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Advanced Statistics I (Winter Term 2022/23)

Problem Set 8

1. Show the following relationships between distributions using the MGF
   (a) binomial distribution nests the Bernoulli distribution
   (b) exponential distribution is a special case of the gamma distribution
   (c) chi square distribution is a special case of the gamma distribution
   (d) (continuous) standard uniform is a special case of the beta distribution
   (e) Poisson distribution can be approximated by a normal distribution for large λ

2. Let $X_1, \ldots, X_n$ be independent and identically distributed with probability density function
$$f(x; \lambda) = \lambda e^{-\lambda x} \mathbb{I}_{(0,\infty)}(x), \qquad \lambda > 0 \ .$$
Define
$$Z_1 = X_1 + X_2 \qquad \text{and} \qquad Z_2 = X_1 - X_2 \ .$$
Derive the moment generating function $M_{\mathbf{Z}}(t)$ for $\mathbf{Z} = (Z_1, Z_2)$.

3. Consider a random variable ω with the following pdf
$$f(\omega) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\omega - \mu)^2}{2\sigma^2}\right) \mathcal{I}_{(-\infty,\infty)}(\omega).$$
   (a) Derive the MGF of ω.
   (b) Assume now $\mu = 0$ and $\sigma = 1$. Find the pdf of the random variable $X = |\omega|$.