

Übung 2

allgemeinster Unifikator:

$$x_1 \mapsto \gamma(x_2)$$

$$x_2 \mapsto x_2$$

$$x_3 \mapsto \sigma(\alpha, \gamma(x_2))$$

weitere Unifikatoren:

$$x_1 \mapsto \gamma(\alpha)$$

$$x_2 \mapsto \alpha$$

$$x_3 \mapsto \sigma(\alpha, \gamma(\alpha))$$

$$x_1 \mapsto \gamma(\gamma(\alpha))$$

$$x_2 \mapsto \gamma(\alpha)$$

$$x_3 \mapsto \sigma(\alpha, \gamma(\gamma(\alpha)))$$

Übung 3

$$\begin{aligned}
 & (\lambda x. (\lambda y. x (\lambda z. yz))) (((\lambda x. \underbrace{(\lambda y. y)}_{GV=\{y\}}) \underbrace{8}_{FV=\emptyset}) (\lambda x. (\lambda y. y) x)) \\
 \Rightarrow_{\beta} & (\lambda x. (\lambda y. x (\lambda z. yz))) ((\lambda y. y) (\lambda x. (\lambda y. \underbrace{y}_{GV=\emptyset}) \underbrace{x}_{FV=\{x\}})) \\
 \Rightarrow_{\beta} & (\lambda x. (\lambda y. x (\lambda z. yz))) ((\lambda y. \underbrace{y}_{GV=\emptyset}) (\underbrace{\lambda x. x}_{FV=\emptyset})) \\
 \Rightarrow_{\beta} & (\lambda x. \underbrace{(\lambda y. x (\lambda z. yz))}_{GV=\{y, z\}}) (\underbrace{\lambda x. x}_{FV=\emptyset}) \\
 \Rightarrow_{\beta} & (\lambda y. (\lambda x. \underbrace{x}_{GV=\emptyset}) (\underbrace{\lambda z. yz}_{FV=\{y\}})) \\
 \Rightarrow_{\beta} & (\lambda y. (\lambda z. yz)) = (\lambda yz. yz)
 \end{aligned}$$

Übung 3

$$\begin{aligned} & (\lambda h. (\lambda x. h(xx)) (\lambda x. h(xx))) ((\lambda x. \underbrace{x}_{GV=\emptyset}) (\underbrace{+15}_{FV=\emptyset})) \\ \Rightarrow_{\beta} & (\lambda h. \underbrace{(\lambda x. h(xx)) (\lambda x. h(xx))}_{GV=\{x\}}) (\underbrace{+15}_{FV=\emptyset}) \\ \Rightarrow_{\beta} & (\lambda x. (+15)(xx)) (\lambda x. (+15)(xx)) \\ \Rightarrow_{\beta} & \underbrace{(\lambda x. (+15)(xx))}_{GV=\{x\}} \underbrace{(\lambda x. (+15)(xx))}_{FV=\emptyset} \\ \Rightarrow_{\beta} & (+15)((\lambda x. (+15)(xx)) (\lambda x. (+15)(xx))) \\ \Rightarrow_{\beta} & \dots \text{ (endlose Rekursion)} \end{aligned}$$

Übung 3

$$\begin{aligned}
 & (\lambda f. \underbrace{(\lambda a. (\lambda b. fab))}_{GV=\{a,b\}}) \underbrace{(\lambda x. (\lambda y. x))}_{FV=\emptyset} \\
 \Rightarrow_{\beta} & (\lambda a. (\lambda b. (\lambda x. \underbrace{(\lambda y. x)}_{GV=\{y\}}) \underbrace{a}_{FV=\{a\}} b)) \\
 \Rightarrow_{\beta} & (\lambda a. (\lambda b. (\lambda y. \underbrace{a}_{GV=\emptyset} \underbrace{b}_{FV=\{b\}}))) \\
 \Rightarrow_{\beta} & (\lambda a. (\lambda b. a)) = (\lambda ab. a)
 \end{aligned}$$

Übung 4

$$\begin{aligned} \langle \text{pow} \rangle \langle 2 \rangle &= (\lambda \textcolor{red}{n} f z. \textcolor{red}{n} (\lambda g x. g(gx))) f z (\lambda h y. h(hy)) \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda h y. h(hy)) (\lambda g x. g(gx))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda y. (\lambda g x. g(gx)) ((\lambda g x. g(gx)) y))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda y. (\lambda x. ((\lambda g x. g(gx)) y) (((\lambda g x. g(gx)) \textcolor{blue}{y}) x)))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda y. (\lambda x. ((\lambda g x. g(gx)) y) ((\lambda \textcolor{red}{x}. y(y\textcolor{red}{x})) \textcolor{blue}{y})))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda y. (\lambda x. ((\lambda g x. g(gx)) \textcolor{blue}{y}) (y(yx))))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda y. (\lambda x. (\lambda \textcolor{red}{x}. y(yx)) (\textcolor{blue}{y}(yx))))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda \textcolor{red}{y}. (\lambda x. \textcolor{red}{y}(y(yx))))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda \textcolor{red}{x}. f(f(f(f\textcolor{red}{x})))) \textcolor{blue}{z}) \\ \Rightarrow_{\beta} & (\lambda f z. f(f(f(fz)))) = \langle 4 \rangle \end{aligned}$$

Übung 4

$$\begin{aligned} \langle \text{pow} \rangle \langle 0 \rangle &= (\lambda \textcolor{red}{n} f z. \textcolor{red}{n} (\lambda g x. g(gx)) f z) (\lambda \textcolor{blue}{h} y. y) \\ &\Rightarrow_{\beta} (\lambda f z. (\lambda \textcolor{red}{h} y. y) (\lambda \textcolor{blue}{g} x. \textcolor{blue}{g}(gx)) f z) \\ &\Rightarrow_{\beta} (\lambda f z. (\lambda \textcolor{red}{y}. \textcolor{red}{y}) \textcolor{blue}{f} z) \\ &\Rightarrow_{\beta} (\lambda f z. f z) = \langle 1 \rangle \end{aligned}$$