Prof. Dr. Kai Carstensen
Chair of Econometrics

Examination in Econometrics II

Summer Term 2021

October 11, 2021, 12:00

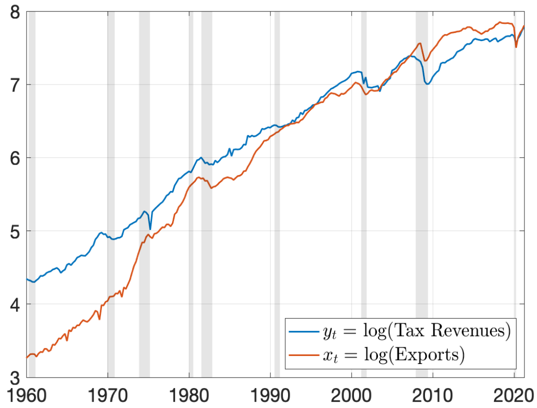
Preliminary remarks:

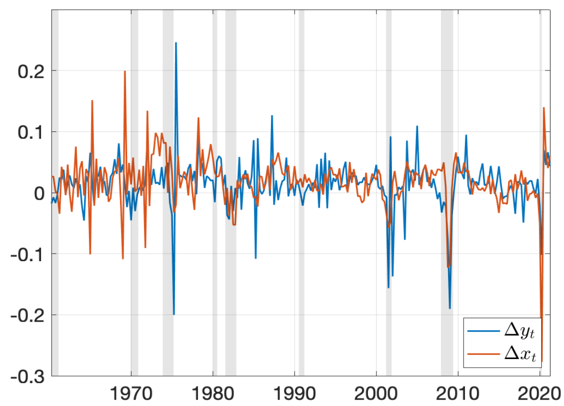
1. Please read these instructions carefully!
2. You have to solve all questions on your own!
3. Conduct each test at the 5% level.
4. Write your name and enrolment (matriculation) number on every sheet of paper!
5. Don't use a pencil!
6. The exam is composed by 3 problems. Check your exam for completeness!
7. Round your solutions to 4 decimal places.
8. You have 60 minutes in total to answer the questions.

Good luck!

Question A (22 credits)

Consider the following quarterly time series of the US macroeconomy:



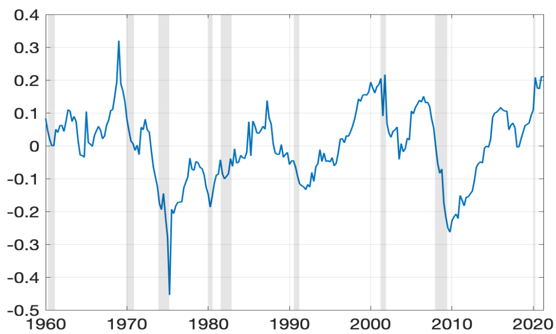


1. (**6P**) Explain which pair of ADF hypotheses (with/out intercept and trend) is the most relevant for y_t and x_t? Carefully state the null and alternative hypotheses and interpret the relevant ADF-t results at the 5% level and sample size $T = 250$.

	y_t = log(Tax Revenues)			x_t = log(Exports)		
Lags	ADF-t	ADF-t^{μ}	ADF-t^{τ}	ADF-t	ADF-t^{μ}	ADF-t^{τ}
1	5.00	-1.36	-1.35	5.15	-2.32	-0.28
2	3.93	-1.30	-1.79	4.10	-2.07	-0.58
3	3.73	-1.37	-1.88	3.64	-2.03	-0.73
4	3.50	-1.40	-1.98	3.37	-2.08	-0.80

2. (**4P**) Based on time series plots, describe the effects of the Great Recession shock of 2008 on the trajectories of y_t and Δy_t.

3. Assume the cointegration regression yields the OLS residuals $\hat{z}_t^* = y_t - \hat{\mu} - \hat{\gamma}_2 x_t$:



   (a) (**6P**) Discuss whether or not the long-run relationship between y_t and x_t seems stable over time. What does it imply to the Engle-Granger test procedure and interpretation of the point estimate $\hat{\gamma}_2$?

   (b) (**6P**) Based on your previous answer, formally state which regression model would you recommend to evaluate the lagging effects of Δx_t on Δy_t. Moreover, how would you determine the appropriate lag orders of Δx_t on Δy_t in the model?

Question B (16 credits)

Consider a simple time trend

$$y_t = \beta_0 + \beta_1 \, t + \varepsilon_t$$

with a white noise process $\varepsilon_t \sim N(0, \sigma^2)$. Consider further an OLS estimator $\hat{\boldsymbol{\beta}}' = (\hat{\beta}_0, \hat{\beta}_1)'$ of the model coefficients based on a sample size of T.

1. Demonstrate how a standard approach for finding an asymptotic distribution of $\hat{\beta}$, as in OLS with stationary regressors, does not work here. In particular, show that the elements in $A \equiv \mathrm{E}(x_t' x_t)$ diverge as $T \to \infty$.

   Hint: note that $\sum_{t=1}^{T} t^2 = T(T+1)(2T+1)/6$.

2. Explain shortly the concept of superconsistency and how it applies here.

Question C (22 credits)

1. Consider the following model

$$\mathbf{y_i} = \theta + u_i \quad i = 1, ..., N \quad u_i \sim N(0, \sigma^2)$$

Using a sample of size N you want to test the hypothesis $H_0 : \theta = 0$ vs. $H_1 : \theta = 1$ with a likelihood-ratio test.

(i) (**4P**) Write down the likelihood function

(ii) (**6P**) Calculate the likelihood-ratio test statistic

Based on a sample size $N = 100$ you obtain the sample moment $\sum_{i=1}^{N} y_i = 55$. You can further assume $\sigma^2 = 1$.

(iii) (**2P**) What is your test decision?

2. Consider a random sample of size N from the logarithmic distribution

$$f(y) = -\frac{p^y}{y} \frac{1}{log(1-p)}, \quad p \in (0, 1), \quad y \in \{1, 2, ...\}$$

(i) (**5P**) Write down the likelihood function

(ii) (**3P**) Derive the score with respect to p

(iii) (**2P**) Describe how you would actually find the ML estimator of p. You can use p_0 as an initial guess for p. You don't need to calculate anything.