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Advanced Statistics I (Winter Term 2023/24)

Calculus Recap

1. Review the *binomial theorem* as well as the *Taylor series* calculus.

2. Illustrate DeMorgan's laws with the help of Venn's diagrams.

3. Find the limits of following expressions using l'Hôspital's rule, if those exist:

   (a) $\lim\limits_{x \to \infty} \dfrac{e^x}{x^2}$

   (b) $\lim\limits_{x \to -\infty} x \exp(x)$

   (c) $\lim\limits_{x \to \infty} \dfrac{3^x}{x^2 + x - 1}$

   (d) $\lim\limits_{x \to \infty} \dfrac{5x^2 + 4}{8x^2 - 3}$

4. Calculate the values of following integrals:

   (a) $\displaystyle\int_0^\infty \lambda e^{-\lambda x} dx$ for $\lambda > 0$

   (b) $\displaystyle\int_{\frac{1}{2}}^2 3x^2 + 5e^x - \frac{1}{x} dx$

   (c) $\displaystyle\int_0^x \sum_{i=2}^n (i-1) s^{i-2} ds \quad$ for $|x| < 1$

5. Review the *integration by parts* theorem and use it to find the following integral:

$$\int e^{x-1} x^3 dx$$

6. If pokémons wanted to form a string quartet, how many combinations would be possible to do this? There are currently 808 listed pokémons. How many possible quartets can there be if the 17 armless pokémons and 34 pokémons that only consist of a head were not allowed to audition?

7. Find all first partial derivatives of following functions:

(a) $f(x, y) = \sqrt{2x^3} + 3\sqrt{xy^2}$

(b) $f(x, y) = \ln\left(\dfrac{y+1}{\sqrt{x}}\right)$

(c) $f(x, y) = a^x(xy - y^2)$

(d) $f(x, y) = \dfrac{2}{(1 + x + y)^3}$