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Examination in Econometrics II

(Summer Term 2020)

14 July 2020

Preliminary remarks:

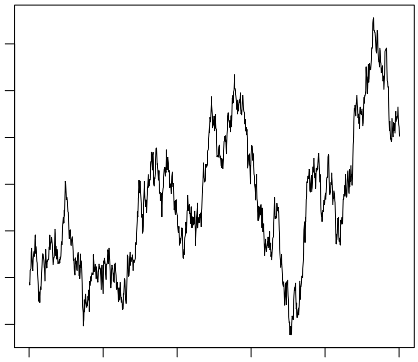
1. Please read these instructions carefully!
2. You have to solve all questions on your own!
3. **Conduct each test at the 5% level**.
4. Write your name and enrolment (matriculation) number on every sheet of paper!
5. Don't use a pencil!
6. The exam problems are printed on 3 pages. Check your exam for completeness!
7. Round your solutions to 4 decimal places.
8. You have 60 minutes in total to answer the questions.

Good luck!

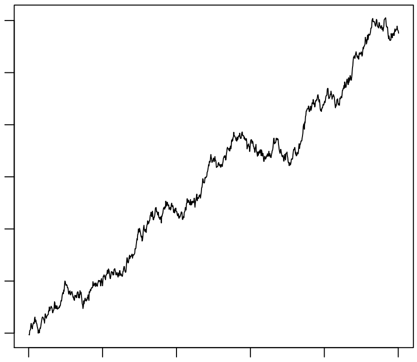
Question A (13 credits)

Consider the following time series

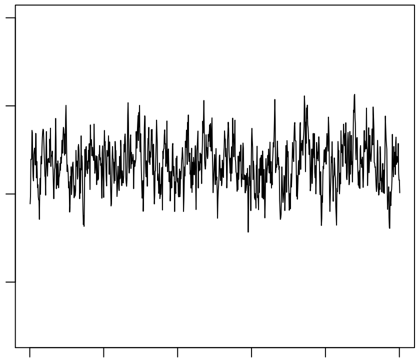
I



II



III



1. (**1P**) Which of the figures depicts a stationary process? Explain your decision briefly.
2. (**6P**) Explain the difference between a stochastic and a deterministic trend. Are such processes depicted in the figures and if yes, in which ones? How would you restore stationarity for a trend-stationary and a difference-stationary process? Explain briefly.
3. (**6P**) Junior data analysts want to analyze the effect of foreign trade on Chinese output growth. They have obtained a time series data set on Chinese GDP and Chinese exports. Using OLS they regress the log of GDP on the log of exports (and an intercept) and obtain an estimate of 1.2 with a t statistic of 3.1.
   (a) Interpret the point estimate.
   (b) Which steps do you have to add to the analysis to be able to trust the point estimate?
   (c) What can you say about statistical significance?

Question B (17 credits)

Historically, the inverted yield curve has been a reliable indicator of economic downturns and recessions. Every time the yield curve inverted for a quarter, i.e. the term spread (difference in interest rates of long- and short-term government bonds) became negative, an economic recession followed. In order to check this for a hypothetical economy, consider the following estimation results for an ADL(2,2) model of the quarterly GDP growth rate y_t (in %) using lags of GDP growth and the term spread TS_t (in percentage points) as regressors:

ADL(2,2): OLS using observations 1982:1 - 2020:1 (T = 153)				
	Coefficient	Std. Error	t-ratio	p-value
const	0.130	0.109	1.196	0.233
y_{t-1}	0.343	0.082	4.163	0.001
y_{t-2}	0.239	0.080	2.957	0.003
TS_t	0.077	0.101	0.761	0.447
TS_{t-1}	-0.155	0.147	-1.052	0.294
TS_{t-2}	0.250	0.034	1.619	0.021

R^2	0.2700	Adjusted R^2	0.2448
F-statistic(5,145)	10.73	P-value (F)	8.711e-09

Breusch-Godfrey LM test for autocorrelation
H_0: no autocorrelation up to AR 8
Test statistic ($T.R^2$) = 2.018 P-value = $P(\chi^2_8 > 2.018) = 0.98$

Obs: y_t and TS_t are weakly stationary variables.

1. (**4P**) Assume the term spread is predetermined at time t but there is a lagged feedback from GDP to the term spread. What does this imply for consistency of the estimated coefficients? Briefly explain your answer.
2. (**3P**) Based on the ADL(2,2) results, briefly discuss if a negative term spread has been a good predictor for economic downturns in this hypothetical economy.
3. (**6P**) What is, according to the ADL(2,2) model, the total effect of a change in the term spread in the first quarter 2020 on GDP growth in the second quarter 2020? Interpret.
4. (**4P**) As a further step, consider the following transformation of the ADL(2,2) model:

$$\Delta y_t = \mu - \alpha(y_{t-1} - \beta TS_{t-1}) - a_2 \Delta y_{t-1} + b_0 \Delta TS_t - b_2 \Delta TS_{t-1} + \varepsilon_t. \tag{1}$$

where $\hat{\alpha} = 0.418$ and $\hat{\beta} = 0.4114$ are both statistically significant at 5% level.
Briefly interpret the terms:
(i) $-\alpha(y_{t-1} - \beta TS_{t-1})$, (ii) $\hat{\alpha}$, and (iii) $\hat{\beta}$.

Question C (30 credits)

Consider the nonlinear regression model with scalar regressor x_i and parameters $\boldsymbol{\theta} = (\delta, \sigma^2)'$

$$y_i = x_i^{\delta} + \varepsilon_i, \qquad u_i \overset{iid}{\sim} N(0, \sigma^2), \qquad x_i > 0.$$

Suppose you have a random sample of $i = 1, \ldots, N$ observations.

1. (**6P**) Write down the log likelihood function $\ell_i(\boldsymbol{\theta})$ and derive the score $\mathbf{s}_i(\boldsymbol{\theta})$. Note that $\partial x_i^{\delta} / \partial \delta = x_i^{\delta} \log(x_i)$.
2. (**7P**) Derive the Hessian $\mathbf{H}_i(\boldsymbol{\theta})$ and its conditional expectation $\mathbf{A}(\mathbf{x}_i, \boldsymbol{\theta})$.
3. (**4P**) Derive the maximum likelihood estimator $\tilde{\sigma}^2$ under the null hypothesis $H_0 : \delta = 0$.
4. (**5P**) Derive the score $\tilde{\mathbf{s}}_i = \mathbf{s}_i(\tilde{\boldsymbol{\theta}})$ and the conditional expectation of the Hessian $\tilde{\mathbf{A}} = \mathbf{A}(\mathbf{x}_i, \tilde{\boldsymbol{\theta}})$ under the null hypothesis $H_0 : \delta = 0$.
5. (**8P**) Derive the LM statistic for the null hypothesis $H_0 : \delta = 0$. Use $\tilde{\mathbf{A}} = N^{-1} \sum_{i=1}^{N} \mathbf{A}(\mathbf{x}_i, \tilde{\boldsymbol{\theta}})$ as the estimator of $\mathbf{A}_o$.