Mathematical Finance: MF

Exercises (for discussion on Monday, 18.12.2023)

Exercise 1. We consider a market $S = (S^0, S^1)$ with time horizon $N = 1$ and deterministic starting value $S_0 = (S_0^0, S_0^1)$. Furthermore let X be a random payoff. Show that the strategy $\varphi = (\varphi^0, \varphi^1)$ with

$$(\varphi_0^1 =)\, \varphi_1^1 = \frac{Cov(\hat{X}, \hat{S}_1^1)}{Var(\hat{S}_1^1)}, \qquad (\varphi_0^0 =)\, \varphi_1^0 = E(\hat{X}) - \varphi_1^1 E(\hat{S}_1^1)$$

minimizes the expression $E\left(\left(\hat{X} - \hat{V}_1(\varphi)\right)^2\right)$.

Exercise 2. In CRR model let S^2 be the fair price process of a *Forward-Start-Option* on S^1. This is an option such that for an $M < N$ the payoff at the end time $N \in \mathbb{N}$ resembles the payoff of a call with strike price S_M^1, hence is

$$S_N^2 = (S_N^1 - S_M^1)^+.$$

Find S_0^2.

Exercise 3. We consider the CRR model form the lecture with interest rate $r = 0.05$, the initial values $S_0^0 = S_0^1 = 100$ and time-horizon $N = 2$. Moreover we assume that the one-period relative value increase and decrease of the stock are given by $u = 1.2$ and $d = 0.9$ which occur with probability p and $1 - p$, respectively. We consider a lookback-option with payoff

$$H := \left(\max_{k=0,\dots,N} S_k^1 - 105\right)^+$$

at time N.

(a) Calculate the fair price process S^2 of the lookback-option.

(b) Calculate the perfect hedge φ for the lookback-option.

Exercise 4. Consider a market with price process $S = (S^0, S^1)$ and time horizon $N = 1$ given by the following tree.

$\begin{pmatrix} S_0^0 \\ S_0^1 \end{pmatrix}$	$\begin{pmatrix} S_1^0 \\ S_1^1 \end{pmatrix}$	ω	$P(\{\omega\})$
	$\begin{pmatrix} 1 \\ 150 \end{pmatrix}$	ω_1	0.2
$\begin{pmatrix} 1 \\ 100 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 120 \end{pmatrix}$	ω_2	0.3
	$\begin{pmatrix} 1 \\ 70 \end{pmatrix}$	ω_1	0.5

(a) Compute the upper and the lower price of a call option on S^1 with maturity 1 and strike 110.

(b) Determine a cheapest superhedge for the call option, that is, a self-financing strategy ϕ with $V_0(\phi) = \pi_U(X)$ and $V_N(\phi) \geq X$..

Submission of the homework until: Thursday, 14.12.2023, 10.00 a.m. via OLAT.