TA Office Hour after class

Sep.28 3:05-4:30

周次	月	星期一	星期二	星期三	星期四	星期五	星期六	星期日
2	2020 九	21	22	23	24	25	26	27 改上周五课程 (补10月2日) 原排课程停上
3		28	29	30				
	十				1 原排课程调到10月10日进行	2 原排课程调到9月27日进行	3 原排课程停上	4 原排课程停上
4		5	6	7	8	9	10 改上周四课程 (补10月1日) 原排课程停上	11

There will be no class for this Thursday (Oct.1)

The next class will be on Oct 5th.

We will have three classes for next week

Oct. 5th, Oct. 8th and Oct.10th.

Review of the previous lesson

- Four basic counting principles
  - Addition
  - Multiplication
  - Substraction
  - Division
- Permutation and combination?
  - If the order **does** matter:
    - **Permutation: P(n,r)**= $n!/(n-r)!$
  - If the order **doesn't** matter:
    - **Combination: C(n,r)**= $n!/r!(n-r)!$.
- Ordered arrangement
  - Without repeating any objects, distinct: **P(n,r)**
  - **Circular permutation**

$$\frac{P(n,r)}{r} = \frac{n!}{r(n-r)!}.$$

Example

- If $S = \{3 \cdot a, 2 \cdot b, 4 \cdot c\}$
  - 8-permutations of S?
  - 8-combinations of S?
- 8-combinations
  - $\{2 \cdot a, 2 \cdot b, 4 \cdot c\}$
  - $\{3 \cdot a, 1 \cdot b, 4 \cdot c\}$
  - $\{3 \cdot a, 2 \cdot b, 3 \cdot c\}$
  - Total: 3

$$x_1 + x_2 + x_3 + x_4 = 20$$

$$x_1 \geq 3, x_2 \geq 1, x_3 \geq 0 \; and \quad x_4 \geq 5$$

$$y_1 = x_1 - 3, y_2 = x_2 - 1, y_3 = x_3 \; and \quad y_4 = x_4 - 5$$

$$y_1 + y_2 + y_3 + y_4 = 20 - 3 - 1 - 5 = 11$$

$$x_1 + x_2 + x_3 + x_4 = 20$$

$$x_1 \leq 5, x_2 \leq 10, x_3 \leq 11, and x_4 \leq 8$$

Complicated!

Inclusion-exclusion principle in Chapter 6

Chapter 6

The Inclusion-Exclusion Principle and Applications

Addition rule

[Addition Rule] Assume that event A can happen in m ways, event B can happen in n ways. Event A **or** B can happen in $m+n$ ways.

Language of set theory

If $|A| = m$, $|B| = n$, $A \cap B = \varnothing$, then $|A \cup B| = m + n$

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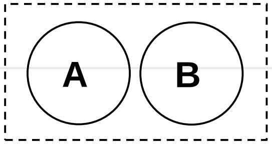
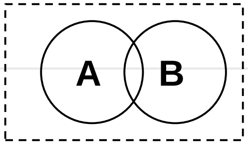




Introduction to Inclusion-Exclusion Principle

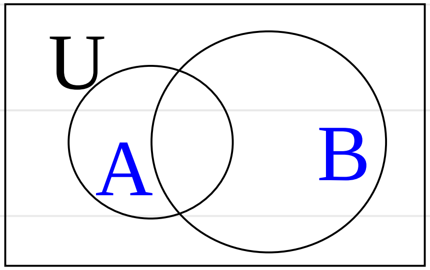
$$\overline{A} = \{x \mid x \in U \text{ 且 } x \notin A\}$$

- **If both A and B are subsets of U, the *complement***

(a) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

(b) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

- **[De Morgan's Law]**



Introduction to Inclusion-Exclusion Principle

A generalization of De Mogan's law: Suppose $A_1, A_2, \ldots. A_n$ are subsets of U

$$So\ (a)\overline{A_1 \cup A_2 \cup \cdots \cup A_n} = \overline{A_1} \cap \overline{A_2} \cap \cdots \cup \overline{A_n}$$

$$(b)\overline{A_1 \cap A_2 \cap \ldots \cap A_n} = \overline{A_1} \cup \overline{A_2} \cup \ldots \cup \overline{A_n}$$

Proof: Use mathematical induction

$\overline{A_1 \cup A_2 \cup \ldots \cup A_n} = \overline{A_1} \cap \overline{A_2} \cap \ldots \cap \overline{A_n}$ is correct

so

$$\overline{A_1 \cup A_2 \cup \ldots \cup A_n \cup A_{n+1}} = \overline{(A_1 \cup \ldots \cup A_n) \cup A_{n+1}}$$

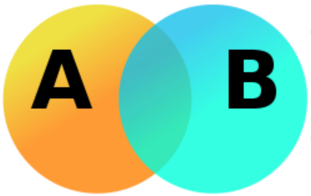
$$= \overline{(A_1 \cup A_2 \cup \ldots \cup A_n} \cap \overline{A_{n+1}}$$

$$= \overline{A_1} \cap \overline{A_2} \cap \ldots \cap \overline{A_n} \cap \overline{A_{n+1}}$$

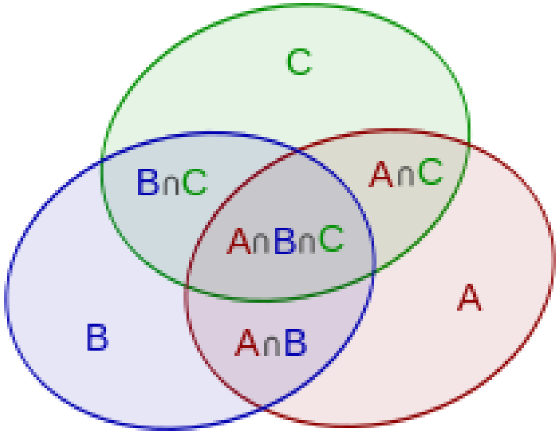
So the theorem is also correct for n+1

Inclusion-Exclusion principle

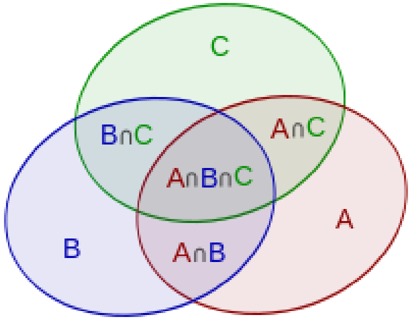
Calculate the number of elements in the union of finite set A and B. $|A \cup B| = |A| + |B| - |A \cap B|$



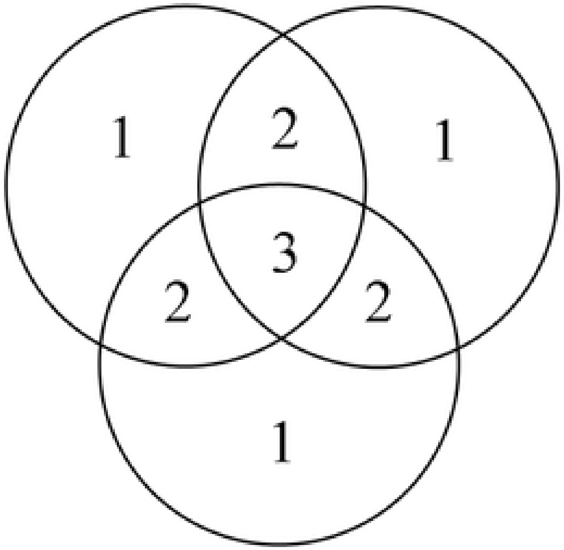
$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$



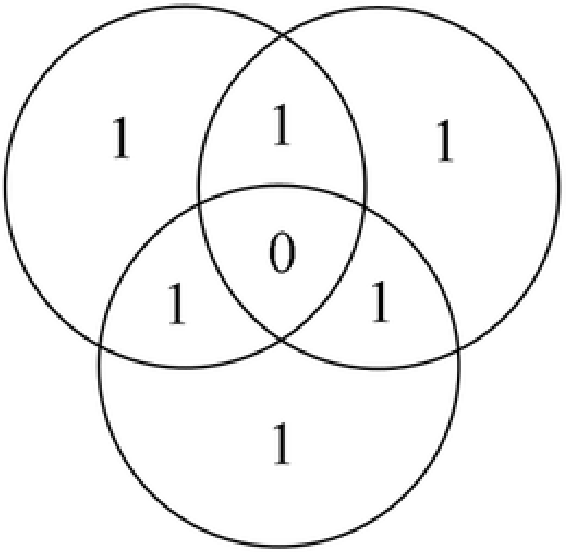
Inclusion-Exclusion Principle



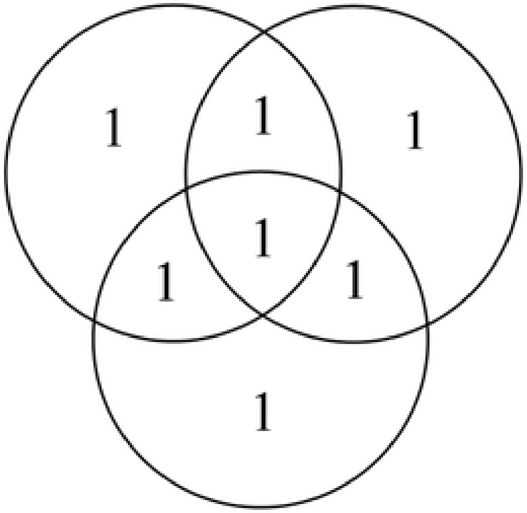
Theorem: $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B|$
$- |A \cap C| - |B \cap C| + |A \cap B \cap C|$



$|A| + |B| + |C|$

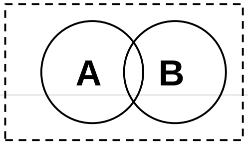


$|A| + |B| + |C|$
$-(|A \cap B| + |A \cap C| + |B \cap C|)$



$|A| + |B| + |C|$
$-(|A \cap B| + |A \cap C| + |B \cap C|)$
$+|A \cap B \cap C|$

Inclusion-Exclusion Principle



- **Inclusion-Exclusion Principle**
  - Inclusion and Exclusion Principle
  - The idea of inclusion-exclusion is:
    - Ignore duplications at first and include all objects which are in it.
    - Then exclude the duplications.
    - Finally the result has no repetitions and no losses.

Inclusion-Exclusion principle

$$|A_1 \cup A_2 \cup \ldots \cup A_n| = \sum_{i=1}^{n} |A_i| - \sum_{i=1}^{n} \sum_{j>i} |A_i \cap A_j|$$

$$+ \sum_{i=1}^{n} \sum_{j>i} \sum_{k>j} |A_i \cap A_j \cap A_k| - \ldots$$

$$+(-1)^{n-1} |A_1 \cap A_2 \cap \ldots \cap A_n|$$

$$|\overline{A}| = N - |A|,$$

$$|\overline{A_1} \cap \overline{A_2} \cap \ldots \cap \overline{A_n}| = N - |A_1 \cup A_2 \cup \ldots \cup A_{n-1} \cup A_n|$$

$$= N - \sum_{i=1}^{n} |A_i| + \sum_{i=1}^{n} \sum_{j>i} |A_i \cap A_j| \quad - \sum_{i=1}^{n} \sum_{j>i} \sum_{k>j} |A_i \cap A_j \cap A_k| + \ldots$$

$$+ (-1)^n |A_1 \cap A_2 \cap \ldots \cap A_n|$$

How to prove it?

Inclusion-Exclusion Principle

$$|\overline{A_1} \cap \overline{A_2}| = |S| - |A_1| - |A_2| + |A_1 \cap A_2|$$

Calculate the number of elements that are in neither A_1 nor A_2.

If x is in neither A_1 nor A_2

1   1-0-0+0 = 1

If x is in A1 but not A2

0   1-1-0+0 = 0

If x is in A2 but not A1

0   1-0-1+0 = 0

If x is in A2 and A1

0   1-1-1+1 = 0

Two sides are equal

$(x+y)^m = C(m,0)x^m + C(m,1)x^{m-1}y + \ldots + C(m,m)y^m$

If $x=1$, $y=-1$:　$0 = C(m,0) - C(m,1) + \ldots + (-1)^m C(m,m)$

$$\left|\overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_m}\right| = |S| - \sum |A_i| + \sum |A_i \cap A_j| - \sum |A_i \cap A_j \cap A_k| + \cdots + (-1)^m \sum |A_1 \cap A_2 \cap \cdots \cap A_m|$$

Calculate the number of elements that no property holds.

X doesn't satisfy any property

1　**1-0-0…+(-1)^{m}0 = 1**

X satisfies 1 property

0　**1-1-0 …+(-1)^{m}0 = 0**

………

X satisfies n properties

n* ≤ *m　**0**

$C(n,0)-C(n,1)+C(n,2)+\ldots+(-1)^m C(n,m)$

$= C(n,0)-C(n,1)+C(n,2)+\ldots+(-1)^n C(n,n) + 0 \ldots + 0$

$=0$

Two sides are equal, calculate the number of elements which satisfies no property in the same way.

INCLUSION EXCLUSION

"One of the **most useful principles** of enumeration in discrete probability and combinatorial theory is the celebrated principle of inclusion–exclusion.
When **skillfully applied**, this principle has yielded the solution to many a combinatorial problem."

Examples

Eg Calculate the number of permutations of *a,b,c,d,e,f,g* which don't contain *ace* or *df*.

Solution：Permutations of 7 letters: |S| = 7!

Assume A is the permutation set in which ace is an element: |A|=5!，

B is the permutation set in which df is an element:|B|=3!，

A∩B is the number of permutations contain ace and df: |A∩B |=4!。

$$|\overline{A}\cap\overline{B}| = |\overline{A\cup B}| = S - |A| - |B| + |A\cap B|$$

$$= 7! - 5! - 6! + 4!$$

Examples

Eg Calculate the number of multiples of 3 or 5 from 1 to 500.

Solution: Let A be the set of multiples of 3 from 1 to 500.
B is the set of multiples of 5 from 1 to 500.

$$|A| = \left\lfloor \frac{500}{3} \right\rfloor = 166, |B| = \left\lfloor \frac{500}{5} \right\rfloor = 100;$$

$$|A \cap B| = \left\lfloor \frac{500}{15} \right\rfloor = 33$$

The number of multiples of 3 or 5 is:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$= 166 + 100 - 33 = 233$$

单选题 1分

设置

Eg Calculate the number of multiples of 6 or 8 from 1 to 500.

A 125

B 135

C 145

D 126

提交

- **Eg** Calculate the number of multiples of 6 or 8 from 1 to 500.
- [500/6]+[500/8]-[500/**24**]

Examples

Find the number of integers between 1 and 1000, inclusive, which are divisible by none of 5, 6, and 8.

Let P_1 be the property that an integer is divisible by 5, P_2 the property that an integer is divisible by 6, and P_3 the property that an integer is divisible by 8.

Let S be the set consisting of the first 1000 positive integers. For i=1, 2, 3 let A_i be the set consisting of those integers in S with property P_i. We wish to find the number of integers in $\left|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}\right|$

$$|A_1| = \left\lfloor \frac{1000}{5} \right\rfloor = 200 \qquad |A_2| = \left\lfloor \frac{1000}{6} \right\rfloor = 166 \qquad |A_3| = \left\lfloor \frac{1000}{8} \right\rfloor = 125$$

lcm(5,6)=30, lcm(5,8)=40, lcm(6,8)=24

$$|A_1 \cap A_2| = \left\lfloor \frac{1000}{30} \right\rfloor = 33 \qquad |A_1 \cap A_3| = \left\lfloor \frac{1000}{40} \right\rfloor = 25 \qquad |A_2 \cap A_3| = \left\lfloor \frac{1000}{24} \right\rfloor = 41$$

lcm(5,6,8)=120

$$|A_1 \cap A_2 \cap A_3| = \left\lfloor \frac{1000}{120} \right\rfloor = 8$$

$$\left|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}\right| = 1000 - (200 + 166 + 125) + (33 + 25 + 41) - (8) = 600$$

Special case for IEP

$$\left|\overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_m}\right| = |S| - \sum |A_i| + \sum |A_i \cap A_j| - \sum |A_i \cap A_j \cap A_k|$$
$$+ \cdots + (-1)^m \sum |A_1 \cap A_2 \cap \cdots \cap A_m|$$

Assume that the size of the set $A_{i1} \cap A_{i2} \cap \ldots \cap A_{ik}$ that occurs in the IEP depends only on k and not on which k sets are used in the intersection. Then

$$\left|\overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_m}\right| = a_0 - \binom{m}{1} a_1 + \binom{m}{2} a_2 - \binom{m}{3} a_3$$
$$+ \cdots + (-1)^k \binom{m}{k} a_k + \cdots + -1)^m a_m$$

where $a_k = |A_{i1} \cap A_{i2} \cap \ldots \cap A_{ik}|$. This is because the *k*th summation that occurs in the IEP contains C(*m*, *k*) summands each equal to a_k.

Example

- How many integers between 0 and 99,999 (inclusive) have digits 2, 5 and 8?
- Let S be the integers between 0 and 99,999.
- P_1 (resp., P_2 and P_3) be the property that an integer does not contain the digit 2 (resp., 5 and 8).
- Let A_i be the set consisting of those integers in S with property P_i. We wish to count the number of integers in

$$\left|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}\right|$$

$$\left|\overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_m}\right| = a_0 - \binom{m}{1} a_1 + \binom{m}{2} a_2 - \binom{m}{3} a_3$$

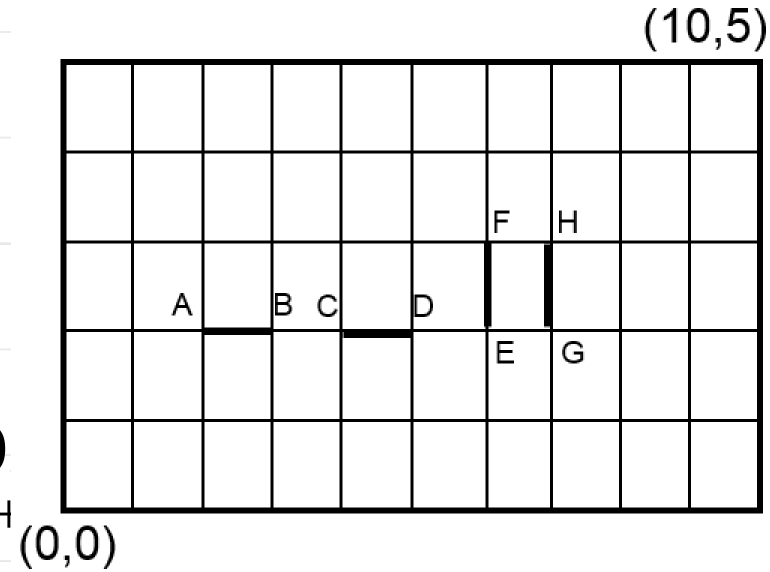
$$+ \cdots + (-1)^k \binom{m}{k} a_k + \cdots + -1)^m a_m$$

- How many integers between 0 and 99,999 (inclusive) have digits 2, 5 and 8?
- S is the 5-permutations of the multiset with 10 elements
  - $\{\infty\{0\}, \infty\{1\}, \infty\{2\}, \ldots, \infty\{9\}\}$
- *resp.*, A_i is the 5-permutations of the multiset with 9 elements
  - $\{\infty\{0\}, \infty\{1\}, \infty\{3\}, \infty\{4\}, \infty\{5\}, \infty\{6\}, \infty\{7\}, \infty\{8\}, ,\infty\{9\}\}$
  - $\{\infty\{0\}, \infty\{1\}, \infty\{2\}, \infty\{3\}, \infty\{4\}, \infty\{6\}, \infty\{7\}, \infty\{8\}, \infty\{9\}\}$
  - $\{\infty\{0\}, \infty\{1\}, \infty\{2\}, \infty\{3\}, \infty\{4\}, \infty\{5\}, \infty\{6\}, \infty\{7\}, \infty\{9\}\}$
- $a_k = |A_{i1} \cap A_{i2} \cap \ldots \cap A_{ik}|$.
  - $a_0 = 10^5$, $a_1 = 9^5$, $a_2 = 8^5$, $a_3 = 7^5$
- the answer is $10^5 - 3 \times 9^5 + 3 \times 8^5 - 7^5$

Applications of Inclusion-Exclusion principals

Eg Lattice Path with barriers:

How many paths go from (0,0) to (10,5) without passing AB, CD, EF, GH?
The coordinates of the points are A(2,2),B(3,2),C(4,2),D(5,2),E(6,2),F(6,3),G(7,2),H(7,3)



Solution: Total number of paths:C(15,5)

Paths that pass AB: $|A_1|$=C(2+2,2)C(7+

Paths that pass CD: $|A_2|$=C(4+2,2)C(5+3,3);

Paths that pass EF: $|A_3|$=C(8,2)C(6,2);

Paths that pass GH: $|A_4|$=C(9,2)C(5,2);

Applications of Inclusion-Exclusion Principle

Paths that pass AB, CD:

$|A_1 \cap A_2| = C(4,2)C(8,3)$;

Paths that pass AB, EF:

$|A_1 \cap A_3| = C(4,2)C(6,2)$;

Paths that pass AB, HG:

$|A_1 \cap A_4| = C(4,2)C(5,2)$;

Paths that pass CD, EF:

$|A_2 \cap A_3| = C(6,2)C(6,2)$;

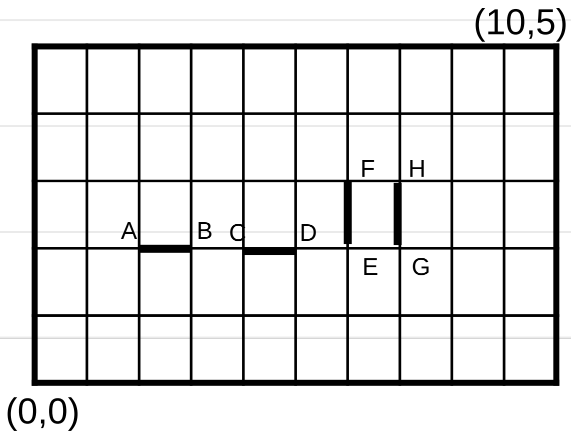
Paths that pass CD, HG:

$|A_2 \cap A_4| = C(6,2)C(5,2)$;

Paths that pass EF, HG:

$|A_3 \cap A_4| = 0$;

$$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}| = 2049$$



Paths that pass AB, CD, EF:

$|A_1 \cap A_2 \cap A_3| = C(4,2)C(6,2)$;

Paths that pass AB, CD, HG:

$|A_1 \cap A_2 \cap A_4| = C(4,2)C(5,2)$;

$|A_2 \cap A_3 \cap A_4| = 0$

$|A_1 \cap A_2 \cap A_3 \cap A_4| = 0$

Applications

- Combinations with repetition
- Derangements
- Permutations with forbidden positions

Combinations with repetition

- Unordered arrangements
  - Without repeating any object: **C(n,r)**
  - With repetition of objects permitted (infinite repetition).
    - $x_1+x_2+\ldots+x_k = r$

$$\binom{r+k-1}{r} = \binom{r+k-1}{k-1}.$$

$$x_1 + x_2 + x_3 + x_4 = 20$$

$$x_1 \geq 3, x_2 \geq 1, x_3 \geq 0 \; and \quad x_4 \geq 5$$

$$y_1 = x_1 - 3, y_2 = x_2 - 1, y_3 = x_3 \; and \quad y_4 = x_4 - 5$$

$$y_1 + y_2 + y_3 + y_4 = 20 - 3 - 1 - 5 = 11$$

$$x_1 + x_2 + x_3 + x_4 = 15$$

$$0 \leq x_1 \leq 5, 0 \leq x_2 \leq 10, 0 \leq x_3 \leq 11, and\, 0 \leq x_4 \leq 8$$

Eg Calculate the number of non-negative integer roots of $x_1+x_2+x_3=15$

Limitation is : $0\le x_1 \le 5$, $0\le x_2 \le 6$, $0\le x_3\le 7$。

Solution： The number of non-negative integer root of $x_1+x_2+\ldots+x_n=b$ is $C(n+b-1,b)$

The number of non-negative roots of $x_1+x_2+x_3=15$ without limitation is $C(15+3-1,15)= C(17,2)$

Assume A1 is the solution when $x_1\ge 6$, $y_1+6+x_2+x_3=15$

$|A1|= C(9+3-1,9)= C(11,2)$

Assume $A2$ is the solution when $x_2\ge 7$, $x_1+y_2+7+x_3=15$

$|A2|= C(8+3-1,8)= C(10,2)$

Assume $A3$ is the solution when $x_3\ge 8$, $x_1+x_2+ y_3+8=15$

$|A3|= C(7+3-1,7)= C(9,2)$

Eg Calculate the number of non-negative solutions of $x_1+x_2+x_3=15$, Limitation is: $0\leq x_1 \leq 5$, $0\leq x_2 \leq 6$; $0\leq x_3\leq 7$

Solution: Without limitation, $x_1+x_2+x_3=15$ has $C(15+3-1,15)= C(17,2)$ non-negative solutions.

$|A1|= C(9+3-1,9)= C(11,2)$

$|A2|= C(8+3-1,8)= C(10,2)$

$|A3|= C(7+3-1,7)= C(9,2)$

$A1\cap A2$: $y_1+6+y_2+7+x_3=15$ $|A1\cap A2| = C(2+3-1,2) = C(4,2)$

$A1\cap A3$: $y_1+6+x_2+y_3+8=15$ $|A1\cap A3| = C(1+3-1,1)= C(3,1)$

$A2\cap A3$: $x_1+y_2+7+y_3+8=15$ $|A2\cap A3| = 1$

$A1\cap A2 \cap A3$: $y_1+6+y_2+7+y_3+8=15$; $|A1\cap A2 \cap A3| = 0$

$|\overline{A1}\cap\overline{A2} \cap\overline{A3}|=C(17,2) - C(11,2) - C(10,2) - C(9,2)$
$+C(4,2) + C(3,1)+1=10$

Determine the number of 10-combinations of the multiset T = {3{a}, 4{b}, 5{c}}.

Combinations with repetition

- Example 1: Determine the number of 10-combinations of the multiset T = {3{a}, 4{b}, 5{c}}.
- Hint: Let $T^* = \{\infty\{a\}, \infty\{b\}, \infty\{c\}\}$, P_1 (resp., P_2, and P_3) be the property that a 10-combination of T^* has more than 3 a's (resp., 4 b's and 5 c's) and A_1 (resp., A_2 and A_3) be the 1—combinations of T^* which have property P_1 (resp., P_2 and P_3). We wish to determine the size of the set

$$\left|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}\right| = |S| - (|A_1| + |A_2| + |A_3|)$$
$$+ (|A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3|)$$
$$- (|A_1 \cap A_2 \cap A_3|)$$

10-combinations of the multiset T = {3{a}, 4{b}, 5{c}}.

- S: |S|=C(10+3-1,10) = 66
- A1:consists of all 10-conbinations of T^* in which *a* occurs at least 4 times.
  - The number of 10-combinations in A_1 equals the number of 6-combinations of T^*.
  - |A1|=C(6+3-1,6) = 28
- A2:consists of all 10-conbinations of T^* in which *b* occurs at least 5 times.
  - |A2|=C(5+3-1,5) = 21
- A3:consists of all 10-conbinations of T^* in which *c* occurs at least 6 times.
  - |A3|=C(4+3-1,4) = 15
- A1∩A2:consists of all 10-conbinations of T^* in which *a* occurs at least 4 times and *b* occurs at least 5 times.
  - |A1∩A2 |=C(1+3-1,1) = 3
- A1∩A3:consists of all 10-conbinations of T^* in which *a* occurs at least 4 times and c occurs at least 6 times.
  - |A1∩A3 |=1
- A2∩A3:
  - |A2∩A3 |=0
- A1 ∩ A2∩A3:
  - | A1 ∩ A2∩A3 |=0

$$\left|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}\right| = 66 - (28 + 21 + 15) + (3 + 1 + 0) - (0) = 6$$

Derangements

- A *derangement* of {1, 2, …, n} is a permutation $i_1 i_2 \ldots i_n$ of {1, 2, …, n} such that $i_1 \neq 1$, $i_2 \neq 2$, …, $i_n \neq n$ (i.e., no integer is in its natural position).
- We denote by D_n the number of derangements of {1, 2, …, n}.
- For n =1, there are no derangements. $D_1=0$
- For n =2, the only derangement is 2 1. $D_2=1$
- For n =3, there are two derangements: $D_3=2$

  2 3 1 and 3 1 2.
- For n = 4, there are 9 derangements: $D_4=9$

2143, 2341, 2413, 3142, 3412, 3421, 4123, 4312, 4321.

Examples

- At a party there are *n* men and *n* women. In how many ways can the *n* women choose male partners for the first dance? How many ways are there for the second dance if everyone has to change partners?
- Answer: for the first dance there are *n!* possibilities.
- For the second dance, the number of possibilities is D_n.

Formulas for Counting D_n

- For $n \geq 1$

$$D_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!}\right)$$

- Proof: A *derangement* of $\{1, 2, \ldots, n\}$ is a permutation $i_1 i_2 \ldots i_n$ of $\{1, 2, \ldots, n\}$ such that $i_1 \neq 1$, $i_2 \neq 2$, …, $i_n \neq n$ (i.e., no integer is in its natural position).
- Let S be the set of all n! permutations
- Let $P_j (j=1,2,\ldots n)$ be the property that, in a permutation, *j* is in its natural position.
- Let A_j denote the set of permutations with property $P_j (j=1,2,\ldots n)$

$$D_n = |\overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_m}|$$

$$D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!})$$

- S is the set of all n! permutations
  - $|S|=n!$
- A_j is the set of permutations with property P_j($j=1,2,\ldots n$) that j is in its natural position
  - $|A_j|=(n-1)!$
- $A_i \cap A_j$ is the set of permutations that i and j is in their natural positions
  - $|A_i \cap A_j|=(n-2)!$
- $a_k = |A_{i1} \cap A_{i2} \cap \ldots \cap A_{ik}|$.
  - $a_k =(n-k)!$
  - $\{i1,i2,\ldots,ik\}$ is a k-combination of $\{1,2\ldots,n\}$
    - $C(n,k)$

$$C(n,i)(n-i)! = \frac{n!}{(n-i)!\,i!}(n-i)! = \frac{n!}{i!}$$

$$|\overline{A_1} \cap \overline{A_2} \cap \ldots \cap \overline{A_n}| = n! - C(n,1)(n-1)!$$

$$+ C(n,2)(n-2)! - \cdots - \pm C(n,n)1!$$

$$= n!(1 - \frac{1}{1!} + \frac{1}{2!} - \cdots \pm \frac{1}{n!})$$

Todo List

- HW Sheet
- No Preclass video
- Next class(Oct.5th)
  - Pigeonhole principle


中秋
快乐
Mid-Autumn
Festival