Completeness Property of the Reals

October 4, 2020

Contents

1 [2.3] Completeness Property of the Reals

1.1 Upper and Lower Bounds [2.3.1]

Upper Bound An upper bound is any value greater than or equal to all elements of a set, e.g. u is an upper bound of A if:

$$\forall s \in S, \exists u \in \mathbb{R} : u \geq s \tag{1}$$

Lower Bound A lower bound is any value less than or equal to all elements of a set, e.g. w is a lower bound of A if:

$$\forall s \in S, \exists w \in \mathbb{R} : w \leq s \tag{2}$$

1.2 Supremea and Infima [2.3.2]

Supremum The suprema of a set is the smallest upper bound value of some set.
This value would be the maximum value of the set if the set had a maximum value.
Let V be the set of all upper bound values, u is a suprema iff:

$$u \leq v, \forall v \in V \tag{3}$$

Infimum The infimum of a set is the largest lower bound value of some set.
This value would be the maximum value of the set if the set had a maximum value.
Let T be the set of all upper bound values, w is a suprema iff:

$$w \leq t, \forall t \in T \tag{4}$$