

① 池化层反向传播。已知 pooling 的 δ^L , 推导 δ^L

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将 δ^L 的所有子矩阵还原回池化前大小 (unsample)

假设池化区 2×2 , δ^L 第 k 个矩阵为:

$$\delta_k^L = \begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix}$$

↓ 还原大小 (unsample)

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 8 & 0 \\ 0 & 4 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(1) 若为 average, 则平均。转换为:

$$\begin{bmatrix} 0.5 & 0.5 & 2 & 2 \\ 0.5 & 0.5 & 2 & 2 \\ 1 & 1 & 1.5 & 1.5 \\ 1 & 1 & 1.5 & 1.5 \end{bmatrix}$$

(2) 若为 max, 则为最大值位置还原 (类似 Relu)

若原最大值位于 左上, 右下, 右上, 左下

$$\text{则} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 \\ 0 & 4 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$f^{L+1} = \text{unsample}(f^L) \odot \sigma'(z^{L+1})$$

$\sigma(z)$ 为池化层激活函数,

$$\sigma(z) = z$$

② 卷积层反向传播推导,

假设 l 层的输出 a^l 是 3×3 矩阵, 第 l 层卷积核 2×2 , 采用 1 像素步长, 则输出 z^l 为 2×2 矩阵. 假设 $b^l = 0$, 则有:

$$a^l * w^l = z^l$$

$$\begin{bmatrix} a_{11}^l & a_{12}^l & a_{13}^l \\ a_{21}^l & a_{22}^l & a_{23}^l \\ a_{31}^l & a_{32}^l & a_{33}^l \end{bmatrix} * \begin{bmatrix} w_{11}^l & w_{12}^l \\ w_{21}^l & w_{22}^l \end{bmatrix} = \begin{bmatrix} z_{11}^l & z_{12}^l \\ z_{21}^l & z_{22}^l \end{bmatrix}$$

$$\Rightarrow z_{11} = a_{11} w_{11} + a_{12} w_{12} + a_{21} w_{21} + a_{22} w_{22}$$

$$z_{12} = a_{12} w_{11} + a_{13} w_{12} + a_{22} w_{21} + a_{23} w_{22}$$

$$z_{21} = a_{21} w_{11} + a_{22} w_{12} + a_{31} w_{21} + a_{32} w_{22}$$

$$z_{22} = a_{22} w_{11} + a_{23} w_{12} + a_{32} w_{21} + a_{33} w_{22}$$

$$\hookrightarrow \nabla a^l = \frac{\partial J}{\partial a^l} = \frac{\partial J}{\partial z^l} \cdot \frac{\partial z^l}{\partial a^l} = f^l \frac{\partial z^l}{\partial a^l}$$

$$\therefore \nabla a_{11} = \frac{\partial J}{\partial z_{11}} \cdot \frac{\partial z_{11}}{\partial a_{11}} = \delta_{11} \cdot w_{11}$$

$$\nabla a_{12} = \frac{\partial J}{\partial z_{12}} \cdot \frac{\partial z_{12}}{\partial a_{12}} + \frac{\partial J}{\partial z_{11}} \cdot \frac{\partial z_{11}}{\partial a_{12}} = \delta_{12} \cdot w_{11} + \delta_{11} w_{12}$$

同理 $\nabla a_{13} = \delta_{12} \cdot w_{12}$

$$\nabla a_{21} = \delta_{11} \cdot w_{21} + \delta_{21} \cdot w_{11}$$

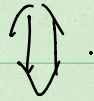
$$\nabla a_{22} = \delta_{11} \cdot w_{22} + \delta_{12} \cdot w_{21} + \delta_{21} \cdot w_{12} + \delta_{22} \cdot w_{11}$$

$$\nabla a_{23} = \delta_{12} \cdot w_{22} + \delta_{22} \cdot w_{12}$$

$$\nabla a_{31} = \delta_{21} \cdot w_{21}$$

$$\nabla a_{32} = \delta_{21} \cdot w_{22} + \delta_{22} \cdot w_{21}$$

$$\nabla a_{33} = \delta_{22} \cdot w_{22}$$



$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \delta_{11} & \delta_{12} & 0 \\ 0 & \delta_{21} & \delta_{22} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} * \underbrace{\begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix}}_{\text{rot } 180(W^L)} = \begin{bmatrix} \nabla a_{11} & \nabla a_{12} & \nabla a_{13} \\ \nabla a_{21} & \nabla a_{22} & \nabla a_{23} \\ \nabla a_{31} & \nabla a_{32} & \nabla a_{33} \end{bmatrix}$$

② 已知 conv 的 δ^L , 推导 δ^{L+1}

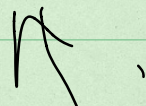
$$a^L = \sigma(z^L) = \sigma(a^{L+1} * w^L + b^L)$$

$$\text{而 } z^{L+1} = a^L * w^{L+1} + b^{L+1} = \sigma(z^L) * w^{L+1} + b^{L+1}$$

计算

$$\delta^{L+1} = \delta^L * \text{rot } 180(w^L) \odot \sigma'(z^{L+1})$$

③ 已知 conv 的 δ^L , 推导 $\frac{\partial J}{\partial w^L}$ 与 $\frac{\partial J}{\partial b^L}$



$$z^L = a^{L-1} * w^L + b^L$$

$$\frac{\partial J}{\partial w^L} = \frac{\partial J}{\partial z^L} \cdot \frac{\partial z^L}{\partial w^L} = \delta^L \cdot \frac{\partial z^L}{\partial w^L}$$

$$= \delta^L * \text{wtl80}(a^{L-1})$$

(同上).

$$\frac{\partial J}{\partial b^L} = \sum_{u,v} (\delta^L)_{u,v} \quad (b \text{ 为向量, 而 } \delta^L \text{ 为标量, 需将 } \delta^L \text{ 全部相加})$$