AGU Ocean Sciences 2024 - Additional Info for Poster AI24B-2373

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1 Technical Description

The spatial scales are estimated by fitting ellipse-shaped (major and minor axes plus rotation angle; L_x, L_y, θ, Equation 3 below) non-stationary anistropic covariance function to the observed anomaly covariances [Paciorek and Schervish, 2006, Karspeck et al., 2012]. The chosen function is the Matérn covariance, which allows a flexible shape (ν) between the exponential ($\nu = 0.5$) and Gaussian ($\nu \to \infty$) limit. ν is usually fixed and is set to the 0.5 exponential limit.

Under this model, the covariance between points x and x' is:

$$c(x, x') = \frac{\sigma\sigma'}{\Gamma(\nu)2^{\nu-1}} \frac{|\Sigma|^{1/4}|\Sigma'|^{1/4}}{|\overline{\Sigma}|^{1/2}} \left(2\overline{\tau}\sqrt{\nu}\right)^{\nu} K_{\nu}\left(2\overline{\tau}\sqrt{\nu}\right) \tag{1}$$

Γ is the gamma function, K_ν is the modified Bessel function of the 2nd kind, and $||$ indicates the determinant. σ and σ' are the standard deviations at x and x'. $\overline{\tau}$ is the normalised (Mahalanobis) distance between the 2 points with the normalisation Σ being the anistropic length scales (L_x (major axis), L_y (minor axis), and θ (rotation angle).

$$\overline{\tau} = \sqrt{(x - x')^T \Sigma^{-1} (x - x')} \tag{2}$$

$$\Sigma = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} L_x & 0 \\ 0 & L_y \end{bmatrix} \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}^T \tag{3}$$

$$\overline{\Sigma} = \frac{\Sigma + \Sigma'}{2} \tag{4}$$

L_x, L_y, θ are determined numerically with maximum likelihood, using a moving window, capturing their spatial non-stationarity. For this estimation, each moving window is fitted with the stationary form of Equation 1 (i.e. without the middle Σ fraction). We seek the best parameters to Equation 4 (enclosed in τ within Equation 1) that minimises the (negative) log-likelihood loss function.

The stationary estimates can then be stitched together to its non-stationary form (aka Equation 1) form ellipse-like covariance and correlation matrices using Equation 1. The Σ fraction essentially averages the length scales and rotation angles between different locations.

2 Useful papers

Paciorek and Schervish 2006:

`https://onlinelibrary.wiley.com/doi/10.1002/env.785`

Karspeck et al 2012:

`https://onlinelibrary.wiley.com/doi/10.1002/qj.900`

References

Christopher J. Paciorek and Mark J. Schervish. Spatial modelling using a new class of nonstationary covariance functions. *Environmetrics*, 17(5):483–506, August 2006. ISSN 1180-4009, 1099-095X. doi: 10.1002/env.785. URL `https://onlinelibrary.wiley.com/doi/10.1002/env.785`.

Alicia R. Karspeck, Alexey Kaplan, and Stephan R. Sain. Bayesian modelling and ensemble reconstruction of mid-scale spatial variability in North Atlantic sea-surface temperatures for 1850-2008. *Quarterly Journal of the Royal Meteorological Society*, 138(662):234–248, January 2012. ISSN 00359009. doi: 10.1002/qj.900. URL `https://onlinelibrary.wiley.com/doi/10.1002/qj.900`.