Übung 2

Induktionsanfang (IA)

Sei `xs = []`.

```
   sum (foo [])
(2)
=  sum []
(6)
=  0
=  2 * 0 - 0
(6)
=  2 * sum [] - 0
(10)
=  2 * sum [] - length []
```

Induktionsvoraussetzung (IV)

Sei `xs :: [Int]`, sodass gilt:

```
sum (foo xs) = 2 * sum xs - length xs
```

Übung 2

Induktionsschritt (IS)

Für alle `x :: Int` zeigen wir, dass

```
sum (foo (x:xs)) = 2 * sum (x:xs) - length (x:xs)
```

gilt:

$$
\begin{aligned}
& \texttt{sum (foo (x:xs))} \\
\overset{(3)}{=} \; & \texttt{sum (x : x : (-1) : foo xs)} \\
\overset{(7)}{=} \; & \texttt{x + sum (x : (-1) : foo xs)} \\
\overset{(7)}{=} \; & \texttt{x + x + sum ((-1) : foo xs)} \\
\overset{(7)}{=} \; & \texttt{x + x + (-1) + sum (foo xs)} \\
\overset{(IV)}{=} \; & \texttt{x + x + (-1) + 2 * sum xs - length xs} \\
= \; & \texttt{2 * (x + sum xs) - (1 + length xs)} \\
\overset{(7)}{=} \; & \texttt{2 * sum (x:xs) - (1 + length xs)} \\
\overset{(11)}{=} \; & \texttt{2 * sum (x:xs) - length (x:xs)}
\end{aligned}
$$

□