Übung 2

allgemeinster Unifikator:

$$x_1 \mapsto \gamma(x_2)$$
$$x_2 \mapsto x_2$$
$$x_3 \mapsto \sigma(\alpha, \gamma(x_2))$$

weitere Unifikatoren:

$$x_1 \mapsto \gamma(\alpha) \qquad\qquad x_1 \mapsto \gamma(\gamma(\alpha))$$
$$x_2 \mapsto \alpha \qquad\qquad x_2 \mapsto \gamma(\alpha)$$
$$x_3 \mapsto \sigma(\alpha, \gamma(\alpha)) \qquad\qquad x_3 \mapsto \sigma(\alpha, \gamma(\gamma(\alpha)))$$

$$(\lambda x.(\lambda y.x(\lambda z.yz)))(((\lambda x.\underbrace{(\lambda y.y)}_{GV=\{y\}})\underbrace{8}_{FV=\emptyset})(\lambda x.(\lambda y.y)x))$$

$$\Rightarrow_\beta \ (\lambda x.(\lambda y.x(\lambda z.yz)))((\lambda y.y)(\lambda x.(\lambda y.\underbrace{y}_{GV=\emptyset})\underbrace{x}_{FV=\{x\}})))$$

$$\Rightarrow_\beta \ (\lambda x.(\lambda y.x(\lambda z.yz)))((\lambda y.\underbrace{y}_{GV=\emptyset})\underbrace{(\lambda x.x)}_{FV=\emptyset}))$$

$$\Rightarrow_\beta \ (\lambda x.\underbrace{(\lambda y.x(\lambda z.yz))}_{GV=\{y,z\}})\underbrace{(\lambda x.x)}_{FV=\emptyset}$$

$$\Rightarrow_\beta \ (\lambda y.(\lambda x.\underbrace{x}_{GV=\emptyset})\underbrace{(\lambda z.yz)}_{FV=\{y\}})$$

$$\Rightarrow_\beta \ (\lambda y.(\lambda z.yz)) = (\lambda yz.yz)$$

Übung 3

$$(\lambda h.(\lambda x.h(xx))(\lambda x.h(xx)))((\lambda x.\underbrace{x}_{GV=\emptyset})\underbrace{(+15)}_{FV=\emptyset})$$

$$\Rightarrow_\beta (\lambda h.\underbrace{(\lambda x.h(xx))(\lambda x.h(xx))}_{GV=\{x\}})\underbrace{(+15)}_{FV=\emptyset}$$

$$\Rightarrow_\beta (\lambda x.(+15)(xx))(\lambda x.(+15)(xx))$$

$$\Rightarrow_\beta (\lambda x.\underbrace{(+15)(xx)}_{GV=\{x\}})\underbrace{(\lambda x.(+15)(xx))}_{FV=\emptyset}$$

$$\Rightarrow_\beta (+15)((\lambda x.(+15)(xx))(\lambda x.(+15)(xx)))$$

$$\Rightarrow_\beta \ ... \ \text{(endlose Rekursion)}$$

Übung 3

$$(\lambda f. \underbrace{(\lambda a.(\lambda b.fab))}_{GV=\{a,b\}}) \underbrace{(\lambda x.(\lambda y.x))}_{FV=\emptyset}$$

$$\Rightarrow_\beta \ (\lambda a.(\lambda b.(\lambda x. \underbrace{(\lambda y.x)}_{GV=\{y\}}) \ \underbrace{a}_{FV=\{a\}} \ b))$$

$$\Rightarrow_\beta \ (\lambda a.(\lambda b.(\lambda y. \underbrace{a}_{GV=\emptyset}) \ \underbrace{b}_{FV=\{b\}}))$$

$$\Rightarrow_\beta \ (\lambda a.(\lambda b.a)) = (\lambda ab.a)$$

Übung 4

$$\text{<pow> <2>} = (\lambda nfz.n(\lambda gx.g(gx))fz)(\lambda hy.h(hy))$$
$$\Rightarrow_\beta (\lambda fz.(\lambda hy.h(hy))(\lambda gx.g(gx))fz)$$
$$\Rightarrow_\beta (\lambda fz.(\lambda y.(\lambda gx.g(gx))((\lambda gx.g(gx))y))fz)$$
$$\Rightarrow_\beta (\lambda fz.(\lambda y.(\lambda x.((\lambda gx.g(gx))y)(((\lambda gx.g(gx))y)x)))fz)$$
$$\Rightarrow_\beta (\lambda fz.(\lambda y.(\lambda x.((\lambda gx.g(gx))y)((\lambda x.y(yx))x)))fz)$$
$$\Rightarrow_\beta (\lambda fz.(\lambda y.(\lambda x.((\lambda gx.g(gx))y)(y(yx))))fz)$$
$$\Rightarrow_\beta (\lambda fz.(\lambda y.(\lambda x.(\lambda x.y(yx))(y(yx))))fz)$$
$$\Rightarrow_\beta (\lambda fz.(\lambda y.(\lambda x.y(y(y(yx)))))fz)$$
$$\Rightarrow_\beta (\lambda fz.(\lambda x.f(f(f(fx))))z)$$
$$\Rightarrow_\beta (\lambda fz.f(f(f(fz)))) = \text{<4>}$$

Übung 4

$$<\mathsf{pow}> <0> = (\lambda nfz.n(\lambda gx.g(gx))fz)(\lambda hy.y)$$
$$\Rightarrow_\beta \ (\lambda fz.(\lambda hy.y)(\lambda gx.g(gx))fz)$$
$$\Rightarrow_\beta \ (\lambda fz.(\lambda y.y)fz)$$
$$\Rightarrow_\beta \ (\lambda fz.fz) = <1>$$