List of questions
Computational Finance
Summer Term 2023

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The following questions are for self-study. The list will be extended and updated over the semester. Some of the questions will be part of the exam (perhaps in slightly modified form).

1. In order to use the binomial model to approximate option prices in the Black-Scholes model, one has to choose certain parameters, i.e., the factors u and d as well as the transition probability p. How are these parameters specified and why?

2. Describe shortly the idea how the price of an American option can be computed in the binomial model.

3. For which of the following option types can one compute prices in the binomial model efficiently or less efficiently, and why: European put option, American put option, down-and-out call option, lookback call option.

4. How does the error of the Monte Carlo estimate depend on the number of simulations? What does *error* mean in this context?

5. How can you provide approximate confidence intervals for Monte Carlo estimates? What does *confidence interval* mean in this context?

6. How can you sample random numbers from a specific distribution if you only have a random number generator for the uniform distribution on $[0, 1]$?

7. What is the aim of variance reduction approaches?

8. When is the control variate approach applicable?

9. What is the stochastic integral $\int_0^t 1_{(T_1,T_2]}(s)\mathrm{d}X(s)$, where

$$1_{(T_1,T_2]}(s) = \begin{cases} 1, & \text{if } T_1 < s \leq T_2, \\ 0, & \text{otherwise.} \end{cases}$$

   What does this mean intuitively if X is a stock price process?

10. What properties distinguish the Brownian motion as a fundamental stochastic process?

11. What is the intuitive meaning of the coefficients in the Itō process representation?

12. What is a martingale? How can you tell from its Itō process representation whether a process is a martingale?

13. What is the quadratic variation of a stochastic process? Where does it occur?

14. Why are stock prices modelled rather by a *geometric* Brownian motion than by a Brownian motion, as it was originally proposed by the Bachelier model?

15. An Ornstein-Uhlenbeck process $(X(t))_t$ satisfies the SDE

$$dX(t) = \theta(\mu - X(t))dt + \sigma dW(t),\ X(0) = x.$$

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    (a) Explain why, unlike the volatility process $\gamma(t)$ in the Heston model, this process can take any value in $\mathbb{R}$.

    (b) The value μ is called the mean-reversion level. Why is that?

    (c) If $x = \mu$, what is $E(X(t)), t \geq 0$?

    (d) Is this process a martingale?

16. Why does one use geometric Brownian motion to model stock prices, and, by contrast, processes like the Ornstein-Uhlenbeck process to model interest and inflation rates (and not, e.g., vice versa)?

17. Give an SDE of an Ito process that is neither a submartingale nor a supermartingale.

18. Consider a self-financing trading strategy with initial endowment v and $\varphi_1(t)$ stocks at time t. Give a formula for the value of this portfolio at time T.

19. What is the general structure of arbitrage free prices of European and American options? Explain the involved mathematical objects.

20. How can one compute, e.g., in the Black-Scholes model, European option prices by numerical integration?

21. Why does the valuation of European options in the Black-Scholes and related models lead to a partial differential equation?

22. How do you obtain the perfect hedging strategy for a European option in the Black-Scholes model from the pricing function?

23. In the Black-Scholes model, prices of American options are described by a linear complementary problem, i.e., a system of four inequalities and equalities, respectively. What do these four equations mean in detail?

24. Can you hedge options in the Heston model perfectly? If yes, how?

25. How do the different approaches to compute sensitivities by Monte Carlo simulation differ?

26. Geometric Brownian motion satisfies the stochastic differential equation

$$\mathrm{d}X(t) = X(t)\left(\mu \mathrm{d}t + \sigma \mathrm{d}W(t)\right).$$

    Can one do better than to simulate this stochastic differential equation via the Euler scheme? What can one do instead?

27. Does the Monte Carlo estimate converge to the true value if the number of samples gets large? When and why? When not and why?

28. Explain where regression is used in the Longstaff-Schwartz method.

29. What are the advantages and disadvantages of the explicit, the implicit and the Crank-Nicolson scheme to solve partial differential equations?

30. How can you obtain error estimates for the finite difference schemes?

31. What are the advantages and disadvantages of the different numerical approaches to compute option prices?

32. How do the finite difference schemes for a standard European call option and a down-and-out call, respectively, differ from each other?

33. What is Richardson extrapolation? Give an example where it can be applied.

34. When is a neural network called deep?

35. What exactly is approximated via NN in the hedging problem?

36. Which optimization problem is solved?