List of questions about the course
Mathematical Finance
Winter Term 19/20

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The following questions are for self-study. The list will be extended and updated over the semester. Some of the questions will be part of the exam.

1. Explain the basic idea how the principle of no arbitrage can be used for pricing in an example of your choice.

2. Explain the intuitive meaning of the following mathematical notions: filtration, adaptedness, conditional expectation, stopping time.

3. You roll a die twice and someone tells you whether both results are > 3 or not. Describe the σ-algebra $\mathcal{F}$ modeling this information and find $E(X|\mathcal{F})$.

4. Decide which of the following rules are stopping times when you have the receive the information from an asset price process:

   (a) Stop when the process has its (overall) maximum value (for the first time).

   (b) Stop the first time the asset price is above 30 (or at terminal time).

   (c) Stop two days after the asset price was below 20 for the first time (or at terminal time).

   (d) Stop two days before the asset price will be below 20 (or at terminal time).

5. In the previous question, give a filtration such that all times are stopping times.

6. Is the time of the all-year-high of the DAX stock index a stopping time?

7. What is a stochastic integral, and where is it used in mathematical finance?

8. Give an example of a process X with the following property (for example, by drawing a tree) or argue why such a process does not exist:

   (a) X is a martingale.

   (b) X is a supermartingale, but no martingale.

   (c) X is a submartingale, but no martingale.

   (d) X is a martingale but no submartingale.

(e) X is a martingale but no supermartingale.

(f) X is both a submartingale and a supermartingale.

(g) X is neither a submartingale nor a supermartingale.

9. What is the stochastic exponential?
10. Why is the stochastic exponential relevant for finance?

11. According to which equation does the value of a self-financing trading strategy change?
12. What is the intuitive meaning of this equation?
13. What are discounted price processes, and why are they used in mathematical finance?
14. What is an arbitrage?
15. What role does the concept of arbitrage play in mathematical finance?
16. State the first fundamental theorem of asset pricing. Explain its relevance.

17. What can be said about the price process of a liquidly traded European option in an arbitrage-free market?
18. How can, in a complete market with trivial numeraire $S^0 = 1$ say, the value $Q(A)$, $A \subseteq \Omega$, be interpreted for the EMM Q?
19. What is a complete market?
20. How can one check whether a given market model is complete?
21. Give reasons why real financial markets are not (always) complete?
22. What is the upper and the lower price of an option? Explain their economic meaning for a bank that wants to trade the option over the counter.
23. What is a cheapest superhedge, and what is its price?
24. What is a perfect hedging strategy?
25. Is it possible that there exist perfect hedging strategies with different prices for an option in an arbitrage-free market?
26. Is it possible that, for a given call option, both initial option prices 10 and 12 do not lead to arbitrage, but 11 does?
27. What are the similarities and differences of futures and forwards from the viewpoint of mathematical finance?

28. A bank trades a forward or an option over the counter. Which risks of loss does the bank face? Which of them are reflected by the mathematical model from the lecture?

29. A bank uses the binomial model to compute the unique fair price of an option sold to a customer. The bank hedges the market risk by the corresponding replicating strategy. Discuss the risks of loss that exist nevertheless in real markets.

30. Why does the fair forward price not depend on valuations and preferences of the market participants?

31. Why is it necessary to consider markets with shortselling constraints when treating American options?

32. What can be said about the price process of a liquidly traded American option in an arbitrage-free market?

33. In which sense can the price of an American option be interpreted as the solution to an optimal stopping problem?

34. What is the value of the stopping problem with maturity N

$$\sup_{\tau} E(X_\tau),$$

when X is a supermartingale? Give an optimal stopping time.

35. How are the price processes of an American and a European call option (with same strike and maturity) related?

36. Explain the concept of calibration.

37. For a market where plain vanilla options are liquidly traded, a bank wants to describe the dynamics of the price process relative to the equivalent martingale measure from the first fundamental theorem of asset pricing. To this end, it uses a model calibrated to option prices. Which evidence could indicate that the model is not appropriate to describe the real market?

38. Why is it problematic, from a theoretical perspective, to recalibrate a given model?

39. Give some definitions that cannot directly be taken over from discrete to continuous time.

40. What are the essential assumptions on the stock price process made in the Black-Scholes model?

41. Discuss strengths and weaknesses of (a) the binomial model and (b) the Black-Scholes model for option pricing.