Let $H \in \mathbb{R}^{n \times n}$ have the form

$$H = \begin{pmatrix} \alpha & z^T \\ z & D \end{pmatrix}$$

where $z = (z_2, z_3, \ldots, z_n)'$, $D = \mathrm{diag}\{d_2, d_3, \ldots, d_n\}$. λ is an eigenvalue of H, which satisfies $\lambda \neq d_j \quad j = 2, 3, \ldots n$ and $\lambda \neq \alpha$. Prove that $f(\lambda) = 0$, where

$$f(\lambda) = \lambda - \alpha - \sum_{j=2}^{n} \frac{z_j^2}{\lambda - d_j}$$