Analytical Tests for MBSLIB

Documentation

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1 Compound Pendulum (1-DoF)

The compound pendulum shown in Figure 1a consists of a fixed base, a revolute joint q and a rigid link modeling the coumpound pendulum with inertia $I = \frac{1}{3}ml^2$, mass m and length l. Gravitation acts in negative y direction of the global reference frame. The joint is actuated by the torque τ. Figure 1b illustrates the model tree of the system with the stated modeling elements as well as a terminating massless endpoint.

1.1 Direct Kinematics

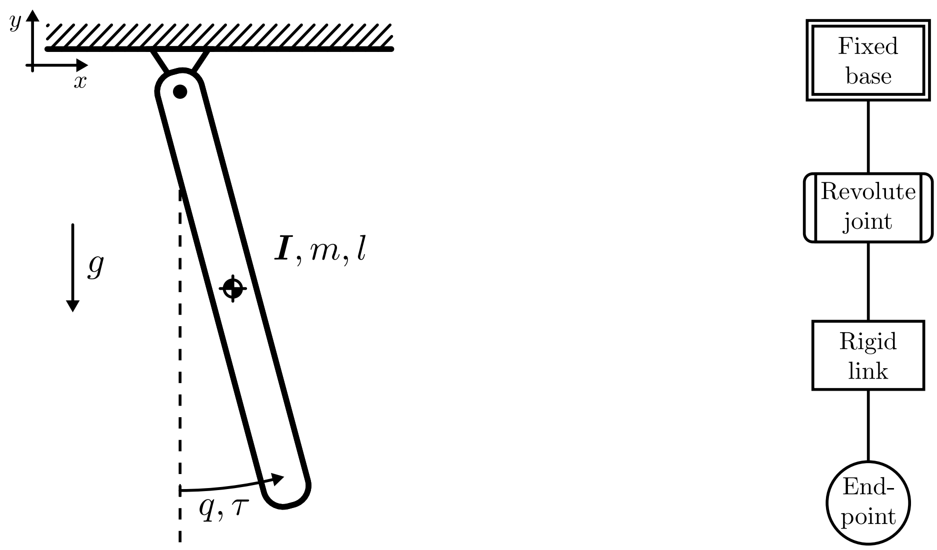
$$x = l\sin(q), \qquad y = -l\cos(q)$$

1.2 Direct Dynamics

$$\ddot{q} = \frac{3(\tau - \frac{1}{2}mgl\sin(q))}{ml^2}$$

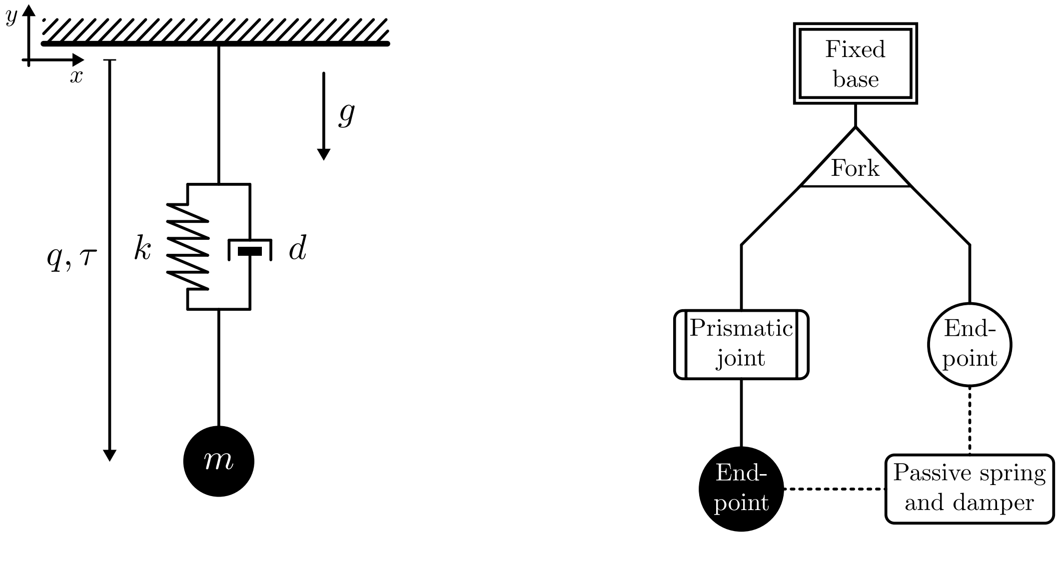
1.3 Inverse Dynamics

$$\tau = \frac{1}{3}ml^2\ddot{q} + \frac{1}{2}mgl\sin(q)$$



(a) **(b)**

Figure 1: Schematic diagram (a) and model tree (b) of the compound pendulum.



(a) **(b)**

Figure 2: Schematic diagram (a) and model tree (b) of the spring pendulum.

2 Spring Pendulum (1-DoF)

The spring pendulum shown in Figure 2a consists of a fixed base, a prismatic joint q and a massless endpoint as well as an endpoint with mass modeling the point mass m. A linear spring and damper act between the two endpoints in parallel to the joint. Gravitation acts in negative y direction of the global reference frame. The joint is actuated by the force τ. Figure 2b illustrates the model tree of the system with the stated modeling elements.

2.1 Direct Kinematics

$$x = 0, \qquad y = -q$$

2.2 Direct Dynamics

$$\ddot{q} = \frac{mg - d\dot{q} - kq + \tau}{m}$$

2.3 Inverse Dynamics

$$\tau = m\ddot{q} + d\dot{q} + kq - mg$$

3 Compound Pendulum on Trolley (2-DoF)

The compound pendulum with an angular spring and damper mounted on a frictionless, massless trolley shown in Figure 3a consists of a fixed base, a prismatic joint q_1 modeling the trolley as well as a revolute joint q_2 and a rigid link modeling the compound pendulum with inertia $I = \frac{1}{3}ml^2$, mass m and length l. The linear spring and damper are modeled by a passive spring damper drive that is connected to joint q_2. Gravitation acts in negative y direction of the global reference frame. The joints are actuated by the generalized forces τ_1 and τ_2. Figure 3b illustrates the model tree of the system with the stated modeling elements as well as a terminating massless endpoint.

3.1 Direct Kinematics

$$
\begin{aligned}
x_1 &= q_1, & y_1 &= 0, \\
x_2 &= q_1 + l\sin(q_2), & y_2 &= -l\cos(q_2)
\end{aligned}
$$

3.2 Direct Dynamics

$$
\begin{aligned}
\ddot{q}_1 &= \frac{2ml^2\sin(q_2)\dot{q}_2^2 + 6d\cos(q_2)\dot{q}_2 + 6k\cos(q_2)q_2 + 3gml\cos(q_2)\sin(q_2) + 4l\tau_1 - 6\cos(q_2)\tau_2}{4ml - 3ml\cos(q_2)^2}, \\
\ddot{q}_2 &= \frac{3ml^2\cos(q_2)\sin(q_2)\dot{q}_2^2 + 12d\dot{q}_2 + 12kq_2 + 6gml\sin(q_2) + 6l\cos(q_2)\tau_1 - 12\tau_2}{3ml^2\cos(q_2)^2 - 4ml^2}
\end{aligned}
$$

3.3 Inverse Dynamics

$$
\begin{aligned}
\tau_1 &= \ddot{q}_1 m + \frac{1}{2}\ddot{q}_2 ml\cos(q_2) - \frac{1}{2}\dot{q}_2^2 ml\sin(q_2) \\
\tau_2 &= \frac{1}{2}\ddot{q}_1 ml\cos(q_2)\frac{1}{3}\ddot{q}_2 ml^2 + \dot{q}_2 d + q_2 k + \frac{1}{2}gml\sin(q_2)
\end{aligned}
$$

4 SCARA Manipulator (2-DoF)

The SCARA manipulator with two rotational degrees of freedom $i = 1, 2$ shown in Figure 4a consists of a fixed base, two revolute joints q_i as well as two rigid links with the masses m_i, lengths l_i, moments of inertia

$$
\boldsymbol{I}_i = \begin{bmatrix} I_{i,xx} & 0 & 0 \\ 0 & I_{i,yy} & 0 \\ 0 & 0 & I_{i,zz} \end{bmatrix}
$$

and centers of mass

$$
\boldsymbol{r}_i = \begin{bmatrix} r_{i,x} \\ r_{i,y} \\ r_{i,z} \end{bmatrix}.
$$

The joints are actuated by the torques τ_i. Gravitation acts in negative y direction of the global reference frame. Figure 4b illustrates the model tree of the system with the stated modeling elements as well as a terminating massless endpoint.

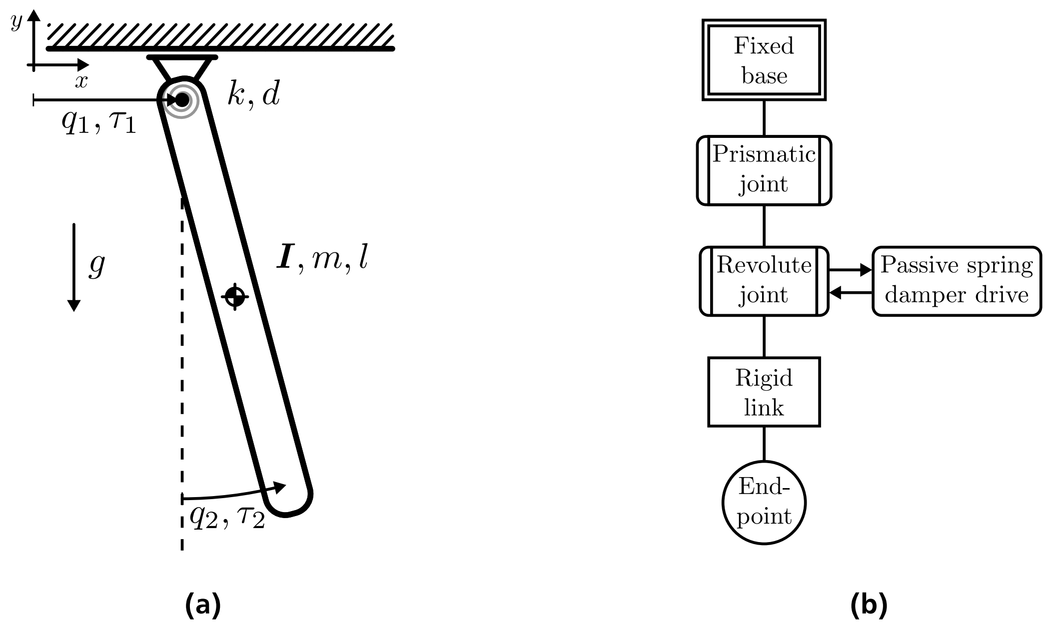


Figure 3: Schematic diagram (a) and model tree (b) of the compound pendulum on trolley.

4.1 Direct Kinematics

$$x_1 = l_1 \cos(q_1), \qquad y_1 = l_1 \sin(q_1),$$
$$x_2 = l_1 \cos(q_1) + l_2 \cos(q_1 + q_2), \qquad y_2 = l_1 \sin(q_1) + l_2 \sin(q_1 + q_2)$$

4.2 Direct Dynamics

$$\begin{aligned}
\ddot{\boldsymbol{q}} &= \boldsymbol{M}(\boldsymbol{q})^{-1}\left(\boldsymbol{\tau} - \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) - \boldsymbol{G}(\boldsymbol{q})\right),\\
\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} &= \begin{bmatrix} M_{1,1}(\boldsymbol{q}) & M_{1,2}(\boldsymbol{q})\\ M_{2,1}(\boldsymbol{q}) & M_{2,2}(\boldsymbol{q})\end{bmatrix}^{-1}\left(\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} - \begin{bmatrix}C_1(\boldsymbol{q},\dot{\boldsymbol{q}})\\ C_2(\boldsymbol{q},\dot{\boldsymbol{q}})\end{bmatrix} - \begin{bmatrix}G_1(\boldsymbol{q})\\ G_2(\boldsymbol{q})\end{bmatrix}\right),\\
&= \frac{1}{\det(\boldsymbol{M}(\boldsymbol{q}))}\begin{bmatrix} M_{2,2}(\boldsymbol{q}) & -M_{1,2}(\boldsymbol{q})\\ -M_{2,1}(\boldsymbol{q}) & M_{1,1}(\boldsymbol{q})\end{bmatrix}\left(\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} - \begin{bmatrix}C_1(\boldsymbol{q},\dot{\boldsymbol{q}})\\ C_2(\boldsymbol{q},\dot{\boldsymbol{q}})\end{bmatrix} - \begin{bmatrix}G_1(\boldsymbol{q})\\ G_2(\boldsymbol{q})\end{bmatrix}\right),\\
&= \frac{1}{M_{1,1}(\boldsymbol{q})M_{2,2}(\boldsymbol{q}) - M_{1,2}(\boldsymbol{q})M_{2,1}(\boldsymbol{q})}\begin{bmatrix} M_{2,2}(\boldsymbol{q}) & -M_{1,2}(\boldsymbol{q})\\ -M_{2,1}(\boldsymbol{q}) & M_{1,1}(\boldsymbol{q})\end{bmatrix}\left(\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} - \begin{bmatrix}C_1(\boldsymbol{q},\dot{\boldsymbol{q}})\\ C_2(\boldsymbol{q},\dot{\boldsymbol{q}})\end{bmatrix} - \begin{bmatrix}G_1(\boldsymbol{q})\\ G_2(\boldsymbol{q})\end{bmatrix}\right)
\end{aligned}$$

4.3 Inverse Dynamics

$$\begin{aligned}
\boldsymbol{\tau} &= \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q})\\
\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} &= \begin{bmatrix} M_{1,1}(\boldsymbol{q}) & M_{1,2}(\boldsymbol{q})\\ M_{2,1}(\boldsymbol{q}) & M_{2,2}(\boldsymbol{q})\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + \begin{bmatrix}C_1(\boldsymbol{q},\dot{\boldsymbol{q}})\\ C_2(\boldsymbol{q},\dot{\boldsymbol{q}})\end{bmatrix} + \begin{bmatrix}G_1(\boldsymbol{q})\\ G_2(\boldsymbol{q})\end{bmatrix}
\end{aligned}$$

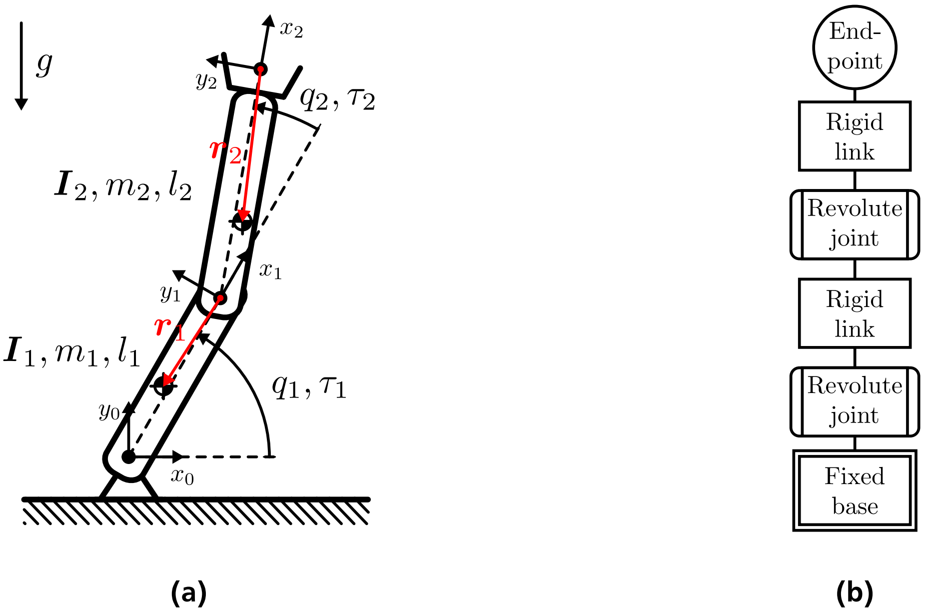


Figure 4: Schematic diagram (a) and model tree (b) of the SCARA manipulator.

4.4 Abbreviations

$$
\begin{aligned}
M_{1,1}(\boldsymbol{q}) &= m_1\left(l_1^2 + r_{1,x}^2 + r_{1,y}^2 + 2l_1 r_{1,x}\right) + m_2\left(l_1^2 + l_2^2 + r_{2,x}^2 + r_{2,y}^2 + 2l_2 r_{2,x} + \right.\\
&\qquad \left. 2l_1 l_2 \cos(q_2) + 2l_1 r_{2,x}\cos(q_2) + 2l_1 r_{2,y}\sin(q_2)\right) + I_{1,zz} + I_{2,zz},\\
M_{1,2}(\boldsymbol{q}) &= m_2\left((r_{2,x}+l_2)^2 + r_{2,y}^2 + l_1 l_2\cos(q_2) + l_1(r_{2,x}\cos(q_2) - r_{2,y}\sin(q_2))\right) + I_{2,zz},\\
M_{2,1}(\boldsymbol{q}) &= M_{1,2}(\boldsymbol{q}),\\
M_{2,2}(\boldsymbol{q}) &= m_2\left((r_{2,x}+l_2)^2 + r_{2,y}^2\right) + I_{2,zz},\\
C_1(\boldsymbol{q},\dot{\boldsymbol{q}}) &= -m_2 l_1 \dot{q}_2 (2\dot{q}_1 + \dot{q}_2)\left(r_{2,y}\cos(q_2) + l_2\sin(q_2) + r_{2,x}\sin(q_2)\right),\\
C_2(\boldsymbol{q},\dot{\boldsymbol{q}}) &= m_2 l_1\left((r_{2,x}+l_2)\sin(q_2) + r_{2,y}\cos(q_2)\right)\dot{q}_1^2,\\
G_1(\boldsymbol{q}) &= m_1 g\left((l_1 + r_{1,x})\cos(q_1) - r_{1,y}\sin(q_1)\right) + m_2 g\left(l_1\sin(q_2)\sin(q_1+q_2) + \right.\\
&\qquad \left. l_1\cos(q_2)\cos(q_1+q_2) + (r_{2,x}+l_2)\cos(q_1+q_2) - r_{2,y}\sin(q_1+q_2)\right),\\
G_2(\boldsymbol{q}) &= m_2 g\left((r_{2,x}+l_2)\cos(q_1+q_2) - r_{2,y}\sin(q_1+q_2)\right)
\end{aligned}
$$