Combinatorics HW 1.2

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1. How many odd numbers between 1000 and 9999 whose digits are distinct with each other?

In order to construct a number between 1000 and 9999, 4 digits would be required.

$$\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }$$

As mentioned by the problem statement, we are looking for 4-digit odd numbers in the given range. Hence, there would be 5 possible digits for the right-most location as numbers 1, 3, 5, 7, and 9 would make it an odd number.

$$\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{5}$$

Accordingly, starting from the left, there would be 8 possible digits for the first location: as the digits are to be distinct with each other, out of the 10 possible values ([0, 9]), an odd integer from the five available options would be used in the right-most location and 0 is unacceptable at it produces a 3-digit number.

$$\underline{8}\ \underline{\ \ }\ \underline{\ \ }\ \underline{5}$$

There would also be 8 possible values for the second location from the right: due to the condition of distinctness, the two already used values cannot be repeated and therefore, the use of the remaining 8 numbers is acceptable.

$$\underline{8}\,\underline{8}\ \underline{\ \ }\ \underline{5}$$

Lastly, with the same logic as the previous step, 7 possible values for the last location exist.

$$\underline{8}\,\underline{8}\,\underline{7}\,\underline{5}$$

Therefore, by the multiplication principal, there exists 8*8*7*5 = <u>2240</u> odd numbers with distinct digits between 1000 and 9999.

2. How many 7-digit numbers are there such that the digits are distinct integers taken from {1, 2, …, 9} and such that the digits 5 and 6 do not appear consecutively in either order?

Let the below 7 empty locations represent the 7 digits of the desired numbers.

$$\_\ \_\ \_\ \_\ \_\ \_\ \_$$

In total, based on the multiplication principal, the number of 7-digit numbers with distinct integers taken from the range 1-9 would be as follows

$$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 = \frac{9!}{2} = 181440$$

Because there are 9 possible integers for the left-most digits. In addition, as we move to the right, the number of possible integers in each location would decrease by 1 compared to the location on its left as the integer that has already been used in the previous locations cannot be used in the current location. Therefore, there are a total of <u>181440</u> different 7-digit numbers with integers from 1-9.

In order to calculate how many of these numbers satisfy the second condition, where 5 and 6 do not appear consecutively in either order, we must first calculate the number of numbers in which 5 and 6 **do** appear consecutively in both orders (5 and 6, and 6 and 5). Hence, we would initially dedicate two of the 7 digits in the number to 5 and 6.

$$(5,6)\ \_\ \_\ \_\ \_\ \_$$

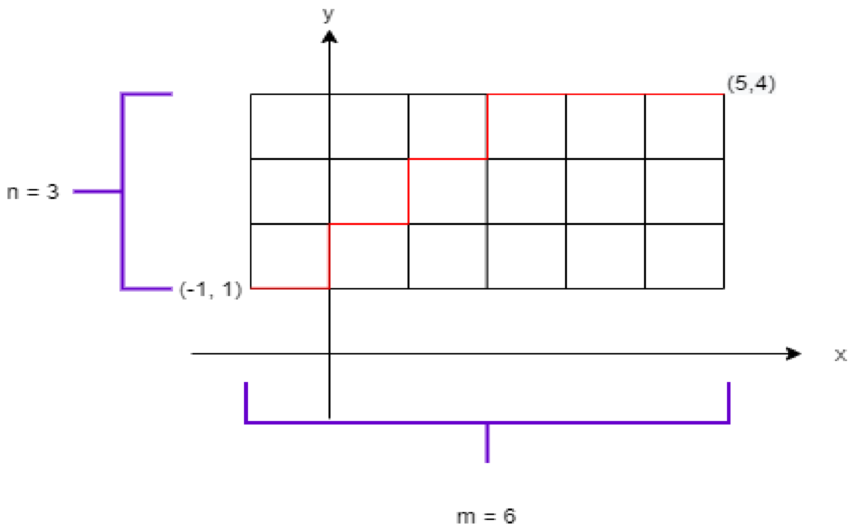
Correspondingly, for the remaining 5 digits we would have p(7, 5) as we should choose 5 digits from the remaining 7 possible integers. In addition, the created group of digits 5 and 6 can move their location 6 times; that is, they could occupy the first and the second digit, the second and the third digit, and so on. Moreover, as these two digits could appear in either order, there would be 2 possible permutations (56 or 65). Therefore, the number of 7-digit numbers in which 5 and 6 appear next to each other in either order would be

$$\frac{7!}{(7-5)!} \times 6 \times 2 = 7! \times 6 = 30240$$

Hence, by the subtraction principal, there would be 181440 – 30240 = 151200 different 7-digits numbers from the given range of integers in which digits 5 and 6 do not appear consecutively in either order.

3. How many different lattice paths from (-1,1) to (5,4)?

The below graph illustrates the problem statement.



In order to go from the point (-1,1) to point (5,4), we have to take m = 6 steps to the right and n = 3 steps to the top, which would result in a total of m + n = 9 steps. Accordingly, we have to choose m of these steps to move towards the right:

$$C(m+n, m) = C(9,6) = \frac{9!}{(9-6)!\,6!} = 84$$

Therefore, there a total of 84 different paths from (-1,1) to (5,4).

4. How many non-repeating 8-strings such that *a* and *b* are not adjacent can be formed with 26 English letters? Please explain the calculation in detail.

Let the below 8 empty locations represent the letter of an 8-string.

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Similar to the approach in question 2, we should first calculate the total number of non-repeating 8-strings using the 26 English letter in the alphabet, which would be p(26, 8) as we are selecting 8 distinct letters from the available 26 letters.

$$p(26,8) = \frac{26!}{(26-8)!} = 62{,}990{,}928{,}000$$

Next, if we dedicated two adjacent locations to letters a and b, the other 6 letters are to be selected from the remaining 24 letters, which gives p(24,6) permutations. The place of the two adjacent locations could be moved 8 -1 = 7 times; that is, letters a and b could be in the first and second location, the second and third location, and so on. Moreover, there are two ways to put a and b together (ab or ba). Hence, the total number of 8-strings with distinct letters in which a and b are adjacent would be

$$p(24,6) \times 7 \times 2 = \frac{24!}{(24-6)!} \times 14 = 1{,}356{,}727{,}680$$

Conclusively, based on the subtraction principal, there would be 62,990,928,000 - 1,356,727,680 = <u>61,634,200,320</u> different 8-strings with distinct English such that letters a and b are not adjacent.