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Chair of Econometrics

Online Examination in Econometrics I

(Winter Term 2019/20)

Examination regulation

August 24, 2020, 16:00

Preliminary remarks:

1. Please read these instructions carefully!
2. You are permitted to use any auxiliary tools.
3. Write your name and enrollment (matriculation) number on every sheet of paper!
4. Don't use a pencil!
5. **Round your solutions to 4 decimal places.**
6. For all tests use a significance level of 5%, if nothing else is specified.
7. You have 60 minutes in total to answer the exam questions.

Good luck!

Problem 1 (18 points)

Consider the following model to study the causal effect of school attendance on the performance of high school students in Germany

$$hsGPA = \beta_0 + \beta_1\, skipped + \beta_2\, size + \beta_3\, seGPA + \beta_4\, pareduc + u, \tag{1}$$

using a random sample of students, where $hsGPA$ denote the average exam grade in high school (lower grade means student performs better), *skipped* is the fraction of lectures missed during high school (i.e., 1 minus the attendance rate), *size* refers to the class size (measured by the student-teacher ratio), $seGPA$ denotes the average exam grade in secondary school, and *pareduc* refers to the sum of parent's education (measured in years).

1. **(3P)** Suppose *skipped* is correlated with the disturbance u. Which assumptions of the OLS estimator are affected? Would you trust in the OLS estimator of β_1 if (i) the sample size is finite, and (ii) if the sample size tends to infinity?
2. **(4P)** Do you think it makes sense to include $seGPA$ in the regression? Briefly explain your answer.
3. **(5P)** You suspect that *skipped* is correlated with students' motivation. Do you think $\hat{\beta}_1$ is likely to have an upward or downward asymptotic bias? Explain!
4. **(6P)** Assume endogeneity in the model solely arises from the fact that $Cov(skipped, u) \neq 0$ due to omitted variables. You are then provided with data on the distance from students' home to school. Describe the estimation approach you would use to consistently estimate β_1 and which conditions are required for that.

Problem 2 (20 points)

Consider the following linear model:

$$y_i = \beta_0 + \beta_1 x_i + u_i, \tag{2}$$

where y_i denotes the amount of private investment (given in log of euros) and x_i represents income (given in log of euros) for a random sample of households $i = 1, \ldots, N$, with $N = 6$. Assume the error term u_i has mean zero and OLS assumptions OLS.1 and OLS.2 are satisfied. Further assume the conditional variances $\mathrm{Var}(u_i|x_i)$ for households 5 and 6, which have income larger than 60,000 euros, are 2 twice as large than for households 1 to 4, which have income below 60,000 euros.

1. **(4P)** Derive the conditional variance matrix $\mathbf{\Omega} = \mathrm{Var}(\mathbf{u}|\mathbf{x})$ for the regression model (2).
2. **(3P)** Explain why the conditional variance structure violates OLS assumption OLS.3 (homoscedasticity). Which property of the OLS estimator is affected?
3. **(9P)** Explain and derive the Weighted Least Squares (WLS) estimator for this example. In particular, show how the observations are re-weighted.
4. **(4P)** Now suppose you don't have any information on the conditional covariance structure, i.e., $\mathbf{\Omega}$ is completely unknown. Which estimation approach would you suggest? Briefly explain.

Problem 3 (22 points)

1. Consider the exponential regression with $m(\boldsymbol{x}, \boldsymbol{\theta}) = \exp(\theta_1 x_1 + \theta_2 x_2)$.
   (a) **(6P)** Set up a quadratic target function $q(\boldsymbol{w}, \boldsymbol{x})$ and find the score.
   (b) **(2P)** Find the marginal effect of x_2 on the dependent variable.

2. Suppose you consistently estimated the parameters of a Cobb-Douglas production function, $Y = AK^{\alpha}L^{\beta}$, by first taking logs and then estimating by OLS which yielded:

$$\log(Y) = -0.05 + 0.38 \log(K) + 0.61 \log(L) + \hat{u}.$$

   Additionally, $\widehat{SE}(\hat{\gamma}_0) = 0.08$ with $\gamma_0 = \log(A)$.
   (a) **(3P)** Show that $\hat{A} \equiv \exp(\hat{\gamma}_0)$ is a consistent estimator for A.
   (b) **(8P)** Derive and calculate the asymptotic standard error of $\hat{A}$. (*Hint:* use the delta method.)
   (c) **(3P)** Breifly explain how you would test for constant returns to scale!