Step 3 For moving link i-1, axis X_{i-1} where i=2,3,4,..n, will be directed towards the common normal axes of joint i and i-1 from i-1 to i. If the joint axes i and i-1 intersect, then axis X_{i-1} will be perpendicular to the intersecting plane and can be directed towards arbitrarily perpendicular axis. The rotation angle θ_i, will be chosen by normal direction of Z_{i-1} axis, which is basically represented between the X_{i-1} and X_i through rotation axis Z_{i-1}. Therefore third axis Y_{i-1} can be evaluated similarly with right hand coordinate rule $Y_{i-1} = Z_{i-1} \times X_{i-1}$.

Step 4 Now the placement of manipulator end effector coordinate frame X_e, Y_e, Z_e will be on the reference point of the gripper. Z_e axis will be directed anywhere in the orthogonal plane of X_e, similar to step three , X_e will be aligned with common normal of Z_{ei-1} and Z_{ei}. But in case of revolute joint axis of last joint, Z_e will be considered as parallel to the previous joint axis. The last axis will be given as right hand coordinate rule $X_e = Z_e \times X_e$

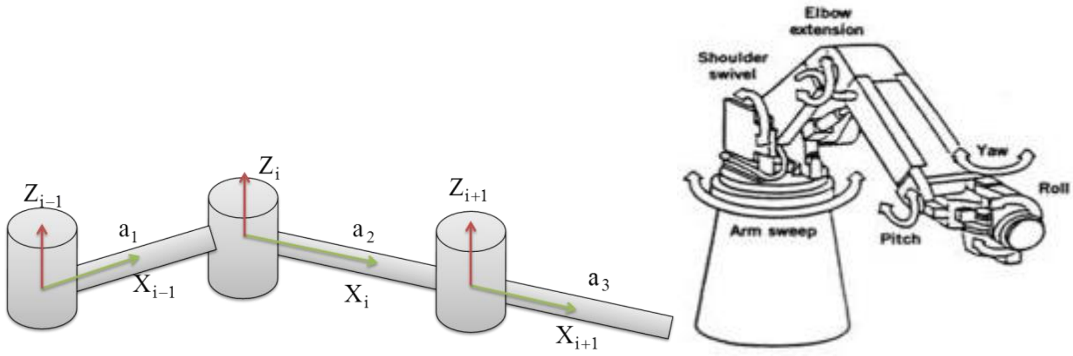
Step 5 Finally after assignment of all coordinate frames for all links i=1, 2, 3, …, n, DH parameters can be evaluated and can be written in tabular form given in the next section and pictorial view is presented in Figure 4.4 and 4.5.

4.2.4 Mathematical modelling of 3-dof revolute manipulator

The mathematical modeling of forward and inverse kinematics of robot manipulator using homogeneous transformation matrix method with DH parameters is presented. The purpose of this application is to introduce to robot kinematics, and the concepts related to both open and closed kinematics chains. The Inverse Kinematics is the opposite problem as compared to the forward kinematics, forward kinematics gives the exact solution but in case of inverse kinematics it gives multiple solutions. The set of joint variables when added that give rise to a particular end effectors or tool piece pose. Figure 4.4 (a) shows the basic joint configuration of 3-dof revolute planar manipulator and Figure 4.4 (b) represents the model of Cincinnati Milacron T3 and used as 3-dof planar manipulator. Figure 4.5 shows the simulation of 3-dof revolute planar manipulator using DH procedure. Position and orientation of the end effectors can be written in terms of the joint coordinates in the following way,

Table 4.2 DH-parameters for 3-dof revolute manipulator

Sl.	θ_i (degree)	d_i (mm)	a_i (mm)	α_i (degree)
1	θ_1	0	a_1	0
2	θ_2	0	a_2	0
3	θ_3	0	a_3	0



(a) (b)

Figure 4.4 Planar 3-dof revolute manipulator

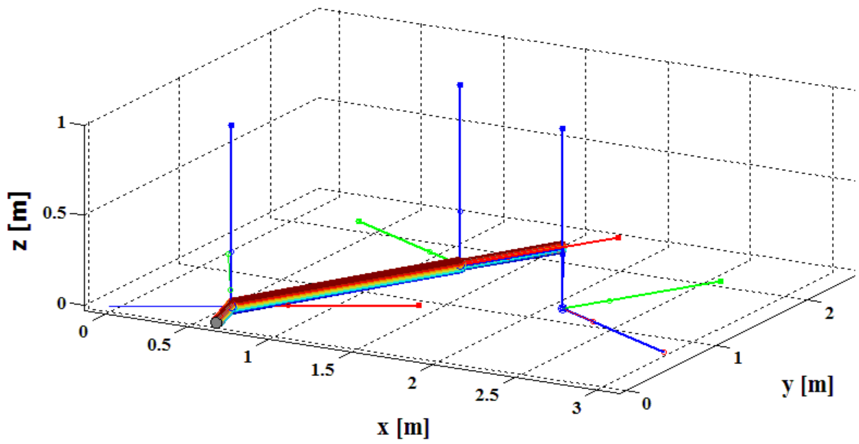


Figure 4.5 Coordinate frames of 3-dof revolute manipulator

Transformation matrix will be given by equation (4.4)

$$A_{i-1,i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i & 0 & a_i\sin\theta_i \\ 0 & 0_i & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{i-1,i} = \begin{bmatrix} c_{123} & -s_{123} & 0 & a_1c_1 + a_2c_{12} + a_3c_{123} \\ s_{123} & c_{123} & 0 & a_1s_1 + a_2s_{12} + a_3s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.6}$$

where, $c_1 = \cos\theta_1$, $s_1 = \sin\theta_1$, $c_{12} = \cos(\theta_1 + \theta_2)$, $s_{12} = \sin(\theta_1 + \theta_2)$, $c_{123} = \cos(\theta_1 + \theta_2 + \theta_3)$ and $s_{123} = \sin(\theta_1 + \theta_2 + \theta_3)$

Therefore forward kinematics is given by,

$$X = a_1c_1 + a_2c_{12} + a_3c_{123} \tag{4.7}$$

$$Y = a_1 s_1 + a_2 s_{12} + a_3 s_{123} \quad (4.8)$$

$$\phi = \theta_1 + \theta_2 + \theta_3 \quad (4.9)$$

ϕ represents orientation of the end effector. All the angles have been measured counter clockwise and the link lengths are assumed to be positive going from one joint axis to the immediately distal joint axis. However, to find the joint coordinates for a given set of end effectors coordinates (x, y, ϕ); one needs to solve the nonlinear equations for θ_1, θ_2 and θ_3.

Inverse kinematics,

$$\theta_2 = a\tan 2(s_2, c_2) = a\tan 2(\pm\sqrt{1-(c_2)^2}, \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1 a_2} \quad (4.10)$$

$$\theta_1 = a\tan 2(y, x) - a\tan 2(k_2, k_1) \quad (4.11)$$

Where,

$k_2 = a_1 + a_2 \cos\theta_2$ and $k_1 = a_2 s_2$

$$\theta_3 = \phi - \theta_1 - \theta_2 \quad (4.12)$$

4.2.5 Mathematical modelling of 4-dof SCARA manipulator

The Denavit-Hartenberg (DH) notation and methodology are used in this section to derive the kinematics of robot manipulator. The coordinate frame assignment and the DH parameters are depicted in Figure 4.5, and listed in Table 4.3 respectively, where to represents the local coordinate frames at the five joints respectively, represents the local coordinate frame at the end-effector, where θi represents rotation about the Z-axis, αi rotation about the X-axis, transition along the Z-axis, and transition along the X-axis.

Table 4.3 The DH Parameters

Sl.	θ_i (degree)	d_i (mm)	a_i (mm)	α_i (degree)
1	θ_1=±120	0	a_1=250	0
2	θ_2=±130	0	a_2=150	180
3	0	d_3=150	0	0
4	θ_4	d_4=150	0	0

The transformation matrix Ai between two neighbouring frames Oi−1 and Oi is expressed in equation (4.1) as,