2 Functional Analysis over Cones

2.1 Order Structures

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We use the book; Variational Methods in Partially Ordered Spaces (author: A.Gopfert, H.Riahi, C.Tammer, and C.Zalinescu).

p.13

As seen in the introduction, we are concerned with certain sets M with order structures. In the sequel we give the basic definitions. As usual, when M is a nonempty set, $M \times M$ is the set of ordered paris elemennts of M:

$$M \times M := \{(x_1, x_2) \mid x_1, x_2 \in M\}.$$

Definition 2.1.1

Let M be a nonempty set and $\mathcal{R}$ a nonempty subset of $M \times M$. Then $\mathcal{R}$ is called a binary relation or an order structure on M, and $(X, \mathcal{R})$ is a set M with order structure $\mathcal{R}$. The fact that $(x_1, x_2) \in \mathcal{R}$ will be denoted by $x_1 \mathcal{R} x_2$. We say that $\mathcal{R}$ is

(a) reflexive if $\forall x \in M : x\mathcal{R}x$,

(b) transitive if$\forall x_1, x_2, x_3 \in M : x_1\mathcal{R}x_2, x_2\mathcal{R}x_3 \Rightarrow x_1\mathcal{R}x_3$,

(c) antisymmetric if $\forall x_1, x_2 \in M : x_1\mathcal{R}x_2, x_2\mathcal{R}x_1 \Rightarrow x_1 = x_2$.