HW Linear Programming

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1. **Solving the following linear program using simplex method**

$$\max Z = 3x_1 + 6x_2 + 2x_3$$

$$\begin{cases} 3x_1 + 4x_2 + x_3 \le 2 \\ x_1 + 3x_2 + 2x_3 \le 1 \\ \\ x_1 \ge 0, x_2 \ge 0, x_3 \ge 0 \end{cases}$$

The above equations could be rewritten in augmented form.

$$\begin{cases} 3x_1 + 4x_2 + x_3 + x_4 = 2 \\ x_1 + 3x_2 + 2x_3 + x_5 = 1 \\ x_1, x_2, x_3, x_4, x_5 \ge 0 \end{cases}$$

Accordingly, the table would be as follows

c_j			3	6	2	0	0	β_i
c_B	x_B	P_0	P_1	P_2	P_3	P_4	P_5	
0	x_4	2	3	4	1	1	0	$\frac{2}{4}$
0	x_5	1	1	3	2	0	1	$\frac{1}{3}$
$-z$		0	3	6↑	2	0	0	

Let x_2 be a basic variable. Accordingly, by modifying the above equations, we have

$$\begin{cases} 9x_1 + 12x_2 + 3x_3 + 3x_4 = 6 \\ -4x_1 - 12x_2 - 8x_3 - 4x_5 = -4 \\ \frac{1}{3}x_1 + x_2 + \frac{2}{3}x_3 + \frac{1}{3}x_5 = \frac{1}{3} \end{cases}$$

Simplifying the top two equation gives

$$\begin{cases} \frac{5}{3}x_1 - \frac{5}{3}x_3 + x_4 - \frac{4}{3}x_5 = \frac{2}{3} \\ \frac{1}{3}x_1 + x_2 + \frac{2}{3}x_3 + \frac{1}{3}x_5 = \frac{1}{3} \end{cases}$$

And the program can be written as

$$\max Z = x_1 - 2x_3 - 2x_5 + 2$$

c_j			3	6	2	0	0	β_i
c_B	x_B	P_0	P_1	P_2	P_3	P_4	P_5	
0	x_4	$\frac{2}{3}$	$\frac{5}{3}$	0	$-\frac{5}{3}$	1	$-\frac{4}{3}$	$\frac{2}{5}$
6	x_2	$\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$	0	$\frac{1}{3}$	1
$-z$		-2	1↑	0	-2	0	-2	

Let x_1 be a basic variable. Accordingly, by modifying the above equations, we have

$$\left\{\begin{array}{c} x_1 - x_3 + \frac{3}{5}x_4 - \frac{4}{5}x_5 = \frac{2}{5} \\ \frac{5}{3}x_1 - \frac{5}{3}x_3 + x_4 - \frac{4}{3}x_5 = \frac{2}{3} \\ -\frac{5}{3}x_1 - 5x_2 - \frac{10}{3}x_3 - \frac{5}{3}x_5 = \frac{-5}{3} \end{array}\right.$$

Simplifying the lower two equation gives

$$\left\{\begin{array}{c} x_1 - x_3 + \frac{3}{5}x_4 - \frac{4}{5}x_5 = \frac{2}{5} \\ x_3 - \frac{1}{5}x_4 + \frac{3}{3}x_5 = \frac{1}{5} \end{array}\right.$$

And the program can be written as

$$\max Z = -x_3 - \frac{3}{5}x_4 - \frac{6}{5}x_5 + \frac{12}{5}$$

c_j			3	6	2	0	0	β_i
c_B	x_B	P_0	P_1	P_2	P_3	P_4	P_5	
3	x_1	$\frac{2}{5}$	1	0	-1	$\frac{3}{5}$	$-\frac{4}{5}$	
6	x_2	$\frac{1}{5}$	0	1	1	$-\frac{1}{5}$	$\frac{3}{5}$	
$-Z$		$-\frac{12}{5}$	0	0	-1	$-\frac{3}{5}$	$-\frac{6}{5}$	

Therefore, the optimal solution would be

$$x_1 = \frac{2}{5}$$
$$x_2 = \frac{1}{5}$$
$$Z = \frac{12}{5}$$