

Computational Statistics 732A89 – Spring 2026

Computer Lab 5

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This computer laboratory is part of the examination for the Computational Statistics course. Create a group report (which is directly presentable, if you are a presenting group), on the solutions to the lab as a PDF file. Be concise and do not include unnecessary printouts and figures produced by the software and not required in the assignments. All R code should be included as an appendix to your report. A typical lab report should contain 2-4 pages of text plus some figures plus the appendix with codes. In the report, refer to all consulted sources and disclose all collaborations. The report has to be written in English.

The report should be handed in via LISAM (or alternatively in case of problems by email) by **23:59 February 24, 2026** at the latest. Notice that there is a deadline for corrections 23:59 07 April 2026 and a final deadline of 23:59 28 April 2026 after which no submissions or corrections will be considered, and you will have to redo the missing labs next year.

The seminar for this lab will take place **March 10, 2026**.

Question 1: Simulation of power curves

The Gumbel distribution (with scale parameter set to 1 and location parameter $\mu + c$ where $c = \log(\log(2)) \approx -0.36651$) has the distribution function

$$F(x) = \exp(-\exp(-(x - \mu - c))), \quad c = \log(\log(2)).$$

The median of a random variable with this distribution is μ .

Assume that $n = 13$ independent observations are made and they follow a Gumbel distribution with location $\mu + c$. To test the null hypothesis $H_0 : \mu = 0$ (median is 0) versus $H_a : \mu > 0$, one can use the Sign test. We want to investigate the power (=probability to reject H_0) of this test in this situation using a simulation study.

- Generate Gumbel random variables using the Inverse Transformation Method.
- Simulate the power of the Sign test for $\mu \in [0, 2]$ using an appropriate grid for μ -values on $[0, 2]$ and an appropriate number of repetitions for each μ (under consideration of the simulation's precision; provide some reasoning for the chosen number of repetitions). Plot the power curve (i.e., power versus μ). For the Sign test, you might use, e.g., the function `SIGN.test` in the package `BSDA`; or alternatively, the function `binom.test` might be used when applied to the number of positive observations.

Question 2: Bootstrap and permutation test

A student frequently travels from Campus Valla to Linköping resecentrum by bus on different days, typically at similar times in the afternoon and using the same bus line. She records the following travel times (in minutes) on different days:

16.5, 14.6, 15.0, 13.9, 15.4, 29.7, 14.8, 15.3, 15.9, 17.0.

Based on these data, we aim to estimate the mean travel time μ and quantify the associated uncertainty by constructing a 95% confidence interval.

- Derive a 95%-bootstrap confidence interval for the mean travel time μ based on the percentile method. Do not use a bootstrap package for this calculation; program the bootstrap on your own. Plot a histogram with the bootstrap distribution for the parameter.
- Derive now 95%-bootstrap confidence intervals for μ using the package `boot` with percentile and BCa method. Further, derive a 95%-confidence interval for μ when assuming that the data is normally distributed.
- Compare the confidence intervals from a. and b. and comment on it. Do you judge that the intervals are similar or do you see relevant differences? What is your overall conclusion from these confidence interval results about the mean?
- On some days, the student traveled in the evening. The recorded travel times (in minutes) were

15.0, 13.0, 12.9, 14.4, 15.1, 13.7.

Let ν denote the mean travel time for these evening trips. What is a reasonable estimate of $\mu - \nu$? Test the null hypothesis that the mean travel times are equal, $H_0 : \mu - \nu = 0$, against the one-sided alternative that the mean travel time in the afternoon is longer than in the evening, $H_1 : \mu - \nu > 0$, using a permutation test. Write your own code to carry out the permutation test without using any external packages. Display a histogram of the permutation-based null distribution and compute the corresponding p-value. How would you interpret the result? (For some discussion of the permutation test for a two sample comparison see Section 9.8 of Givens and Hoeting, 2013.)