Performance Analysis and Comparison of ZF and MRT Based Downlink Massive MIMO Systems

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***Abstract*—In this paper the performances of zero-forcing (ZF) and maximum ratio transmission (MRT) are analyzed and compared in a downlink massive multiple-input multiple-output system. The system employs a large number of base station antennas serving multiple user terminals within the same cell. The achievable sum rate and the required downlink transmit power using each of the precoding schemes are derived, analyzed and compared under the same conditions and assumptions. Simulation results are found to coincide with the theoretical results, and show that ZF performs better than MRT under the same conditions.**

Keywords-Massive MIMO, zero-forcing(ZF), maximum ratio transmission(MRT), achievable sum rate, downlink transmit power.

I. Introduction

With todays advances in mobile communication systems, the need for data services has never been greater. The bandwidth of wireless communication systems is often limited by the cost of the radio spectrum required. Any increase in data rate, which can be realized without increasing the bandwidth, but with a reduction in power consumption, makes the system more spectrally and power efficient and less costly [1]. In order to meet the need, multiple-input multiple-output (MIMO) communication systems have been a hot topic of research over the past several years, due to its ability to greatly increase spectral and energy efficiencies[2].

With the advancement of technology comes an increase in demand for spectral capacity, and massive MIMO technique has been proposed to improve the performance of MIMO. Massive MIMO is essentially a multiuser MIMO technique with lots of base station antennas in which a large number of antennas are served simultaneously [2]. Massive MIMO reaps all the benefits of conventional MIMO in a much greater scale in terms of spectral efficiency, energy efficiency, reliability, and interference minimization [3]. In addition to these benefits, it also takes advantage of the large number of antennas to simplify multiuser processing, and makes thermal noise and fast fading vanish [3].

The huge potential of massive MIMO has attracted the interest of many researchers with much focus on spectral efficiency and power efficiency in cellular communication systems [2]-[4]-[5]. Linear precoding/beamforming schemes play an important role in massive MIMO signal processing. In [6], the authors analyzed the performance of the downlink massive MIMO in terms of spectral efficiency, energy efficiency and link reliability using ZF precoding. The authors in [7] compared the matrix and vector normalization for downlink ZF and MRT precoding and analyzed the ergodic performance of such precoding in a cell-boundary users scenario. In [8], the authors compared the eigen beamforming (BF) and regularized zero-forcing (RZF) performance in terms of achievable data rate in a multi-cell downlink scenario. The author in [9] analyzed the spectral efficiency of a single-cell downlink massive MIMO system with ZF, MRT and MMSE (minimum mean-square error).

Although the above works provide good results about the performances of the linear precoding schemes, they did not provide the comparison of ZF and MRT performances in terms of achievable data rate and transmit power at the same time, under the same conditions and in a single-cell downlink scenario. The author in [9], who analyzed the performance of such precoding schemes in terms of spectral efficiency in a single-cell downlink scenario, fixed the same value of signal-to-interference-to-noise ratio for both precoders; which should not be done for a good analysis. This is because according to equation (3) the signal-to-interference-to-noise ratio is a function of the transmit beamforming vector, and ZF and MRT have different beamforming vectors according to equations (4) and (6).

This paper analyzes and compares the performance; in terms of achievable data rate and total transmit power, of ZF and MRT precoding schemes in a single-cell downlink massive MIMO system, under the same assumptions. A fixed number of mobile users are assumed to be uniformly distributed in the cell with equal power allocation. After formulating the system model, the achievable data rate is derived for each of the linear precoding schemes under the same assumptions. The total downlink transmit power is also derived for each of them. Simulations for both data rate and transmit power are done for both schemes. The various

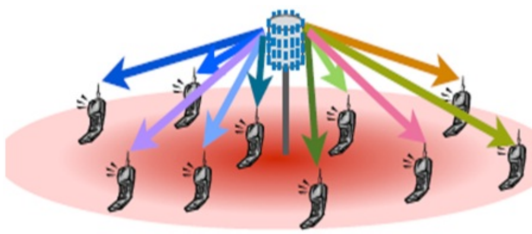

Figure 1. A single-cell downlink massive MIMO system

results are analyzed in order to compare the performance of the techniques in the given system.

The remaining parts of this paper are organized as follows. Section II looks at the system model and ZF and MRT transmit beamforming. The achievable data rate with each of the precoding schemes is presented in section III. The total downlink transmit power for each scheme is derived in section IV. Section V provides numerical results and analysis. Conclusion and future work are dealt with in VI.

II. SYSTEM MODEL

A. Channel Model

As shown in Figure 1, the system is a single-cell downlink where one base station (BS) equipped with M antennas serves K single-antenna mobile users with the same time-frequency resource. The channel is a Rayleigh fading MIMO channel with the assumption of perfect channel state information. Let h_k denote the channel vector between the BS and the k_{th} user. The system channel vector between the BS and all the users is h with H as channel matrix. The elements of H are independent and identically distributed (iid) complex Gaussian variables with zero mean and unit variance. The linear precoding or beamforming vector of kth user is denoted by w_k, and the system beamforming matrix is W.

The receiver vector is given by:

$$y = \sqrt{P_d}Hx + n = \sqrt{P_d}HWS + n \tag{1}$$

Where P_d is the total downlink transmit power, x is the transmitter vector, S is the receiver signal matrix, and n is the additive Gaussian white noise (AGWN). H is a $K \times M$ matrix, and W is an $M \times K$ matrix. The signal received by the k_{th} user after using the linear precoding/ beamforming scheme is given by

$$y_k = \sqrt{P_d}h_k w_k s_k + \sqrt{P_d}\sum_{i=1}^{k} h_k w_i s_i + n \tag{2}$$

where $\sqrt{P_d}h_k w_k s_k$ is the desired signal, $\sqrt{P_d}\sum_{i=1}^{k} h_k w_i s_i$ is the interference, and n is the noise.

The received signal-to-interference-plus-noise ratio of the kth user can then be expressed as [10]

$$SINR_K = \frac{P_d|h_k w_k|^2}{P_d\sum_{i=1}^{k}|h_k w_i|^2 + 1} \tag{3}$$

which is a function of the transmit beamforming vector.

B. Transmit beamforming

Two conventional linear precoding/beamforming schemes are used in this work: the Zero-Forcing beamformer (ZF) and the Maximum Ratio Transmission (MRT). The two beamformers have different SINR according to equation (3).

C. Zero-forcing (ZF) precoding

ZF is one technique of linear precoding in which the inter-user interference can be cancelled out at each user [9]. The ZF precoding employed by the BS is written as

$$W = H^H(HH^H)^{-1} \tag{4}$$

For large values of M and K, the related signal-to-interference-plus-noise ratio of the kth user is given as [10]

$$SINR_K^{zf} = P_d(\frac{M-K}{K}) \tag{5}$$

D. Maximum ratio transmission (MRT) precoding

MRT is one technique of linear precoding which maximizes the signal gain at the intended user [9]. The MRT precoding employed by the BS is written as

$$W = H^H \tag{6}$$

For large values of M and K, the related signal-to-interference-plus-noise ratio of the kth user is given as [10]

$$SINR_K^{mrt} = \frac{P_d M}{K(P_d+1)} \tag{7}$$

From equations (5) and (7), it is obvious that for the same available downlink transmit power and same value of M and K in ZF and MRT cases, the two schemes have different signal-to-interference-to-noise ratio. Therefore, the same value should not be assigned to it (SINR) for both precoders for a good performance comparison.

III. ACHIEVABLE DATA RATE

One of the methods to quantify the system performance is the achievable data rate. The achievable data rate follows the Shannon theorem. This theorem gives the maximum rate at which the transmitter can transmit over the channel. In this section, we describe the achievable data rate with ZF and MRT according to the system under consideration, with the assumption that the total downlink power is fixed and equally divided among all the users. From Shannon theorem, the channel capacity over Additive White Gaussian Noise channel is derived by [10] as

$$R = log_2(1+SNR)(bits/s/Hz) \tag{8}$$

Where SNR is the signal-to-noise ratio. Channel state information (CSI) is an important issue in multiuser communication systems. Typically, the transmitter transmits multiple data streams to each user simultaneously and selectively with CSI [11]. All the receivers send the channel estimation feedback to the transmitter on the reverse link, so the transmitter obtains CSI. Hence, the transmitter communicates with all the receivers with perfect CSI [9]. And as shown in equation (2), the signal received by the each user consists of additive noise and interference between the users themselves. Then, the achievable data rate per user in a single-cell downlink massive MIMO system, with perfect channel state information is given as

$$R_k = log_2(1 + SINR_k) \tag{9}$$

and for K number of users, the achievable sum rate is given as

$$R_{sum} = Klog_2(1 + SINR_k) \tag{10}$$

A. The Achievable Data Rate with ZF

From (10), the achievable data rate with ZF can be deduced as

$$R_{sum}^{zf} = Klog_2(1 + SINR_k^{zf}) \tag{11}$$

Substituting (5) into (11), gives

$$R_{sum}^{zf} = Klog_2[1 + P_d(\frac{M-K}{K})] \tag{12}$$

B. The Achievable Data Rate with MRT

Similarly, the achievable data rate with MRT can be deduced from (10) as

$$R_{sum}^{mrt} = Klog_2(1 + SINR_k^{mrt}) \tag{13}$$

Substituting (7) into (13) gives

$$R_{sum}^{mrt} = Klog_2[1 + \frac{P_d M}{K(P_d + 1)}] \tag{14}$$

Equations (12) and (14) show that for the same available downlink transmit power and a fixed number of mobile users; as the number of transmit antennas increases with $M \gg K$, ZF achieves higher data rate than MRT.

IV. REQUIRED DOWNLINK TRANSMIT POWER

The energy efficiency of a communication system depends on the required transmit power. The system is more energy efficient when less transmit power is required to achieve the targeted information rate on condition that the quality of service is satisfied. In this section, we derive the total downlink transmit power required by ZF and MRT to achieve the same data sum rate according to the system under consideration. For the derivations we denote

$$\alpha = \frac{M}{K} \tag{15}$$

And then

$$\alpha - 1 = \frac{M-K}{K} \tag{16}$$

A. Total Downlink Transmit Power with ZF

Substituting (16) into (12) and considering the fact that the targeted data rate is the same for both precoding schemes, we can write

$R_{sum} = Klog_2[1 + P_d^{zf}(\alpha - 1)]$

$\Rightarrow ln[1 + P_d^{zf}(\alpha - 1)] = ln2(\frac{R_{sum}}{K})$

Taking exponential of both sides we have

$$P_d^{zf} = \frac{e^{\frac{ln2R_{sum}}{k}} - 1}{\alpha - 1} \tag{17}$$

Substituting (16) into (17) gives

$$P_d^{zf} = K[\frac{e^{\frac{ln2R_{sum}}{k}} - 1}{M-K}] \tag{18}$$

which is the total downlink transmit power required with ZF.

B. Total Downlink Transmit Power with MRT

With the same analogy as for ZF, we substitute (15) into (14) and we can write

$$R_{sum} = Klog_2(1 + \frac{P_d^{mrt}\alpha}{P_d^{mrt} + 1}) \tag{19}$$

Taking exponential of both sides we have

$$P_d^{mrt} = \frac{[e^{\frac{ln2R_{sum}}{K}} - 1]}{\alpha - [e^{\frac{ln2R_{sum}}{K}} - 1]} \tag{20}$$

Substituting (15) into (20) gives

$$P_d^{mrt} = \frac{[e^{\frac{ln2R_{sum}}{K}} - 1]}{\frac{M}{K} - [e^{\frac{ln2R_{sum}}{K}} - 1]} \tag{21}$$

$$P_d^{mrt} = \frac{K[e^{\frac{ln2R_{sum}}{K}} - 1]}{M - K[e^{\frac{ln2R_{sum}}{K}} - 1]} \tag{22}$$

Since the total downlink transmit power is assumed to be equally divided among all the users, we also assumed that the total achievable data rate is equally shared among the users; and three different cases are studied for our analysis, considering Equations (18) and (22).

Case 1: the total power is required to achieve 1 bit per second per Hertz for each user. That is

$R_{sum} = K$

and for $R_{sum} = K$,$e^{\frac{ln2R_{sum}}{K}} = e^{ln2} = 2$

$$\Rightarrow M - K[e^{\frac{ln2R_{sum}}{K}} - 1] = M - K \tag{23}$$

Based on this analysis and from equations (18), (22) and (23), we can conclude that $P_d^{zf} = P_d^{mrt}$. For the total achievable data rate equally shared among the users, as the

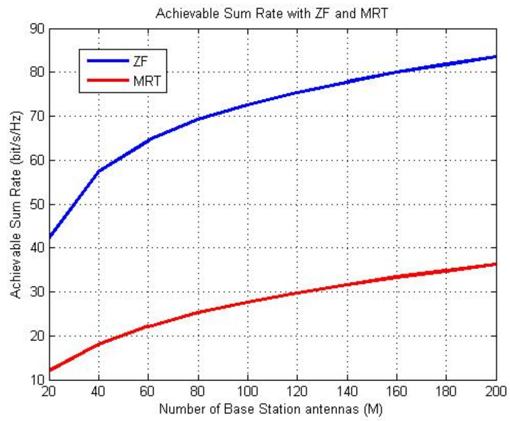

Figure 2. Performance comparison of achievable sum rate versus the number of transmit antennas, with K=10 users and the total available transmit power P_d = 15dB

number of transmit antennas increases with $M \gg K$, the same total downlink transmit power is required with ZF and MRT to achieve 1 bit per second per Hertz for each user.

Similarly:

Case 2: the total power is required to achieve more than 1 bit per second per Hertz for each user. That is

$R_{sum} >$ K which gives

$P_d^{mrt} > P_d^{zf}$

Based on this analysis, we conclude that, for the total achievable data rate equally shared among the users, as the number of transmit antennas increases with M>>K, MRT requires more power than ZF to achieve more than 1 bit per second per Hertz for each user. It means that ZF is more power efficient to achieve higher data rate.

Case 3: the total power is required to achieve less than 1 bit per second per Hertz for each user.That is:

$R_{sum} <$ K which gives

$P_d^{mrt} < P_d^{zf}$

Based on this analysis, we conclude that, for the total achievable data rate equally shared among the users, as the number of transmit antennas increases with M>>K, MRT requires less power than ZF to achieve less than 1 bit per second per Hertz for each user. It means that MRT is more power efficient to achieve lower data rate.

V. Numerical Results and Analysis

In order to validate the theoretical results in sections III and IV, numerical results are provided by simulations in this section. With available BS downlink transmit power equal to 15dB after all losses, and the number of mobile users fixed to 10, Figure 2 depicts the achievable sum rate versus the number of transmit antennas for both precoding schemes. It shows that as the number of BS antennas increases, the achievable sum rate for each of the schemes also increases. Furthermore, a comparison of the two performances shows

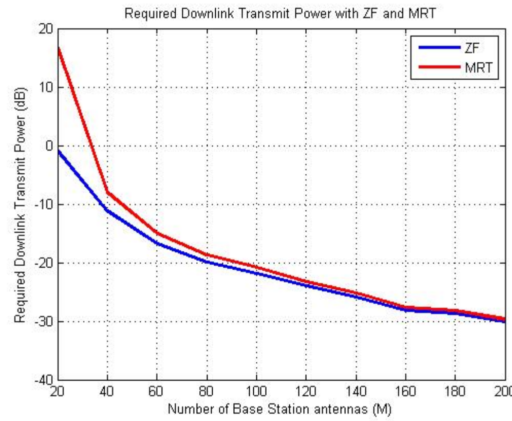

Figure 3. Performance comparison of total downlink transmit power required to achieve more than 1bit per second per Hertz for each user, with the number of users K=10 and the targeted sum rate R_{sum} = 15bits/s/Hz

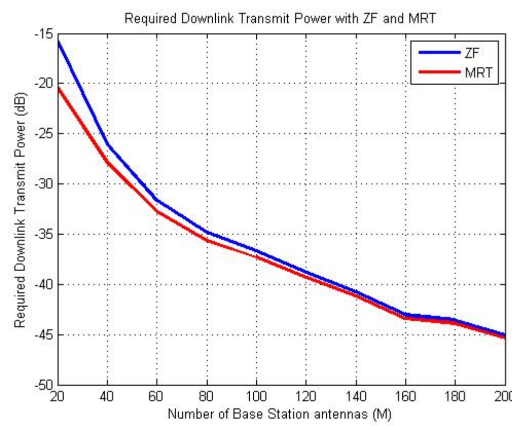

Figure 4. Performance comparison of total downlink transmit power required to achieve less than 1bit per second per Hertz for each user, with the number of users K=10 and the targeted sum rate R_{sum} = 5bits/s/Hz

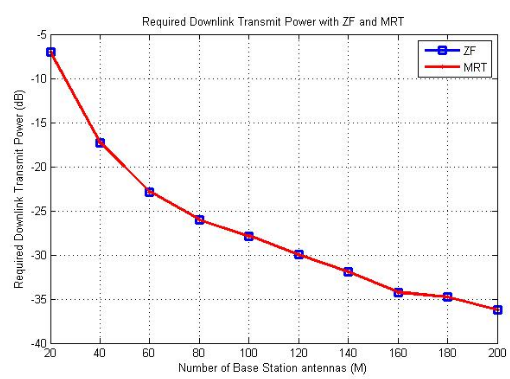

Figure 5. Performance comparison of total downlink transmit power required to achieve 1bit per second per Hertz for each user, with the number of users K=10 and the targeted sum rate R_{sum} = 10bits/s/Hz

Figure 6. Power efficiency comparison of ZF and MRT, with K=10 users and M=200 base station antennas

that ZF achieves much higher sum rate than MRT. The average sum rate achieved with ZF can be estimated to be more than a double of that achieved with MRT for 20 to 200 base station antennas. This validates the theoretical results obtained in section III. Therefore, we conclude that ZF achieves higher data rate than MRT in a single-cell downlink massive MIMO system where the available BS transmit power is assumed to be equally divided among all the users.

Figures 3, 4 and 5 show the required downlink transmit power versus the number of transmit antennas for both precoding schemes. In figure 3 the power is required to achieve more than 1 bit per second per Hertz for each user and the targeted sum rate is 15 bits/s/Hz and is assumed to be equally shared among 10 mobile users. In figure 4 the power is required to achieve less than 1 bit per second per Hertz for each user and the targeted sum rate is 5 bits/s/Hz and is assumed to be equally shared among 10 mobile users. In figure 5 the power is required to achieve 1 bit per second per Hertz for each user and the targeted sum rate is 10 bits/s/Hz and is assumed to be equally shared among 10 mobile users.

The results in the three figures show that as the number of BS antennas increases, the required downlink transmit power for each of the schemes decreases. Furthermore, a comparison of the performances of both schemes for 20 to 200 BS antennas shows that ZF requires less power than MRT in figure 3, MRT requires less power than ZF in figure 4, and both schemes require the same power in figure 5. This also validates the theoretical results obtained in section IV. Therefore, we conclude that ZF requires less power than MRT to achieve higher data rate, MRT requires less power than ZF to achieve lower data rate, and both require the same power to achieve 1 bit per second per Hertz for each user in a single-cell downlink massive MIMO system where the sum rate is assumed to be equally shared among all the users.

Figure 6 shows the achievable sum rate versus the required downlink transmit power for both schemes. The results show that for the same available downlink transmit power ranging from 0 dB to 20 dB, ZF achieves higher data rate than MRT. The achievable rate is higher than the number of mobile users; which means that more than 1 bit per second per Hertz is achieved for each user. This also validates the theoretical results obtained in section IV. Therefore, we conclude that ZF is more power efficient than MRT to achieve high data rate.

VI. Conclusion

This paper provides the performance analysis and comparison of ZF and MRT precoding schemes in a single-cell downlink massive MIMO system. The key performance parameters studied are the achievable sum rate and the total downlink transmit powers which are theoretically derived for both schemes under the same assumptions and according to the system. Simulation and theoretical results show that ZF achieves higher data rates than MRT. ZF is more power efficient than MRT to achieve high data rate. Therefore, we conclude that ZF is the best choice in a single-cell downlink massive MIMO system. In future work we will consider multi-cells and both down and up links for more performance analysis.

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References

[1] John Fitzpatrick, Simulation of a Multiple Input Multiple Output Wireless System, A thesis of Dublin City University, April 2004

[2] Erik G. Larsson, Fredrik Tufvesson, Ove Edfors and Thomas L. Marzetta, Massive MIMO for next Generation Wireless Systems, IEEE Communication Magazine, Vol.52, no. 2, 2014, pp.186-195

[3] Robert W. Heath Jr., What is the Role of MIMO in Future Cellular Networks:Massive? Coordinated?mmWave?, A lecture Presentation on massive MIMO, The University of Texas at Austin, 2013

[4] Hien Quoc Ngo, Erik G. Larsson, and Thomas L. Marzetta, Energy and Spectral Efficiency of Very Large Multiuser MIMO Systems, IEEE Transactions on Communications, Vol.61, no.4, 2013, pp.1436-1449

[5] Jinkyu Kang, Joonhyuk Kang, Namjeong Lee, Byung Moo Lee and Jongho Bang, Minimizing Transmit Power for Cooperative Multicell System with Massive MIMO, The 10th Annual IEEE Cosumer Communications and Networking Conference (CCNC), 2013, pp.438-442

[6] Long Zhao, Kan Zheng, Hang Long Hui Zhao and Wenbo Wang, Performance Analysis for Downlink Massive MIMO System with ZF precoding, Transactions on Emerging Telecommunications Technologies, Vol.8, no.3, 2014, pp.390-398

[7] Yeon-Geun Lim, Chan-Byoung Chae, and Giuseppe Caire, Performance Analysis of Massive MIMO for Cell-Boundary Users, IEEE Communication Magazine, In Press, Available at: http//arxiv:2309.7817v1 [cs.IT], 30 September 2013

[8] Jakob Hoydis, Stephan Ten Brink, and Merouane Debbah, Comparison of Linear Precoding Schemes for Downlink Massive MIMO, IEEE ICC - Communications Theory, 2012, pp.2135-2139

[9] Eakkamol Pakdeejit, Linear Precoding Performance of Massive MU-MIMO Downlink System, A thesis of Linkopeng University, May 2013

[10] Sooyong Choi, Special Topics: Massive MIMO, A lecture presentation on Massive MIMO, Yonsei University, June 2012

[11] T. Marzetta and B. Hochwald, Fast Transfer of Channel State Information in Wireless Systems, IEEE Transactions on Signal Processing, Vol. 54, no. 4, 2006, pp. 1268-1278