**Combinatorics HW 2.2**

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**Find the 2020th permutation with nine numbers of 1-9 in lexicographic order? The first permutation is 123456789.**

Since we are using numbers 1-9 in the lexicographic order, there would be 9! possible permutations. Hence, we should start scanning from the left to find out the correct order of numbers that corresponds to the 2020$^{th}$ permutation. Since 8! = 40320 and 7! = 5040, then the first two digits from the left would be similar to the first permutation as there are 5040 and 40320 permutations respectively before the second and first digit change, which are all far away from the 2020$^{th}$ permutation. Hence, we have

$$12 - - - - - - -$$

$$0 \times 8! + 0 \times 7! + \cdots = 2020$$

Accordingly, 6! = 720 and the highest multiple of 6! that is lower than 2020 would be 2*6! = 1440. In this case, since numbers 1 and 2 have already been used, then this digit would be 5 as the permutations of 3 and 4 have occurred prior to it.

$$125 - - - - - -$$

$$0 \times 8! + 0 \times 7! + 2 * 6! + \cdots = 2020$$

Therefore, the remaining sum would be 2020 – 1440 = 580. For the fourth digit, the highest multiple of 5! = 120 that is lower than 580 is 4 (4*120 = 480). Since, numbers 1,2, and 5 have been used for the previous location, in order for this position to have 4 possible values, number 8 has to be chosen. In this way, the number of permutations of 3, 4, 6, and 7 would be counted before 8 is reached, which corresponds to the required multiplication (4*5!).

$$1258----$$

$$0 \times 8! + 0 \times 7! + 2 * 6! + 4 * 8! + \cdots = 2020$$

The remaining sum is 580 – 480 = 100. The highest multiple of 4! lower than 100 is 4. This would occur if 9 is selected; in this case, the permutations of 3, 4, 6, and 7 occur prior to 9.

$$12589----$$

$$0 \times 8! + 0 \times 7! + 2 * 6! + 4 * 5! + 4 * 4! + \cdots = 2020$$

Correspondingly, the remaining sum would be 100 – 96 = 4. As 3! = 6, the only possible multiple for this factorial would be 0. Hence, number 3 with no prior permutations is chosen.

$$125893---$$

$$0 \times 8! + 0 \times 7! + 2 * 6! + 4 * 5! + 4 * 4! + 0 * 3! + \cdots = 2020$$

Therefore, the remainder would still be 4. The highest multiple for 2! Would be 2 and it occurs if 7 is placed in the next location (4 and 6 prior to 7).

$$1258937--$$

$$0 \times 8! + 0 \times 7! + 2 * 6! + 4 * 5! + 4 * 4! + 0 * 3! + 2 * 2! + \cdots = 2020$$

Now that the remaining sum is 0, there is only one acceptable order for the remaining numbers 4 and 6, and that is 46; That is, if 6 is selected in the next position, the permutations of 4 would be prior to it, which results in a 1*1! addition to the sum.

$$125893746$$

$$0 \times 8! + 0 \times 7! + 2 * 6! + 4 * 5! + 4 * 4! + 0 * 3! + 2 * 2! + 0 * 1! = 2020$$

Hence, the 2020$^{th}$ permutation of numbers 1-9 with the first permutation being 123456789 is <u>125893746</u>.