周次	月 \ 星期	一	二	三	四	五	六	日
夏季学期	2020 八	10	11	12	13	14	15	16
		17	18	19	20	21	22	23
		24	25	26	27	28	29	30
		31						
	九		1	2	3	4	5	6
		7	8	9	10	11	12	13
1		14	15	16	17	18	19	20
2		21	22	23	24	25	26	27
3		28	29	30				
	十				1	2	3	4
4		5	6	7	8	9	10	11
5		12	13	14	15	16	17	18
6		19	20	21	22	23	24	25
7		26	27	28	29	30	31	
								1
8		2	3	4	5	6	7	8

10.19 Magic sequences
10.22 Linear Programming
10.26 Linear programming
10.29 Summary
11.2 Algorithm Design
11.5 Exam

Generating Function

- Given an infinite sequence of numbers: $h_0,h_1,h_2,\ldots,h_n,\ldots$
- The generating function is defined to be the infinite series

  $$G(x) = h_0+h_1x+h_2x^2+\ldots+h_nx^n+\ldots$$

- A **generating function** is a formal power series in one indeterminate, whose coefficients encode information about a sequence of numbers h_n that is indexed by the natural numbers.
- A finite sequence: $h_0,h_1,h_2,\ldots,h_m$
  - $h_0,h_1,h_2,\ldots,h_m,0,0,\ldots$
  - $G(x)= h_0+h_1x+h_2x^2+\ldots+h_mx^m$
- The generating function of the infinite sequence 1,1,1,…,1,… (h_i=1)
  - $g(x) = 1+x+x^2+\ldots+x^n+\ldots$

    $$= \frac{1}{1-x}$$

Linear Homogeneous Recurrence Relation

$F_n - F_{n-1} - F_{n-2} = 0$ $\qquad$ $h(n) - 3h(n-1) + 2h(n-2) = 0$

$x^2 - x - 1 = 0$ $\qquad$ $x^2 - 3x + 2 = 0$

Def if sequence $\{a_n\}$ satisfies:

$$a_n + C_1 a_{n-1} + C_2 a_{n-2} + \cdots + C_k a_{n-k} = 0,$$

$$a_0 = d_0, a_1 = d_1, \cdots, a_{k-1} = d_{k-1},$$

$C_1, C_2, \cdots C_k$ and $d_0, d_1, \cdots d_{n-1}$ are constants, $C_k \neq 0$, then this expression is called a k[th]-order linear homogeneous recurrence relation of $\{a_n\}$.

$$\boxed{C(x) = x^k + C_1 x^{k-1} + \cdots + C_{k-1} x + C_k}$$

Characteristic Polynomial

Summary of Linear Recurrence Relation

According to the non-zero roots of C(x)

1) k distinct non-0 real roots $C(x)=(x-a_1)(x-a_2)\cdots(x-a_k)$

$$a_n = l_1 a_1^n + l_2 a_2^n + \cdots + l_k a_k^n$$

In which $l_1, l_2, \cdots, l_k$, are undetermined coefficients.

2)A pair of conjugate complex root $\alpha_1 = \rho e^{i\theta}$ and $\alpha_2 = \rho e^{-i\theta}$:

$$\boldsymbol{a_n = A\rho^n \cos n\theta + B\rho^n \sin n\theta}$$

In which A， B are undetermined coefficients.

3)Has root α_1 with multiplicity of k.

$$(A_0 + A_1 n + \cdots + A_{k-1} n^{k-1})\alpha_1^n$$

In which $A_0, A_1, \cdots, A_{k-1}$ are k undetermined coefficients.

5 Magical Sequences

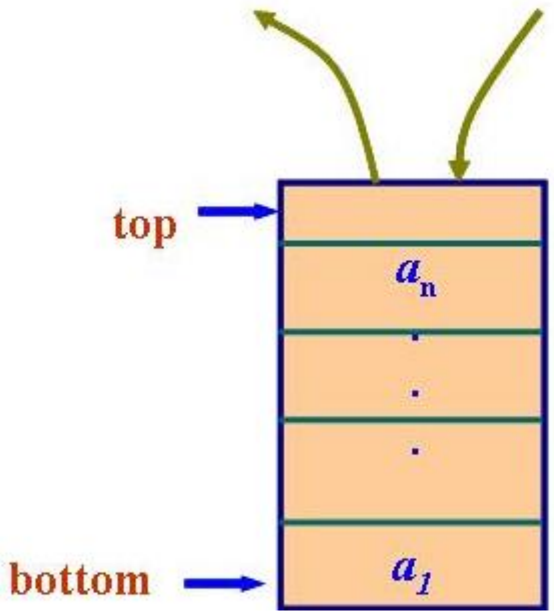
5-1 Catalan Numbers

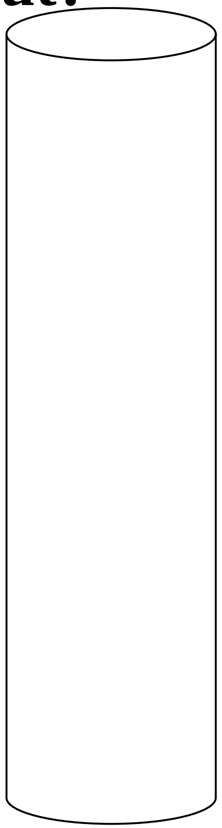
组合数学 Combinatorics

清华大学 马昱春

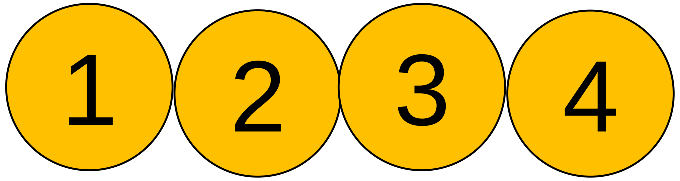
Amazing World of Computers

- **One stack (of infinite size) has the “push” sequence: 1, 2, 3, … n. How many ways can the numbers be popped out?**





1, 2, 3, 4

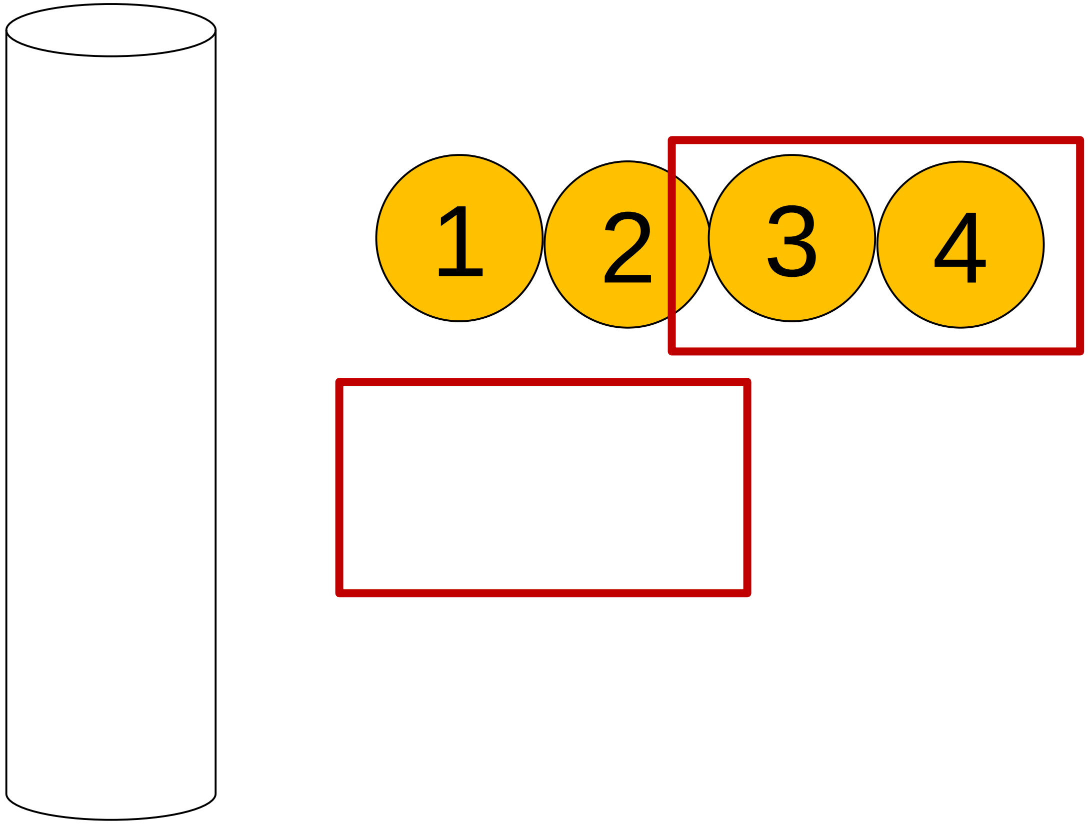


Push 1, Push 2, Pop 2, Pop 1, Push 3, Pop 3, Push 4, Pop 4

Amazing World of Computers

- **One stack (of infinite size) has the "push" sequence: 1, 2, 3, … n. How many ways can the numbers be popped out?**

Partition into steps when the stack was first empty.



The first time (Seq. 1) when the stack is empty, k elements are popped out.

Partition the n sequences into two sub-sequences, where one is Seq. 1~k-1 where there are k-1 elements, and the other is Seq. k+1~n, where there are n-k elements.

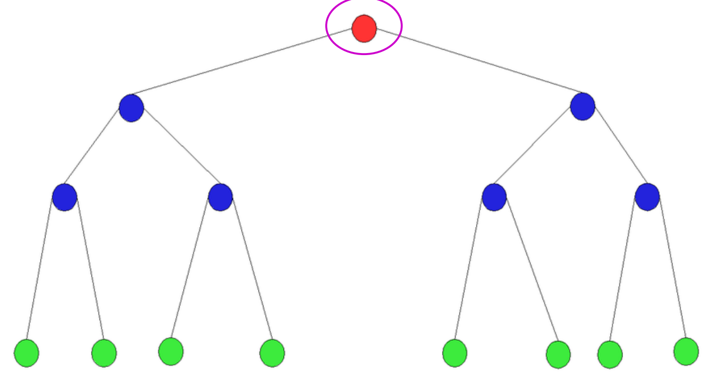
If f(n) is the sequence with n elements, then:

$f(n) = f(k-1) * f(n-k)$

$k = 1 \sim n$

$$f(n) = f(0)*f(n-1) + f(1)*f(n-2) + \ldots\ldots + f(n-2)*f(1) + f(n-1)*f(0)$$

Binary Tree



- ***How many different shapes can a binary tree with n nodes have?***
- The root will obviously contain one node. Assume T(i, j) stands the left subtree of the root containing i nodes, and right subtree with j nodes.
- Not counting the root, there are n-1 nodes that could be structured as: $T(0, n\text{-}1), T(1, n\text{-}2), \ldots T(n\text{-}1, 0)$.
- Suppose the solution is f(n):

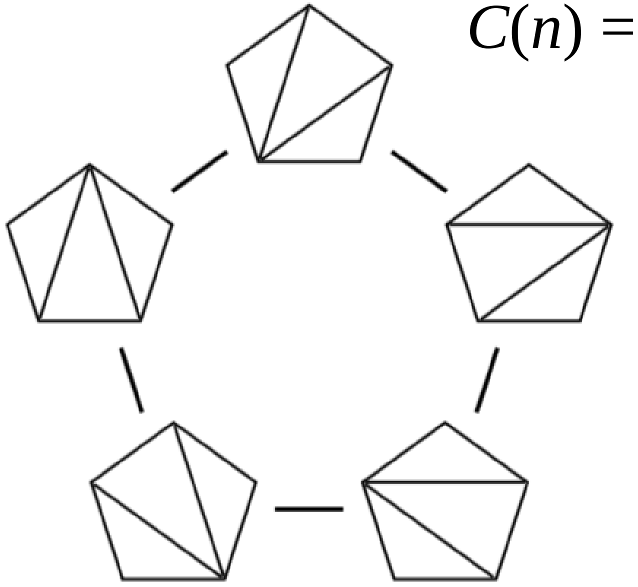
$$f(n) = f(0)*f(n-1) + f(1)*f(n-2) + \ldots\ldots + f(n-2)*f(1) + f(n-1)*f(0)$$

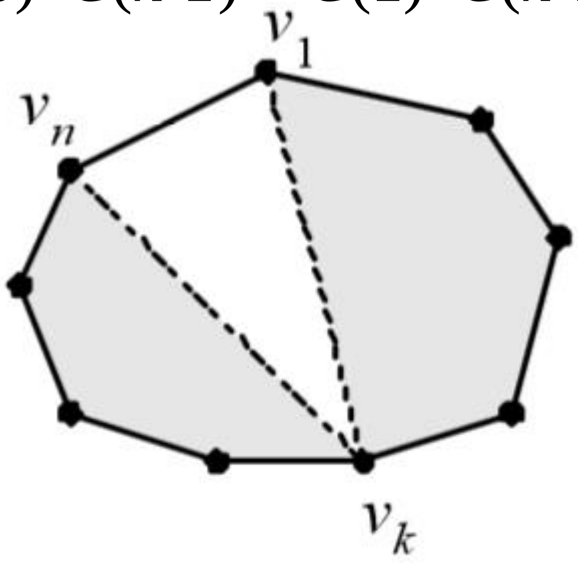
- If $f(0) = 1$， then $f(1) = 1$, $f(2) = 2$, $f(3) = 5$。

Catalan Numbers

- In 1751, in a letter to Christian Goldbach, Euler discussed about the following problem:
  - How many triangulations of a n-gon are there?

$$C(n) = C(0)*C(n\text{-}1) + C(1)*C(n\text{-}2) + \ldots\ldots + C(n\text{-}2)*C(1) + C(0)*C(n\text{-}2)$$





$$\frac{2 \cdot 6 \cdot 10 \cdot 14 \cdot 18 \cdot 22 \cdots (4n-10)}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot (n-1)}$$


Christian Goldbach (1690 – 1764)

History

- In1758, Johann Segner gave a recurrence formula for this question

$$C_n = \sum_{i=0}^{n-1} C_i C_{n-i-1}.$$

- In 1838， this became a hot research topic
  - Gabriel Lame gave a complete proof and a simpler expression.
  - Eugène Charles Catalan discovered the connection to parenthesized expressions during his exploration of the Towers of Hanoi puzzle.

  …

  - In 1900 Eugen Netto in his book named these numbers *Catalan* numbers.

History

- According to research papers in 1988 and 1999, the first person to discover Catalan numbers wasn’t Euler.
  - In 1753, Euler described the Catalan sequence while dividing a polygon into triangles.
  - In 1730, however, Minggantu, a Chinese mathematician (Mongolian) during Qing dynasty, had used the Catalan sequence. See “Ge Yuan Mi Lu Jie Fa”. This research was completed and published by his student in 1774.

Catalan Numbers

- Named after the Belgian mathematician Eugene Charles Catalan (1814–1894)
- OEIS A000108
- 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420, 24466267020, 91482563640, 343059613650, 1289904147324, 4861946401452, ...

$$C(n) = C(0)*C(n\text{-}1) + C(1)*C(n\text{-}2) + \ldots\ldots + C(n\text{-}2)*C(1) + C(n\text{-}1)*C(0)$$

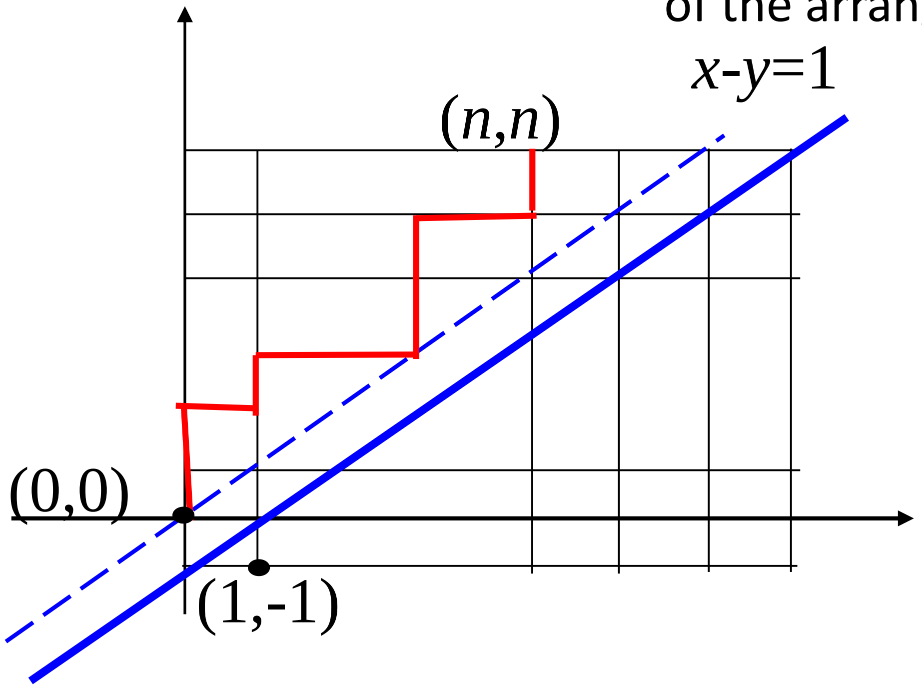
$$
\begin{array}{rcl}
C_1 & = & C_0C_0 \\
C_2 & = & C_1C_0 + C_0C_1 \\
C_3 & = & C_2C_0 + C_1C_1 + C_0C_2 \\
C_4 & = & C_3C_0 + C_2C_1 + C_1C_2 + C_0C_3 \\
\cdots & & \cdots
\end{array}
$$

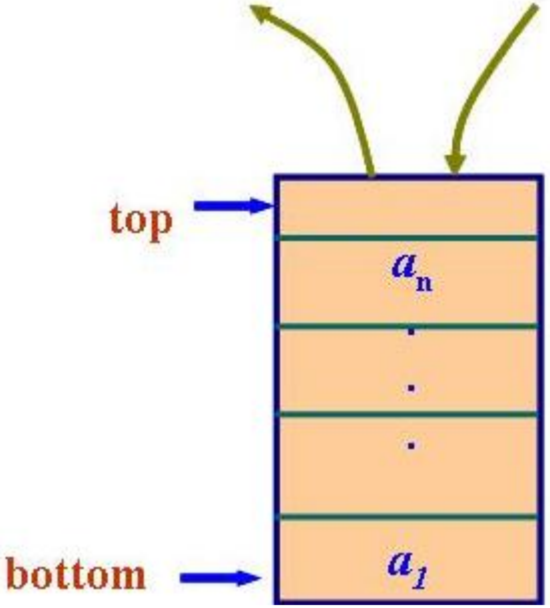
$C(n) = C(0)*C(n\text{-}1) + C(1)*C(n\text{-}2) + \text{.......} + C(n\text{-}2)*C(1) + C(n\text{-}1)*C(0)$

Stack and Lattice Paths

$$C_n = \sum_{i=0}^{n-1} C_i C_{n-i-1}.$$

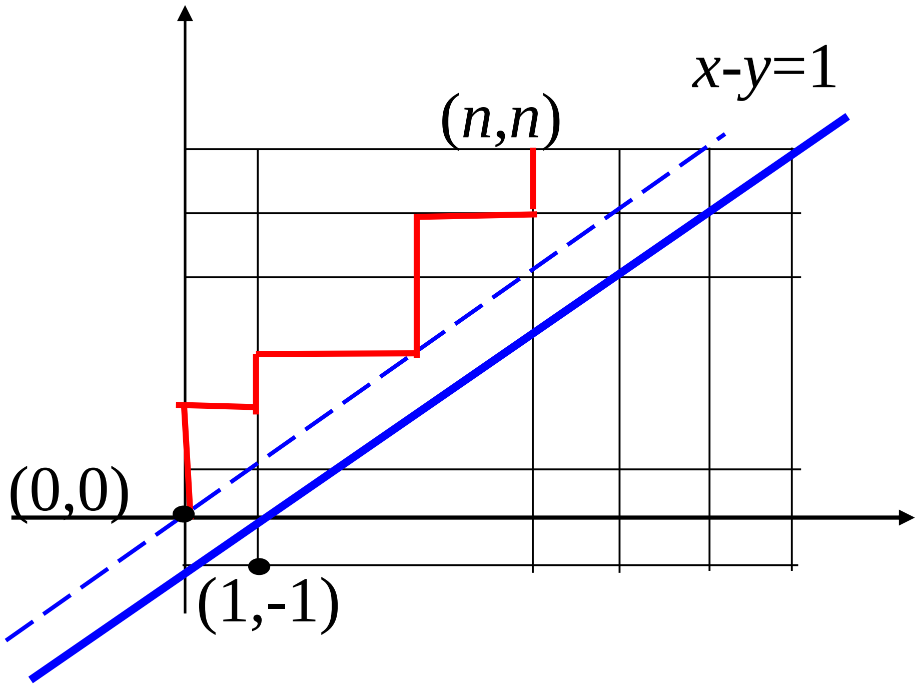
8 people is lining up to purchase tickets, 4 people are holding 10 Yuan, 4 people are holding 20 Yuan, the ticket price is 10 Yuan. The ticket booth does not have money at the first place, find out how many different possible ways of the arrangement of 8 people that they can successful purchase the tickets.





Catalan Numbers?

Dyck Path



Limit the line to either move one grid down or one grid to the right. $x - y = 1$;

The problem turns into a lattice path going from (0,0) to (n,n) without touching the x-y=1 line.

The symmetry point of (0,0) about $x - y = 1$ is (1,-1).

Using the method used in the last question, the number of lattice paths equal to:

$$C_n = C(2n, n) - C(2n, n-1)$$

$$= \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!\,n!} = \prod_{k=2}^{n} \frac{n+k}{k}$$

Proof using generating functions

$$C_n = \sum_{i=0}^{n-1} C_i C_{n-i-1}.$$

- Generating function $$c(x) = \sum_{n=0}^{\infty} C_n x^n.$$

$$c(x)^2 = \underbrace{C_0C_0}_{C_1} + (\underbrace{C_1C_0 + C_0C_1}_{C_2})x + (\underbrace{C_2C_0 + C_1C_1 + C_0C_2}_{C_3})x^2 + \ldots\ldots$$

$$c(x)^2 = C_1 + C_2x + C_3x^2 + \ldots\ldots$$

$$c(x) = C_0 + C_1x + C_2x^2 + \ldots\ldots$$

$$\boxed{\begin{aligned} C_1 &= C_0C_0 \\ C_2 &= C_1C_0 + C_0C_1 \\ C_3 &= C_2C_0 + C_1C_1 + C_0C_2 \\ C_4 &= C_3C_0 + C_2C_1 + C_1C_2 + C_0C_3 \\ \ldots & \qquad \ldots \end{aligned}}$$

$\mathbf{C_0=1}$

$$c(x) = 1 + xc(x)^2;$$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$(1+x)^{\alpha} = \sum_{n=0}^{\infty} \frac{\alpha(\alpha-1)\ldots(\alpha-n+1)}{n!} x^{n} \qquad \alpha \in R$$

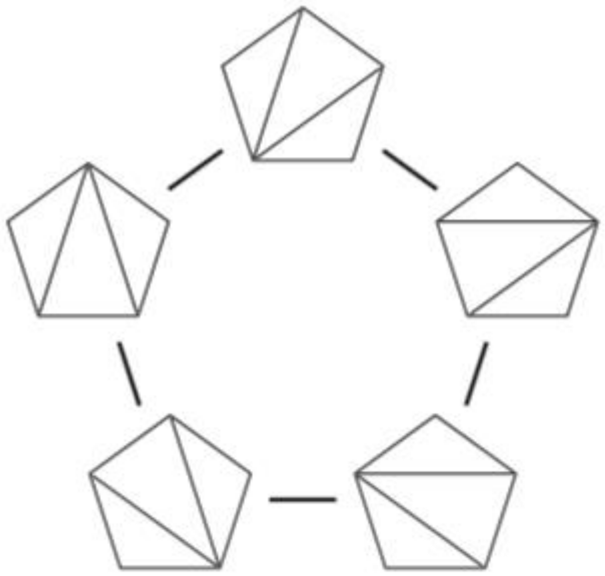
$$c(x) = 1 + xc(x)^2;$$

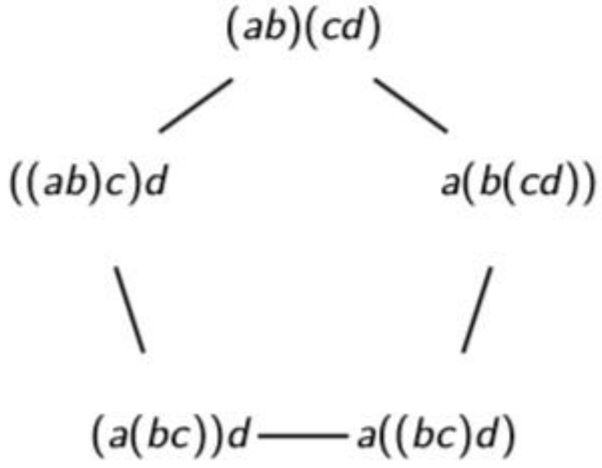
$$\lim_{x\to 0+} c(x) = C_0 = 1 \qquad c(x) = \frac{1-\sqrt{1-4x}}{2x}$$

$$c(x) = \sum_{n=0}^{\infty} \binom{2n}{n} \frac{x^n}{n+1}.$$

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

Same problem





If we remove the parenthesis, what will the calculation be like?

$$((2-1)\cdot(-(1+2))\cdot 4)\div((1+2)\cdot 2)$$

$$2-1\cdot -1+2\cdot 4\div 1+2\cdot 2$$

$$(()(()))(())$$

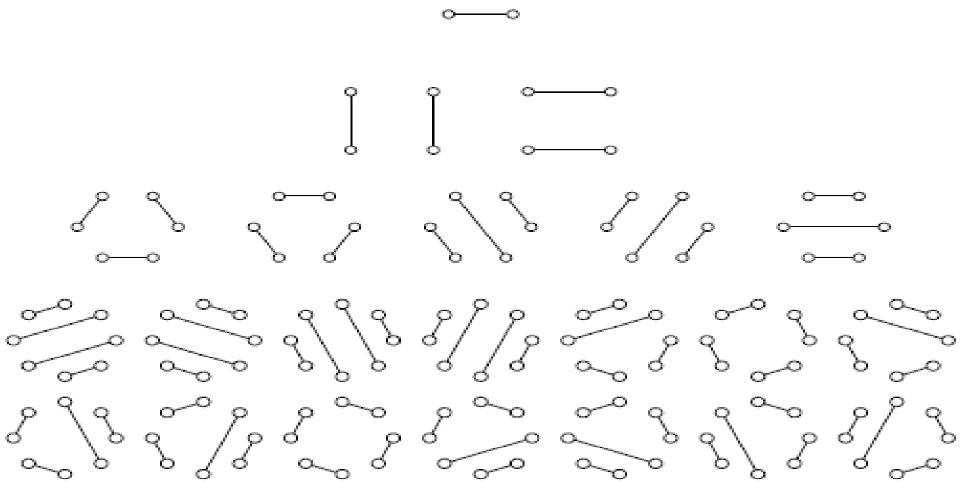
- How many pairs of parenthesis can be correctly matched?
- C_n is the number of Dyck words of length 2n.
- Here, a Dyck word is a string consisting of n X's and n Y's such that no initial segment of the string has more Y's than X's. For example, the following are Dyck words of length 6.
  XXXYYY XYXXYY XYXYXY XXYYXY XXYXYY

Dyck Language

- Dyck Language is a very interesting language in the field of Computer Science. In the theory of formal languages of CS, Mathematics, and Linguistics, Dyck language is the language consisting of balanced strings of parentheses [and].
- It is important in the parsing of expressions that must have a correctly nested sequence of parentheses, such as arithmetic or algebraic expressions.
- It is named after the mathematician Walther von Dyck.

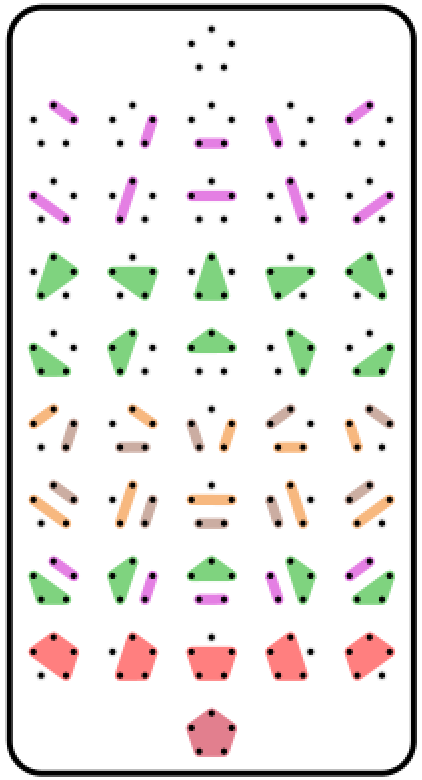
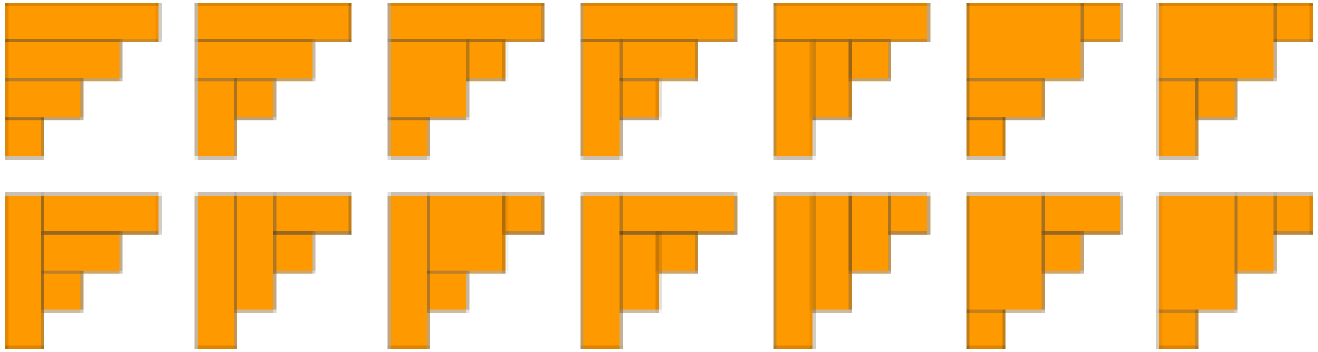
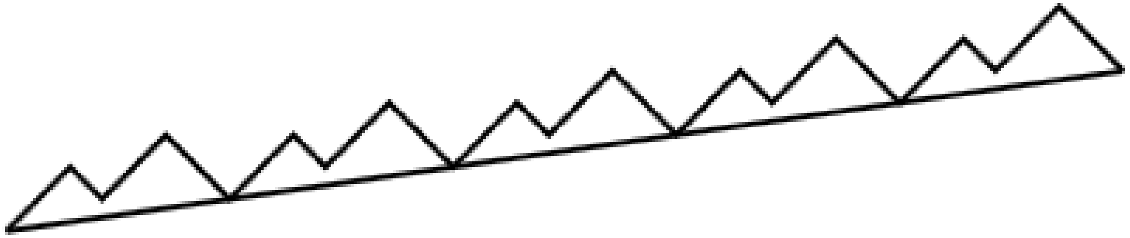
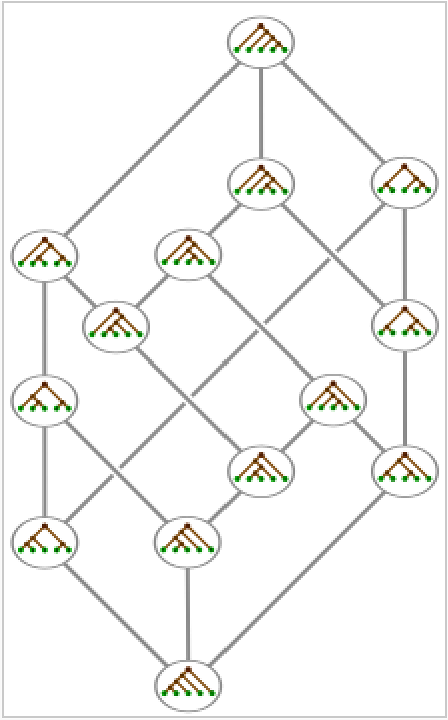
Hands across the table

- If n people are seated around a circular table, in how many ways can all of them be simultaneously shaking hands with another person at the table in such a way that none of the arms cross each other?
- The result is Catalan sequence. This is called the Hands across the table" problem. There's also a romantic movie with the same name!



Catalan Numbers

- Hot research topic in the 20th century
  - M.Kuchinski found 31 structures that could be enumerated by Catalan numbers.
  - As of 08/21/2010, R. Stanley counts 190 structures counted by Catalan numbers.

组合数学 Combinatorics

5 Magical Sequences

5-2 Exponential Generating Functions

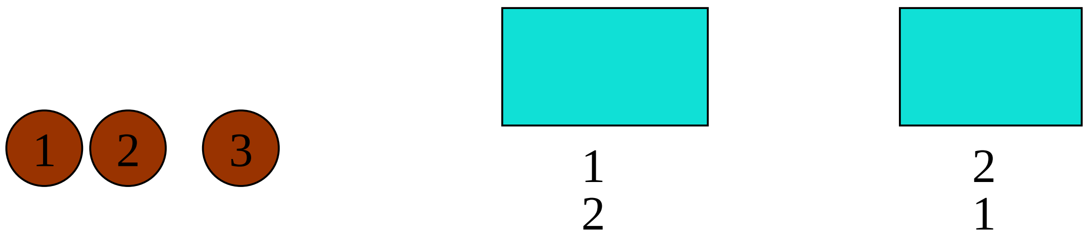
Tsinghua University
Yuchun Ma

Exponential Generating Functions

- Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots + \frac{n(n-1)\cdots(n-k+1)}{k!}x^k + \cdots$$

- Generating function of C(*n*,*k*)
- The coefficient is the combination of selecting k terms from n



$$C(n,k)k! = P(n,k)$$

$$(1+x)^n = (1+x)(1+x)(1+x)(1+x)\ldots\ldots(1+x)(1+x)$$

$$x_0 \quad 1 \quad x_1 \quad x_2 \quad \ldots \quad x_{k\text{-}1} \quad 1$$

$$(1+x)^n = \sum_{k=0}^{\infty} C(n,k)x^k = \sum_{k=0}^{\infty} \frac{C(n,k)k!}{k!}x^k = \sum_{k=0}^{\infty} P(n,k)\frac{x^k}{k!}$$

To compute permutations, and not combinations, use these terms: $\{1, x, \frac{x^2}{2!}, \ldots, \frac{x^n}{n!}\}$

Full r-permutations

- How many different permutations do the 8 letters in “pingpang” have?
- Adding indices to differentiate: $p_1p_2n_1n_2g_1g_2ia$

$$\binom{8}{2\quad 2\quad 2\quad 1\quad 1} = \frac{8!}{2!2!2!}$$

Full r-permutations

- $3a_1$, $2a_2$, $3a_3$
  - How many combinations of length k?

$$G(x) = (1 + x + x^2 + x^3)(1 + x + x^2)(1 + x + x^2 + x^3)$$

$$= (1 + 2x + 3x^2 + 3x^3 + 2x^4 + x^5) \cdot (1 + x + x^2 + x^3)$$

$$= 1 + 3x + 6x^2 + 9x^3 + 10x^4 + 9x^5 + 6x^6 + 3x^7 + x^8$$

$$G(x)=1+3x+6x^2+9x^3+10x^4+9x^5+6x^6+3x^7+x^8$$

$3a_1, 2a_2, 3a_3$; How many combinations of length 4?

- From the coefficient of x^4, we get 10 combinations of length 4 from these 8 elements. Expanding the expression yields:

$$(1+x_1+x_1^2+x_1^3)(1+x_2+x_2^2)(1+x_3+x_3^2+x_3^3)$$

$$=[1+(x_1+x_2)+(x_1^2+x_1x_2+x_2^2)+(x_1^3+x_1^2x_2+x_1x_2^2)$$
$$+(x_1^3x_2+x_1^2x_2^2)+x_1^3x_2^2]\cdot(1+x_3+x_3^2+x_3^3)$$
$$=1+(1+x_1+x_2+x_3)$$
$$+(x_1^2+x_1x_2+x_2^2+x_1x_3+x_2x_3+x_3^3)$$
$$+(x_1^3+x_1^2x_2+x_1x_2^2+x_1^2x_3+x_1x_2x_3+x_2^2x_3+x_1x_3^2+x_2x_3^2+x_3^3)$$
$$+(\underline{x_1x_3^3+x_2x_3^3+x_1^2x_3^2+x_1x_2x_3^2+x_2^2x_3^2+x_1^3x_3+x_1^2x_2x_3+x_1x_2^2x_3}$$
$$\underline{+x_1^3x_3+x_1^2x_2^2})+\cdots$$

Full r-permutations

$3a_1$, $2a_2$, $3a_3$; How many permutations of length 4?

$x_1^2 x_3^2$ using 2 of each as an example, the different permutations equal to:

$$\frac{4!}{2!2!}=6$$

$a_1a_1a_3a_3$, $a_1a_3a_1a_3$, $a_3a_1a_3a_1$, $a_1a_3a_3a_1$,
$a_3a_3a_1a_1$, $a_3a_1a_1a_3$, 6 ways. Similarly for one a_1 and 3 a_3, the different permutations equal: $\frac{4!}{3!}=4$

$a_1a_3a_3a_3$, $a_3a_1a_3a_3$, $a_3a_3a_1a_3$, $a_3a_3a_3a_1$,

can be solved for, in a similar manner.

$$x_1x_3^3 + x_2x_3^3 + x_1^2x_3^2 + x_1x_2x_3^2 + x_2^2x_3^2 + x_1^3x_3$$
$$+ x_1^2x_2x_3 + x_1x_2^2x_3 + x_1^3x_3 + x_1^2x_2^2$$

Therefore, the total no. of different permutations:

$$4!(\frac{1}{1!3!} + \frac{1}{1!3!} + \frac{1}{2!2!} + \frac{1}{1!1!2!} + \frac{1}{2!2!} + \frac{1}{3!1!} + \frac{1}{2!1!1!} + \frac{1}{1!2!1!} + \frac{1}{3!1!} + \frac{1}{2!2!})$$

$$= 4!(\frac{4}{3!} + \frac{3}{2!2!} + \frac{3}{2!}) = 4!\frac{4\cdot 2!\cdot 2! + 3\cdot 3! + 3\cdot 2!\cdot 3!}{2!2!3!}$$

$$= 16 + 18 + 36 = 70$$

$$G(x)=(1+x+x^2+x^3)(1+x+x^2)(1+x+x^2+x^3)$$

For easier computation, using the aforementioned property:

$$G_e(x)=(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!})(1+\frac{x}{1!}+\frac{x^2}{2!})(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!})$$

$$=(1+2x+2x^2+\frac{7}{6}x^3+\frac{5}{12}x^4+\frac{1}{12}x^5)\cdot(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3)$$

$$=1+3x+\frac{9}{2}x^2+\frac{14}{3}x^3+\frac{35}{12}x^4+\frac{17}{12}x^5+\frac{35}{72}x^6+\frac{8}{72}x^7+\frac{1}{72}x^8$$

$$G_e(x)=1!+\frac{3}{1!}x+\frac{9}{2!}x^2+\frac{28}{3!}x^3+\frac{70}{4!}x^4+\frac{170}{5!}x^5+\frac{350}{6!}x^6+\frac{560}{7!}x^7+\frac{560}{8!}x^8$$

$$\boxed{=4!(\frac{4}{3!}+\frac{3}{2!2!}+\frac{3}{2!})=16+18+36=70}$$

- Generating Function

Combinations

For a seq. $a_0, a_1, a_2, \cdots$, construct:

$$G(x) = a_0 + a_1 x + a_2 x^2 + \cdots,$$

G(x) is known as the generating function of $a_0, a_1, a_2, \cdots$

- Exponential Generating Function

Permutations

$$G_e(x) = a_0 + \frac{a_1}{1!}x + \frac{a_2}{2!}x^2 + \frac{a_3}{3!}x^3 + \cdots + \frac{a_k}{k!}x^k + \cdots$$

G(x) is known as the exponential generating function of $a_0, a_1, a_2, \cdots$

$$\boxed{e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}\dots}$$

Is the exponential generating function of {1,1,…,1}

Exponential Generating Function

Full r-permutations: If there are n_1 a_1's, n_2 a_2's, …, n_k a_k's, then: Constructing a permutation of length n, with different permutations equals:

$$\frac{n!}{n_1!n_2!\cdots n_k!} \qquad n = n_1 + n_2 + \cdots + n_k$$

r-permutations: If there are n_1 a_1's, n_2 a_2's, …, n_k a_k's, then: From n elements choosing r permutations, where all permutations are different equal to p_r. Therefore, for the sequence, p_0, p_1, … , p_n, the exponential generating function is:

$$G_e(x) = (1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^{n_1}}{n_1!})$$

$$\cdot (1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^{n_2}}{n_2!}) \cdots (1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^{n_k}}{n_k!})$$

Exponential Generating Function

Exponential Generating Function is perfect for counting permutations of multisets.

Example： Using the digits 1, 2, 3 and 4, construct a five digit number where 1 doesn’t appear more than 2 times, but has to appear at least once; 2 can’t appear more than once; 3 can appear up to 3 times but can also not appear in the number; 4 can only appear an even number of times. How many numbers can satisfy these conditions?

1 doesn't appear more than twice, but has to appear at least once; 2 can't appear more than twice; 3 can appear up to 3 times but can also not appear in the number; 4 can only appear an even number of times.

An r digit number that satisfies the condition is a_r; the exponential generating function for the seq. $a_0, a_1, \ldots, a_{10}$ is:

$$G_e(x) = \left(\frac{x}{1!}+\frac{x^2}{2!}\right)(1+x)\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}\right)\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}\right)$$

$$= \left(x+\frac{3}{2}x^2+\frac{1}{2}x^3\right)\left(1+x+x^2+\frac{2}{3}x^3+\frac{7}{24}x^4+\frac{1}{8}x^5+\frac{x^6}{48}+\frac{x^7}{144}\right)$$

$$= x+\frac{5}{2}x^2+3x^3+\frac{8}{3}x^4+\frac{43}{24}x^5+\frac{43}{48}x^6+\frac{17}{48}x^7+\frac{1}{288}x^8+\frac{1}{48}x^9+\frac{1}{288}x^{10}$$

$$= \frac{x}{1!}+5\frac{x^2}{2!}+18\frac{x^3}{3!}+64\frac{x^4}{4!}+215\frac{x^5}{5!}+645\frac{x^6}{6!}+1785\frac{x^7}{7!}$$

$$+140\frac{x^8}{8!}+7650\frac{x^9}{9!}+12600\frac{x^{10}}{10!}$$

There are 215 5-digit numbers that satisfy the conditions.

Example

With digits 1, 3, 5, 7, 9, how many n-digit numbers are there, where 3 and 7 appear an even number of times, and 1, 5, 9 don't have any conditions.

Assume an r-digit number that satisfies the conditions is a_r. Then, the exponential generating function of the seq. of a_1, a_2, …, a_3 is:

$$G_e(x) = \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right)^2 \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)^3$$

$$G_e(x) = \left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots\right)^2\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\right)^3$$

<u>Since</u> $e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots,\ e^{-x} = 1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots,$

$$\therefore \qquad 1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots = \frac{1}{2}(e^x+e^{-x}).$$

$$G_e(x) = \frac{1}{4}(e^x+e^{-x})^2 e^{3x} = \frac{1}{4}(e^{2x}+2+e^{-2x})e^{3x}$$

$$= \frac{1}{4}(e^{5x}+2e^{3x}+e^x) = \frac{1}{4}\left(\sum_{n=0}^{\infty}\frac{5^n}{n!}x^n+2\sum_{n=0}^{\infty}\frac{3^n x^n}{n!}+\sum_{n=0}^{\infty}\frac{x^n}{n!}\right)$$

$$= \frac{1}{4}\sum_{n=0}^{\infty}(5^n+2\cdot 3^n+1)\frac{x^n}{n!}.$$

$$\therefore \qquad a_n = \frac{1}{4}(5^n+2\cdot 3^n+1).$$

Plan for the class

Homework sheet

OJ

周次 \ 月 \ 星期 日	月	一	二	三	四	五	六	日
夏季学期	2020 八	10	11	12	13	14	15	16
		17	18	19	20	21	22	23
		24	25	26	27	28	29	30
		31						
	九		1	2	3	4	5	6
		7	8	9	10	11	12	13
1		14	15	16	17	18	19	20
2		21	22	23	24	25	26	27
3		28	29	30				
	十				1	2	3	4
4		5	6	7	8	9	10	11
5		12	13	14	15	16	17	18
6		19	20	21	22	23	24	25
7		26	27	28	29	30	31	
								1
8		2	3	4	5	6	7	8

10.19 Magic sequences

10.22 Linear Programming

10.26 Linear programming

10.29 Summary

11.2 Algorithm Design

11.5 Exam

Thank you