Review: A Semi-Bregman Proximal Alternating Method for a Class of Nonconvex Problems: Local and Global Convergence Analysis

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1 Introduction

We consider the following non-convex and non-smooth block optimization model:

$$\begin{aligned} \min_{x\in\mathbb{R}^n} \quad & \Psi(x,y) \coloneqq F(x) + \Phi(y) + Q(x,y) \\ \text{s.t.} \quad & x \in \mathbb{R}^n, \quad y \in \mathbb{R}^m \end{aligned} \tag{1}$$

In the above model, F and Φ are non-smooth functions, and Q is a smooth function. This model has been studied in various fields in recent years, mostly non-convex optimization. In this paper, we focus our attention on the following class of smooth coupling functions:

$$Q(x,y) = \frac{\rho}{2}\|q(x) - y\| \tag{2}$$

where $\rho > 0$ and $q\colon \mathbb{R}^n \to \mathbb{R}^m$ is a continuously differentiable mapping.

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