LEAKY INTEGRATE AND FIRE NEURON MODEL:

Theory:

An Action Potential is a sharp voltage spike elicited by stimulating a neuron with a current that exceeds a certain threshold value. The current amplitude is increased gradually, at a threshold amplitude, the voltage response does not increase proportionally. It shows a sharp, disproportionate increase. Once the membrane voltage reaches a threshold value, it increases further rapidly to maximum value and drops again rapidly to a value that is less than resting value, before returning to the baseline value after a delay.

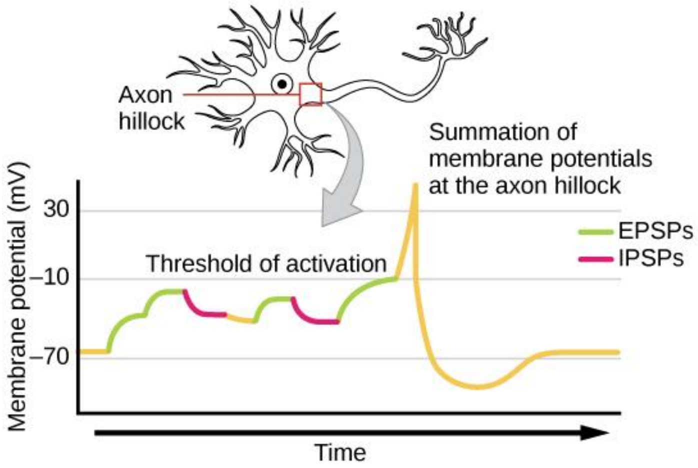

Fig: Site of action potential generation and the membrane potential variation during AP

LIF model:

This is the simplest model of spike/action potential generation. An electric circuit implementation of it consists of a capacitance charged by a current, and discharged whenever the voltage across the capacitance exceeds a limit.

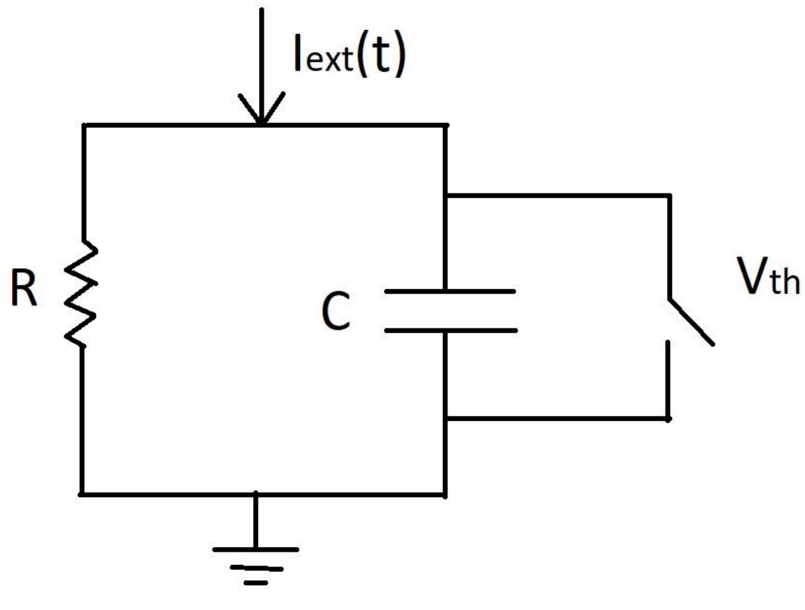

Fig1: Leaky Integrate and Fire circuit diagram

Applying the Kirchoff's current law of electric circuits, the external current, I_{ext}, going into the circuit may be expressed as,

$$C\frac{dV}{dt} + \frac{V}{R} = I_{ext}(t) \tag{1}$$

Capacitance is discharged whenever, V >θ, a threshold value. It is assumed that whenever the capacitor is charged, the "neuron" emits a spike. Note that in this model, there is no nonlinear, explosive build of excitation reaching a peak producing an action potential. The spike generated in this neuron model is more notional. This has always been one of the points of criticism about the leaky integrate and fire model.

If the capacitance starts off at 0 voltage, let us consider the time taken by the capacitance to reach the threshold, θ.
If I_{ext} is a constant current, I_0, voltage variation while the capacitance is charging may be expressed as,

$$V(t) = RI_0\left(1 - \exp\left(-\frac{t}{\tau}\right)\right)$$

Where τ the time-constant of the circuit equals RC. Since the charging stops at V =θ, the time taken, T, to reach this threshold is given by setting V(t) =θ, or,

$$\theta = RI_0\left(1 - \exp\left(-\frac{T}{\tau}\right)\right)$$

Or,

$$T = \tau \ln\left(\frac{RI_0}{RI_0 - \theta}\right)$$

Since there is a spike every time the capacitance discharges, spike frequency, f, is the reciprocal of T.

$$f = 1/\tau \ln\left(\frac{RI_0}{RI_0 - \theta}\right)$$

A key property of a real neuron reproduced by the above model is thresholding effect. Note that the model exhibits firing only when R I_0 >θ. But as I_0 increases beyond R/θ, f increases indefinitely (Fig. 5.4.2.2), instead of saturating as it happens in a real neuron.

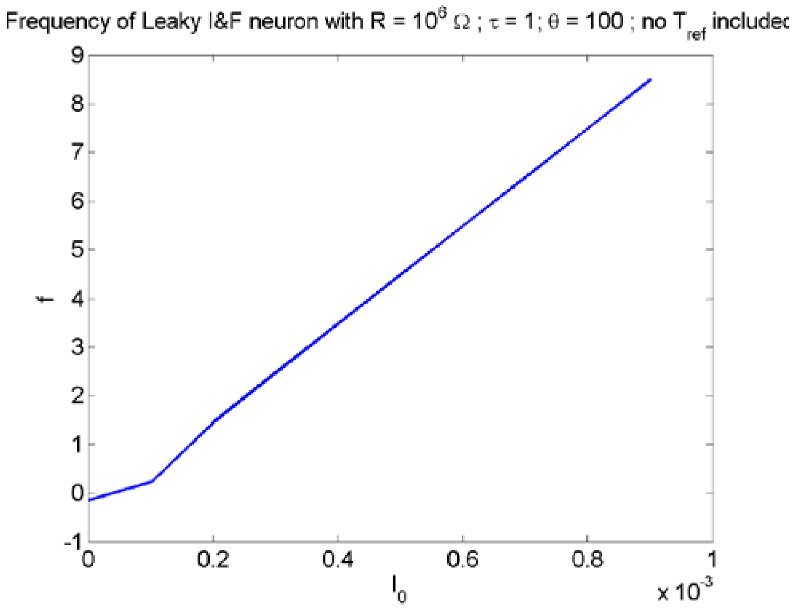

Fig2: frequency vs I_o without inclusion of absolute refractory period.

In order to restore the saturation property, the condition of absolute refractory period is introduced into the above model. Accordingly the capacitance can begin to be charged only a little while, T_{ref}, after the external current is applied. Therefore, the time taken to charge is incremented as,

$$T = \tau \ln\left(\frac{RI_0}{RI_0 - \theta}\right) + T_{ref}$$

With the inclusion of absolute refractory period, the plot of I_o vs f, shows saturating behaviour (Fig. 3).

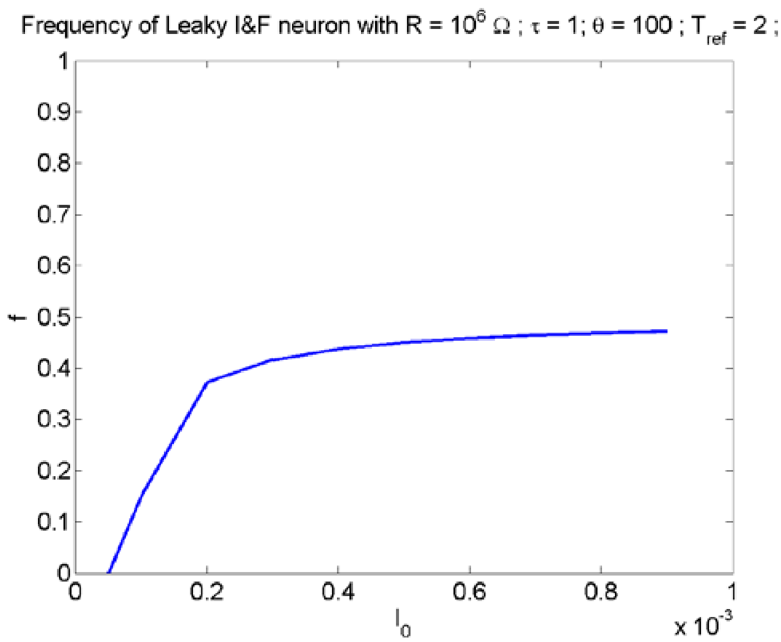

Fig3: frequency vs I_o with inclusion of absolute refractory period.