HW W5-2

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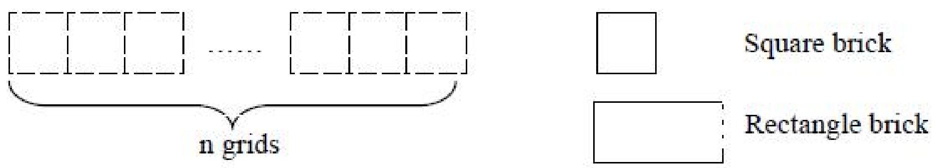
1. **A worker is tiling a road with n square grids, there are two different kinds of bricks available, one is the square brick and the other is the rectangle brick which covers 2 grids. Please count the total different ways the road can be tiled.**



Let a_n denote the number of arrangement. We would consider 2 scenarios:

1) The first tile is filled with a square brick. In this case, n-1 grids would be left.
2) The first tile is filled with a rectangle brick, leaving n-2 grids to be filled.

Accordingly, we would consider these two scenarios for the remaining grids, which shows a recursive behavior in how the grids are filled. In order to fill in 1 grid, we would need 1 square brick, giving $a_1 = 1$. For 2 grids, we can have 1 rectangle brick or 2 square bricks, giving $a_2 = 2$. For 3 grids, we can use 3 square bricks, 1 rectangle brick and a square brick, or a square brick and a rectangle brick, , giving $a_3 = 3$. Hence, assuming $a_0 = 1$ since the only way to tile 0 grids is by not placing any tiles, it can be observed that the number of ways to fill in the grids follow the Fibonacci sequence, with a shift in values corresponding to each index (i.e. $a_n = F_{n+1}$). Therefore we have

$$a_n = a_{n-1} + a_{n-2} \ => a_n - a_{n-1} - a_{n-2} = 0$$

Giving characteristic equation $x^2 - x - 1 = 0$ with roots $\frac{1+\sqrt{5}}{2}$ and $\frac{1-\sqrt{5}}{2}$.

Therefore, $a_n = A[\frac{1+\sqrt{5}}{2}]^n + B[\frac{1-\sqrt{5}}{2}]^n$, where

$$a_0 = 1 \text{ so } A + B = 1$$

$$a_1 = 1 \text{ so } A[\frac{1+\sqrt{5}}{2}] + B[\frac{1-\sqrt{5}}{2}] = 1$$

Solving the above equations results in A=$\frac{1+\sqrt{5}}{2}$ and B=$\frac{1-\sqrt{5}}{2}$. Therefore,

$$a_n = [\frac{1+\sqrt{5}}{2}]^{n+1} + [\frac{1-\sqrt{5}}{2}]^{n+1}$$

Where a_n is the number of ways to tile a road consisting of n grids.

2. **How many different ways to color n grids in a line with red, white or blue colors but no two adjacent grids are colored with red?**

Let a_n denote the number of arrangement. There would be two scenarios:

1) The first grid and the third grid are colored with white. In this case, there would be a 1-1 correspondence between the arrangements of the 4th grid to nth grid and the arrangements for remaining n-2 grids: $2a_{n-2}$.

2) The first grid is white and the third grid is colored with either blue or red. In this case, there would be a 1-1 correspondence between the arrangements of the 3rd grid to nth grid and the arrangements for remaining n-1 grids: $2a_{n-1}$.

This gives $a_n = 2a_{n-1} + 2a_{n-2}$. Accordingly, $a_0 = 1$ and $a_1 = 3$, and the characteristic equation $C(x) = x^2 - 2x - 2 = 0$ with roots $\frac{2+\sqrt{12}}{2}$ and $\frac{2-\sqrt{12}}{2}$.

$$a_n = A[\frac{2+\sqrt{12}}{2}]^n + B[\frac{2-\sqrt{12}}{2}]^n$$

$$a_0 = 1 \text{ so } A + B = 1$$

$$a_1 = 3 \text{ so } A[\frac{2+\sqrt{12}}{2}] + B[\frac{2-\sqrt{12}}{2}] = 3$$

Giving $A = \frac{2+\sqrt{3}}{2\sqrt{3}} = \frac{3+2\sqrt{3}}{6}$ and B=$\frac{3-2\sqrt{3}}{6}$. Therefore,

$$a_n = \frac{3+2\sqrt{3}}{6}[\frac{2+\sqrt{12}}{2}]^n + \frac{3-2\sqrt{3}}{6}[\frac{2-\sqrt{12}}{2}]^n$$

$$a_n = \frac{3+2\sqrt{3}}{6}[1+\sqrt{3}]^n + \frac{3-2\sqrt{3}}{6}[1-\sqrt{3}]^n$$