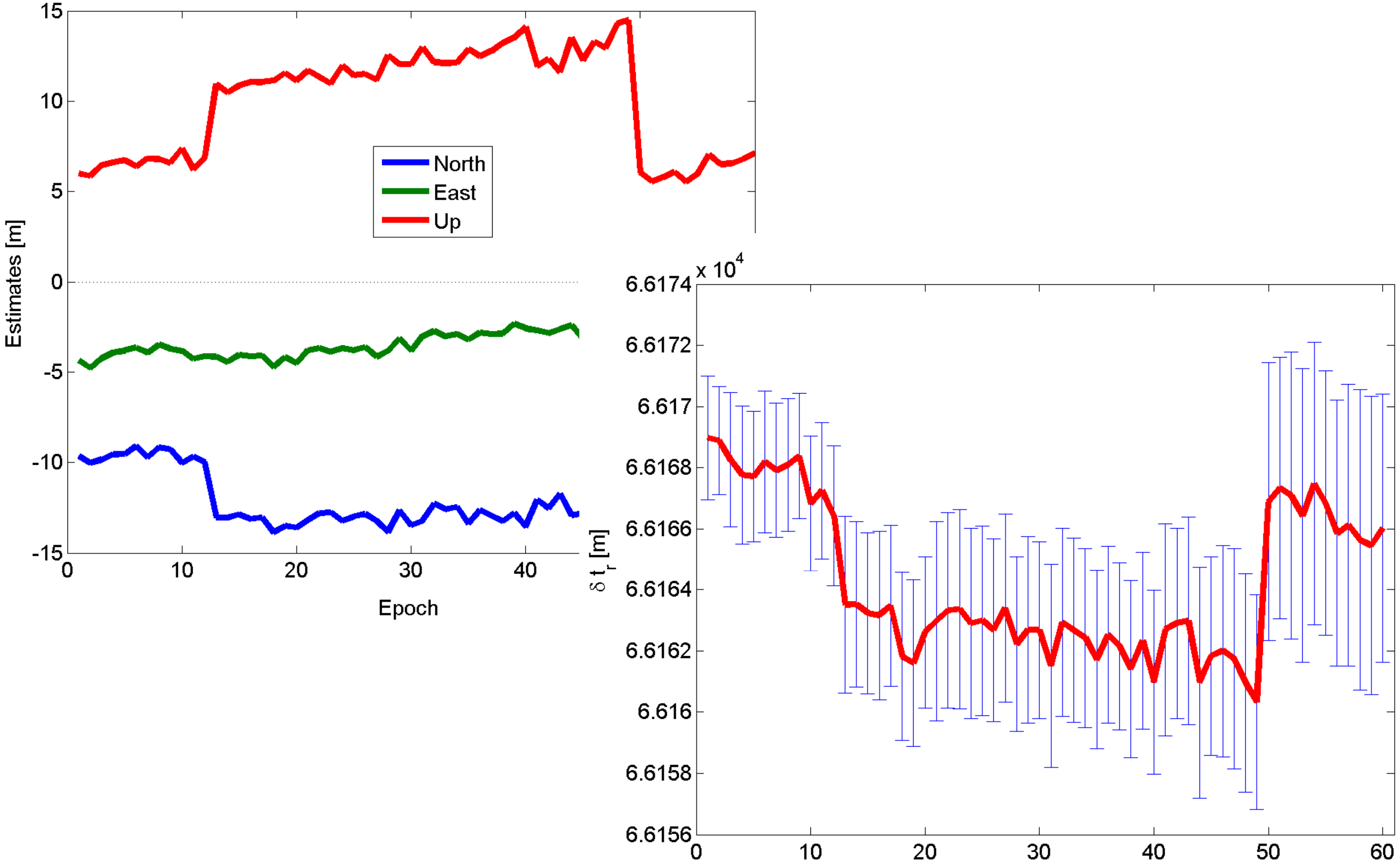
Exercise 2: Positioning with Pseudoranges – Part A



Simplified Pseudorange Observation Equation

$$P_r^s = \rho_r^s + c\,\delta t_r - c\,\delta t^s$$

P_r^s	pseudorange observation
ρ_r^s	geometric distance
δt^s	satellite clock correction
δt_r	receiver clock correction
c	speed of light

$$\rho_r^s = \sqrt{(x^s - x_r)^2 + (y^s - y_r)^2 + (z^s - z_r)^2}$$

Light travel time correction (1)

Correct for satellite motion during signal travel time

$$\rho_r^s(t) = \left| \mathbf{r}^s(t - \Delta t_r^s) - \mathbf{r}_r(t) \right|$$

With
$$\Delta t_r^s = \frac{\tilde{\rho}_r^s}{c}$$

$$\tilde{\rho}_r^s = \sqrt{(\tilde{x}^s - x_r)^2 + (\tilde{y}^s - y_r)^2 + (\tilde{z}^s - z_r)^2}$$

i.e., compute satellite ephemeris at epoch $t - \Delta t_r^s$.

$$\mathbf{r}^s(t - \Delta t_r^s) = \mathbf{r}^s(t) - \mathbf{v}^s(t) \cdot \Delta t_r^s$$

This correction is already applied in the exercise data.

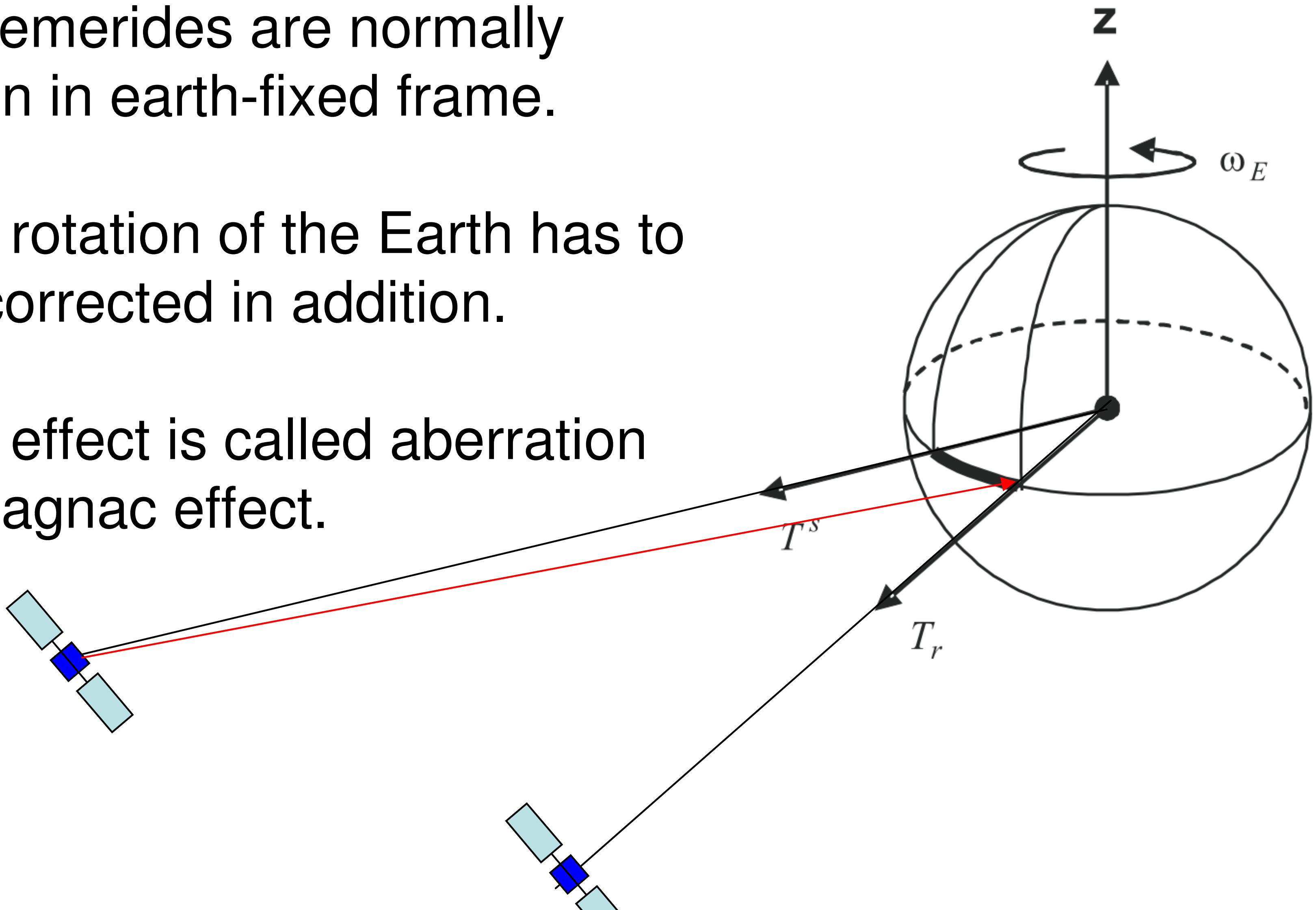
Light travel time correction (2)

Correct for Earth rotation during signal travel time

Ephemerides are normally given in earth-fixed frame.

The rotation of the Earth has to be corrected in addition.

The effect is called aberration or Sagnac effect.



Light travel time correction (2)

Correct satellite coordinates

$$\begin{aligned} x^s &= \tilde{x}^s \cos(d\Omega_E) + \tilde{y}^s \sin(d\Omega_E) \\ y^s &= -\tilde{x}^s \sin(d\Omega_E) + \tilde{y}^s \cos(d\Omega_E) \\ z^s &= \tilde{z}^s \end{aligned}$$

with $d\Omega_E = \Delta t_r^s \cdot \omega_E = \dfrac{\tilde{\rho}_r^s \cdot \omega_E}{c}$

$\tilde{x}^s,\ \tilde{y}^s,\ \tilde{z}^s$ raw satellite coordinates

$x^s,\ y^s,\ z^s$ corrected satellite coordinates

Recompute ρ_r^s with corrected satellite coordinates for further usage

Least Squares Adjustment

1. Compute the partial derivatives of the observation equation w.r.t. the unknowns:

$$\frac{\partial P_r^s}{\partial x_r} = \ldots, \qquad \frac{\partial P_r^s}{\partial y_r} = \ldots, \qquad \frac{\partial P_r^s}{\partial z_r} = \ldots, \qquad \frac{\partial P_r^s}{\partial \delta t_r} = \ldots$$

2. Compute "observed minus computed" based on the observations, the known satellite positions and satellite clock errors, and the a priori coordinates

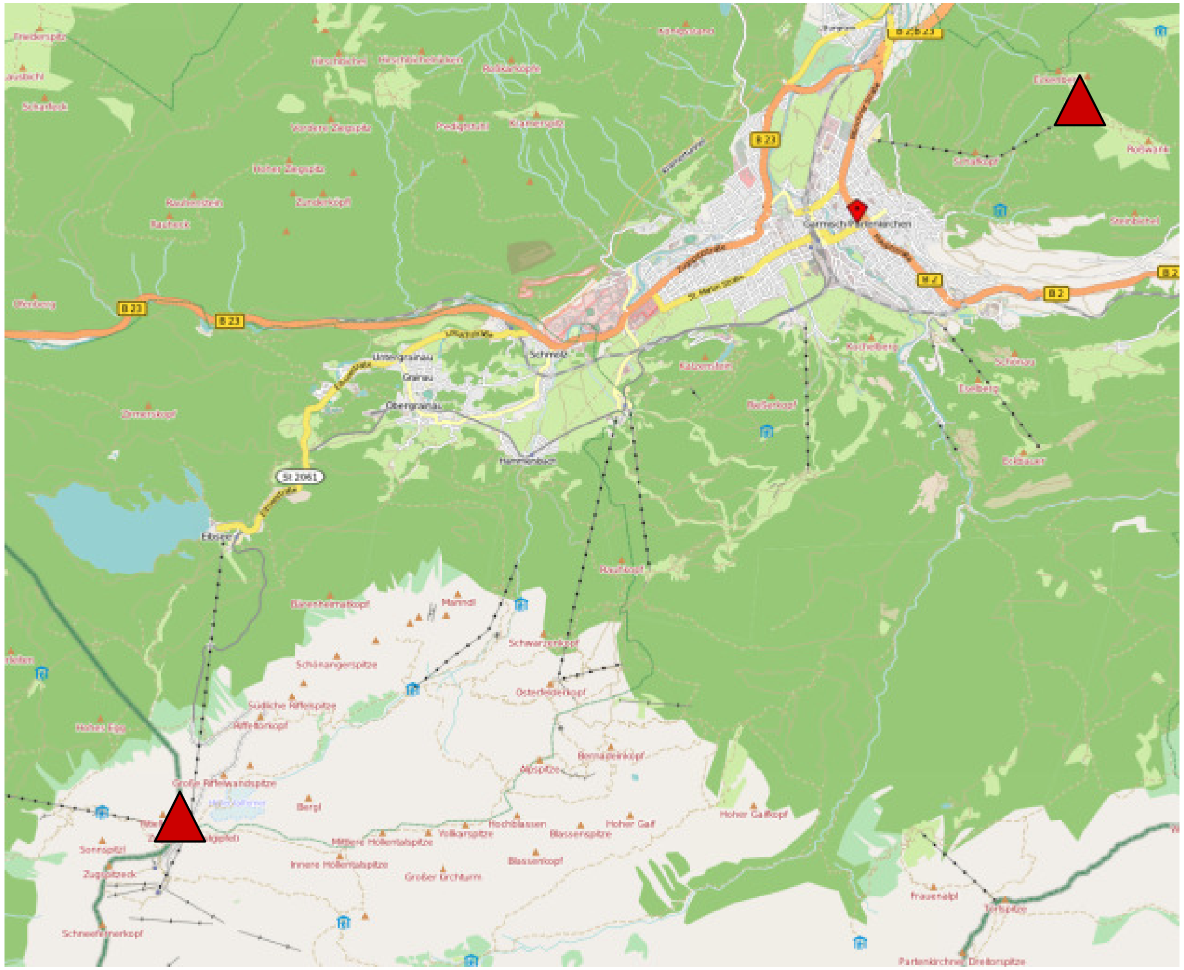
3. Solve for the unknown parameters with a least squares adjustment

4. Compute formal errors and residuals

GNSS Data (1)

74 observation epochs with 30s sampling for two stations close to Garmisch; each tracking 7 satellites:

- Wank: **`WANK`**
- Zugspitze: **`ZUGS`**



GNSS Data (2)

`Epochs.txt`	Epochs and satellite numbers
`WANK_C1`	C/A code observations for station WANK
`WANK_SATT`	satellite clock corrections for station WANK
`WANK_SATX`	satellite positions for station WANK, x-component
`WANK_SATY`	satellite positions for station WANK, y-component
`WANK_SATZ`	satellite positions for station WANK, z-component
`ZUGS_C1`	C/A code observations for station ZUGS
`ZUGS_SATT`	satellite clock corrections for station ZUGS
`ZUGS_SATX`	satellite positions for station ZUGS, x-component
`ZUGS_SATY`	satellite positions for station ZUGS, y-component
`ZUGS_SATZ`	satellite positions for station ZUGS, z-component
`ZUGSepo`	epochs and satellite numbers for station ZUGS

Exercise 2: Program structure

1. Read satellite positions and observations
2. Loop over all observation epochs (start with a few epochs)
3. For a given epoch:
    1. Correct satellite positions for light travel time.
    2. Compute the derivatives of the observation equation w.r.t. the estimated parameters.
    3. Set up the grand design matrix A and the observation vector l for all observations of the epoch.
    4. Compute the normal equation matrix and solve it.

Tasks

1. Plot the time series of estimated parameters and their formal errors.
2. Assess accuracy and precision of the estimated parameters.
3. Which effects have been neglected and which order of magnitude do they have?

Exercise 2: Positioning with Pseudoranges – Part B

Single Set of Coordinates

- Modify your positioning program to estimate one set of coordinates from all epochs (receiver clock corrections still have to be estimated every epoch)

Simple troposphere model

Relativistic correction

Differential Positioning with coordinate differences

Tropospheric Correction

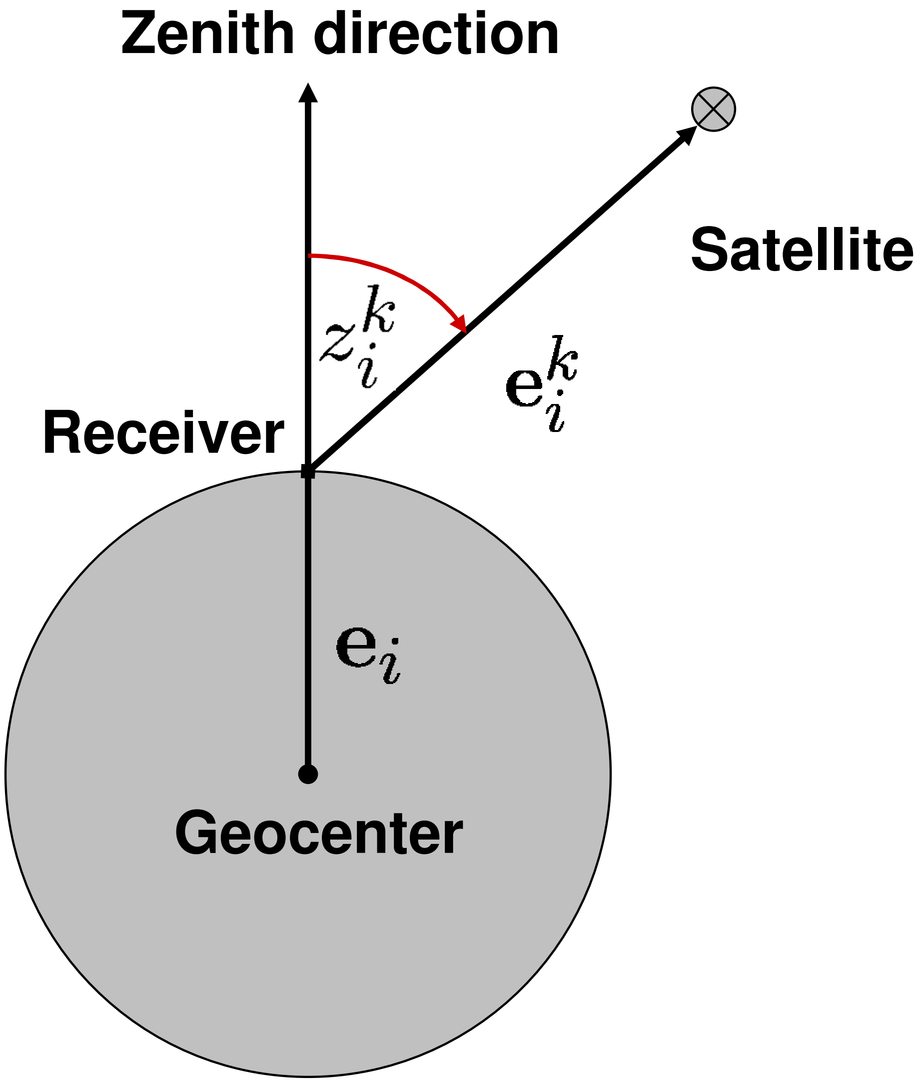
Very simple model only depending on the zenith angle

$$T_i^k(z_i^k) = \frac{T_0}{\cos z_i^k}$$

$T_0 = 2.3$ m tropospheric zenith path delay
z_i^k topocentric zenith angle

- What is the impact on station coordinates when modelling tropospheric delays?

Topocentric zenith angle



The topocentric zenith angle can be computed from the unit vector from the geocenter to the receiver $\mathbf{e}_i$ and from the receiver to the satellite $\mathbf{e}_i^k$

$$\cos z_i^k = \mathbf{e}_i \cdot \mathbf{e}_i^k$$

Topocentric zenith angle

$$\cos z_i^k = \left\{ \frac{x_i}{\sqrt{x_i^2 + y_i^2 + z_i^2}} \cdot \frac{x^k - x_i}{\sqrt{(x^k - x_i)^2 + (y^s - y_i)^2 + (z^k - z_i)^2}} + \right.$$

$$+ \frac{y_i}{\sqrt{x_i^2 + y_i^2 + z_i^2}} \cdot \frac{y^k - y_i}{\sqrt{(x^k - x_i)^2 + (y^k - y_i)^2 + (z^k - z_i)^2}} +$$

$$\left. + \frac{z_i}{\sqrt{x_i^2 + y_i^2 + z_i^2}} \cdot \frac{z^k - z_i}{\sqrt{(x^k - x_i)^2 + (y^k - y_i)^2 + (z^k - z_i)^2}} \right\}$$

Relativistic Correction

Due to eccentric satellite orbits the onboard clock exhibits periodic relativistic variations due to varying velocity (special relativity) and varying height in the Earth potential.

$$\Delta t_{rel}^{s} = -2\mathbf{r}^{s} \cdot \dot{\mathbf{r}}^{s} / c^{2}$$

- What is the impact on station coordinates when relativistic corrections are introduced?

Inertial satellite velocity

Inertial satellite velocity $\dot{\mathbf{r}}^s$ may be computed from position differences

$$\mathbf{v}^s \cong (\mathbf{r}^s(t_i) - \mathbf{r}^s(t_{i-1})) / (t_i - t_{i-1})$$

with sampling of 30sec

Satellite positions are given in Earth-fixed frame. The velocity has thus to be corrected for Earth rotation

$$\dot{\mathbf{r}}^s = \mathbf{v}^s + \mathbf{\Omega} \times \mathbf{r}^s$$

or

$$\dot{x}^s = v_x^s - \Omega\, y^s$$

$$\dot{y}^s = v_y^s + \Omega\, x^s$$

$$\dot{z}^s = v_z^s$$

Differential Positioning

1. Repeat the positioning for the second station Zugspitze
2. Form coordinate differences:
   - from estimated coordinates
   - from a priori coordinates
   - compare these differences
3. Why have the results improved?