Discuss your findings.

c) Check how sensitive your results are to changes in the instrument set.

d) Show that a log-linearization under certainty equivalence yields the approximate condition

$$E\left[log\beta - \gamma\Delta log(C_{t+1}) + r_{t+1}|I_t\right] = 0$$