CS 302

Modelling and Simulation

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Lab 3

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1 Modeling Influenza

The influenza model can be considered as a three compartment model.

Here the susceptible people get infected when they come in contact with the infectious people. The infectious people become immune to influenza after recovery.
Mathematically we can describe the above problem with the following equations.

$$dS/dt = -\beta SI$$

$$dI/dt = (\beta SI) - (\alpha I)$$

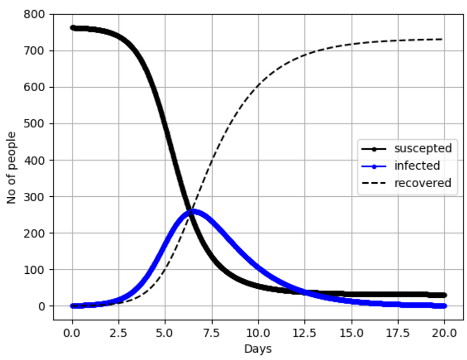
$$dR/dt = \alpha I$$

where,
β is the rate of infection
α is the recovery rate
Whenever a susceptible comes in contact with the infectious we say that there is a contact.Let us assume that number of contacts occurring between a susceptible and a infectious are "c".
Also not all contacts result in the spread of infection. Let us define the probability "p" with which the contact results in spread of infection. So the rate at which the infection spreads can be defined as

$$rate\_of\_infection = \beta = cp$$

1.1

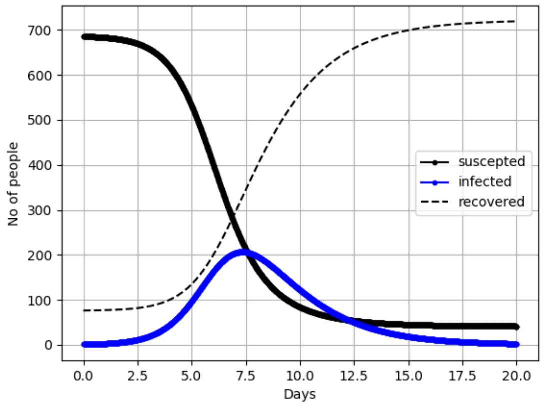
Considering the following initial values:
Susceptible(t=0) = 762
Infected(t=0) = 1
Recovered(t=0) = 0
$\beta = 0.00218$
$\alpha = 0.5$
We obtain the following figure:



From the figure obtained by running the simulation with the equations above we observe that the number of susceptible decreases slowly at first before experiencing a rapid decline and then decreases at very low rate. In contrast, the number of recovered, which is initially 0, has a curve that is similar to the logistic curve. When the number of susceptible decreases sharply, the infected increase to their maximum. Afterwards, as the number of infected decreases, the number of recovered rises. The maximum number of infected people are 259.

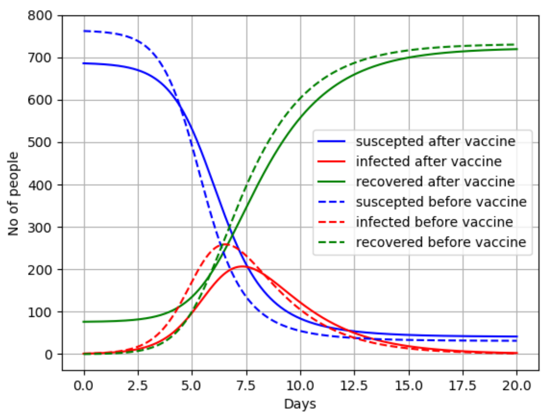
1.2

Now let us say that we vaccinate 15% of the susceptible people each day. On vaccination we assume that the immunization begins immediately.
We can say from our intuition that the spread of the influenza will be less as each day the susceptible people decreases due to vaccination.



From the figure obtained based on our model we can confirm that our intuition was correct. As we can see that the total number of infected people is less and recovered is more as compared to previous figure.Here the total number of infected people is 207.

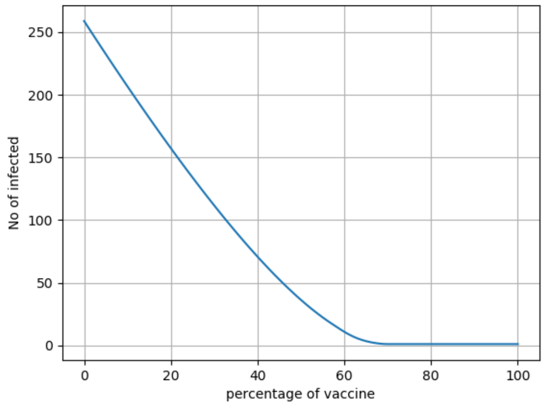
1.3



Here we can clearly see the effect of vaccination. We can see that before vaccination the number of infected were more as compared to number of infected after vaccination. Initially at time t=0, we are vaccinating 10% of the susceptible so numbered of recovered will be 76 at time t=0.

1.4

By seeing the above figures we can make a assumption that as we increase the rate of vaccination we can curb the spread of disease.



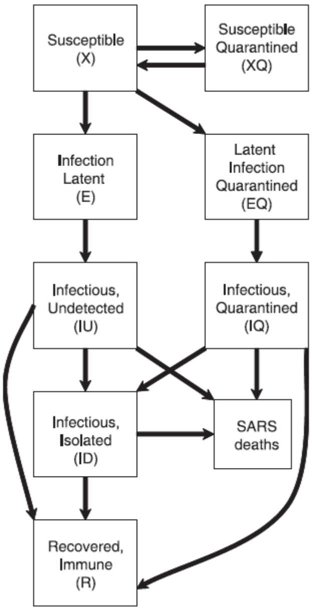
If we plot the graph of rate of vaccination versus the number of people infected we can see that as the vaccination rate increases from 0% to 100% the number of infected also decreases. In-fact wen we reach 70% vaccination rate or above the number of infected almost becomes equal to zero.

1.5

Now, Lets say that the government has limited budget which it wants to distribute towards different measures such as vaccination, generating health care awareness, etc. So there is a way in which government can optimally distribute the allocated fund.
Saying that the government will allocate 100% of the funds in vaccination is useless because we can see from the graph that number of people infected becomes almost 0 when the vaccination rate is 70%. So we can say that government can allocate maximum up-to 70-72% in vaccination and beyond that allocating the fund is unnecessary.
Now the remaining fund the government can allocate in creating awareness. As we know that β (the rate of infection) comprises of the product of number of contact("c") and the probability that the contact will cause infection("p"). So by creating awareness of maintaining cleanliness and developing healthy habits of washing hands, etc we can reduce the probability factor. So that even if the number of contacts remain the same the spread of the disease decreases.

2 Modeling SARS

For modeling SARS we can consider the mathematical model provided by Marc Lipsitch and others [1].



We have the following equations:

$\frac{dX}{dt} = \frac{-kbI_uX}{N_0} + r_QX_Q$

$\frac{dX_Q}{dt} = \frac{-qk(1-b)I_uX}{N_0} - r_QX_Q$

$\frac{dE}{dt} = \frac{kb(1-q)I_uX}{N_0} - pE$

$\frac{dE_Q}{dt} = \frac{qkbI_uX}{N_0} - pE_Q$

$\frac{dI_U}{dt} = pE - (v+m+w)I_U$

$\frac{dI_D}{dt} = w(I_U + I_Q) - (v+m)I_D$

$\frac{dI_Q}{dt} = pE - (v+m+w)I_Q$

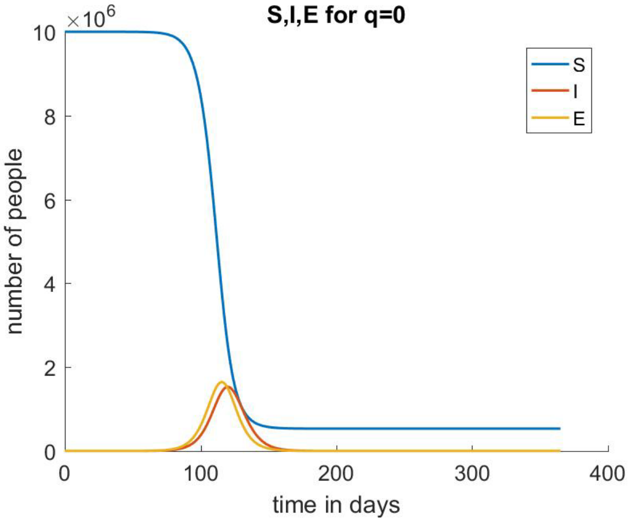
$\frac{dR}{dt} = v(I_U + I_D + I_Q)$

$\frac{d(dead)}{dt} = m(I_U + I_D + I_Q)$

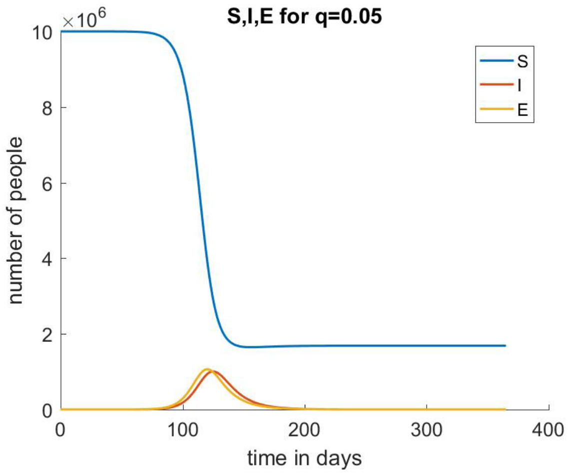
Here,
k = daily number of contacts
b = probability of transmission per contact
$\frac{1}{p}$ = mean time for progression from latently infected to infectious
v = per capita recovery rate
m = per capita death rate
w = mean daily rate at which infectious case are detected and isolated
q = fraction of infected contacts quarantined before they become infectious
Thus the mean contact of infectiousness is $\frac{1}{v+m+w}$.

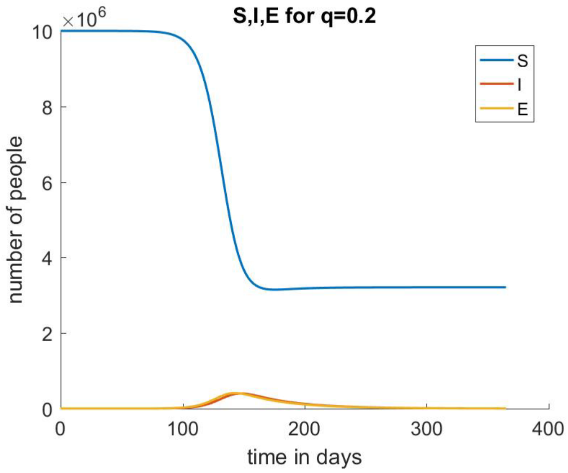
2.1

The following graphs are obtained by taking the following values:
k = 10/day
b = 0.06
1/p = 5 days
v = 0.04
m = 0.0975
w = 0.0625
1/u = 10 days
N0 = 10,000,000 people
q = 0 - upward



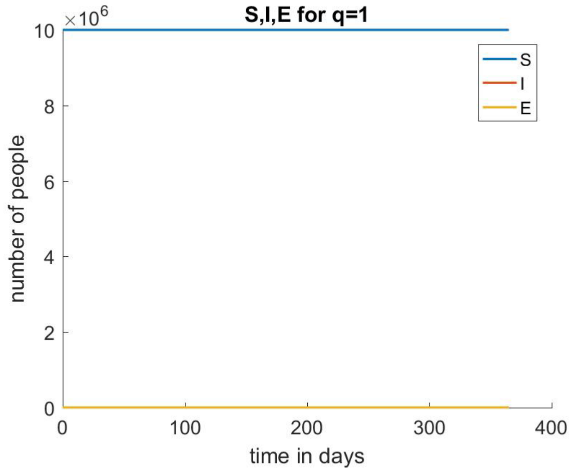
Here we can see that, for q=0, there is no quarantine happening, So all the exposed will be exposed who do not know about the disease. They will definitely become infected soon after that and will infect other susceptible.



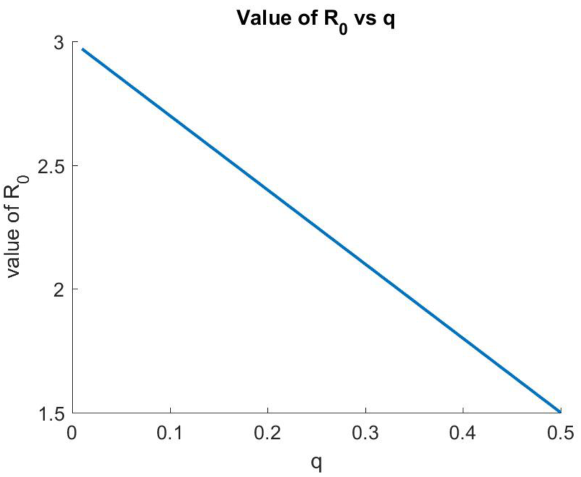


As the value of q increases, the maximum number of infected will become smaller and smaller. There will be a q value for which the value of R0 will become less than 1 initially and then no spreading will happen at

all.
Here the mean contact of infectiousness is $\frac{1}{(v+m+w)} = 5$ days
$R_0 = \frac{kb(1-q)}{v+m+w}$



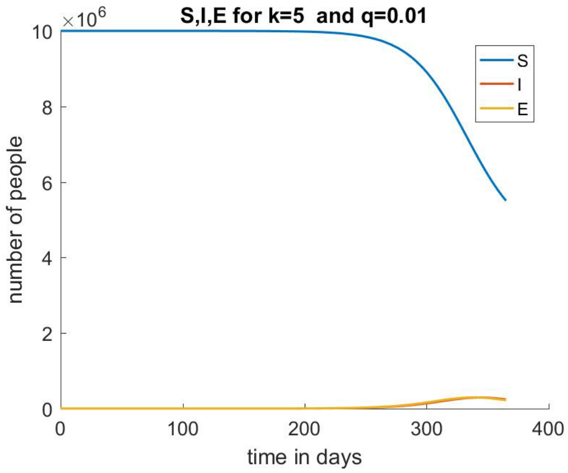
Q=1 is an extreme case where no infected will be unaware of the disease and no epidemic can happen at all. This is very hypothetical scenario where all the susceptible will be quarantined as they get exposed.



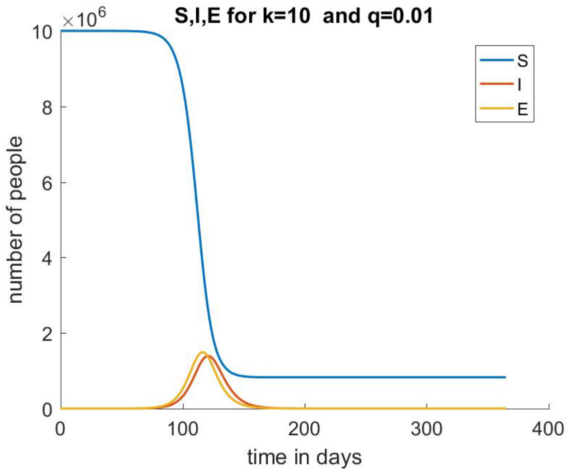
The above graph shows the variation in R_0 with q. Which concludes that if we quarantine before the people gets exposed the reproduction number decreases.

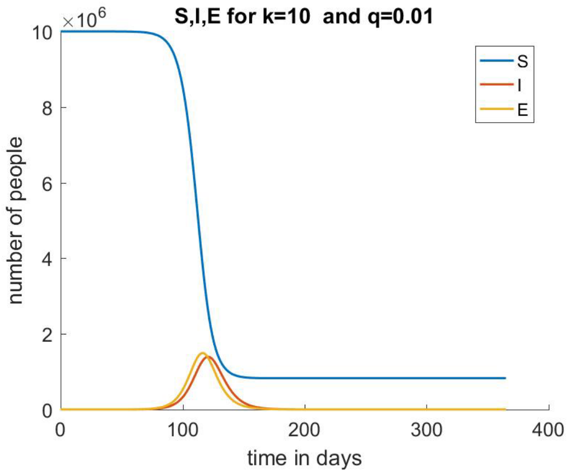
2.2

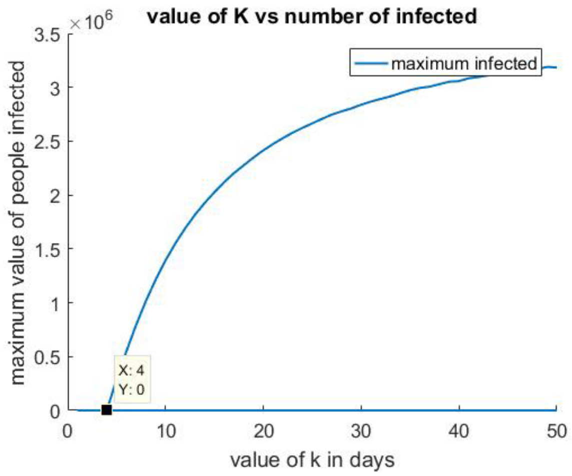
Now, the following graphs are obtained by fixing the value of q and changing the value of k from 5 to 20. We also have:
$R0 = \frac{k(1-q)b}{v+m+w}$
In the above equation we can see that $R_0 \propto k$, so for lesser value of k the value of R will be less.



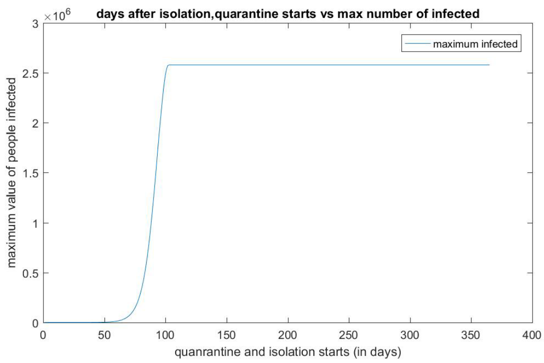
For k = 4 and k = 5 the value of R_0 becomes greater than 1. $R_0 = 1.4850$ In the above graph the epidemic spread is happening but it is not to its full potential.



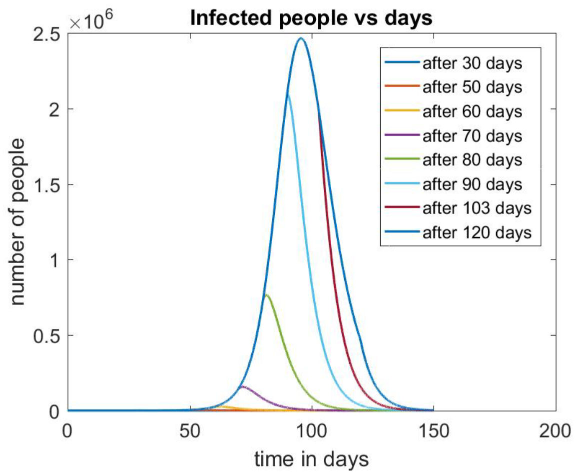




2.3



Here, we are introducing q and w, so they reduce the value of reproduction number to less than 1. Now after the simulation starts we can not compute value of R, but we can get intuition about reproduction number. Here we can see that after 103 days there is no point of introducing w and q. Because other factors make the value of R to less than 1 and there is no epidemic at all. So maximum value of infected do not effect after 103 days.



Here we can see that as we introduce the value of q and w and make R less than 1 the infected immediately starts decreasing. After 103 days the infected anyway starts to decrease.

References

[1] Marc Lipsitch et al. "Transmission Dynamics and Control of Severe Acute Respiratory Syndrome". In: *Science* 300.5627 (2003). DOI: `https://doi.org/10.1126/science.1086616`.