Combinatorics HW recurrence relations – 1

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1. **Please prove the following equation of Fibonacci sequence F_i:**

$$F_1 + F_3 + F_5 + \cdots + F_{2n-1} = F_{2n}$$

Since $F_n = F_{n-1} + F_{n-2}$, where n>2 and $F_1 = F_2 = 1$, we can prove by induction that assuming $\sum_{i=1}^{n} F_{2i-1} = F_{2n}$, then we would have that $\sum_{i=1}^{n+1} F_{2i-1} = F_{2n+2}$. By taking out F_{2n+1} of the second sum , we get

$$\sum_{i=1}^{n+1} F_{2i-1} = \sum_{i=1}^{n} F_{2i-1} + F_{2n+1}$$

Which based on the assumption gives

$$\sum_{i=1}^{n+1} F_{2i-1} = F_{2n} + F_{2n+1} = F_{2n+2}$$

Hence, it is proven by mathematical induction that the sum of odd-indexes of the Fibonacci sequence until F_{2n-1} is F_{2n}.

2. **Please provide the corresponding characteristic equation for the following recurrence relation:**

$$a_n = 2a_{n-1} + 4a_{n-2} - 5a_{n-3}$$

The above equation can be written as $a_n - 2a_{n-1} - 4a_{n-2} - 5a_{n-3} = 0$ to form the k^{th} order linear homogeneous recurrence relation of $\{a_n\}$. Accordingly, the characteristic equation is

$$C(x) = x^3 - 2x^2 - 4x - 5 = 0$$

3. **Solve the recurrence relation $h_n=2h_{n-1}+8h_{n-2}$,n≥2, $h_1=1$, $h_2=10$.**

The above equation can be written as $h_n - 2h_{n-1} - 8h_{n-2} = 0$ to obtain the characteristic equation $C(x) = x^2 - 2x - 8 = (x-4)(x+2) = 0$. Hence 4 and -2 are derived as the roots of C(x), which allows h_n to be re-written as $h_n = A(4)^n + B(-2)^n$. By observing the provided values of h_1 and h_2, we can get

$$h_1 = 4A - 2B = 1$$
$$h_2 = 16A + 4B = 10$$

Multiplying the first equation by 2, gives

$$2h_1 = 8A - 4B = 2$$
$$h_2 = 16A + 4B = 10$$

Giving 24A=12, which makes A=0.5 and B=0.5. Accordingly, we get $h_n = \frac{1}{2}[(4)^n + (-2)^n]$.