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Advanced Statistics I (Winter Term 2023/24)

Problem Set 10

1. Are the following distributions member of the exponential class of distributions?
   (a) Beta
   (b) (continuous) uniform

2. Suppose the random Variable X has the following pdf

$$f(x) = C \exp\left(-\frac{1}{2}\lambda |x|^{\gamma}\right), \quad \lambda > 0,\ \gamma > 0$$

   (a) Show that $f(x)$ is a member of the generalized normal family of distributions and give C.
   (b) Is the pdf a member of the exponential class?
   (c) Show that any member of the generalized normal family is also member of the location-scale family.

3. Let the sequence of random variables $\{Y_n\}$ follow a Normal distribution and $\{Z_n\}$ a Gamma distribution with
   (a) $Y_n \sim \mathcal{N}\left(\mu, [\sigma^2 + \sqrt{n}]/[n+1]\right)$
   (b) $Z_n \sim \Gamma\left(n, \frac{1}{2+n}\right)$

   Examine which type of convergence applies for Y_n and Z_n and give their asymptotic distributions.
   Hint: For Z_n you can use that $\Gamma(\alpha, \beta) \to \mathcal{N}(\alpha\beta, \alpha\beta^2)$ for large α.

4. Let the random variable X_i following a Poisson distribution

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}\mathbb{I}_{\{0,1,2,\ldots\}}(x)$$

   have stochastically independent realizations. The latter are used to estimate λ^2 by means of $\overline{X}_n^2 = (\sum_{i=1}^n X_i/n)^2$.

   (a) Check if $\mathrm{E}(\bar{X}_n^2) = \lambda^2$ and $\lim_{n\to\infty} \mathrm{E}(\overline{X}_n^2) = \lambda^2$ are fulfilled.
   (b) Does $\mathrm{plim}(\overline{X}_n^2) = \lambda^2$ hold?