Problem Set 9: GMM Estimation

Review the Concepts and Proofs

1. Show that the GMM estimator is consistent.

2. Use a mean value expansion to find the asymptotic distribution of the GMM estimator.

3. Explain Hansen's J test. Why is it impossible to test just identifying moment conditions for validity?

Exercises

1. Show that the variances (= the elements on the main diagonal of the variance matrix) of the efficient GMM estimator are smaller than, or equal to, those of any other GMM estimator that uses the same moment conditions. (Hint: use the approach proposed in problem 8.5 on page 234 of the textbook.)

2. Find the asymptotic variance matrix $\mathbf{V}$ of the GMM estimator with general weighting matrix $\mathbf{\Xi}$ in the case of just identifying moment conditions and compare it to the asymptotic variance matrix of the efficient GMM estimator. Explain your findings.

3. Consider the binary choice model $\mathrm{E}(y_i|\mathbf{x}_i) = G(\mathbf{x}_i\theta_o)$ with general link function $G(\cdot)$.

   (a) Find the asymptotic variance matrix $\mathbf{V}$ of the GMM estimator that uses the orthogonality condition $\mathrm{E}(\mathbf{x}_i'u_i) = 0$, where $u_i = y_i - G(\mathbf{x}_i\theta_o)$.

   (b) How does this asymptotic variance matrix look like in the logit model, i.e., for the link function $G(z) = \exp(z)/[1 + \exp(z)]$? Compare it to the asymptotic variance matrix of the CML estimator. Explain your findings.

   (c) Based on the insights gained from the previous problem, how do you have to specify the orthogonality condition for a general link function $G(\cdot)$ such that the asymptotic variance matrix of the GMM estimator coincides with the asymptotic variance matrix of the CML estimator?

Empirical Exercises

1. A standard version of the consumption-based CAPM says that

$$\mathrm{E}\left[\beta\left(\frac{C_t}{C_{t+1}}\right)^{\gamma}(1+r_{t+1})-1\middle|\mathcal{I}_t\right]=0,$$

where r_t is the real rate of return, C_t is consumption, $\mathcal{I}_t$ is the information set available in period t, $0<\beta<1$ is a time discount factor, and γ measures the curvature of the utility function (and $1/\gamma$ is the elasticity of intertemporal substitution). Your data file provides quarterly US time series data from 1960Q1 to 2016Q4. It includes real personal consumption expenditures of nondurable goods, C (chain index, seasonally adjusted, per capita), the associated price index, P (seasonally adjusted), and the nominal 3-month Treasury Bill rate, r.

(a) Load the data into Stata. Prepare them to obtain C_t/C_{t+1} and r_t.

(b) Estimate β and γ by means of GMM with identity weighting matrix, using the above orthogonality condition. Start with a baseline instrument set that includes a vector of ones, the first four lags of consumption growth $\Delta\log(C_t)$, inflation $\Delta\log(P_t)$, and the real rate r_t. Discuss your findings.

(c) Check how sensitive your results are to changes in the instrument set.

(d) Show that a log-linearization under certainty equivalence yields the approximate condition

$$\mathrm{E}\left[\log(\beta)-\gamma\Delta\log(C_{t+1})+r_{t+1}|\mathcal{I}_t\right]=0.$$

Estimate this equation by GMM using an identity weighting matrix and the baseline instrument set. Compare your results.

(e) (*) Repeat (a)-(c) using your own Matlab script.