Izhikevich neuron model:

The three models visited above in this chapter have certain common features. They differ in the ion channel composition; they are derived from different cell models by different steps of simplification. But all three share certain features of dynamics. All the three models have:

- Resting state for I = 0

- Spikes/oscillations/limit cycles for a sufficiently large I.

- An N-shaped V-nullcline. The shape of the second nullcline is linear in the FN model, and sigmoidal in ML and INAPK model.

- The intersection between the V-nullcline and the second nullcline near the U-shaped, lower arch of the V-nullcline plays a crucial role in neuron dynamics.

When the second nullcline intersects the V-nullcline in the 1st branch there is a resting state; when the intersection occurs in the middle branch there are oscillations.

Consider the role of the remaining part of the state space. In the FN model for example, when the external current I = 0. When the initial voltage (V(0)) is less than 'a', the system returns to the resting potential, 0. But if V(0) > a, the neuron state (V, w) increases until it touches the 3rd branch of the V-nullcline, climbs up towards the maximum of the V-nullcline, turns leftwards at the maximum, proceeds up to the 1st branch of the V-nullcline, before it slides down the 3rd branch to the resting state. Thus, the remaining part of the phase-plane determines the downstroke and the peak of the action potential.

When the intersection point shifts slightly from the left of the minimum of V-nullcline (where the system exhibits excitability) to the right of the minimum (where the system oscillates), the shape and size of the action potential are about the same. Thus, in the oscillatory regime too, the part of the phase-space other than the shaded portion shown in fig.1 merely contributes to downstroke and the peak value of the action potential.

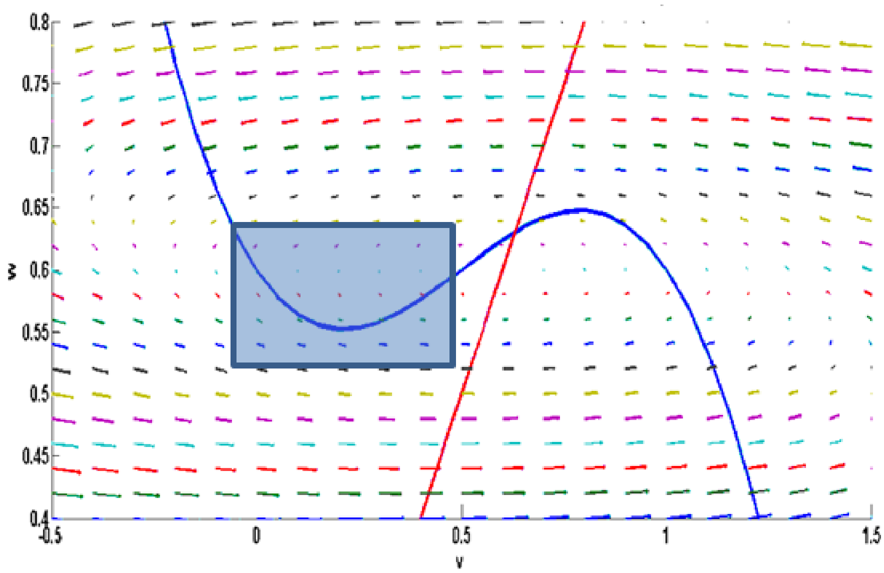

Figure-1: Area of interest for the upstroke: Phase plane analysis- Oscillations (FN model) at a=0.5; b = 0.1; r=0.1; I_a=0.6

Similar comments can be said about the dynamics in the other two models also.
Thus, the above three systems can be reduced to a more general, simpler system as follows. The U-shaped, lower arch of the V-nullcline is approximated by a quadratic function. Since only the shape of the second null-cline at its intersection with V-nullcline matters, and not its shape elsewhere, the second nullcline is approximated by a straight line. Dynamics far from the intersection point are implemented by a simple resetting once the membrane voltage hits a peak value or a minimum.
The quadratic approximation of the V-nullcline at its minimum is given as,

$$u = u_{max} + p(V - V_{max})^2$$

The linear approximation of the second nullcline may be expressed as,

$$u = s(V - V_0)$$

The dynamics of the reduced system then becomes,

$$\tau_V \frac{dV}{dt} = -u + u_{max} + p(V - V_{max})^2 \qquad 1$$

$$\tau_u \frac{du}{dt} = -u + s(V - V_0) \qquad 2$$

Where τ_V and τ_u denote the time-scales of V- and u-dynamics. Since V and u denote the fast and slow variables respectively, $\tau_V << \tau_u$.
If the V-dynamics of Eqn.1 above is implemented without any auxiliary conditions, V blows up to infinity in a finite time 't'. This can be shown easily on integrating Eqn.1 as:

$$V = c_1 * \tan(c_1 t)$$

Therefore, the downstroke of the action potential is modelled by resetting V(t) when it reaches the peak value V_{max} as follows.

$$(V, u) \leftarrow (V_{reset}, u + u_{reset}) \qquad when, V = V_{max}$$

When the membrane voltage exceeds V_{max}, both V and u are reset instantaneously as specified by the above equation.
Eqns.1,2 may be rewritten in a simpler form as,

$$\frac{dv}{dt} = I + f(v) - u$$

$$\frac{du}{dt} = a(bv - u)$$

If, $v \geq 1, \quad v \leftarrow c, u \leftarrow u + d$

a,b,c,d= constants;

The last set of eqns is generally referred to as the Izhikevich neuron model in current literature. Its merit lies in the low computational cost, and the ability to reproduce firing patterns of a large variety of neurons (E.M. Izhikevich et al, 2004, 2004).