Collision detection

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September 2023

1 Collision between 2 particles

$$\begin{aligned}
d(t) &= \|x_1(t) - x_2(t)\| - r1 - r2 \\
x_1'(t) &= x_1(t_{i-1}) + (x_1(t_i) - x_1(t_{i-1}))t, t \in [0,1] \\
d'(t) &= \|x_1'(t) - x_2'(t)\| - r1 - r2 \\
d'(t) &= \|x_1(t_{i-1}) + (x_1(t_i) - x_1(t_{i-1}))t - x_2(t_{i-1}) - (x_2(t_i) - x_2(t_{i-1}))t\| - r1 - r2 \\
&= \|g(t)\| - r1 - r2
\end{aligned} \tag{1}$$

$$\begin{aligned}
d'(t) = 0 &\iff \|g(t)\| + r1 + r2 = 0 \\
&\iff \|g(t)\|^2 = (r1 + r2)^2 \\
&\iff \|g(t)\|^2 - (r1 + r2)^2 = 0
\end{aligned} \tag{2}$$

$$\begin{aligned}
g(t) &= x_1(t_{i-1}) + (x_1(t_i) - x_1(t_{i-1}))t - x_2(t_{i-1}) - (x_2(t_i) - x_2(t_{i-1}))t \\
&= (g_x(t); g_y(t)) \\
g_x(t) &= x_{1_x}(t_{i-1}) + (x_{1_x}(t_i) - x_{1_x}(t_{i-1}))t - x_{2_x}(t_{i-1}) - (x_{2_x}(t_i) - x_{2_x}(t_{i-1}))t \\
&= x_{1_x}(t_{i-1}) - x_{2_x}(t_{i-1}) + t((x_{1_x}(t_i) - x_{1_x}(t_{i-1})) - (x_{2_x}(t_i) - x_{2_x}(t_{i-1})) \\
&= c_{x_1} + c_{x_2}t,\ c_{x_i} \in \mathbb{R}
\end{aligned} \tag{3}$$

$$\begin{aligned}
\|g(t)\|^2 - (r1 + r2)^2 &= g_x(t)^2 + g_y(t)^2 - (r1 + r2)^2 \\
&= (c_{x_1} + c_{x_2}t)^2 + (c_{y_1} + c_{y_2}t)^2 - (r1 + r2)^2 \\
&= c_{x_1}^2 + 2c_{x_1}c_{x_2 t} + c_{x_2}^2 t^2 + c_{y_1}^2 + 2c_{y_1}c_{y_2}t + c_{y_2}^2 t^2 - (r1 + r2)^2 \\
&= (c_{x_2}^2 + c_{y_2}^2)t^2 + +2(c_{x_1}c_{x_2} + c_{y_1}c_{y_2})t + c_{y_1}^2 + c_{x_1}^2 - (r1 + r2)^2 \\
&= at^2 + bt + c
\end{aligned} \tag{4}$$

We have :

$a = (x_{1_x}(t_i) - x_{1_x}(t_{i-1}) - (x_{2_x}(t_i) - x_{2_x}(t_{i-1})))^2 + (x_{1_y}(t_i) - x_{1_y}(t_{i-1}) - (x_{2_y}(t_i) - x_{2_y}(t_{i-1})))^2$

$b = 2(((x_{1_x}(t_{i-1}) - x_{2_x}(t_{i-1})) \cdot (x_{1_x}(t_i) - x_{1_x}(t_{i-1}) - (x_{2_x}(t_i) - x_{2_x}(t_{i-1}))) + (x_{1_y}(t_{i-1}) - x_{2_y}(t_{i-1})) \cdot (x_{1_y}(t_i) - x_{1_y}(t_{i-1}) - (x_{2_y}(t_i) - x_{2_y}(t_{i-1}))$

$c = (x_{1_x}(t_{i-1}) - x_{2_x}(t_{i-1}))^2 + (x_{1_y}(t_{i-1}) - x_{2_y}(t_{i-1}))^2 - (r1 + r2)^2$

$\Delta = b^2 - 4ac$
If $\Delta < 0$ there is no collision
If $\Delta = 0$ there is a collision and it happens at $t = t_0$ (the root)
If $\Delta > 0$ there is a collision and we take the first root in $[0, 1]$: $t = t_1$

2 Collision between a particle and a border

A border is either horizontal or vertical and has only one coordinate here (x or y).

2.1 Vertical border

Let the wall x coordinate be x_w.

$$
\begin{aligned}
d(t) &= \|x_{1_x}(t) - x_w\| - r1 \\
x'_{1_x}(t) &= x_{1_x}(t_{i-1}) + (x_{1_x}(t_i) - x_{1_x}(t_{i-1}))t, t \in [0,1] \\
d'(t) &= \|x'_{1_x}(t) - x_w\| - r1 \\
&= |x'_{1_x}(t) - x_w| - r1 \\
&= |x_{1_x}(t_{i-1}) + (x_{1_x}(t_i) - x_{1_x}(t_{i-1}))t - x_w| - r1 \\
&= |(x_{1_x}(t_i) - x_{1_x}(t_{i-1}))t + x_{1_x}(t_{i-1}) - x_w| - r1 \\
&= |c_1 t + c_2| - r1
\end{aligned} \tag{5}
$$

2.2 Horizontal border

Let the wall y coordinate be y_w.

$$
\begin{aligned}
d(t) &= \|x_{1_y}(t) - y_w\| - r1 \\
x'_{1_y}(t) &= x_{1_y}(t_{i-1}) + (x_{1_y}(t_i) - x_{1_y}(t_{i-1}))t, t \in [0,1] \\
d'(t) &= \|x'_{1_y}(t) - y_w\| - r1 \\
&= |x'_{1_y}(t) - y_w| - r1 \\
&= |x_{1_y}(t_{i-1}) + (x_{1_y}(t_i) - x_{1_y}(t_{i-1}))t - y_w| - r1 \\
&= |(x_{1_y}(t_i) - x_{1_y}(t_{i-1}))t + x_{1_y}(t_{i-1}) - y_w| - r1 \\
&= |c_1 t + c_2| - r1
\end{aligned} \tag{6}
$$

2.3 Conclusion for both cases

$$
\begin{aligned}
d'(t) = 0 &\iff |c_1 t + c_2| - r1 = 0 \\
&\iff (c_1 t + c_2)^2 = r1^2 \\
&\iff c_1^2 t^2 + c_1 c_2 t + c_2^2 - r1^2 = 0 \\
&\iff at^2 + bt + c = 0
\end{aligned} \tag{7}
$$

$\Delta = b^2 - 4ac$
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