Exploring the ages of rotating stars using galactic dynamics: a novel approach to calibrating gyrochronology

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ABSTRACT

Due to a lack of precise ages and rotation periods for old main-sequence stars, the rotational evolution of FGKM dwarfs is mostly unconstrained after around 2-3 billion years. However, recent evidence suggests that the relationship between rotation period and color/temperature/mass does not remain constant over large timescales, contradicting a long-held assumption that underlies many gyrochronology relations. In this work we use the kinematic ages of populations of K dwarfs to reveal the evolving relationship between rotation period and effective temperature. We find that groups of stars, selected to be the same age, do *not* have the same velocity dispersion at all temperatures; cooler stars have greater velocity dispersions than hotter stars. Two different scenarios could be responsible for producing this observation: either the rotation period-color relation changes over time, or stars of different masses experience different rates of dynamical heating. If we assume that mass-dependent heating does not affect the data, our results indicate that the slope of the period-color relation actually *inverts*: at young ages early K stars rotate more rapidly than late K stars, but beyond ~ 2 Gyr they rotate more slowly.

1. Introduction

1.1. Gyrochronology

It has long been known that the rotation periods of FGKM dwarfs increase over time (Skumanich 1972). This characteristic of main-sequence stars allows them to be dated via their rotation periods in a practice known as gyrochronology, which is useful since the ages of main-sequence stars are extremely difficult to measure via the traditional age-dating method of isochrone placement. It is also well established that stars of the same age but different *masses* have different rotation periods (citation), thought to be caused by the deeper convective zones, and therefore stronger magnetic dynamos (and more efficient magnetic braking) in lower-mass stars. However, an underlying assumption behind many empirical gyrochronology relations (*e.g.* Barnes 2003, 2007; Mamajek and Hillenbrand 2008; Meibom et al. 2011; Angus et al. 2015; Angus *et al.* in prep) is that the relationships between rotation period and photometric color[1], and rotation period and age are *separable*. In other words, the period-color relation is the same at all ages and the period-age relation is the same at all colors. It was recently shown that old field stars rotate more rapidly than a simple, separable gyrochronology relation would predict (Angus et al. 2015; van Saders et al. 2016, 2018; Metcalfe and Egeland 2019), and that a mass-dependent modification to the classical Skumanich (1972) spin-down law is required to reproduce the data (van Saders et al. 2016, 2018). An even more recent analysis of middle-aged open clusters provides further indication that the exponent of the period-age relation is mass-dependent. New rotation period measurements of the 1.1 Gyr open cluster, NGC 6811, reveal a flattened relationship between rotation period and color (Curtis et al. 2019). In this cluster, the G dwarfs rotate at the same rate as the K dwarfs.

Unfortunately, the open clusters with rotation period measurements are mostly young – currently the oldest is 2.5 Gyr (NGC 6819). Without rotation periods for precisely dated old stars, it is extremely difficult to calibrate the relationship between rotation period and color at old ages. For this reason, we used a population-based stellar age indicator, velocity dispersion, to investigate the period-color relations at old ages.

1.2. Kinematics as an age proxy

Stars are thought to be born in the thin disk of the Milky Way, orbiting the center of the galaxy with a low out-of-plane, or vertical, velocity (W, or v_z). Young stars have relatively small vertical velocities, but gain momentum in the vertical direction over time. Although the cause of orbital heating is not well understood, interactions with giant molecular clouds and spiral arms are thought to play an important role (citations). Although the velocity of any individual star will only provide a weak age constraint, the velocity dispersion of a group of stars can indicate whether, on average, that group is old or young relative to other groups. In this work we compare the velocity dispersions of groups of stars to ascertain which groups are older and which younger and draw conclusions based on the implied relative ages of populations. Since the age-velocity dispersion relations (AVRs) are themselves still actively being calibrated, it is difficult to directly compare gyrochronal ages with kinematic ones.

[1] As the directly observable quantity, color is usually used as a mass-proxy, and empirical gyrochronology relations are usually calibrated in color, rather than mass or effective temperature.

However, regardless of the exact relation between velocity dispersion and stellar age, it is expected to be a monotonic relationship, therefore velocity dispersion can be used effectively to *rank* groups of stars by age.

Vertical action is a better age indicator than vertical velocity, however both vertical action (J_z) and vertical velocity ($v_{\mathbf{z}}$/W) can only be calculated with full 6-dimensional position and velocity information. Unfortunately, most stars with measured rotation periods do not have radial velocity (RV) measurements because they are relatively faint *Kepler* targets (∼11th-18th magnitudes). For this reason, we used an alternative age proxy: velocity in the direction of galactic latitude, $v_{\mathbf{b}}$. The *Kepler* field is positioned at low galactic latitude (b=5-20°), so $v_{\mathbf{b}}$ is a close approximation to $v_{\mathbf{z}}$.

In this paper, we tested the Angus *et al.* (in prep) gyrochronology relation, which was calibrated using the period-color relation of Praesepe (in *Gaia* $G_{BP} - G_{RP}$ color) and the period-age relation of Praesepe and the Sun. The large number of Praesepe members with precise rotation periods from the *K2* mission (Douglas et al. 2017; Rebull et al. 2017), spanning spectral types F through early M, makes it the ideal cluster for calibrating the period-color relation of stars at 650 Myrs. Although only calibrated using Praesepe (∼ 650 Myr) and the Sun (4.56 Gyr), the Angus *et al.* (in prep) gyrochronology relation predicts accurate ages for members of NGC 6819, a 2.5 Gyr open cluster. However there are no rotation periods for K or M dwarfs in this cluster – its coolest members with rotation periods are G dwarfs. The period-color relation of Praesepe is nearly identical to the period-color relation of the Hyades, a cluster of around the same age: ∼ 650 years (Douglas et al. 2016, 2017; Rebull et al. 2017). However, Praesepe and the Hyades do *not* have the same period-color relation as NGC 6811, a 1.1 Gyr cluster (Curtis et al. 2019). The G dwarfs in NGC 6811 rotate at the same rate as the K dwarfs and it appears as though the K stars have 'stalled' – their spin-down has been halted. The Angus *et al.* (in prep) gyrochronology relation assumes that the rotation period-color relation of Praesepe is applicable to stars of all ages: the same polynomial relation fit to Praesepe is used to describe the period-color relation for all stars. However, the NGC 6811 cluster suggests that this assumption does not hold for K dwarfs past around 1 Gyr. If the period-color relation of the Angus *et al.* (in prep) gyrochronology model *were* a perfect model for the rotational evolution of stars, then groups of stars selected to be similar ages using this relation should fall on an isochrone on a CMD. Unfortunately, uncertainties on *Gaia* photometry and parallaxes, plus variations in metallicity and extinction, blur out the main sequence enough that differences between stars in different age and rotation bins are not easily discernable on the CMD, although there is still a general age and rotation period gradient on the CMD, as seen in figures 1 and 2. For this reason, we chose to use *kinematic* 'isochrones', instead of magnitude and color-based isochrones: although kinematics as an age indicator is not necessarily as well calibrated to an absolute age scale as CMD position, it can be very sensitive to *differences* in the ages of stellar populations.

Isochrones and stellar evolution tracks are highly dependent on choices made about input physics and assumptions. As a result, different sets of models can have very different shapes on the CMD, particularly at low masses. In fact, only empirical models, not physical ones, are currently able to reproduce the CMD positions of M dwarfs. Instead of relying on CMD position to age-date groups of stars, we opted to explore age trends via kinematics. Kinematic age-dating has the advantage of being relatively model independent, or at least, having a very simple model: that velocity dispersion increases over time. This means that it is relatively easy to rank groups of stars by age: older groups have a larger velocity dispersion.

2. Method

2.1. The data

We crossmatched the McQuillan et al. (2014) catalog of stellar rotation periods, measured from *Kepler* light curves, with the *Gaia* DR2 catalog. Reddening and extinction from dust was calculated for each star using the Bayestar dust map implemented in the dustmaps *Python* package (M. Green 2018). We estimated effective temperatures from dereddened *Gaia* $G_{BP} - G_{RP}$ color, using an 8th-order polynomial relation calibrated using .... stars ask Jason for details.

$$\mathrm{T_{eff}} = 8960 - 4802C + 1931C^2 - 2446C^3 + 2669C^4 - 1324C^5 + 301C^6 - 26C^7, \quad (1)$$

where C is *Gaia* $G_{BP} - G_{RP}$.

To explore the age of this stellar population from a rotation-period standpoint, it was first necessary to remove visual binaries and subgiants from the sample. The rotation-period evolution of these two types of stars is generally different to that of single stars which more usually follow a Skumanich-like spin-down law. We removed visual binaries and subgiants from the sample by applying cuts to the color-magnitude diagram (CMD), shown in figure 1. We fit a 6th-order polynomial to the main sequence and raised it by 0.22 dex, to approximate the division between single stars and visual binaries. We removed all stars above this line from the sample. We also removed subgiants by eliminating stars brighter than 6th magnitude in *Gaia* G-band.

We removed stars with negative parallaxes and parallax signal-to-noise ratios below 10 from the sample. We also removed a small number of stars fainter than 16th magnitude. We used the `Pyia` (Price-Whelan 2018) and `astropy` (Astropy Collaboration et al. 2013; Price-Whelan et al. 2018) *Python* packages to calculate stellar velocities. `Pyia` has built-in functionality for calculating velocity samples from the full *Gaia* uncertainty covariance matrix via Monte Carlo sampling. It therefore not only incorporates uncertainties on the *Gaia* positions parallaxes and proper motions, it also accounts for the *covariance* between these properties. We removed stars with absolute $v_{\mathbf{b}}$ uncertainties greater than 1 kms^{-1} from the sample.

Fig. 1.— A *Gaia* color magnitude diagram showing the McQuillan et al. (2014) sample with extinction-corrected magnitudes, colored by rotation period. We excluded photometric binaries and subgiants from our analysis by removing stars above the two dashed lines.

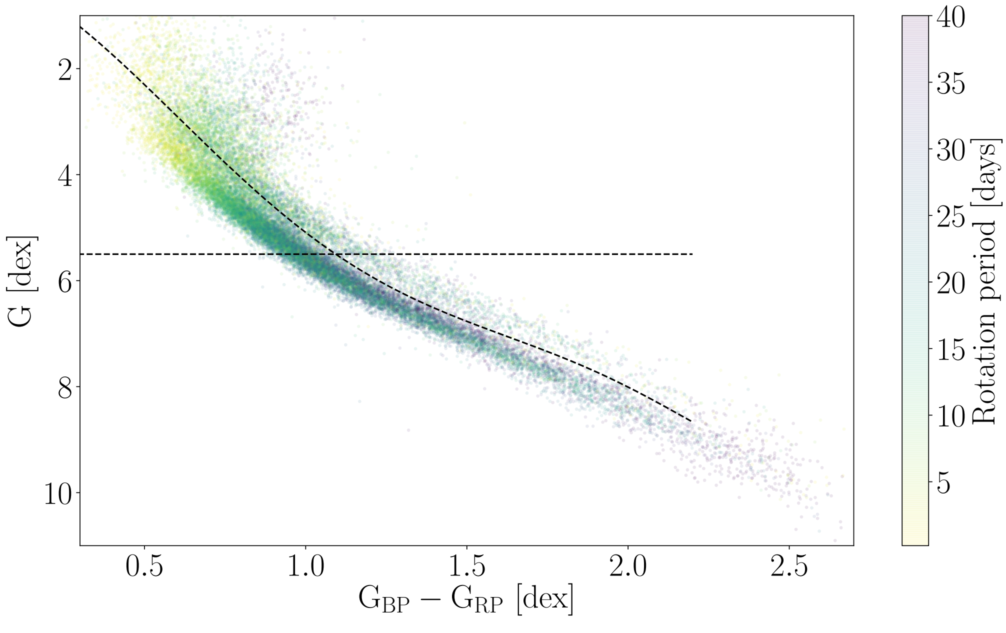
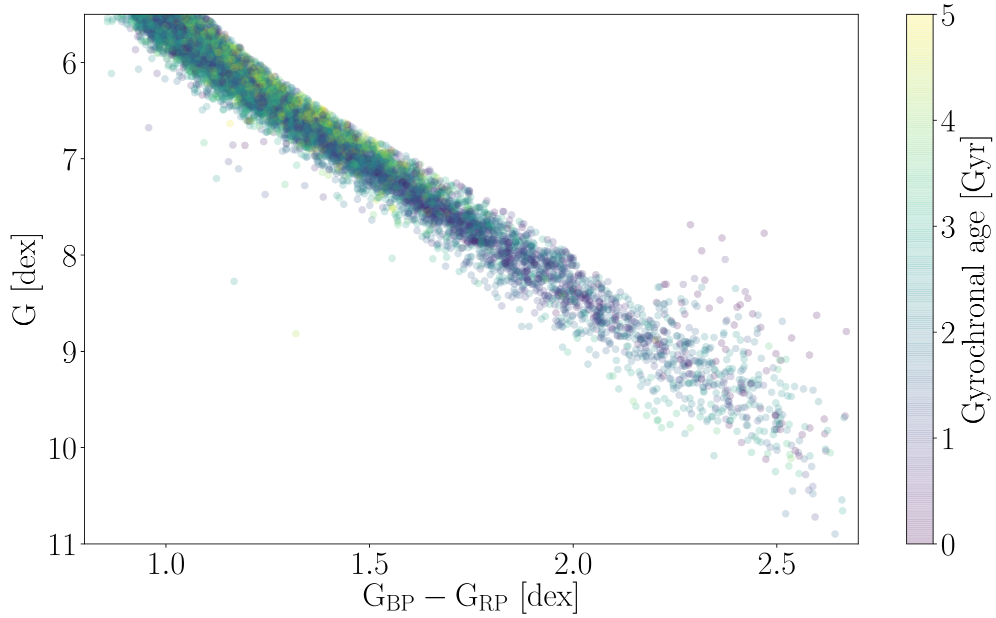


Fig. 2.— Dereddened main sequence stars with McQuillan et al. (2014) rotation periods on the *Gaia* CMD with visual binaries removed. Points are colored by their gyrochronal age, according to the Angus *et al.* (in prep) gyrochronology relation. A general age gradient is visible across the main sequence.



3. Results

3.1. Selecting groups of coeval stars

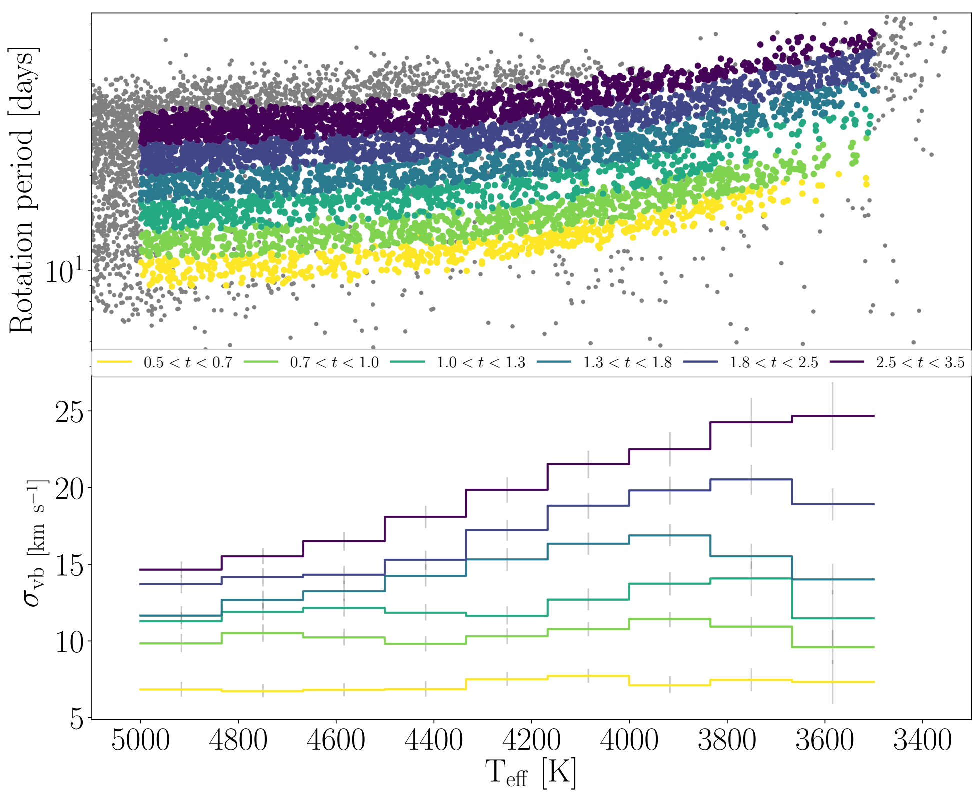
To explore the relationship between rotation period, $T_{\rm eff}$ and age, we selected groups of stars within different age ranges (where age was calculated using the Angus *et al.* in prep, gyrochronology relation), and calculated the velocity dispersion: the standard deviation of velocities, $\sigma_{v\mathbf{b}}$, as a function of effective temperature for each age group. Since the processes that produce dynamical heating in the galactic disk are expected to generate Gaussian-distributed velocities, we performed sigma-clipping on stars in each age and temperature bin, to remove non-Gaussian outliers. Ages were calculated using dereddened *Gaia* $G_{BP} - G_{RP}$ color, however throughout this paper we show rotation periods as a function of *effective temperature*. We chose to use effective temperatures and not colors in this analysis as it is the linear quantity and therefore easier to divide into bins of roughly equal numbers of stars.

The top panel of figure 3 shows the full McQuillan et al. (2014) sample (excluding visual binaries and subgiants) in grey, with coeval groups shown in color. The color of the points corresponds to the age ranges specified in the legend (in Gyr), which also apply to the lines in the lower panel. The bottom panel shows the velocity dispersion, $\sigma_{v\mathbf{b}}$ of each age group, as a function of effective temperature. We only included stars within a temperature range of 5000 - 3500 K in our analysis, as hotter stars are more likely to have stopped magnetic braking (van Saders et al. 2016), which could bias the results. M dwarfs were not included in our analysis because such faint stars cannot be observed at large heights above the plane (because of the low galactic latitude of the *Kepler* field, stars at high-Z are more distant), which introduces a mass-dependent velocity bias: cooler populations of stars are skewed towards lower velocity dispersions. The coolest temperature bins in figure 3 have low velocity dispersions, indicating that this effect may already become important at temperatures lower than $\sim$ 4000 K.

Overall, figure 3 shows that velocity dispersion increases with gyrochronal age across all temperatures. The consistent velocity dispersion of young stars as a function of temperature shows that the Praesepe-calibrated gyrochronology relation accurately predicts the relative ages of *young* field stars. However, although the youngest age groups have a relatively constant velocity dispersions across temperatures, the oldest age groups do not. This indicates *either* that the shape of the period-color relation does not remain constant over time, *i.e.* it flattens out, *or* that cool stars experience more efficient dynamical heating than hot stars.

Mass-dependent dynamical heating could occur because lower-mass stars experience greater velocity changes when gravitationally perturbed. However, mass-dependent orbital heating has not yet been unambiguously observed in the galactic disk because of the strong

Fig. 3.— Top: rotation period vs effective temperature for stars in the McQuillan et al. (2014) catalog. The full catalog, with subgiants and visual binaries removed is shown in grey, and stars selected to be in different age groups (between 3500 and 5000 K) are overlayed in color. These age groups were selected using the Angus *et al.* (in prep) gyrochronology relation. The legend in the center of the figure lists the age range, in Gyr, of each group. Bottom: velocity dispersion vs effective temperature for each age group. The color of the line corresponds to the color of the group shown in the top panel. If the gyrochronal model were correct at all ages, and the stars in each group were the same age across temperatures, the velocity dispersion would be constant as a function of $T_{\rm eff}$. However, the velocity dispersions of the oldest age groups increase $T_{\rm eff}$, indicating the Angus *et al.* (in prep) gyrochronology model underpredicts the the ages of late-K dwarfs relative to the ages of early K dwarfs at old ages. An alternative explanation could be that the gyrochronology relation is correct and *mass-dependent heating* is responsible for the greater velocity dispersions of cooler stars.



anti-correlation between stellar mass and stellar age. Less massive stars do indeed have larger velocity dispersions, however they are also older on average (citations). This mass-age degeneracy is highly reduced in M dwarfs because their main-sequence lifetimes are longer than the age of the Universe, however no evidence for mass-dependent heating has been detected in these low mass stars (Faherty et al. 2009).

The $v_{\mathbf{b}}$ AVR is not directly comparable to the $v_{\mathbf{z}}$ AVR, so, to draw a comparison with results from the literature, we calculated a $v_{\mathbf{z}}$ AVR for the subset of 290 stars in our sample with *Gaia* RVs. We measured an AVR exponent of 0.47 $\pm$ 0.1, which falls within the range of values (0.45-0.53) reported from measurements of F and G stars in the Solar neighborhood (Holmberg et al. 2009; Aumer and Binney 2009; Aumer et al. 2016). It is a little lower than the values of 0.56 $\pm$ 0.14 (for low-z) and 0.51 $\pm$ 0.15 (for high-z stars), calculated using *LAMOST* (LAMOST citation) K giants (Yu and Liu 2018). K giants are more massive than K dwarfs (how much more massive?), so this slightly higher value contradicts the mass-dependent heating hypothesis. Add some words about selection functions...

To investigate further, we calculated the exponent of the ($v_{\mathbf{b}}$) AVR for each temperature bin in figure 3. Figure 4 shows the significant rise the in AVR exponent as a function of effective temperature. The hottest stars in our sample ($T_{\rm eff}$ = 4833-5000 K) have a ($v_{\mathbf{b}}$) AVR exponent of 0.39 $\pm$ 0.02 and the coolest ($T_{\rm eff}$ = 3500-3667 K) have an exponent of 0.74 $\pm$ 0.02. The masses of stars in our sample differ by less than a factor of two, so mass-dependent heating cannot be entirely responsible for this large spread in heating rates. Need to look into this further.

If we assume that the rise in velocity dispersions at cooler effective temperatures is caused by an inaccurate period-color relation rather than mass-dependent dynamical heating, we can estimate what the shape of the period-color relations *should* be. Figure 3 indicates that the period-color relation flattens out, so we applied the same analysis to groups of stars with similar rotation periods, equivalent to a completely flat period-color relation. The top panel of figure 5 shows the McQuillan et al. (2014) sample with stars in different period ranges plotted in different colors. The bottom panel shows the velocity dispersion of each group as a function of effective temperature. Once again, the velocity dispersion increases with rotation period overall. For the most rapidly rotating groups of stars, velocity dispersion decreases with $T_{\rm eff}$ as expected given the positively sloped period-color relation of Praesepe and other young clusters: late K dwarfs rotate more slowly than early K dwarfs of the same age. Between rotation periods of 15 and 25 days, the temperature dependence of the velocity dispersion starts to disappear, indicating that the period-color relation becomes flat: late K dwarfs rotate at the same rate as early K dwarfs of the same age. At long rotation periods, the velocity dispersion still increases with effective temperature, although the increase is

Fig. 4.— The exponent of the ($v_{\mathbf{b}}$) AVR as a function of effective temperature.

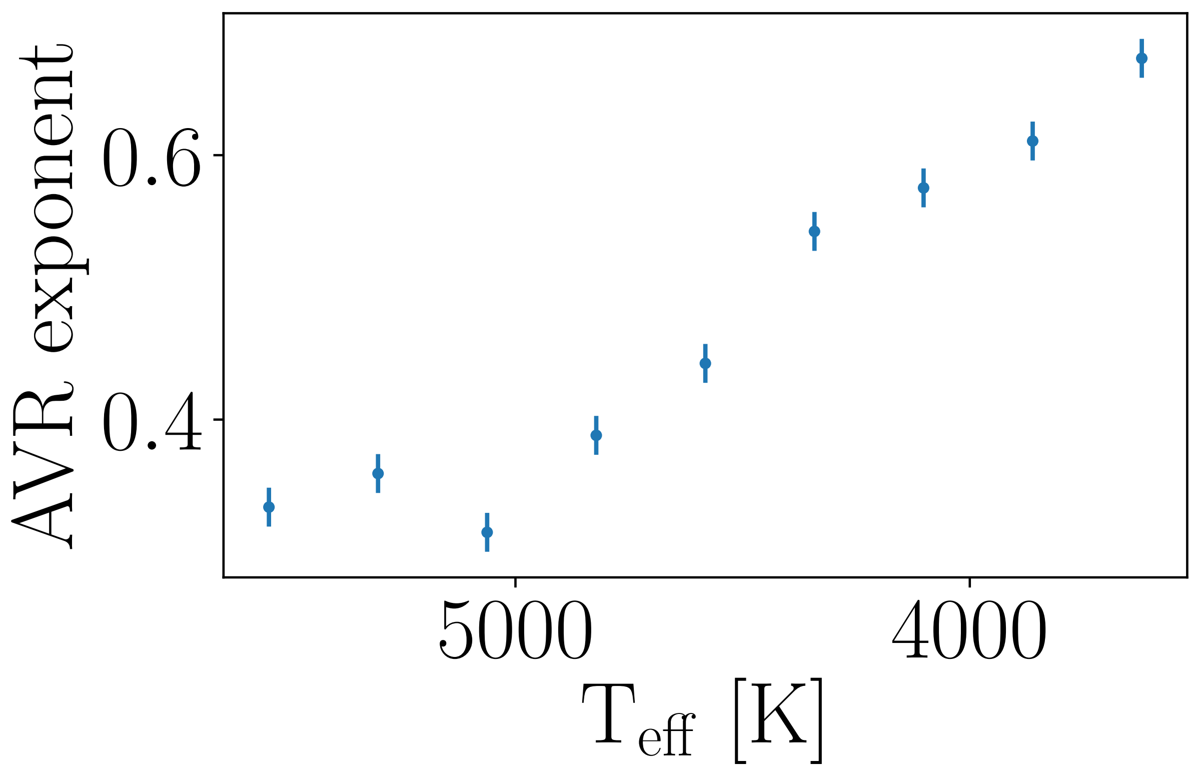
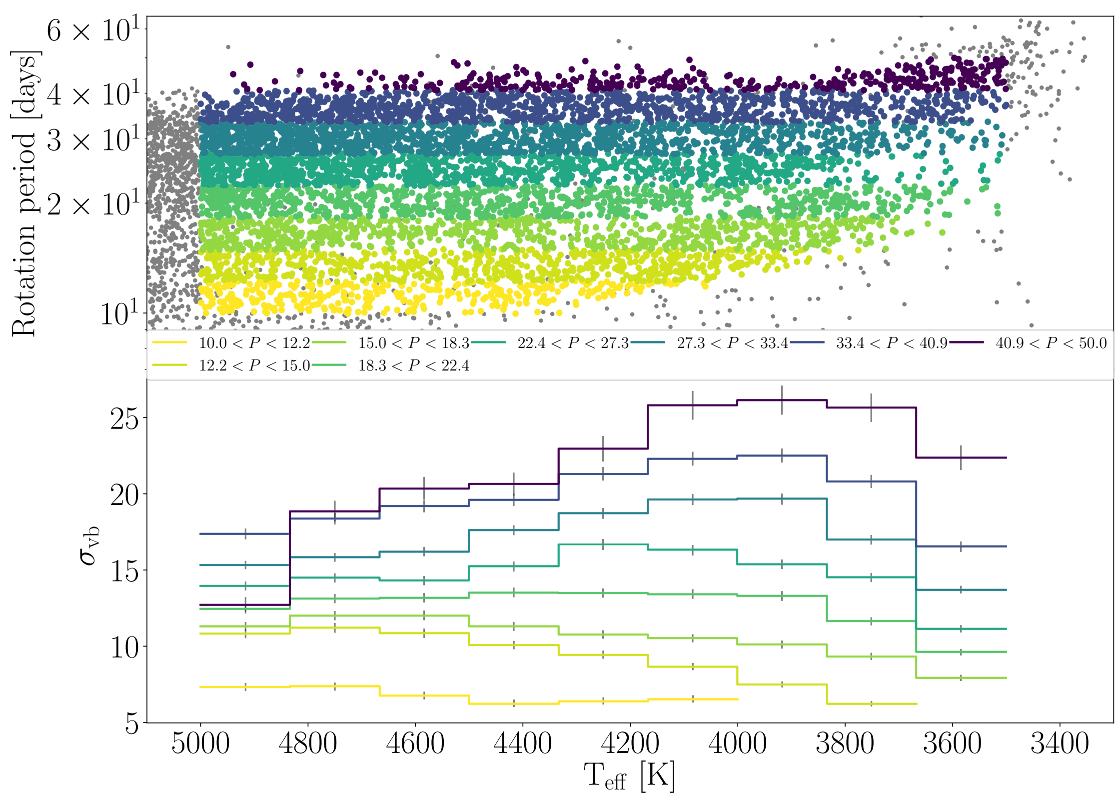


Fig. 5.— This figure is similar to figure 3, with stars divided into period groups rather than age groups. The velocity dispersion is more constant across effective temperatures for the most slowly rotating stars, compared to the stars selected with the Angus *et al.* (in prep) gyrochronology model, indicating that the gyrochronology models flatten out, and possibly even invert, at old ages.

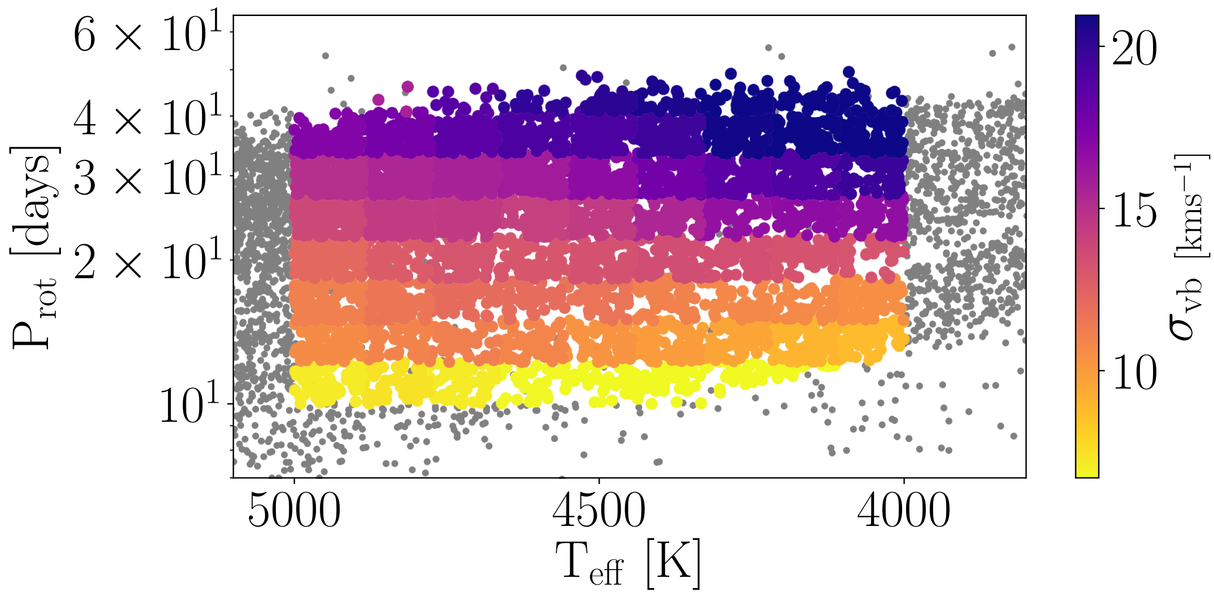


much more modest compared to the oldest stars in figure 3. This suggests that rotation period may *decrease* with effective temperature, *i.e.* late K dwarfs may rotate *more* rapidly than early K dwarfs. This would be a paradigm shift for gyrochronology, stellar spin-down rate is thought to be directly tied to magnetic field strength. The deeper convection zones of later type stars generate stronger magnetic fields which *should* lead to more efficient angular momentum loss.

Figure 6 shows the stars in our sample on a plot of period vs effective temperature, coloured by their ($v_{\mathbf{b}}$) velocity dispersion, where velocity dispersion was calculated in period and temperature bins (made apparent by the visible grid-like structure). If we assume that mass dependent heating does *not* affect this sample, velocity dispersion can be interpreted as an age proxy and stars bins of similar colour in figure 6 are similar ages. Interpreted this way, it appears as though isochrones slope downwards in period-temperature space at old ages. This figure also suggests that the oldest stars with measured rotation periods are cool, which may be because cooler stars remain active for longer, and their rotation periods can therefore be measured at older ages.

Unfortunately, we still cannot rule out the possibility that mass-dependent heating is acting on these stars which could be responsible for some, if not all, of the increased velocity dispersion at cooler effective temperatures.

Fig. 6.— The rotation periods vs effective temperatures of stars in the McQuillan et al. (2014) sample, colored by the velocity dispersions of stars in restricted period and temperature ranges.



4. Conclusion

We found that old groups of K dwarfs selected to be coeval using the Angus *et al.* (in prep) gyrochronology relation do *not* have the same velocity dispersion across all temperatures. This indicates that either the (Angus *et al.* in prep) period-color relation is wrong, or that mass-dependent dynamical heating is affecting this data set. We presented three pieces of evidence that suggest mass-dependent heating cannot be responsible for all of the increasing velocity dispersion.

- No strong evidence for mass-dependent heating has been found among M, L and T-type stars. These low-mass stars live longer than the current age of the Universe and therefore do not suffer from a mass-age degeneracy (Faherty et al. 2009).
- Literature measurements of the ($v_{\mathbf{z}}$) AVR exponent are slightly higher than the value measured from our data set, but calculated using more massive stars. Mass-dependent dynamical heating should however produce a *smaller* AVR exponent for higher-mass stars. We acknowledge that the stars in the McQuillan et al. (2014) sample, analyzed in this work, are subject to different selection biases than those used to calculate AVRs in the literature (Holmberg et al. 2009; Aumer and Binney 2009; Yu and Liu 2018), so drawing direct comparisons between them does not provide a reliable test.
- The differences in the ($v_{\mathbf{b}}$) AVR exponent between the hottest and coolest K dwarfs in our sample are extremely large, spanning values between 0.4 and 0.75, yet the difference in mass is less than a factor of two. Such a large spread in AVR exponents cannot be explained by mass-dependent heating alone.

Words about the significance of an inverted period-color relation.

This work was partly developed at the KITP conference 'Better stars, better planets'.

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This work made use of the gaia-kepler.fun crossmatch database created by Megan Bedell

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