Übung 2

$$
\begin{aligned}
& <\text{pow}> <2> = (\lambda n f z.n(\lambda g x.g(g x))f z)(\lambda h y.h(h y)) \\
\Rightarrow_\beta\ & (\lambda f z.(\lambda h y.h(h y))(\lambda g x.g(g x))f z) \\
\Rightarrow_\beta\ & (\lambda f z.(\lambda y.(\lambda g x.g(g x))((\lambda g x.g(g x))y))f z) \\
\Rightarrow_\beta\ & (\lambda f z.(\lambda y.(\lambda x.((\lambda g x.g(g x))y)(((\lambda g x.g(g x))y)x)))f z) \\
\Rightarrow_\beta\ & (\lambda f z.(\lambda y.(\lambda x.((\lambda g x.g(g x))y)((\lambda x.y(y x))x)))f z) \\
\Rightarrow_\beta\ & (\lambda f z.(\lambda y.(\lambda x.((\lambda g x.g(g x))y)(y(y x))))f z) \\
\Rightarrow_\beta\ & (\lambda f z.(\lambda y.(\lambda x.(\lambda x.y(y x))(y(y x))))f z) \\
\Rightarrow_\beta\ & (\lambda f z.(\lambda y.(\lambda x.y(y(y(y x)))))f z) \\
\Rightarrow_\beta\ & (\lambda f z.(\lambda x.f(f(f(f x))))z) \\
\Rightarrow_\beta\ & (\lambda f z.f(f(f(f z)))) = <4>
\end{aligned}
$$

Übung 2

$$
\begin{aligned}
& <\text{pow}> <0> = (\lambda n f z.n(\lambda g x.g(g x))f z)(\lambda h y.y) \\
\Rightarrow_\beta \; & (\lambda f z.(\lambda h y.y)(\lambda g x.g(g x))f z) \\
\Rightarrow_\beta \; & (\lambda f z.(\lambda y.y)f z) \\
\Rightarrow_\beta \; & (\lambda f z.f z) = <1>
\end{aligned}
$$