Modelling and Simulation
Lab Assignment - 4

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I. Abstract

Modelling Malaria:
Using multi-compartment model for modelling malaria and noting the effect of other measures to curb the spread of malaria.

II. Problem Statement

Malaria involves relationship between humans and mosquitoes. Not all mosquitoes spread malaria. While we do not go into the details of this part, the part that is important to us from a modelling perspective is that out of the total mosquito population only a fraction have the potential to spread malaria. Amongst these we can think of the ones which are already infected and the ones which are not infected (susceptible). Amongst humans all have equal chance of being infected with malaria (all are susceptible). A bite from an infected mosquito can lead to malaria in humans. Alternatively, if a susceptible mosquito bites an infected human, they can become infected.

- We work under the following assumptions Since life expectancy of humans is much larger than that of mosquito, we assume that the human population is closed with no births, immigration and deaths except from malaria.
- As soon as an infected mosquito bites a susceptible human, it becomes infected.
- As soon as a susceptible mosquito bites an infected human it becomes infected.
- Because of their relatively short life expectancy mosquito's births and deaths are considered.

We therefore consider the following processes in malaria spread:

Humans:

- susceptible humans get infected due to mosquito bite from an infected mosquito.
- Infected humans can recover and become susceptible again.
- Infected humans can die due to malaria.
- Infected humans can become immune.

Mosquitoes:

- Susceptible mosquitoes can become infected when they bite an infected human.
- Mosquitoes can be born. All mosquitoes are born susceptible.
- All mosquitoes can die natural deaths.

III. Building the Model

Based on the above assumptions we propose the following model for malaria:

IV. Building Differential Equations

From the model above we can identify following rates and variables:

- Susceptible Humans: S_h
- Infected Humans: I_h
- Immune Humans: Im_h
- Susceptible Mosquitoes: S_m
- Infected Mosquitoes: I_m
- Probability of mosquito biting a susceptible/infected human: p
- Conversion rate of Infected human to Susceptible human: r_1
- Death rate of the Infected human: d_1
- Immunity rate (Infected human to Immune human): r_2
- Birth rate of the mosquito: b
- Death rate of mosquitoes (both infected and Susceptible death rate are same): d_2
- Total population of humans: $S_h + I_h + Im_h = H$
- Total population of mosquitoes: $S_m + I_m = M$

Using the above variables and rates we can build following differential equations:

1. $S'_h = -pS_h\left(\frac{I_m}{M}\right) + r_1 I_h$
2. $I'_h = pS_h\left(\frac{I_m}{M}\right) - r_1 I_h - r_2 I_h - d_1 I_h$
3. $Im'_h = r_2 I_h$
4. $S'_m = b(S_m + I_m) - pS_m\left(\frac{I_h}{H}\right) - d_2 S_m$
5. $I'_m = pS_m\left(\frac{I_h}{H}\right) - d_2 I_m$

IV. Implementing the Model

The following results are obtained by implementing the above proposed model, for the given values:

i. Initial Values (at t=0)

- $S_h = 300$
- $I_h = 1$
- $Im_h = 0$
- $S_m = 300$
- $I_m = 0$

ii.Constants:

- $r_1 = 0.3$
- $r_2 = 0.01$
- $d_1 = 0.005$
- $b = 0.01$
- $d_2 = 0.005$
- $p = 0.3$

When the simulation is run for 250 days:

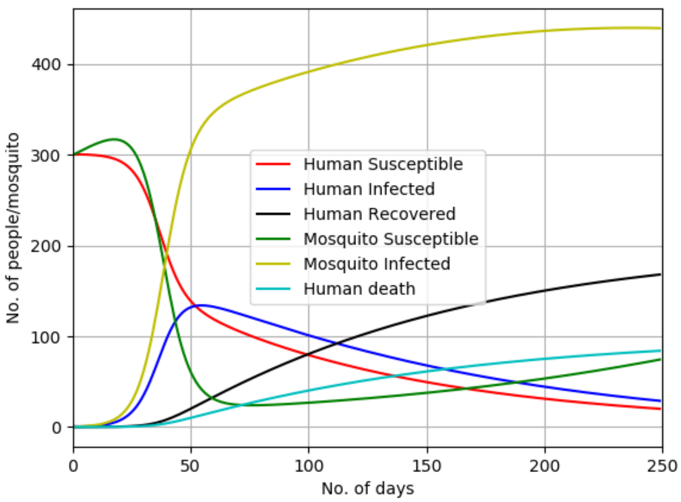


Figure 1.

When the simulation is run for 600 days:

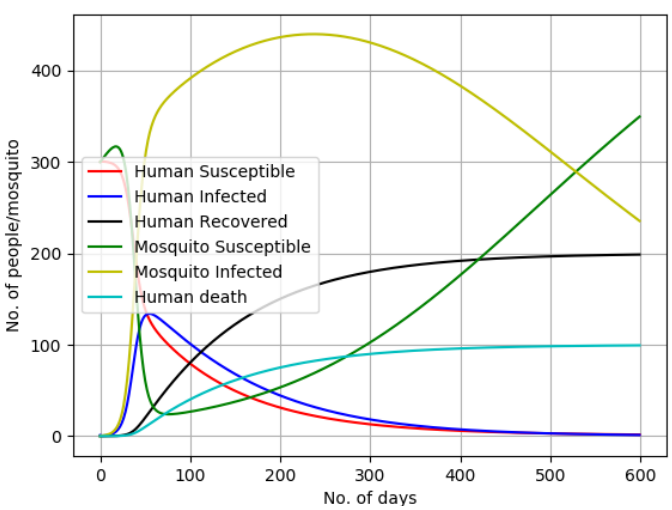


Figure 2.

The number of susceptible mosquitoes initially decreases sharply till 50 days as those are converted to infected mosquitoes due to contact with infected humans. After some time, number of infected mosquitoes decreases slowly due to death and that of susceptible mosquitoes increases due to birth. The number of infected human increases rapidly initially till 50 days then reaches saturation and decreases because of immune and deaths. Number of infected humans is always less than susceptible humans. Also, the number of susceptible humans decreases rapidly initially (due to infection) till 50 days and then slowly (due to deaths and immune). Number of immune increases continuously and becomes almost constant after stable state of no spread in malaria disease. Infected mosquitoes become zero after nearly 600 days. The total number of humans is reduced to one-third of initial population (nearly 200).

V. Effects of Different Measures

A. *Taking in account the effect of fumigation:*

By taking in the account the effect of fumigation the differential equation number (4) and (5) changes as follows:

- $S'_m = b(S_m + I_m) - pS_m\left(\frac{I_h}{H}\right) - d_2S_m - fS_m$
- $I'_m = pS_m\left(\frac{I_h}{H}\right) - d_2I_m - fI_m$

The effect of fumigation is introduced by f which is the death rate of mosquitoes due to fumigation.

Taking the value of f to be 0.009 we have the following result:

When the simulation is rum for 250 days:

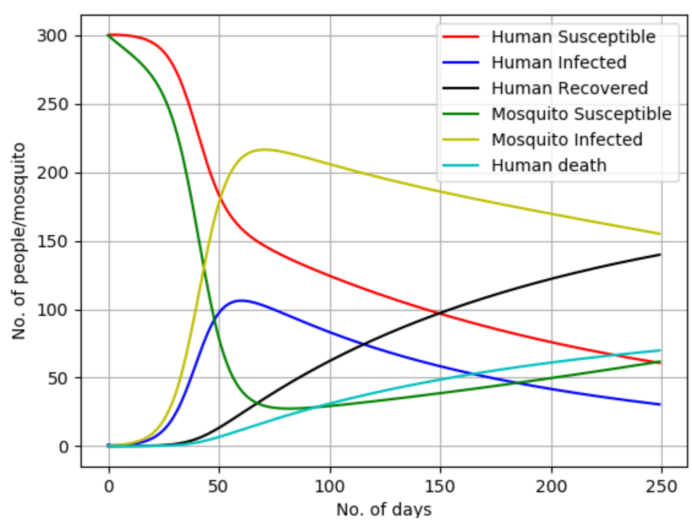


Figure 3.

When the simulation is run for 600 days:

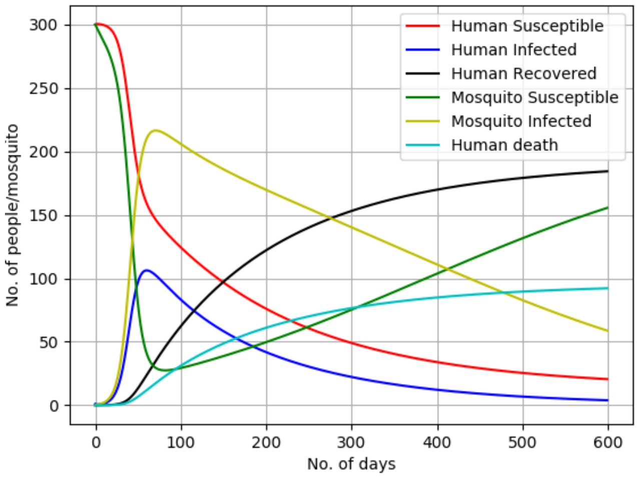


Figure 4.

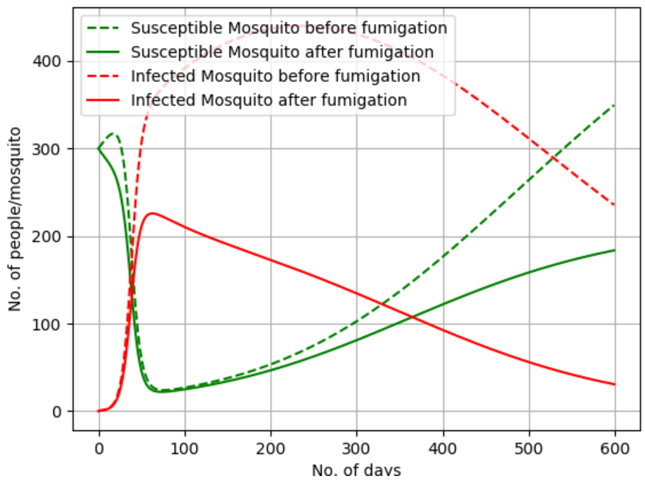


Figure 5.

By comparing figure 3 and figure 4 with figure 1 and figure 2 respectively we observe that due to fumigation the number of infected mosquitoes decreases. Also, the maximum number of infected mosquitoes reached is less compared to case without fumigation. However, by comparing the figure 2 and 4 we see that at the end of the simulation (i.e. at t=600) the number of susceptible humans is more.

B. Taking in account the effect of Vaccination:

By taking in the account the effect of vaccination the differential equation number (1) and (3) changes as follows:

- $S_h' = -pS_h\left(\frac{I_m}{M}\right) + r_1 I_h - vS_h$
- $Im_h' = r_2 I_h + vS_h$

The effect of vaccination is introduced by v which is the death rate of mosquitoes due to fumigation.

Taking the value of v to be 0.009 we have the following result:

When the simulation is rum for 250 days:

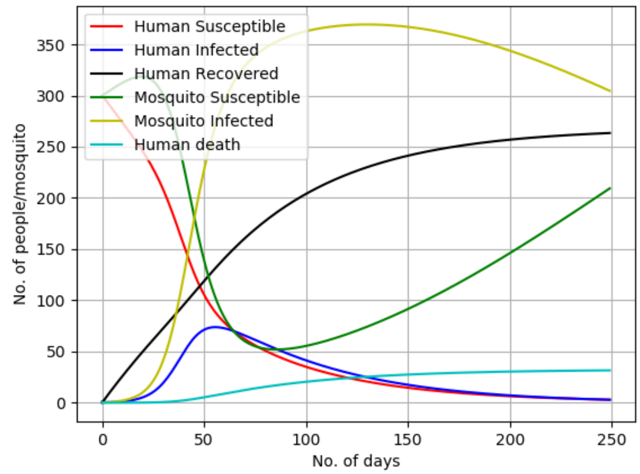


Figure 6.

When the simulation is run for 600 days:

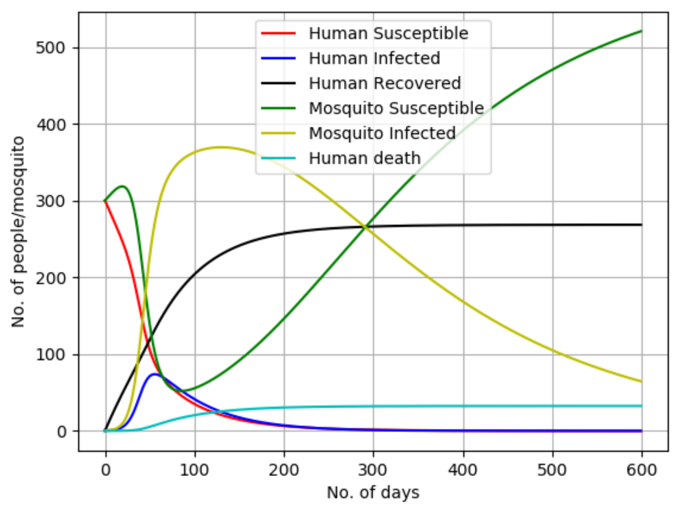
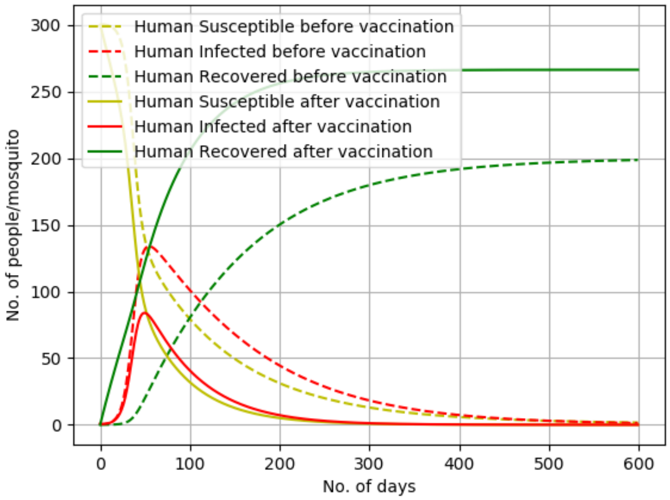


Figure 7.



Due to vaccination, maximum number of infected people is decreased and also the spread of malaria stops faster, this is because susceptible humans are converted to immune humans due to vaccination and hence a smaller number of infected humans. Also, the number of human death decreases due to vaccination.