

Übung 2

$$\begin{aligned} <\text{pow}> <2> &= (\lambda \textcolor{red}{n} f z. \textcolor{red}{n} (\lambda g x. g(gx))) f z (\lambda h y. h(hy)) \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda h y. h(hy)) (\lambda g x. g(gx))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda y. (\lambda g x. g(gx)) ((\lambda g x. g(gx)) y))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda y. (\lambda x. ((\lambda g x. g(gx)) y) (((\lambda g x. g(gx)) \textcolor{blue}{y}) x)))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda y. (\lambda x. ((\lambda g x. g(gx)) y) ((\lambda \textcolor{red}{x}. y(y\textcolor{red}{x})) \textcolor{blue}{y})))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda y. (\lambda x. ((\lambda g x. g(gx)) \textcolor{blue}{y}) (y(yx)))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda y. (\lambda x. (\lambda \textcolor{red}{x}. y(yx)) (\textcolor{blue}{y}(yx)))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda \textcolor{red}{y}. (\lambda x. \textcolor{red}{y}(y(yx))))) f z \\ \Rightarrow_{\beta} & (\lambda f z. (\lambda \textcolor{red}{x}. f(f(f(f\textcolor{red}{x})))) \textcolor{blue}{z}) \\ \Rightarrow_{\beta} & (\lambda f z. f(f(f(fz)))) = <4> \end{aligned}$$

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$$\begin{aligned} \langle \text{pow} \rangle \langle 0 \rangle &= (\lambda \textcolor{red}{n} f z. \textcolor{red}{n} (\lambda g x. g(gx)) f z) (\lambda \textcolor{blue}{h} y. y) \\ &\Rightarrow_{\beta} (\lambda f z. (\lambda \textcolor{red}{h} y. y) (\lambda \textcolor{blue}{g} x. \textcolor{blue}{g}(gx)) f z) \\ &\Rightarrow_{\beta} (\lambda f z. (\lambda \textcolor{red}{y}. \textcolor{red}{y}) \textcolor{blue}{f} z) \\ &\Rightarrow_{\beta} (\lambda f z. f z) = \langle 1 \rangle \end{aligned}$$