Modelling MSEIRS

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I. Abstract

We try to model MSEIRS epidemic model with cellular automaton and compartment deterministic model.

II. Introduction

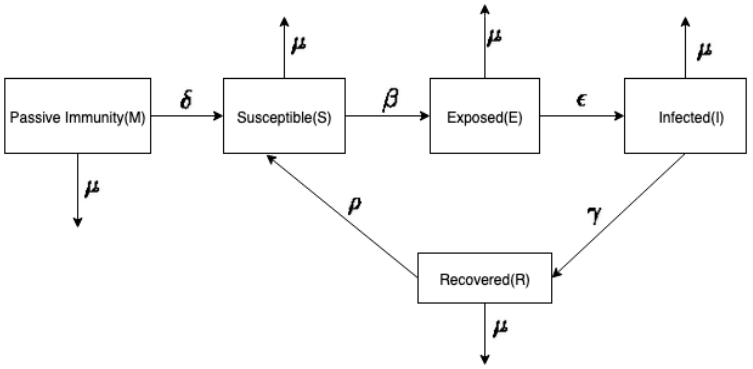
Epidemic spread of different infectious diseases have been modeled by many researchers. After making these models, these models can be applied to relevant diseases. SIR model is the basic and the first most popular version of epidemic spread model. In this paper, Kermack et. al. showed mathematical details of the SIR model and more. Johnson, this paper explains the details, assumptions and other small details in an easy way. After this basic model, there are many different variants of this model are available and studied for different assumptions to include in the modeling. These models include SIR model with births and deaths, SEIS model, SEIR model, MSIR model, MSEIR model etc.

The MSEIRS model includes the case when an individual is born with natural passive immunity, specifically maternal passive immunity. In the maternal passive immunity, either the fetus is born with immunity or it gets immune with breast feed of the human milk. These dis- ease includes immunity in Hepatitis B, polio, TB, Pertussis (Whooping cough) etc. But, in the MSEIRS model the recovered can become susceptible again. In the real-world Hepatitis B and other disease cannot happen again. That is why we are considering Pertussis disease in our discussion. The parameters are estimated by keeping the disease in mind. A person with whooping cough can infect another person when he breathes, sneezes or coughs. The disease can be cured with vaccine. It is a highly contagious infection.

III. Model

We model this problem in

1. Deterministic compartment model
2. Cellular automaton.



First, we will discuss the deterministic compartment model. Here, we are considering standard assumption of the epidemic compartment model. M(t) denotes the number of people with maternal passive immunity. B is death rate of non-infected people. An immune can become susceptible again with the rate δ.

$$\frac{dM}{dt} = B + (-\delta - \mu_1)M$$

S(t) denotes the susceptible, the number people who can become infected due to contact of any infected person, but not became infected yet. Here, the inflow term is simply the immune becoming susceptible term and out- flow terms include death rate and the interaction term. The interaction term depends on the number of susceptible and infected multiplied by the term β, which is the rate at which this interaction makes the susceptible quarantined.

$$\frac{dS}{dt} = B + \delta M - \frac{\beta SI}{N} - \mu_1 S + \rho R$$

E(t) denotes the number of people in the quarantined state i.e. The number of people that are in the latent period of the disease. Here, the inflow term is coming from the susceptible and the outflow is death with rate μ_2 and going into infected category with the rate ϵ. Here μ_2 is greater than μ_1 because we have assumed that death rate of infected as well as people in latent period of disease is higher than normal death rate.

$$\frac{dE}{dt} = \frac{\beta SI}{N} - \epsilon E - \mu_2 E$$

I(t) denotes infected, the number of people that can infect the susceptible person, when come in contact. These people themselves are infected with the disease. They are the spreaders of the infectious disease. An infected can recover or die with rate γ and μ_2 respectively.

$$\frac{dI}{dt} = \epsilon E - \gamma I - \mu_2 I$$

R(t) is the recovered category, the number of people that are removed from the infected category due to recovery from the disease. As the name suggests, it is MSEIRS model and that is why the recovered can become susceptible again.

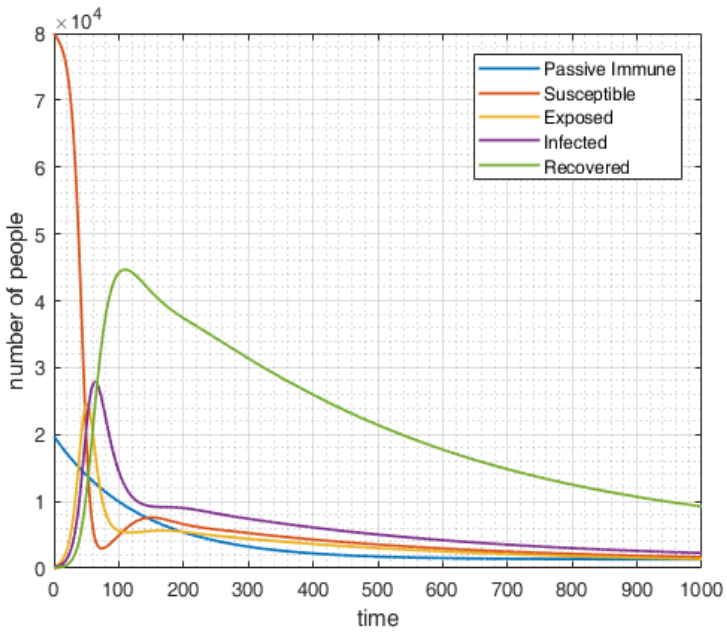
$$\frac{dR}{dt} = \gamma I - \mu_1 R - \rho R$$

Parameter	Information	Values
B	Number of births per day	10
δ	Passive immune becomes susceptible rate	0.0055 (180 days)
μ_1	Death rate of non-infected	0.002
μ_2	Death rate of infected	0.004
β	A susceptible meet an infected at contact rate of 10 it can go in latent period with probability 0.05	0.5
ϵ	Exposed to Infected	0.0833 (12 days)
γ	Infected to Recovered	0.04761 (21 days)
ρ	Recovered to Susceptible	0.011 (90 days)

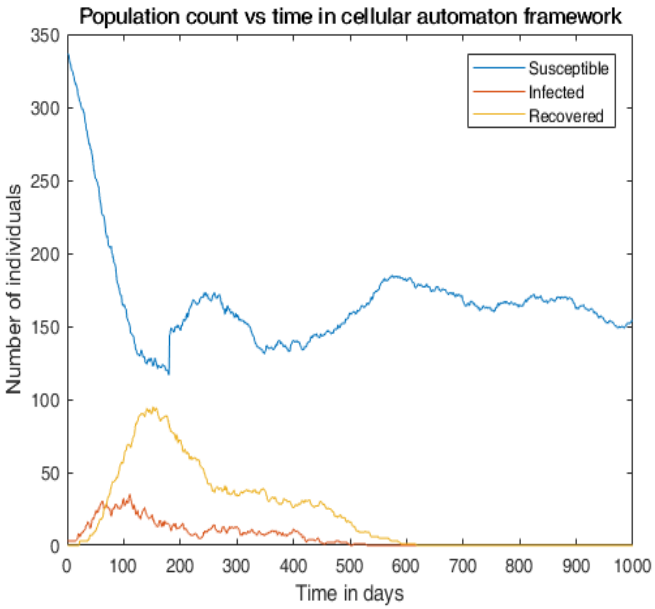
Now, we will describe the cellular automaton model. The rules for the automaton are given below.

- All the infectious neighbour of a susceptible individual can infect the susceptible with the probability $\frac{\beta}{10}$. β is the product of the contact rate and probability that the susceptible becomes the infected. The contact rate is 10, that is why the probability is $\frac{\beta}{10}$.
- A passive immune becomes susceptible again after some days, say x. To accommodate it in the automata, a time step counter of each position was maintained. After 180 time stamps a passive immune will become susceptible.
- Exposed becomes infected in $\frac{1}{\epsilon}$ days. By using the same logic in the above rule, after 12 times stamps of being in the state of quarantine an individual becomes an infected.
- To include natural death in the model, each cell, at every time stamp, irrespective of the its state can die with probability 0.002.
- To include birth in the model, we have to have some position where the infant can be added. That is why, the dead state is made available. A dead state can become alive with probability 0.002.
- The model includes that if the cell is born, then it can be in susceptible state or in maternal passive state with probability 0.5.

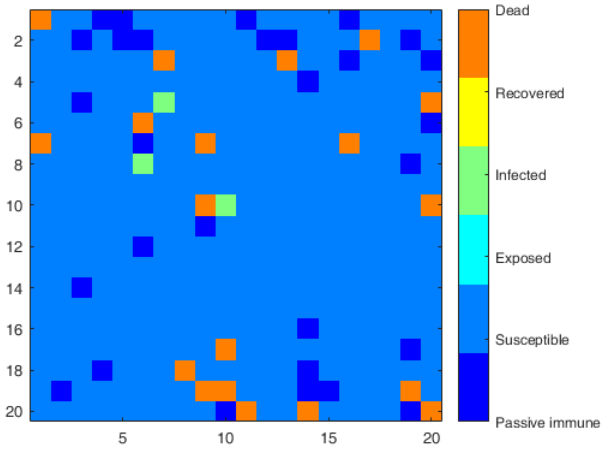
IV. Results



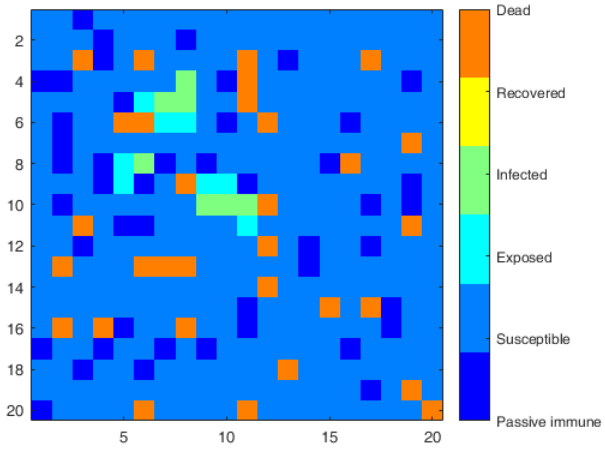
As we can see, the model is giving the expected results. The epidemic settles after approximately 1000 days. The number of infected do not die off because it receives a supply of susceptible from the recovers. The number of infected decreases and increases after sometime because after sometime the recovers to susceptible conversion kicks in.



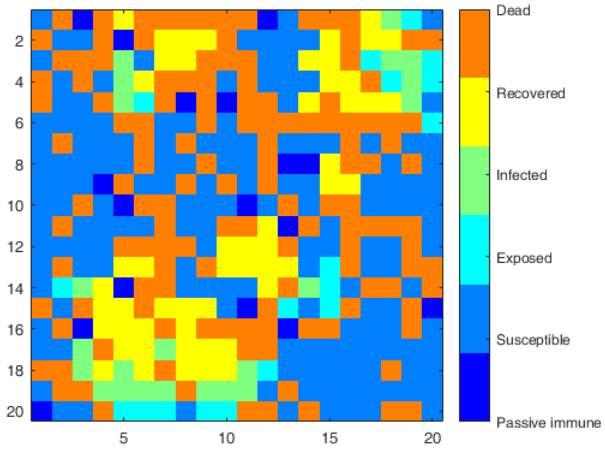
It has stochastic properties, because the automaton does not have the information of the whole grid. It only has local properties. That is why the curve is not smooth.



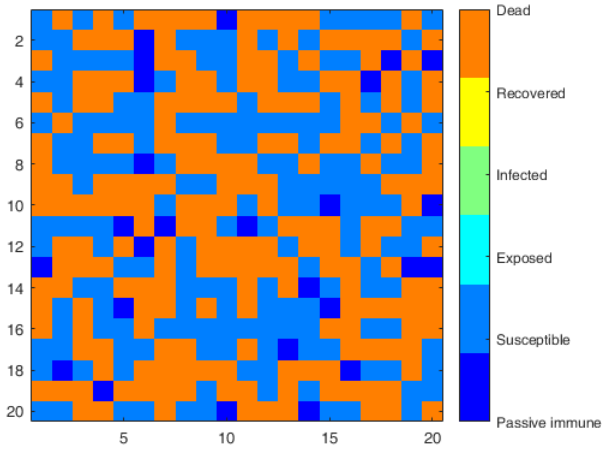
At time = 1



At time = 15



At time = 250



At time = 850

Here is the link to the video of cellular automation:
https://drive.google.com/open?id=1W3Nhjhj-oiRoroHtDrwCCTUJhnkEI1qx

V. CONCLUSION

From the above cellular automata model, we can see that the epidemic eventually dies away. In the model, we have tried to include all the setting that the deterministic model provides. But some things which do not depend on the local property, but depend on the overall number of people in the study, cannot be included in the cellular automaton. We can see that the model cannot include the fact that there are no people other than susceptible in the spatial world, but it remains producing passive immunes. The rules for the automaton were good enough to produce the same results as in the deterministic model.

VI. REFERENCES

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[2] Bernhard Pfeifer, and Karl Kugler et. Al. A Cellular Automaton Framework for Infectious Disease Spread Simulation
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