Computational Statistics 732A89 – Spring 2026
Computer Lab 1

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January 20, 2026

This computer laboratory is part of the examination for the Computational Statistics course. Create a group report (which is directly presentable, if you are a presenting group), on the solutions to the lab as a PDF file. Be concise and do not include unnecessary printouts and figures produced by the software and not required in the assignments. All R code should be included as an appendix to your report. A typical lab report should contain 2-4 pages of text plus some figures plus the appendix with codes.
In the report, refer to all consulted sources and disclose all collaborations. The report has to be written in English.
The report should be handed in via LISAM (or alternatively in case of problems by email) by **23:59 January 27, 2026** at the latest. Notice that there is a deadline for corrections 23:59 07 April 2026 and a final deadline of 23:59 28 April 2026 after which no submissions or corrections will be considered, and you will have to redo the missing labs next year.
The seminar for this lab will take place **February 11, 2026**.

Question 1: Maximization of a function in one variable

Consider the function

$$g(x) = \frac{\log(x+1)}{x^{3/2}+1}.$$

a. Plot the function $g(x)$ in the interval $[0,4]$. What is your guess for the maximum point?

b. Compute the derivative $g'(x)$ of $g(x)$; recall the quotient rule that for $g(x) = \frac{u(x)}{v(x)}$, the derivative is $g'(x) = \frac{u'(x)v(x)-u(x)v'(x)}{(v(x))^2}$. Plot g' in $[0,4]$, and add a horizontal reference line at 0 to the plot.

c. Write your own `R` function applying the bisection method to g' to find a local maximum of g for a user-selected starting interval.

d. Write your own `R` function applying the secant method to g' to find a local maximum of g for a user-selected pair of starting values.

e. Run the functions in c. and d. for different starting intervals/pairs of starting values and check when they converge to the true maximum and when not. Discuss why. Compare the two methods also in terms of number of iterations used and programming effort required.

f. When you just should program one of them: Would you use bisection or secant, here? In general, for another function $g(x)$ to be maximized: When would you switch and use the other algorithm?

Question 2: Application for optimization of a function of one variable

Given a cubic regression model $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \epsilon_i$ for four observations $x_1 = -1, x_2 = -a, x_3 = a, x_4 = 1$, the model can be written in matrix notation as:

$$Y = \mathbf{X}\beta + \epsilon,$$

with

$$\mathbf{X} = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & -a & a^2 & -a^3 \\ 1 & a & a^2 & a^3 \\ 1 & 1 & 1 & 1 \end{pmatrix}, \qquad \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}.$$

The matrix $\mathbf{X}^\top \mathbf{X}$ is called information matrix for the parameter-vector β.

a. Program the function $f(a)$ in R which is the determinant of the information matrix depending on the parameter a, that is $f(a) = \det(\mathbf{X}^\top \mathbf{X})$.

b. Plot f for $a \in [0, 1]$. Reflect about the cases $a = 0$ and $a = 1$: what is their interpretation and of $f(0)$ and $f(1)$?

c. In R, one can use the function `optimize` to find a local optimum of a function within a given interval. Use this function to compute the local maximum of $f(a)$ on the interval $[0, 1]$.