

Calibrating gyrochronology using Galactic kinematics

RUTH ANGUS,^{1,2,3} YUXI (LUCY) LU,^{3,1} DAN FOREMAN-MACKEY,² ADRIAN M. PRICE-WHELAN,² JASON CURTIS,¹ AND
EMILY CUNNINGHAM²

¹*Department of Astrophysics, American Museum of Natural History, 200 Central Park West, Manhattan, NY, USA*

²*Center for Computational Astrophysics, Flatiron Institute, 162 5th Avenue, Manhattan, NY, USA*

³*Department of Astronomy, Columbia University, Manhattan, NY, USA*

ABSTRACT

Gyrochronology, the method of inferring the age of a star from its rotation period, could provide ages for billions of stars over the coming decade of time-domain astronomy. However, the gyrochronology relations remain poorly calibrated due to a lack of precise ages for old, cool main-sequence stars. Now however, with proper motion measurements from Gaia, Galactic kinematics can be used as an age proxy, and the magnetic and rotational evolution of stars can be examined in detail. We demonstrate that kinematic ages, inferred from the velocity dispersions of groups of stars, beautifully illustrate the time and mass-dependence of the gyrochronology relations. We use the kinematic ages of field stars, plus benchmark clusters and asteroseismic stars, to calibrate a new empirical Gaussian process gyrochronology relation, that fully captures the complex rotational evolution of cool dwarfs over a range of masses and ages. We use cross validation to demonstrate that this relation accurately predicts ages for GKM dwarfs.

Keywords: Stellar Rotation — Stellar Evolution — Stellar Activity — Stellar Magnetic Fields — Low Mass Stars — Solar Analogs — Milky Way Dynamics

1. INTRODUCTION

Low mass dwarfs are the most common stars in the Milky Way, and their ages could reveal the evolution of Galactic stellar populations and planetary systems. However, the ages of GKM stars are difficult to measure because their luminosities and temperatures evolve slowly on the main sequence. Fortunately, rotation-dating, or ‘gyrochronology’ provides a promising means to measure precise ages for these cool dwarfs. (*e.g.* [Schatzman 1962](#); [Weber & Davis 1967](#); [Kraft 1967](#); [Skumanich 1972](#); [Kawaler 1988](#); [Pinsonneault et al. 1989](#); [Barnes 2003, 2007](#); [Mamajek & Hillenbrand 2008](#); [Barnes 2010](#); [Meibom et al. 2011, 2015](#); [van Saders et al. 2016](#)). The rotation periods of GKM stars evolve relatively rapidly, and a fully calibrated gyrochronology model that captures the time and mass-dependence of stellar spin down could provide ages that are precise to within 20% for millions of Milky Way stars in the time-domain era ([Epstein & Pinsonneault 2014](#); [Najita et al. 2016](#); [Angus et al. 2019](#)). However, gyrochronology models are not yet reliably calibrated, especially for low-mass and old stars.

A lack of low-mass and old calibration stars has previously limited the mass and age coverage of gyrochronology relations, which require precise age and rotation period measurements. Historically the calibration sample has been limited to open clusters and asteroseismic stars, which can be precisely dated with isochrone fitting and main sequence turn off, or precise oscillation frequency analysis. The rotation periods of both cluster and asteroseismic stars can be measured with precise time-series photometry. Magnetically active regions create inhomogeneous surface features and produce periodic variability in their integrated, broad-band emission. The photometric rotation periods of thousands of stars have been measured with the *Kepler/K2* and *TESS* space missions ([Borucki et al. 2010](#); [Howell et al. 2014](#); [Ricker et al. 2015](#)).

For the purposes of calibrating gyrochronology, open clusters provide good mass coverage for young stars: rotation periods have been measured for F to mid M dwarfs up to ages of around 700 Myr. In contrast, asteroseismic stars provide reasonable age coverage for hot stars: ages and photometric surface rotation periods have been measured for F, G and early K dwarfs up to ages of 10 Gyr. However, neither asteroseismology nor cluster analysis can provide rotation periods and ages for old, late K and M dwarfs. In addition, cluster and asteroseismic stars generally provide sparse coverage of the rotation period-effective temperature plane, and cannot reveal the detailed evolution of stellar

rotation rates. As a result, most empirical gyrochronology relations are only reliable for G dwarfs up to Solar age, K dwarfs up to 2-3 Gyr, and early M dwarfs up to < 1 Gyr.

The rotational evolution of cool dwarfs is not well understood because few old M dwarfs with rotation periods have age measurements. However, as we showed in Angus *et al.* (2020), the *kinematic* ages of field stars observed by *Kepler*, can provide a calibration sample with broad mass and age coverage. Although the *Kepler* sample does not include late M dwarfs, it can still be used to extend gyrochronology relations to much older ages for late K and early M dwarfs. By adding the ages and rotation periods of thousands of field stars to the open cluster and asteroseismic calibration sample, we can calibrate a gyrochronology relation that is applicable to FGK and early M dwarfs between the ages of ~ 500 Myr and 8 Gyr.

1.1. Core-envelope decoupling

In Angus *et al.* (2020) we demonstrated that Galactic kinematics can be used to explore the evolution of stellar rotation. We showed that velocity dispersion, an established age proxy in the Galactic thin disk, increases smoothly as a function of rotation period, indicating that rotation period increases with age as expected. Using velocity dispersion as an age proxy, we also showed that old K dwarfs spin down more slowly than G dwarfs: their rotational evolution appears to ‘stall’ after around 1 Gyr, in a manner that reflects the behavior of K dwarfs observed in open clusters (Curtis *et al.* 2019). At young ages ($\sim 0.5 - 1$ Gyr), K dwarfs spin more slowly than G dwarfs of the same age, because their deeper convection zones generate stronger magnetic fields, which leads to more efficient magnetic braking. However, at old ages ($\gtrsim 1$ Gyr) K dwarfs rotate at the same rate or more rapidly than contemporary G dwarfs. The leading explanation for this phenomenon is that angular momentum is transferred from the core to the surface over longer timescales for lower-mass stars (Spada & Lanzafame 2019), *i.e.* they experience a more extended phase of ‘core-envelope decoupling’.

A period of core-envelope decoupling is necessary to explain the observed rotation periods of stars in extremely young open clusters (1-10 Gyr) (*e.g.* Irwin *et al.* 2007; Bouvier 2008; Denissenkov *et al.* 2010; Spada *et al.* 2011; Reiners & Mohanty 2012; Gallet & Bouvier 2013). During this phase there is little transfer of angular momentum between radiative core and convective envelope and, as wind-braking removes angular momentum from the envelope, it decelerates while the core continues to spin rapidly. Over time however, angular momentum is transported across the interface between the two zones, and momentum from the rapidly spinning interior surfaces, inhibiting the deceleration of the outer envelope. Currently, the rotation periods of field and cluster stars can only be reproduced by semi-empirical models with a mass-dependent timescale for core-envelope coupling (Spada & Lanzafame 2019; Curtis *et al.* 2019, Angus *et al.*, 2020).

1.2. Using kinematics as an age proxy

The star forming molecular gas clouds observed in the Milky Way have a low out-of-plane, or vertical, velocity (*e.g.* Stark & Brand 1989; Stark & Lee 2005; Aumer & Binney 2009; Martig *et al.* 2014; Aumer *et al.* 2016). In contrast, the vertical velocities of older stars are observed to be larger in magnitude on average (Strömberg 1946; Wielen 1977; Nordström *et al.* 2004; Holmberg *et al.* 2007, 2009; Aumer & Binney 2009; Casagrande *et al.* 2011; Ting & Rix 2019; Yu & Liu 2018). There are two possible explanations for this observed increase in velocity dispersion with age: either stars are born kinematically ‘cool’ and their orbits are heated over time via interactions with giant molecular clouds (see Sellwood 2014, for a review of secular evolution in the MW), or stars formed kinematically ‘hotter’ in the past (*e.g.* Bird *et al.* 2013). Either way, the vertical velocity dispersions of thin disk stars are observed to increase with stellar age. This behavior is codified by Age-Velocity dispersion Relations (AVRs), which typically express the relationship between age and velocity dispersion as a power law: $\sigma_v \propto t^\beta$, with free parameter, β (*e.g.* Holmberg *et al.* 2009; Yu & Liu 2018). These expressions can be used to infer the ages of groups of stars from their velocity dispersions, as we do in this paper (see section 2).

Kinematic ages have been used to explore the evolution of cool dwarfs for over a decade. West *et al.* (2004, 2006) found that the fraction of magnetically active M dwarfs decreases over time, by using the vertical distances of stars from the Galactic mid-plane as an age proxy, and West *et al.* (2008) used kinematic ages to calculate the expected activity lifetime for M dwarfs of different spectral types. Faherty *et al.* (2009) used tangential velocities to infer the ages of M, L and T dwarfs, and showed that dwarfs with lower surface gravities tended to be kinematically younger, and Kiman *et al.* (2019) used velocity dispersion as an age proxy to explore the evolution of H α equivalent width (a magnetic activity indicator), in M dwarfs.

AVRs are usually calibrated in Galactocentric velocity coordinates (v_x , v_y , v_z or UVW), and these velocities can only be calculated with full 6D positional and velocity information, however most *Kepler* rotators do not have RV measurements¹. In Angus *et al.* (2020) we used velocity in the direction of Galactic latitude (v_b) as a stand-in for v_z because, in the *Galactic* coordinate system, velocities can be calculated from 3D positions and 2D proper motions. The *Kepler* field lies at low Galactic latitude, so v_b is a close approximation to v_z . Though v_b velocity dispersion does not equal v_z velocity dispersion, it still increases monotonically over time and provides accurate age rankings for *Kepler* stars. Unfortunately however, given that AVRs are calibrated in *Galactocentric* coordinates (v_x , v_y , v_z), we could not directly translate v_b velocity dispersions to ages.

In this paper, our aim was to use kinematic ages to calibrate a new gyrochronology relation, for which four main steps were required. Firstly, we inferred *vertical* velocity, v_z , for each star without an RV measurement by marginalizing over missing RVs using a hierarchical Bayesian model (see section 2.2). Secondly, we calculated velocity dispersion for every star using a moving, or rolling dispersion method (see section 2.3). Thirdly, these velocity dispersions were converted into ages using an AVR (Yu & Liu 2018, section ??). Finally, we used a Gaussian process model to capture the complexities of stellar rotational evolution and calibrated a new gyrochronology relation using our kinematic ages, plus benchmark cluster and asteroseismic stars in section ??.

¹ Although RVs for most will be released in *Gaia* DR3

2. METHOD

2.1. Data

This study focuses on stellar rotation in the original *Kepler* field. This is partly because *Kepler* provides the largest sample of published, homogeneously measured rotation periods, and partly because its low Galactic latitude allows us to marginalize over missing RV measurements and approximate vertical velocity, v_z .

We combined two large rotation period catalogs constructed from original *Kepler* data, from [McQuillan et al. \(2014\)](#) and [Santos et al. \(2019\)](#). These two studies used different techniques to measure rotation periods from *Kepler* light curves: autocorrelation functions and wavelets respectively. The [Santos et al. \(2019\)](#) study was specifically focused on cooler stars: K and M dwarfs, and includes a larger number of rotation periods for these stars. The combined catalogs provide a total of over 38,000 rotation periods.

To calculate kinematic ages, an estimate of vertical velocity, v_z , is required. The ideal way to calculate v_z and similarly, v_x and v_y , is to use 6D positional and velocity information. Many stars in the *Kepler* field do not have RV measurements and an alternative approach must be taken to infer their vertical velocities (see section 2.2). However, a large number of *Kepler* rotators, over 10,000 of 34,000 *do* have RV measurements from *Gaia* DR2 and *LAMOST*. Figure 1 shows rotation period vs effective temperature for all stars in the [McQuillan et al. \(2014\)](#) and [Santos et al. \(2019\)](#) catalogs, plotted in grey. Stars with RV measurements are colored by their vertical velocity dispersion (see section 2.3 to see how we calculated velocity dispersion). [Discuss what this plot shows.](#)

Although RVs are available for a significant number of *Kepler* rotators (almost one in three), few stars cooler than 4000 K have RV measurements. This is due to the faint limits of the *Gaia* DR2 and *LAMOST* surveys (although RV measurements for fainter targets will be available in *Gaia* DR3). Given that magneto-rotational evolution is poorly understood for M dwarfs, the cool stars with missing RVs are arguably the ones we care most about. For this reason, we attempted to compensate for the lack of RV measurements by inferring vertical velocities for stars without RVs, to fill in the low-temperature region of figure 1 in section 2.2.

2.2. Inferring 3D velocities (marginalizing over missing RV measurements)

It has been demonstrated that the dispersion in vertical velocity, v_z for a group of stars increases with the age of that group (citations). However, velocities in Galactocentric coordinates, v_x , v_y and v_z , can only be calculated with full 6-D position and velocity information, *i.e.* proper motions, position and radial velocity. In [Angus et al. \(2020\)](#) we showed that kinematic ages can be used to explore rotational evolution and showed, in the appendix of that paper, that velocity, v_b in the Galactic frame, which can be calculated without an RV measurement, can be used as an approximation to v_z for *Kepler* stars. This is because the *Kepler* field of view lies at relatively low Galactic latitudes, ($\sim 5 - 20^\circ$), so the z -direction is similar to the b -direction for *Kepler* stars. However, v_b is only a close approximation to v_z at extremely *low* latitudes, and even in the *Kepler* field, kinematic ages calculated with v_b instead of v_z are systematically larger because of extra noise introduced by the imperfect translation between v_b and v_z . In this work, we *infer* v_z by marginalizing over missing RV measurements.

For each star in our sample, we inferred v_x , v_y , and v_z from the 3D positions and 2D proper motions provided in the *Gaia* DR2 catalog ([Brown et al. 2011](#)). We also simultaneously inferred distance, (instead of using inverse-parallax), to model velocities (see *e.g.* [Bailer-Jones 2015](#); [Bailer-Jones et al. 2018](#)).

Using Bayes rule, the posterior probability of the parameters given the data can be written:

$$p(v_{xyz}, D | \mu_\alpha, \mu_\delta, \alpha, \delta, \pi) = p(\mu_\alpha, \mu_\delta, \alpha, \delta, \pi | v_{xyz}, D) p(v_{xyz}) p(D), \quad (1)$$

where D is distance, α is Right Ascension (RA), δ is declination (dec), π is parallax, μ_α is proper motion in RA, and μ_δ is proper motion in dec. For each star in the *Kepler* field, we explored the posteriors of these four parameters using the *PyMC3* Hamiltonian Monte Carlo (HMC) sampler ([citations](#)).

We used the mean and covariance of the distance and velocity distributions of *Kepler* targets *with* RV measurements to determine the 4D multivariate Gaussian prior over $\log(\text{distance})$ and velocities.

We tuned the *PyMC3* model for 1500 steps, with a target acceptance fraction of 0.9. The model was then run for 1000 steps with 4 chains.

Figure 2 shows the v_x , v_y and v_z velocities we inferred, compared with those calculated from measured RVs. 2000 stars are shown, which were chosen at random from the *Kepler* rotators with RV measurements. The *Kepler* field is oriented almost along the y -axis of the Galactocentric coordinate system. As a result, x and z -direction velocities

Figure 1. Vertical velocity dispersion as a function of rotation period and effective temperature for *Kepler* stars with measured rotation periods. Colored points show stars with RV measurements from *Gaia* or *LAMOST*, with their color indicating their velocity dispersion. Faint grey points show the combined [McQuillan et al. \(2014\)](#) and [Santos et al. \(2019\)](#) samples, including stars without RV measurements. The coolest stars in this sample do not have RVs because they are faint.

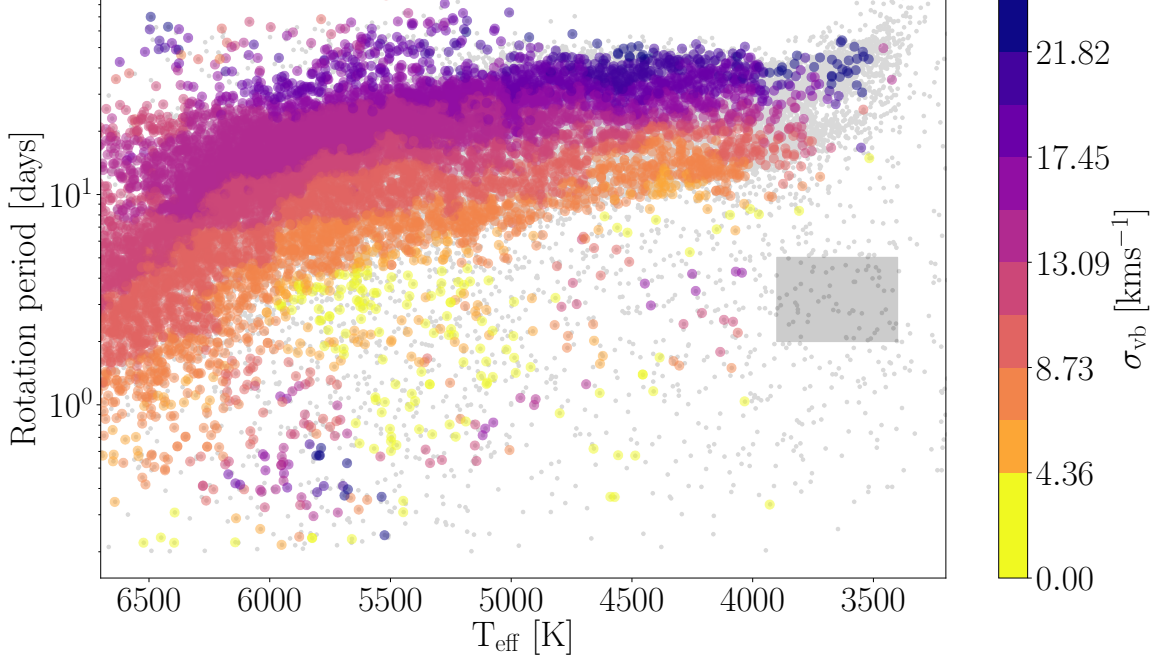
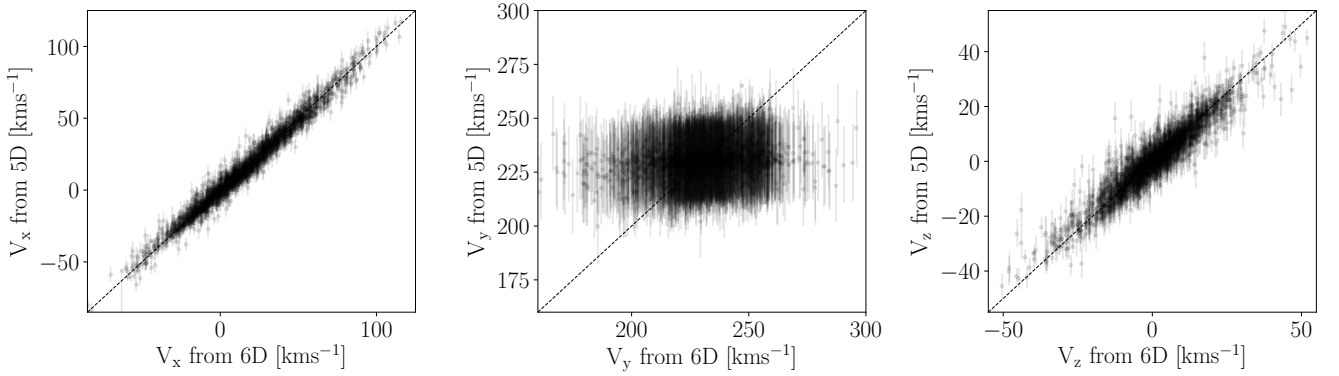


Figure 2. Vertical velocities calculated with full 6D information vs vertical velocities inferred without RV, for all 3000 [McQuillan et al. \(2014\)](#) stars with *Gaia* RV measurements.



of *Kepler* stars are extremely well-constrained with proper motion alone, but v_y is almost completely unconstrained without an RV. Figure 2 shows that v_x and v_z velocities inferred without RV measurements are extremely similar to those calculated with RVs.

2.3. Calculating velocity dispersions

A kinematic age can be calculated from the velocity dispersion, *i.e.* standard deviation, of a group of stars. These velocity dispersions can then be converted into an age using an AVR (*e.g.* [Holmberg et al. 2009](#); [Yu & Liu 2018](#)).

Kinematic ages represent the *average age* of a group of stars and are most informative when stars are grouped by age. If a group of stars have similar ages, their kinematic age will be close the age of each individual. On the other hand, the kinematic age of a group with large age variance will not provide much information about the ages of individual stars. Velocity distributions themselves do not reveal whether a group of stars have similar or different ages, since either case the velocities are Gaussian-distributed. Fortunately however, we can group *Kepler* stars by age using the implicit assumption that underpins gyrochronology: that stars with the same rotation period and color are the same age. We discuss the implications of this assumption and cases where it doesn't apply in the Discussion of this paper (section ??).

In this paper, we calculated the kinematic age of *each individual star* in our sample, by grouping it with its neighbors in $\log(P_{\text{rot}})$ - T_{eff} space. This method is similar to calculating a rolling, or running standard deviation and allowed us to assign a unique age to each star. However, ages calculated this way are tightly correlated, and their correlation depends strongly on window-size.

We tested two methods of grouping stars: K-nearest neighbors, and bins in $\log(P_{\text{rot}})$ and T_{eff} . In the K-nearest neighbors method, each star was grouped with the K-nearest stars in $\log(P_{\text{rot}})$ - T_{eff} space. Groups created this way spanned a small $\log(P_{\text{rot}})$ - T_{eff} range where the stellar number density was large, and a large range where the number density was small. In other words, the number of stars was fixed but the window-size changed. In the fixed range method, stars were grouped within a fixed $\log(P_{\text{rot}})$ - T_{eff} window. This method created groups with large numbers of stars in densely populated regions of the $\log(P_{\text{rot}})$ - T_{eff} plane, and small numbers of stars in sparsely populated regions, *i.e.* the number of stars changed but the window-size was fixed. To choose the best method, and to optimize for the parameters of each (K and window-size), we conducted a set of tests.

2.4. Converting velocity dispersion to age with an AVR

We used the Yu & Liu (2018) AVR to convert velocity dispersion to age. This relation was calibrated using the ages and velocities of red clump stars. They divided their sample into metal rich and poor subsets, and calibrated separate AVRs for each, plus a global AVR. Their AVR is a power law:

$$\sigma_{vz} = \alpha t^\beta, \quad (2)$$

where α and β take values (6.38, 0.578) for metal rich stars (3.89, 1.01) for metal poor stars, and (5.47, 0.765) for all stars.

We used $1.5\times$ the Median Absolute Deviation (MAD) of velocities, which is a robust approximation to the standard deviation and is less sensitive to outliers. Velocity outliers could be binary stars or could be generated by underestimated parallax or proper motion uncertainties.

2.5. Comparing kinematic ages with asteroseismic and cluster ages

2.6. A Gaussian process gyrochronology relation

3. RESULTS

4. DISCUSSION

5. CONCLUSION

This work was partly developed at the 2019 KITP conference ‘Better stars, better planets’. Parts of this project are based on ideas explored at the Gaia sprints at the Flatiron Institute in New York City, 2016 and MPIA, Heidelberg, 2017. This work made use of the *gaia-kepler.fun* crossmatch database created by Megan Bedell.

Some of the data presented in this paper were obtained from the Mikulski Archive for Space Telescopes (MAST). STScI is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS5-26555. Support for MAST for non-HST data is provided by the NASA Office of Space Science via grant NNX09AF08G and by other grants and contracts. This paper includes data collected by the Kepler mission. Funding for the *Kepler* mission is provided by the NASA Science Mission directorate.

This work has made use of data from the European Space Agency (ESA) mission *Gaia* (<https://www.cosmos.esa.int/gaia>), processed by the *Gaia* Data Processing and Analysis Consortium (DPAC, <https://www.cosmos.esa.int/web/gaia/dpac/consortium>). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the *Gaia* Multilateral Agreement.

REFERENCES

- Angus, R., Morton, T. D., & Foreman-Mackey *et al*, D. 2019, *AJ*, 158, 173, doi: [10.3847/1538-3881/ab3c53](https://doi.org/10.3847/1538-3881/ab3c53)
- Aumer, M., Binney, J., & Schönrich, R. 2016, *MNRAS*, 462, 1697, doi: [10.1093/mnras/stw1639](https://doi.org/10.1093/mnras/stw1639)
- Aumer, M., & Binney, J. J. 2009, *MNRAS*, 397, 1286, doi: [10.1111/j.1365-2966.2009.15053.x](https://doi.org/10.1111/j.1365-2966.2009.15053.x)
- Bailer-Jones, C. A. L. 2015, *PASP*, 127, 994, doi: [10.1086/683116](https://doi.org/10.1086/683116)
- Bailer-Jones, C. A. L., Rybizki, J., & Foesneau *et al*, M. 2018, *AJ*, 156, 58, doi: [10.3847/1538-3881/aacb21](https://doi.org/10.3847/1538-3881/aacb21)
- Barnes, S. A. 2003, *ApJ*, 586, 464, doi: [10.1086/367639](https://doi.org/10.1086/367639)
- . 2007, *ApJ*, 669, 1167, doi: [10.1086/519295](https://doi.org/10.1086/519295)
- . 2010, *ApJ*, 722, 222, doi: [10.1088/0004-637X/722/1/222](https://doi.org/10.1088/0004-637X/722/1/222)
- Bird, J. C., Kazantzidis, S., & Weinberg *et al*, D. H. 2013, *ApJ*, 773, 43, doi: [10.1088/0004-637X/773/1/43](https://doi.org/10.1088/0004-637X/773/1/43)
- Borucki, W. J., Koch, D., & Basri *et al*, G. 2010, *Science*, 327, 977, doi: [10.1126/science.1185402](https://doi.org/10.1126/science.1185402)
- Bouvier, J. 2008, *A&A*, 489, L53, doi: [10.1051/0004-6361/200810574](https://doi.org/10.1051/0004-6361/200810574)
- Brown, T. M., Latham, D. W., & Everett *et al*, M. E. 2011, *AJ*, 142, 112, doi: [10.1088/0004-6256/142/4/112](https://doi.org/10.1088/0004-6256/142/4/112)
- Casagrande, L., Schönrich, R., & Asplund *et al*, M. 2011, *A&A*, 530, A138, doi: [10.1051/0004-6361/201016276](https://doi.org/10.1051/0004-6361/201016276)
- Curtis, J. L., Agüeros, M. A., & Douglas *et al*, S. 2019, arXiv e-prints. <https://arxiv.org/abs/1905.06869>
- Denissenkov, P. A., Pinsonneault, M., & Terndrup *et al*, D. M. 2010, *ApJ*, 716, 1269, doi: [10.1088/0004-637X/716/2/1269](https://doi.org/10.1088/0004-637X/716/2/1269)
- Epstein, C. R., & Pinsonneault, M. H. 2014, *ApJ*, 780, 159, doi: [10.1088/0004-637X/780/2/159](https://doi.org/10.1088/0004-637X/780/2/159)
- Faherty, J. K., Burgasser, A. J., & Cruz *et al*, K. L. 2009, *AJ*, 137, 1, doi: [10.1088/0004-6256/137/1/1](https://doi.org/10.1088/0004-6256/137/1/1)
- Gallet, F., & Bouvier, J. 2013, *A&A*, 556, A36, doi: [10.1051/0004-6361/201321302](https://doi.org/10.1051/0004-6361/201321302)
- Holmberg, J., Nordström, B., & Andersen, J. 2007, *A&A*, 475, 519, doi: [10.1051/0004-6361/20077221](https://doi.org/10.1051/0004-6361/20077221)
- . 2009, *A&A*, 501, 941, doi: [10.1051/0004-6361/200811191](https://doi.org/10.1051/0004-6361/200811191)
- Howell, S. B., Sobeck, C., & Haas *et al*, M. 2014, *PASP*, 126, 398, doi: [10.1086/676406](https://doi.org/10.1086/676406)
- Irwin, J., Hodgkin, S., & Aigrain *et al*, S. 2007, *MNRAS*, 377, 741, doi: [10.1111/j.1365-2966.2007.11640.x](https://doi.org/10.1111/j.1365-2966.2007.11640.x)
- Kawaler, S. D. 1988, *ApJ*, 333, 236, doi: [10.1086/166740](https://doi.org/10.1086/166740)
- Kiman, R., Schmidt, S. J., & Angus *et al*, R. 2019, *AJ*, 157, 231, doi: [10.3847/1538-3881/ab1753](https://doi.org/10.3847/1538-3881/ab1753)
- Kraft, R. P. 1967, *ApJ*, 150, 551, doi: [10.1086/149359](https://doi.org/10.1086/149359)
- Mamajek, E. E., & Hillenbrand, L. A. 2008, *ApJ*, 687, 1264, doi: [10.1086/591785](https://doi.org/10.1086/591785)
- Martig, M., Minchev, I., & Flynn, C. 2014, *MNRAS*, 443, 2452, doi: [10.1093/mnras/stu1322](https://doi.org/10.1093/mnras/stu1322)
- McQuillan, A., Mazeh, T., & Aigrain, S. 2014, *ApJS*, 211, 24, doi: [10.1088/0067-0049/211/2/24](https://doi.org/10.1088/0067-0049/211/2/24)
- Meibom, S., Barnes, S. A., & Latham *et al*, D. W. 2011, *ApJL*, 733, L9, doi: [10.1088/2041-8205/733/1/L9](https://doi.org/10.1088/2041-8205/733/1/L9)
- Meibom, S., Barnes, S. A., & Platais *et al*, I. 2015, *Nature*, 517, 589, doi: [10.1038/nature14118](https://doi.org/10.1038/nature14118)
- Najita, J., Willman, B., & Finkbeiner *et al*, D. P. 2016, arXiv e-prints, arXiv:1610.01661. <https://arxiv.org/abs/1610.01661>
- Nordström, B., Mayor, M., & Andersen *et al*, J. 2004, *A&A*, 418, 989, doi: [10.1051/0004-6361:20035959](https://doi.org/10.1051/0004-6361:20035959)
- Pinsonneault, M. H., Kawaler, S. D., & Sofia *et al*, S. 1989, *ApJ*, 338, 424, doi: [10.1086/167210](https://doi.org/10.1086/167210)
- Reiners, A., & Mohanty, S. 2012, *ApJ*, 746, 43, doi: [10.1088/0004-637X/746/1/43](https://doi.org/10.1088/0004-637X/746/1/43)
- Ricker, G. R., Winn, J. N., & Vanderspek *et al*, R. 2015, *Journal of Astronomical Telescopes, Instruments, and Systems*, 1, 014003, doi: [10.1117/1.JATIS.1.1.014003](https://doi.org/10.1117/1.JATIS.1.1.014003)
- Santos, A. R. G., García, R. A., & Mathur *et al*, S. 2019, *ApJS*, 244, 21, doi: [10.3847/1538-4365/ab3b56](https://doi.org/10.3847/1538-4365/ab3b56)
- Schatzman, E. 1962, *Annales d'Astrophysique*, 25, 18
- Sellwood, J. A. 2014, *Reviews of Modern Physics*, 86, 1, doi: [10.1103/RevModPhys.86.1](https://doi.org/10.1103/RevModPhys.86.1)
- Skumanich, A. 1972, *ApJ*, 171, 565, doi: [10.1086/151310](https://doi.org/10.1086/151310)
- Spada, F., & Lanzafame, A. C. 2019, arXiv e-prints, arXiv:1908.00345. <https://arxiv.org/abs/1908.00345>
- Spada, F., Lanzafame, A. C., & Lanza *et al*, A. F. 2011, *MNRAS*, 416, 447, doi: [10.1111/j.1365-2966.2011.19052.x](https://doi.org/10.1111/j.1365-2966.2011.19052.x)
- Stark, A. A., & Brand, J. 1989, *ApJ*, 339, 763, doi: [10.1086/167334](https://doi.org/10.1086/167334)
- Stark, A. A., & Lee, Y. 2005, *ApJL*, 619, L159, doi: [10.1086/427936](https://doi.org/10.1086/427936)
- Strömberg, G. 1946, *ApJ*, 104, 12, doi: [10.1086/144830](https://doi.org/10.1086/144830)
- Ting, Y.-S., & Rix, H.-W. 2019, *ApJ*, 878, 21, doi: [10.3847/1538-4357/ab1ea5](https://doi.org/10.3847/1538-4357/ab1ea5)
- van Saders, J. L., Ceillier, T., & Metcalfe *et al*, T. S. 2016, *Nature*, 529, 181, doi: [10.1038/nature16168](https://doi.org/10.1038/nature16168)
- Weber, E. J., & Davis, Jr., L. 1967, *ApJ*, 148, 217, doi: [10.1086/149138](https://doi.org/10.1086/149138)
- West, A. A., Bochanski, J. J., & Hawley *et al*, S. L. 2006, *AJ*, 132, 2507, doi: [10.1086/508652](https://doi.org/10.1086/508652)
- West, A. A., Hawley, S. L., & Bochanski *et al*, J. J. 2008, *AJ*, 135, 785, doi: [10.1088/0004-6256/135/3/785](https://doi.org/10.1088/0004-6256/135/3/785)
- West, A. A., Hawley, S. L., & Walkowicz *et al*, L. M. 2004, *AJ*, 128, 426, doi: [10.1086/421364](https://doi.org/10.1086/421364)
- Wielen, R. 1977, *A&A*, 60, 263

Yu, J., & Liu, C. 2018, MNRAS, 475, 1093,
doi: [10.1093/mnras/stx3204](https://doi.org/10.1093/mnras/stx3204)