Set-valued Fan-Takahashi inequalities via scalarization

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In convex analysis and optimization theory, Fan-Takahashi minimax inequality plays a key role to solve equilibrium problems. Let X be a nonempty compact convex subset of a Hausdorff topological vector space and $f\colon X \times X \to \mathbb{R}$. Fan-Takahashi minimax inequality is: if f satisfies the following conditions:

(a) for each fixed $y \in X$, $f(\cdot, y)$ is lower semicontinuous,

(b) for each fixed $x \in X$, $f(x, \cdot)$ is quasi concave,

(c) $f(x, x) \leq 0$ for all $x \in X$,

then there exists $\bar{x} \in X$ such that $f(\bar{x}, y) \leq 0$ for all $y \in X$.

A quarter century ago, Georgiev and Tanaka [2] extended the minimax inequality to the form of set-valued maps. After that, Kuwano, Tanaka, and Yamada [4] constructed the result of four types set-valued minimax inequalities with set relations [3], which are binary relations depending on a given convex cone. However, this result is limited to the case of the specific scalarization functions. To obtain more practical results, we need to generalize convexity properties for set-valued maps. In addition, Dechboon and Tanaka [1] proposed generalized continuity to inherit properties of cone continuity for set-valued maps.

The aim of this talk is to explain the background of Fan-Takahashi minimax inequality and to generalize the convexity properties for set-valued maps and to apply them to the set-valued minimax inequalities.

References

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