

Exploring the ages of rotating stars using galactic dynamics: a novel approach to calibrating gyrochronology

Ruth Angus *et al.*

ABSTRACT

The rotational evolution of cool dwarfs is mostly unconstrained after around 2-3 billion years, and the gyrochronology relations are poorly calibrated beyond this time due to a lack of precise ages and rotation periods for old main-sequence stars. In this work we use the velocities of populations of K dwarfs to reveal the rotational evolution of these cool stars at old ages. We find that groups of stars, selected to be the same age using a gyrochronology relation calibrated to the Praesepe cluster, do *not* have the same velocity dispersion at all temperatures; cooler stars have greater velocity dispersions than hotter stars. We interpret this to mean that these cooler stars are older than the hotter ones, which implies that the period-color relation of Praesepe is not appropriate for old stars. Furthermore, our results indicate that the slope of the period-color relation actually *inverts*: at young ages early K stars rotate more rapidly than late K stars, but beyond ~ 2 Gyr they rotate more slowly. These observations could be reproduced by mass-dependent dynamical heating, where lower-mass stars experience a greater rate of dynamical heating than higher-mass stars, however we find no evidence for mass-dependent heating within K and M dwarfs in the *Kepler* field.

1. Introduction

1.1. Gyrochronology

It is well established that the rotation periods of FGKM dwarfs increase over time (Skumanich 1972). This characteristic of main-sequence (MS) stars allows them to be dated via their rotation periods in a practice known as gyrochronology, which is useful since the ages of MS stars are extremely difficult to measure via the traditional age-dating method of isochrone placement. It is also well established that stars of the same age but different masses have different rotation periods (citation), thought to be caused by the deeper

convective zones, and therefore stronger magnetic dynamos (and more efficient magnetic braking) in lower-mass stars. However, an underlying assumption behind many empirical gyrochronology relations is that the relationships between rotation period and photometric color¹, and rotation period and age are *separable*, meaning that the period-color relation is the same at all ages and the period-age relation is the same at all colors (*e.g.* Barnes 2003, 2007; Mamajek and Hillenbrand 2008; Meibom et al. 2011; Angus et al. 2015; Angus *et al.* 2019). It was recently shown that old field stars rotate more rapidly than a simple, separable gyrochronology relation would predict (Angus et al. 2015; van Saders et al. 2016, 2018; Metcalfe and Egeland 2019), and that a mass-dependent modification to the classical Skumanich (1972) spin-down law is required to reproduce the data (van Saders et al. 2016, 2018). An even more recent analysis of the 1.1 Gyr open cluster, NGC 6811, provides further indication that the exponent of the period-age relation is mass and time-dependent. This cluster has a flattened relationship between rotation period and color: the G dwarfs rotate at the same rate as the K dwarfs (Curtis et al. 2019).

In this paper, we tested the Angus *et al.* (2019) gyrochronology relation, which was calibrated using the period-color relation of Praesepe (in *Gaia* $G_{BP} - G_{RP}$ color) and the period-age relation of Praesepe and the Sun. The large number of Praesepe members with precise rotation periods from the *K2* mission (Douglas et al. 2017; Rebull et al. 2017), spanning spectral types F through early M, makes it the ideal cluster for calibrating the period-color relation of stars at 650 Myrs. This relation accurately describes the rotation periods of F and G stars up to around 2.5 Gyr (the age of NGC 6819 – the oldest cluster with available rotation periods), but over-predicts the rotation periods of K dwarfs in the 1.1 Gyr NGC 6811 cluster. No reliable age estimates exist for K dwarfs with rotation periods older than 1.1 Gyr, so the rotational evolution of these stars beyond this age is a mystery. For this reason, we used a population-based stellar age indicator, velocity dispersion, to investigate the period-color relations at old ages.

1.2. Kinematics as an age proxy

Stars are thought to be born in the thin disk of the Milky Way (MW), orbiting the center of the galaxy with a low out-of-plane, or vertical, velocity (W , or v_z). Young stars have relatively small vertical velocities, but gain momentum in the vertical direction over time. Although the cause of orbital heating is not well understood, interactions with giant

¹As the directly observable quantity, color is usually used as a mass-proxy, and empirical gyrochronology relations are usually calibrated in color, rather than mass or effective temperature.

molecular clouds, spiral arms and the galactic bar are thought to play an important role (see Sellwood 2014, for a review of secular evolution in the MW). Although the velocity of any individual star will only provide a weak age constraint, the velocity dispersion of a group of stars can indicate whether, on average, that group is old or young relative to other groups. In this work we compare the velocity dispersions of groups of stars to ascertain which groups are older and which younger and draw conclusions based on the implied relative ages of populations. Since the age-velocity dispersion relations (AVRs) are themselves still actively being calibrated, it is difficult to directly compare gyrochronal ages with kinematic ones. However, regardless of the exact relation between velocity dispersion and stellar age, it is expected to be a monotonic relationship, therefore velocity dispersion can be used effectively to *rank* groups of stars by age.

Vertical action is a better age indicator than vertical velocity (although still only a weak age indicator for individual stars, Beane et al. 2018), however both vertical action (J_z) and vertical velocity (v_z/W) can only be calculated with full 6-dimensional position and velocity information. Unfortunately, most stars with measured rotation periods do not have radial velocity (RV) measurements because they are relatively faint *Kepler* targets ($\sim 11^{\text{th}}\text{--}18^{\text{th}}$ magnitudes). For this reason, we used velocity in the direction of galactic latitude, v_b . The *Kepler* field is positioned at low galactic latitude ($b=5\text{--}20^\circ$), so v_b is a close (although not perfect – see section 2) approximation to v_z .

1.3. The data

We used the publicly available *Kepler-Gaia* DR2 crossmatched catalog² to combine the McQuillan et al. (2014) catalog of stellar rotation periods, measured from *Kepler* light curves, with the *Gaia* DR2 catalog. Reddening and extinction from dust was calculated for each star using the Bayestar dust map implemented in the `dustmaps` *Python* package (M. Green 2018), and `astropy` (Astropy Collaboration et al. 2013). We estimated effective temperatures from dereddened *Gaia* $G_{BP} - G_{RP}$ color, using an 8th-order polynomial relation calibrated using stars [ask Jason for details](#).

$$T_{\text{eff}} = 8960 - 4802C + 1931C^2 - 2446C^3 + 2669C^4 - 1324C^5 + 301C^6 - 26C^7, \quad (1)$$

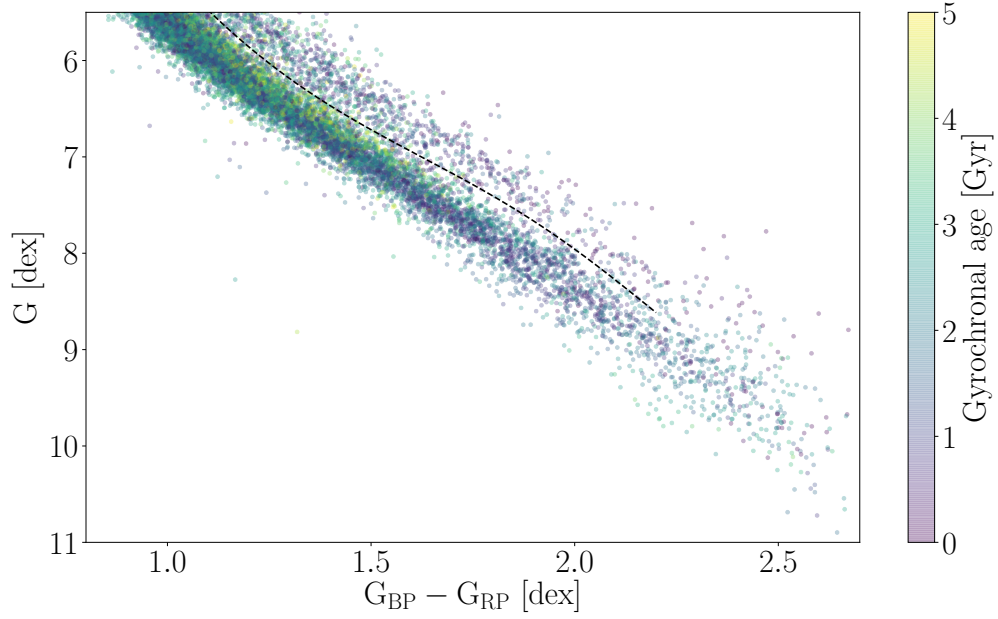
where C is *Gaia* $G_{BP} - G_{RP}$.

We removed visual binaries and subgiants from the sample by applying cuts to the color-magnitude diagram (CMD), shown in figure 1. We fit a 6th-order polynomial to the main sequence and raised it by 0.27 dex to approximate the division between single stars and visual binaries and removed all stars above this line from the sample. We also removed subgiants by eliminating stars brighter than 6th magnitude in *Gaia* G-band.

The `Pyia` (Price-Whelan 2018) and `astropy` (Astropy Collaboration et al. 2013; Price-Whelan et al. 2018) *Python* packages were used to calculate stellar velocities. `Pyia` has built-in functionality for calculating velocity samples from the full *Gaia* uncertainty covariance matrix via Monte Carlo sampling. It therefore not only incorporates uncertainties on the *Gaia* positions parallaxes and proper motions, it also accounts for the *covariance* between these properties. We removed stars with negative parallaxes, parallax signal-to-noise ratios less than 10, stars fainter than 16th magnitude, stars with absolute v_b uncertainties greater than 1 kms^{-1} and stars with galactic latitudes greater than 15° from the sample. Because v_b is only a close approximation to v_z at low galactic latitudes, and because latitude is correlated with stellar mass (lower mass stars are older and tend to be preferentially located at high b), we found that including stars with $b > 15^\circ$ led to a larger number of v_b outliers at low stellar mass. 15° was found to be an adequate compromise between maintaining a close relationship between v_b and v_z and keeping a large enough sample size to provide meaningful results. We also found that the anti-correlation between stellar mass and b is negligible for stars cooler than 5000 K. For this reason, and because the rotational evolution of old, G stars does not follow a simple Skumanich-like spin-down law (van Saders et al. 2016), we restricted our sample to temperatures between 4000 and 5000 K, *i.e.* K dwarfs.

²Available at gaia-kepler.fun

Fig. 1.— Dereddened MS *Kepler* stars with McQuillan et al. (2014) rotation periods on the *Gaia* CMD. We excluded visual binaries by removing stars above the dashed line. Points are colored by their gyrochronal age, according to the Angus et al. (2019) gyrochronology relation. A general age gradient is visible across the main sequence.



2. Results

2.1. Selecting groups of coeval stars

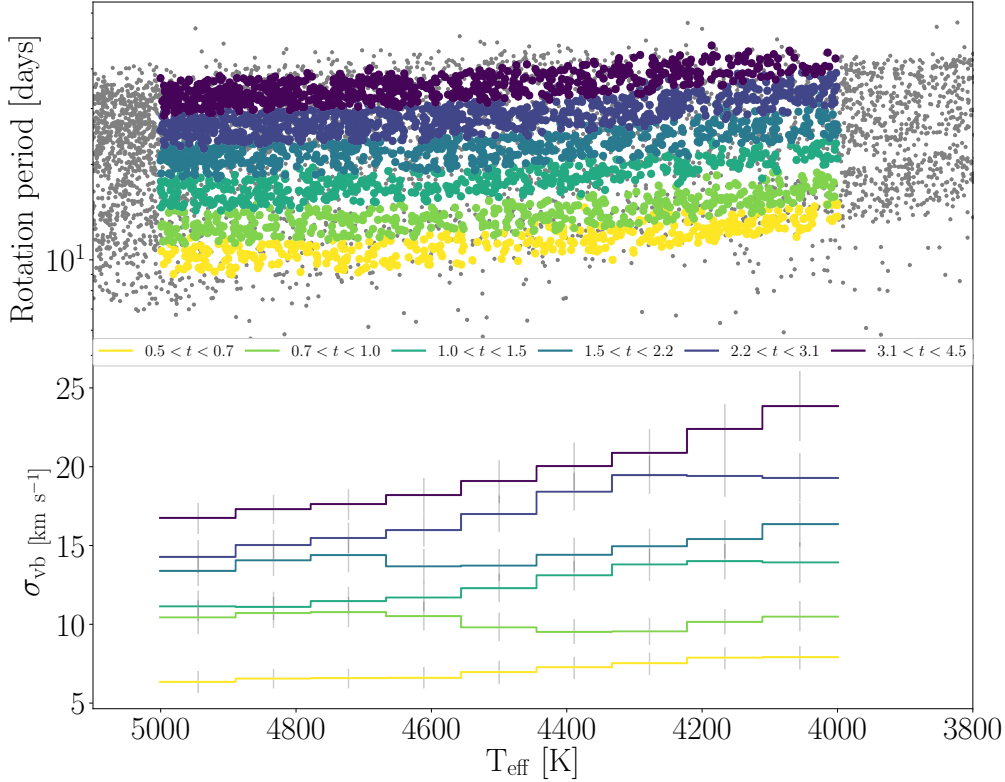
To explore the relationship between rotation period, T_{eff} and age, we selected groups of stars within different age ranges (where age was calculated using the Angus *et al.* 2019, gyrochronology relation), and calculated the velocity dispersion: the standard deviation of velocities, $\sigma_{v\mathbf{b}}$, as a function of effective temperature for each age group. We performed 3σ sigma-clipping on the stellar velocities in each age and temperature bin to remove non-Gaussian outliers. Without sigma-clipping, we found that a small number of high velocity outliers at the low-temperature end of our sample substantially raised the velocity dispersion for cooler stars. Ages were calculated using dereddened *Gaia* $G_{BP} - G_{RP}$ color, however throughout this paper we show rotation periods as a function of *effective temperature*. We chose to use effective temperatures and not colors in this analysis as it is the linear quantity and therefore easier to divide into bins of roughly equal numbers of stars.

The top panel of figure 2 shows the full McQuillan et al. (2014) sample (excluding visual binaries and subgiants) in grey, with coeval groups shown in color. The color of the points corresponds to the age ranges specified in the legend (in Gyr), which also apply to the lines in the lower panel. The bottom panel shows the velocity dispersion, $\sigma_{v\mathbf{b}}$ of each age group, as a function of effective temperature. M dwarfs were not included in our analysis because such faint stars cannot be observed at large heights above the plane (because of the low galactic latitude of the *Kepler* field, stars at high-Z are more distant), which introduces a mass-dependent velocity bias: cooler populations of stars are skewed towards lower velocity dispersions. The coolest temperature bins in figure 2 have low velocity dispersions, indicating that this effect may already become important at temperatures lower than ~ 4000 K.

Overall, figure 2 shows that velocity dispersion increases with gyrochronal age across all temperatures. The constant velocity dispersion of young stars as a function of temperature shows that the Praesepe-calibrated gyrochronology relation accurately predicts the relative ages of *young* field stars. However, although velocity dispersion is constant across temperature for young stars, it increases as a function of temperature for old stars. This indicates that the shape of the period-color relation flattens out over time.

An alternative explanation for this phenomenon is that lower-mass stars experience greater velocity changes when gravitationally perturbed and are more efficiently dynamically heated. Mass-dependent orbital heating has not been unambiguously observed in the galactic disk because of the strong anti-correlation between stellar mass and stellar age. Less massive stars do indeed have larger velocity dispersions, however they are also older on average. This mass-age degeneracy is highly reduced in M dwarfs because their main-sequence lifetimes

Fig. 2.— Top: rotation period vs effective temperature for stars in the McQuillan et al. (2014) catalog. The full catalog, with subgiants and visual binaries removed is shown in grey, and stars selected to be in different age groups (between 4000 and 5000 K) are overlayed in color. These age groups were selected using the Angus *et al.* (2019) gyrochronology relation. The legend in the center of the figure lists the age range, in Gyr, of each group. Bottom: velocity dispersion vs effective temperature for each age group. The color of the line corresponds to the color of the group shown in the top panel. If the gyrochronal model were correct at all ages, and the stars in each group were the same age across temperatures, the velocity dispersion would be constant as a function of T_{eff} . However, the velocity dispersions of the oldest age groups increase with T_{eff} , indicating the Angus *et al.* (2019) gyrochronology model underpredicts the the ages of late-K dwarfs relative to the ages of early K dwarfs at old ages.



are longer than the age of the Universe, however no evidence for mass-dependent heating has been detected in these low mass stars (Faherty et al. 2009). To investigate further, we selected late K and M dwarfs observed by both *Kepler* and *Gaia*, whose MS lifetimes exceed around 11 Gyrs and are therefore representative of the initial mass function. We selected all *Kepler* targets with dereddened *Gaia* $G_{BP} - G_{RP}$ colors greater than 1.2 (corresponding to an effective temperature $\lesssim 4800$ K) and absolute *Gaia* G -band magnitudes < 4 . We also eliminated visual binaries by removing stars above a 6th order polynomial, fit to the MS on the *Gaia* CMD. We then applied the same quality cuts as described in section 1.3. To search for evidence of mass-dependent heating we calculated the (v_b) velocity dispersion of stars in effective temperature bins. Again, we applied sigma clipping to remove high velocity outliers before calculating the standard deviation of stars in each bin. Figure 3 shows velocity and velocity dispersion as a function of effective temperature (calculated by transforming dereddened *Gaia* colors using equation 1). Velocity dispersion very slightly *decreases* with decreasing temperature, (the opposite of the trend expected for mass-dependent heating) however the slope is only inconsistent with zero at the 1.3σ level. This trend may be due to a selection bias: cooler stars are fainter and therefore typically closer, with smaller heights above the galactic plane and smaller velocities. The essential point here is that the increasing velocity dispersion with age and decreasing temperature that is evident in figure 2 is not a result of an intrinsically larger velocity dispersion for low-mass stars. We performed the same analysis on the 537 stars in this sample with RVs in *vertical* velocity (v_z) and found the slope of the velocity dispersion-temperature relation was consistent with zero.

Figure 3 indicates that mass-dependent heating does not strongly affect the McQuillan et al. (2014) sample of rotating *Kepler* stars. For this reason, we assume that age difference is the only significant cause of velocity dispersion differences between groups. In other words, (v_b) velocity dispersion is a reliable age proxy for the McQuillan et al. (2014) sample.

Figure 4 shows the stars in our sample of rotating dwarfs on a plot of period vs effective temperature, coloured by their (v_b) velocity dispersion. Here, velocity dispersion was calculated for groups of stars in uniformly-spaced bins in $\log_{10}(\text{period})$ and temperature. If we assume that mass dependent heating doesn’t affect this sample, and v_b at low galactic latitudes is an unbiased tracer of v_z , v_b velocity dispersion can be interpreted as an age proxy and stars plotted in a similar color in figure 4 are similar ages. Interpreted this way, it appears as though rotation period *decreases* with decreasing effective temperature at old ages. This is a paradigm shift for gyrochronology because stellar spin-down rate is thought to be directly tied to magnetic field strength, and the deeper convection zones of cooler stars generate stronger magnetic fields which *should* lead to more efficient angular momentum loss.

Something like this has been observed in the 1.1 Gyr NGC 6811 cluster, where the

Fig. 3.— Top: Stellar velocity (v_b) as a function of T_{eff} for *Kepler* K and M dwarfs. Vertical lines indicate different T_{eff} -groupings used to calculate velocity dispersion. Pink stars were not included in velocity dispersion calculations as they were either removed as outliers during a sigma clipping process, or they lie at the sparsely populated, extremely cool end of the temperature range. Velocity dispersion and T_{eff} are slightly correlated, indicating that mass-dependent heating does not significantly affect low-mass stars in the *Kepler* field.

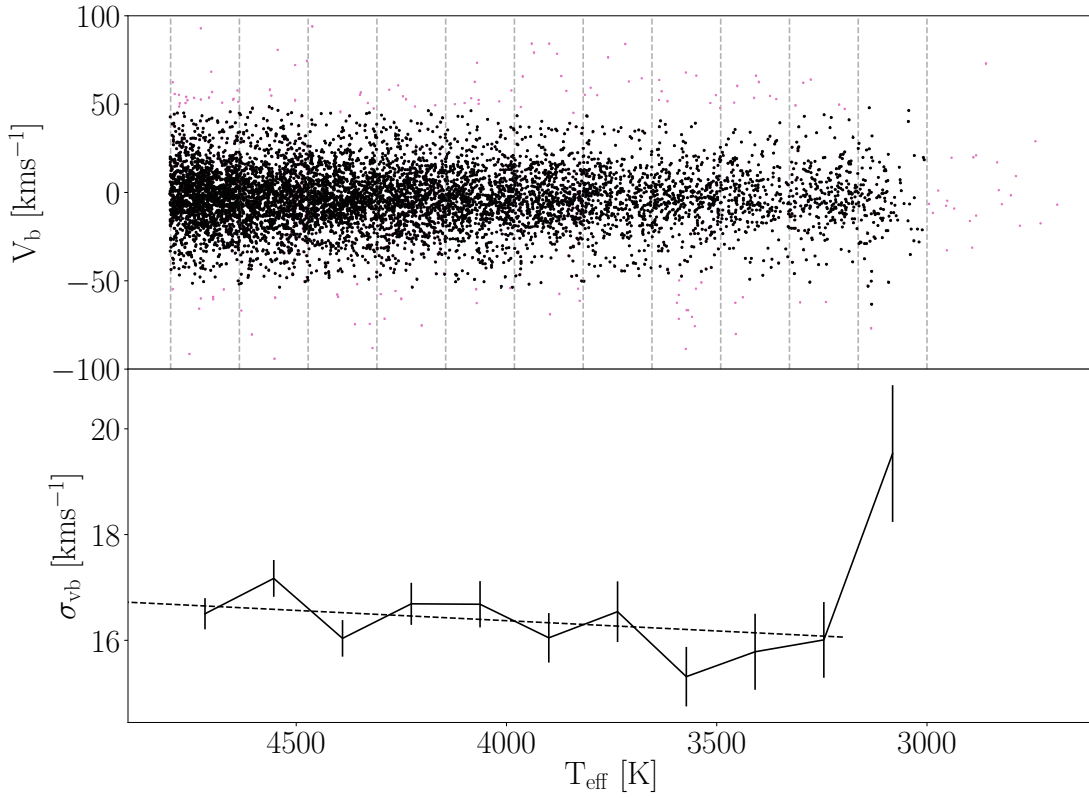
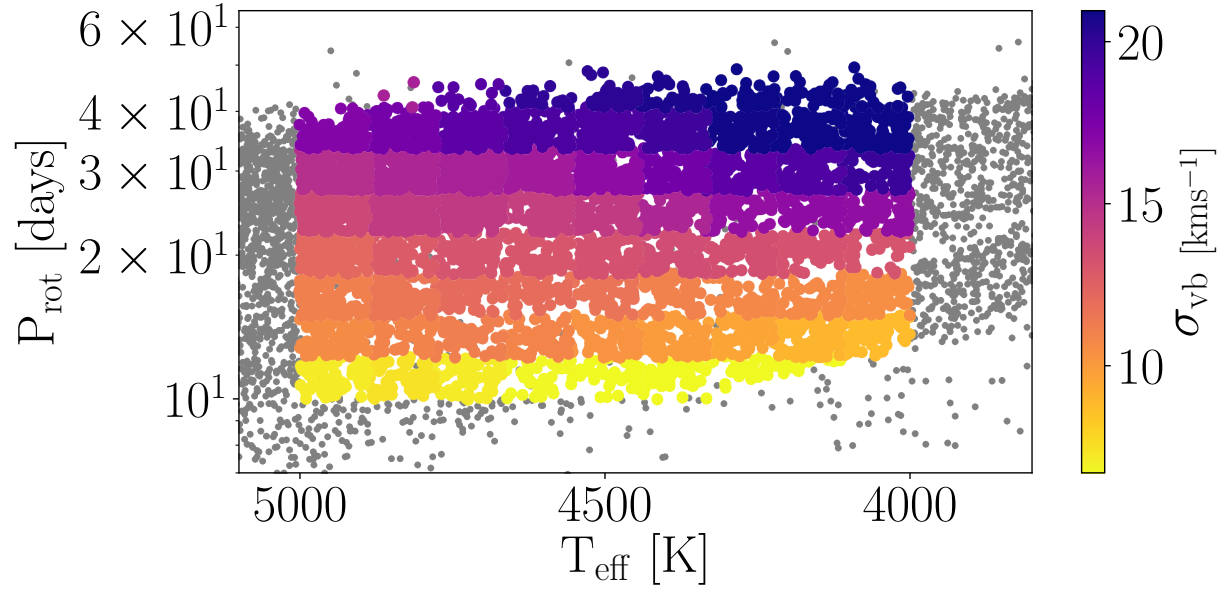


Fig. 4.— Rotation period vs effective temperature for stars in the McQuillan et al. (2014) sample, colored by the velocity dispersions of stars in restricted period and temperature ranges.



rotation periods of mid-K dwarfs are faster than expected; their rotational evolution appears to have stalled. NGC 6811 straddles the rotation period gap and may be the ‘missing link’ of gyrochronology that connects the two periods of stellar spin-down: the early stage where the period-color relation for K dwarfs has a positive slope and the late stage where it has a negative slope. In this case the period gap may delineate the transition between these two regimes and is the point at which stellar magnetic dynamos likely undergo a dramatic structural shift.

Figure 4 also indicates that the oldest stars with measured rotation periods are cool, which may be because cooler stars remain active for longer, and their rotation periods can therefore be measured at older ages.

3. Conclusion

We found that old groups of K dwarfs selected to be coeval using the Angus *et al.* (2019) gyrochronology relation do *not* have the same velocity dispersion across all temperatures.

Words about the significance of an inverted period-color relation.

How does the weakened van Saders braking law fit into all this?

Talk about the fact that the stars in each bin are not necessarily the same age.

Dispersion could be dependent on the selection function. We cut stars at high b , because we find that these do lead to a bias. We find no evidence for increased velocity dispersion with decreasing $teff$ in either v_b or v_z . We find that sigma clipping is necessary for v_b but not v_z and do not find that the outliers are distributed towards higher galactic latitudes.

This work was partly developed at the KITP conference ‘Better stars, better planets’.

Some of the data presented in this paper were obtained from the Mikulski Archive for Space Telescopes (MAST). STScI is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS5-26555. Support for MAST for non-HST data is provided by the NASA Office of Space Science via grant NNX09AF08G and by other grants and contracts. This paper includes data collected by the Kepler mission. Funding for the *Kepler* mission is provided by the NASA Science Mission directorate.

This work made use of the `gaia-kepler.fun` crossmatch database created by Megan Bedell

This work has made use of data from the European Space Agency (ESA) mission *Gaia* (<https://www.cosmos.esa.int/gaia>), processed by the *Gaia* Data Processing and Analysis Consortium (DPAC, <https://www.cosmos.esa.int/web/gaia/dpac/consortium>). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the *Gaia* Multilateral Agreement.

REFERENCES

- R. Angus, S. Aigrain, D. Foreman-Mackey, and A. McQuillan. Calibrating gyrochronology using Kepler asteroseismic targets. *MNRAS*, 450:1787–1798, June 2015. doi: 10.1093/mnras/stv423.
- Ruth Angus *et al.* Towards precise stellar ages: combining isochrone fitting with empirical gyrochronology. *AJ*, 2019.
- Astropy Collaboration, T. P. Robitaille, E. J. Tollerud, P. Greenfield, M. Droettboom, E. Bray, T. Aldcroft, M. Davis, A. Ginsburg, A. M. Price-Whelan, W. E. Kerzendorf, A. Conley, N. Crighton, K. Barbary, D. Muna, H. Ferguson, F. Grollier, M. M. Parikh, P. H. Nair, H. M. Unther, C. Deil, J. Woillez, S. Conseil, R. Kramer, J. E. H. Turner, L. Singer, R. Fox, B. A. Weaver, V. Zabalza, Z. I. Edwards, K. Azalee Bostroem, D. J. Burke, A. R. Casey, S. M. Crawford, N. Dencheva, J. Ely, T. Jenness, K. Labrie, P. L. Lim, F. Pierfederici, A. Pontzen, A. Ptak, B. Refsdal, M. Servillat, and O. Streicher. Astropy: A community Python package for astronomy. *A&A*, 558:A33, October 2013. doi: 10.1051/0004-6361/201322068.
- M. Aumer and J. J. Binney. Kinematics and history of the solar neighbourhood revisited. *MNRAS*, 397:1286–1301, August 2009. doi: 10.1111/j.1365-2966.2009.15053.x.
- S. A. Barnes. On the Rotational Evolution of Solar- and Late-Type Stars, Its Magnetic Origins, and the Possibility of Stellar Gyrochronology. *ApJ*, 586:464–479, March 2003. doi: 10.1086/367639.
- S. A. Barnes. Ages for Illustrative Field Stars Using Gyrochronology: Viability, Limitations, and Errors. *ApJ*, 669:1167–1189, November 2007. doi: 10.1086/519295.
- Angus Beane, Melissa K. Ness, and Megan Bedell. Actions Are Weak Stellar Age Indicators in the Milky Way Disk. *ApJ*, 867(1):31, Nov 2018. doi: 10.3847/1538-4357/aae07f.
- J. L. Curtis, M. A. Agüeros, S. T. Douglas, and S. Meibom. A Temporary Epoch of Stalled Spin-Down for Low-Mass Stars: Insights from NGC 6811 with Gaia and Kepler. *arXiv e-prints*, May 2019.
- S. T. Douglas, M. A. Agüeros, K. R. Covey, and A. Kraus. Poking the Beehive from Space: K2 Rotation Periods for Praesepe. *ApJ*, 842:83, June 2017. doi: 10.3847/1538-4357/aa6e52.
- Jacqueline K. Faherty, Adam J. Burgasser, Kelle L. Cruz, Michael M. Shara, Frederick M. Walter, and Christopher R. Gelino. The Brown Dwarf Kinematics Project I. Proper

- Motions and Tangential Velocities for a Large Sample of Late-Type M, L, and T Dwarfs. *AJ*, 137(1):1–18, Jan 2009. doi: 10.1088/0004-6256/137/1/1.
- J. Holmberg, B. Nordström, and J. Andersen. The Geneva-Copenhagen survey of the solar neighbourhood. III. Improved distances, ages, and kinematics. *A&A*, 501:941–947, July 2009. doi: 10.1051/0004-6361/200811191.
- Gregory M. Green. dustmaps: A Python interface for maps of interstellar dust. *The Journal of Open Source Software*, 3(26):695, Jun 2018. doi: 10.21105/joss.00695.
- E. E. Mamajek and L. A. Hillenbrand. Improved Age Estimation for Solar-Type Dwarfs Using Activity-Rotation Diagnostics. *ApJ*, 687:1264–1293, November 2008. doi: 10.1086/591785.
- A. McQuillan, T. Mazeh, and S. Aigrain. Rotation Periods of 34,030 Kepler Main-sequence Stars: The Full Autocorrelation Sample. *ApJS*, 211:24, April 2014. doi: 10.1088/0067-0049/211/2/24.
- S. Meibom, S. A. Barnes, D. W. Latham, N. Batalha, W. J. Borucki, D. G. Koch, G. Basri, L. M. Walkowicz, K. A. Janes, J. Jenkins, J. Van Cleve, M. R. Haas, S. T. Bryson, A. K. Dupree, G. Furesz, A. H. Szentgyorgyi, L. A. Buchhave, B. D. Clarke, J. D. Twicken, and E. V. Quintana. The Kepler Cluster Study: Stellar Rotation in NGC 6811. *ApJ*, 733:L9, May 2011. doi: 10.1088/2041-8205/733/1/L9.
- Travis S. Metcalfe and Ricky Egeland. Understanding the Limitations of Gyrochronology for Old Field Stars. *ApJ*, 871(1):39, Jan 2019. doi: 10.3847/1538-4357/aaf575.
- A. M. Price-Whelan, B. M. Sipőcz, H. M. Günther, P. L. Lim, S. M. Crawford, S. Conseil, D. L. Shupe, M. W. Craig, N. Dencheva, A. Ginsburg, J. T. VanderPlas, L. D. Bradley, D. Pérez-Suárez, M. de Val-Borro, (Primary Paper Contributors, T. L. Aldcroft, K. L. Cruz, T. P. Robitaille, E. J. Tollerud, (Astropy Coordination Committee, C. Ardelean, T. Babej, Y. P. Bach, M. Bachetti, A. V. Bakanov, S. P. Bamford, G. Barentsen, P. Barmby, A. Baumbach, K. L. Berry, F. Biscani, M. Boquien, K. A. Bostroem, L. G. Bouma, G. B. Brammer, E. M. Bray, H. Breytenbach, H. Buddelmeijer, D. J. Burke, G. Calderone, J. L. Cano Rodríguez, M. Cara, J. V. M. Cardoso, S. Cheedella, Y. Copin, L. Corrales, D. Crichton, D. D’Avella, C. Deil, É. Depagne, J. P. Dietrich, A. Donath, M. Droettboom, N. Earl, T. Erben, S. Fabbro, L. A. Ferreira, T. Finethy, R. T. Fox, L. H. Garrison, S. L. J. Gibbons, D. A. Goldstein, R. Gommers, J. P. Greco, P. Greenfield, A. M. Groener, F. Grollier, A. Hagen, P. Hirst, D. Homeier, A. J. Horton, G. Hosseinzadeh, L. Hu, J. S. Hunkeler, Ž. Ivezić, A. Jain, T. Jenness, G. Kanarek, S. Kendrew, N. S. Kern, W. E. Kerzendorf, A. Khvalko, J. King,

- D. Kirkby, A. M. Kulkarni, A. Kumar, A. Lee, D. Lenz, S. P. Littlefair, Z. Ma, D. M. Macleod, M. Mastropietro, C. McCully, S. Montagnac, B. M. Morris, M. Mueller, S. J. Mumford, D. Muna, N. A. Murphy, S. Nelson, G. H. Nguyen, J. P. Ninan, M. Nöthe, S. Ogaz, S. Oh, J. K. Parejko, N. Parley, S. Pascual, R. Patil, A. A. Patil, A. L. Plunkett, J. X. Prochaska, T. Rastogi, V. Reddy Janga, J. Sabater, P. Sakurikar, M. Seifert, L. E. Sherbert, H. Sherwood-Taylor, A. Y. Shih, J. Sick, M. T. Silbiger, S. Singanamalla, L. P. Singer, P. H. Sladen, K. A. Sooley, S. Sornarajah, O. Streicher, P. Teuben, S. W. Thomas, G. R. Tremblay, J. E. H. Turner, V. Terrón, M. H. van Kerkwijk, A. de la Vega, L. L. Watkins, B. A. Weaver, J. B. Whitmore, J. Woillez, V. Zabalza, and (Astropy Contributors). The Astropy Project: Building an Open-science Project and Status of the v2.0 Core Package. *AJ*, 156:123, September 2018. doi: 10.3847/1538-3881/aabc4f.
- Adrian Price-Whelan. adrn/pyia: v0.2. Apr 2018. doi: 10.5281/zenodo.1228136.
- L. M. Rebull, J. R. Stauffer, L. A. Hillenbrand, A. M. Cody, J. Bouvier, D. R. Soderblom, M. Pinsonneault, and L. Hebb. Rotation of Late-type Stars in Praesepe with K2. *ApJ*, 839:92, April 2017. doi: 10.3847/1538-4357/aa6aa4.
- J. A. Sellwood. Secular evolution in disk galaxies. *Reviews of Modern Physics*, 86(1):1–46, Jan 2014. doi: 10.1103/RevModPhys.86.1.
- A. Skumanich. Time Scales for CA II Emission Decay, Rotational Braking, and Lithium Depletion. *ApJ*, 171:565, February 1972. doi: 10.1086/151310.
- J. L. van Saders, T. Ceillier, T. S. Metcalfe, V. Silva Aguirre, M. H. Pinsonneault, R. A. García, S. Mathur, and G. R. Davies. Weakened magnetic braking as the origin of anomalously rapid rotation in old field stars. *Nature*, 529:181–184, January 2016. doi: 10.1038/nature16168.
- J. L. van Saders, M. H. Pinsonneault, and M. Barbieri. Forward Modeling of the Kepler Stellar Rotation Period Distribution: Interpreting Periods from Mixed and Biased Stellar Populations. *ArXiv e-prints*, March 2018.
- Jincheng Yu and Chao Liu. The age-velocity dispersion relation of the Galactic discs from LAMOST-Gaia data. *MNRAS*, 475(1):1093–1103, Mar 2018. doi: 10.1093/mnras/stx3204.