Painleve-Kuratowski convergence
with base of topological space

Ryota Iwamoto

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We use the book; Variational Methods in Partially Ordered Spaces (author: A.Gopfert, H.Riahi, C.Tammer, and C.Zalinescu).

p.24

Definition 2.1.20

Let X be a linear space endowed with a topology τ. We say that $(X,\, \tau)$ is a topological linear space or topological vector space (t.l.s or t.v.s for short) if both operations on X (the addition and the multiplication by scalers) are continuous; in this case τ is called a linear topology on X.

Since these operations are defined on product spaces, we call that for two topological spaces $(X_1,\, \tau_1)$ and $(X_2,\, \tau_2)$, there exists a unique topology on $X_1 \times X_2$, denoted by $\tau_1 \times \tau_2$, with the property that

$$\mathcal{B}(x_1, x_2) \coloneqq \{U_1 \times U_2 \mid U_1 \in \mathcal{N}_{\tau_1}(x_1), U_2 \in \mathcal{N}_{\tau_2}(x_2)\}$$

is a neighborhood base of (x_1, x_2) w.r.t (= with regard as) $\tau_1 \times \tau_2$ for every $(x_1, x_2) \in X_1 \times X_2$; $\tau_1 \times \tau_2$ is called the product topology on $X_1 \times X_2$. Of course, in Definition 2.1.20 the topology on $X \times X$ is $\tau \times \tau$, and the topology on $\mathbb{R} \times X$ is $\tau_0 \times \tau$, where τ_0 is the usual topology of $\mathbb{R}$. It is easy to see that when $(X,\, \tau)$ is a topological linear space, $a \in X$ and $\lambda \in \mathbb{R}\backslash\{0\}$, the mappings T_a, $H_\lambda : X \to X$ defined by $T_a(x) = a+x$, $H_\lambda \coloneqq \lambda x$, are bijective and continuous with continuous inverse i.e., they are homeomorphisms.