Tsinghua University Combinatorics Final Exam –Fall 2018 (25 Points)

Write your name on the top right corner on each page.
Write all the answers on the answer sheets. An answer with no explanation will receive no credit. Write the formula with proper explanation. The exact number value of factorials or permutations or combinations are not necessary.
[Total time: 1.5 hours]

Name : Student ID:

1. 1) When generating the permutations of {1, 2, 3, 4, 5, 6, 7}, try to determine the **next** permutation of "2637145" in lexicographic order. ( ) (1 point)

   2) Count the number of permutations $i_1i_2i_3i_4i_5i_6i_7$ of {1, 2, 3, 4, 5, 6, 7}, where i1 $\neq$ 1; i2 $\neq$ 2, 3; and i3 $\neq$ 4, 5。(3 points)

2. How many **even** numbers between 1 and 1000 which are relative prime to 105? (4 points)

3. 1) How many ways to put 8 identical apples into 4 different boxes that no empty box is allowed? (2 points)

   2) Please figure out the corresponding generating function for the problem of how many ways to put *n* identical apples into *m* identical boxes that no empty box is allowed? (2 points)

4. How many ways to compose a binary strings (made up of 0s and 1s) of length *n* that do not contain "010" or "101"? (7 points)

5. Transform the following problems into augmented form and solve it by simplex Method and show at least 2 tables.(6 points)

$$Min \quad z = x_1 + 3x_2 - x_3$$

$$\begin{cases} 2x_1 + x_2 + 3x_3 \le 5 \\ -x_1 + x_2 - 6x_3 \ge -8 \\ x_1 + 3x_2 - x_3 \le 7 \\ x_1 \le 1, x_2 \ge -3 \end{cases}$$