INSTITUTO SUPERIOR TÉCNICO

Traffic Engineering

Lab Project

Analysis of queues and Poisson distributions

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1 Goal

Simulate, analyze and discuss the properties of Poisson arrival processes and its application to queue processing.

The full first lab project will cover two lab sessions. Please note that before the end of the second session the work must be assessed on site by the teaching staff, otherwise the final report will not be considered for grading. The written report including lab results (see section 4 below) must be delivered until Monday, 16 Oct, on the Fenix System.

2 Poisson process simulation

2.1 Introduction

The probability of observing a given number of events on a unitary period in a Poisson arrival process is given by

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

The Poisson process is closely related to the exponential distribution. As it is well known, the time *t* between two successive events is a continuous random variable with distribution given by the probability density function

$$f(t) = \lambda e^{-\lambda t}$$

A pseudo random variable δt with exponential distribution can be generated following the according to the following procedure:

1. Generate a random number *u* with a uniform distribution on [0,1).
2. Apply the transformation

$$\delta t = -\frac{\log(1-u)}{\lambda}$$

2.2 Arrival process generation

The first goal of this lab is to write a computer program in a language of your choice in order to simulate a Poisson process:

1. Generate a sequence of N=50 events with time interval δt between successive events. Generate δt according to a exponential distribution of parameter λ. Make λ=5 for initial tests

2. Use the above sequence to make a histogram (bar chart) of the number of events occurring in a unitary time interval. The use of the Matlab / Octave *hist()* function is forbidden, develop your own version!

   Why is the *hist()* function forbidden? We noticed that many students use the *hist()* function without fully understand its behavior, therefore achieving wrong results. Since the implementation of the *hist()* function just requires a few lines of code, the mandatory implementation will help you to understand what is happening…

3. Use any plotting software (*matlab*, *gnuplot, excel*, …) to display the above histogram against the theoretical Poisson distribution. The theoretical distribution and the experimental one (histogram) must be displayed in the same plot.

4. Repeat for different values of λ. Use as reference at least the following values for λ: {0.5, 1.0, 5.0, 10.0}. Try your program with values of N in the range 10 to 20000. Check the results. Discuss.

2.3 Superposition of Poisson processes

1. Modify the program above to generate 3 independent sequences with different λ values;
2. Generate a single sequence that combines the 3 processes above;
3. Use the sequence above to generate an histogram of the combined process;
4. Is the resulting sequence a Poisson process?

3 M/M/1 queue simulation

Simulate a M/M/1 queue. You may re-use some of the code developed before.

Assume that the arrival process at the queue is Poisson with parameter λ. Assume that events on the queue are processed on a FIFO basis, and that each event at the top of the queue takes t seconds to be processed. t is a random variable with an exponential distribution with parameter μ. Check the average queue size for different values of λ and μ and compare it with the theoretical value. Simulate the queue in the cases $\mu < \lambda$, $\mu = \lambda$ and $\mu > \lambda$. Discuss the results.

4 Assignment report

The written report of this assignment must include:

1. Discussion of used algorithms and developed software;
2. Graphics, tables and other results of the simulations which may be relevant for this experiment;
3. Discussion of achieved results;
4. Software source (supplied as an attached archive in any standard format: zip, tar, tgz, rar…).