BCSE 204L- Design and Analysis of Algorithms
Practice Problem Sheet (Dynamic Prograaming)
Rod-cutting Problem

Practice makes you Perfect

Rod Cutting Problem :

Given a rod of length n units and the price of a rod of length i units, p_i, $i = 1, 2, ..., n$, task is to determine the maximum revenue r_n obtainable by cutting up the rod and selling the pieces. One can sell the rod of length n assuch (without any cutting) or one can cut the rod into small pieces and the sellall the pieces at the respective prices given. One has to compute the best wayto cut the rod so that maximum revenue is obtained.

1. Describe the stages involved in the design of a dynamic programming pseudocode for a problem. You are expected to take a problem (your choice) and illustrate all the stages of the dynamic-programming-design with the problem chosen by you. The problem chosen by you should be different from the ones discussed in the class.

2. Given a problem P, Explain the required properties that has to be ex- hibited in the probelm so that dynamic programming based pseudocode is feasible for the problem P. In other words, you are expected to take a problem (of your choice) and describe the 'thought-process' involved in taking a decision 'whether the problem (chosen by you) is solvable by a dynamic-programming-pseudocode or not?'.

3. MyFibonacci series is one which is defined as follows:

   $fiba(1) = 0,\ \ fiba(2) = 1,\ \ fiba(3) = 2\ \ fiba(n) = fiba(n-1)+fiba(n-2)+fiba(n-3)$

   Given n, design a pseudocode to compute $fiba(n)$, using the dynamic-programming (top-down approach). Analyse your algorithm with the required components.

4. Given a rod of length n, and price p_i, $i = 1, 2, ..., n$, where p_i is a rod of length i units, design a brute-force algorithm to solve the 'Rod Cutting Problem' described above. Your algorithm should return the maximum- revenue that is obtainable by selling the rod of length n after cutting them into smaller pieces. Anayse your algorithm with the required components.

5. Design a pseudocode to solve the 'Rod Cutting Problem' that uses the dynamic programming strategy (top-down strategy). Your pseudocode should return the maximum-revenue that is obtainable by selling the rod of length n after cutting them into smaller pieces. Anayse your algorithm with the required components.

6. Design a recursive pseudocode to solve the 'Rod Cutting Problem' that uses the dynamic programming strategy (bottom-up strategy). Your pseudocode should return the maximum-revenue that is obtainable by selling the rod of length n after cutting them into smaller pieces.

7. . Design a pseudocode to solve the 'Rod Cutting Problem'. Given the length of the rod n and the prices p_i, your pseudocode should return the maximum revenue along with the lengths of the smaller pieces (decompo-sition of n) . You can follow any strategy for the purpose.

8. We modify the 'Rod Cutting Problem' as follows:

   Length of the given rod is greater than or equal to 8.

   You have to cut the rods in such a way that the length of smaller piece is either of length 3 or of length 5.

   If there is a wastage of rod (a piece whose is length is less than 3 cannot be cut further due to the restriction), there will be a penaltyof Re 1 for each unit length.

   We call the above problem as 'Modified Rod cutting problem'. Design a pseudocode that will return the maximum revenue that can be obtained by selling the rod, given the length of the rod and the price p_i, $i = 1, 2, ..., n$.

9. Given a rod of length n units, the price of a rod of length i units, p_i, $i = 1, 2, ..., n$ and another positive integer k($k \leq n$), Design a pseudocode to compute the maximum-revenue that is obtainable by selling the rod of length n after cutting them into k smaller pieces. For each possible value of k, your code should return the maximum-revenue obtainable by selling the k pieces. Analyse the algorithm with the required components.

10. Illustrate the differences between the Divide-Conquer-Combine strategy and the dynamic programming strategy, by taking a problem (of your choice)