

Here are some additional descriptions for these three algorithms of this homework.

## Conductance

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In graph theory, the conductance of a cut  $(S, \bar{S})$  in a graph is defined as:

$$\varphi(S) = \frac{\sum_{i \in S, j \in \bar{S}} a_{ij}}{\min(a(S), a(\bar{S}))}$$

Where the  $a_{ij}$  are the entries of the adjacency matrix for  $G$ , so that  $a(S)$  is the total number (or weight) of the edges incident with  $S$ .  $a(S) = \sum_{i \in S} \sum_{j \in V} a_{ij}$

In this assignment, you can classify edges into two categories, by the lowest bit value of the source vertex. Given an edge  $e(u, v)$ , it's red if  $u \& 1 = 0$ , otherwise is black. A edge is crossover, if its source and target vertices belong to different categories.

So, the conductance value of a graph need be calculated in this assignment is:

$$conductance = \frac{\#crossover\ edges}{\min\{|\#red\ edges|, |\#black\ edges|\}}$$

## PageRank-Delta

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PageRank-Delta is similar to PageRank, but only update vertices whose PageRank value has changed by more than some  $\Delta$ -fraction.

In this assignment, each iteration of PageRank Delta:

$$\begin{aligned} Rank(A) &= Rank(A) + Delta(A) \\ Delta(A) &= 0.85 * \left( \frac{Delta(B)}{L(B)} + \frac{Delta(C)}{L(C)} + \dots \right) \end{aligned}$$

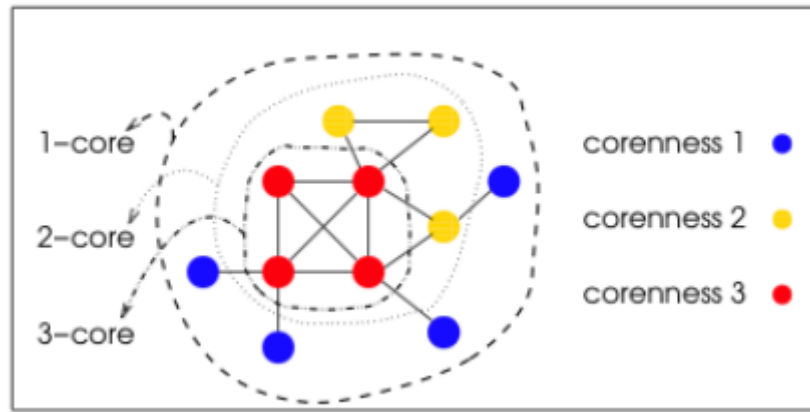
where  $Delta(A)$  is added, if  $\frac{Delta(A)}{Rank(A)}$  is larger than *propagation\_threshold*.

## k-Cores

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A  $k$ -core of a graph  $G$  is a maximal connected subgraph of  $G$  in which all vertices have degree at least  $k$  (the degree here is the degree in the final subgraph). Equivalently, it is one of the connected components of the subgraph of  $G$  formed by repeatedly deleting all vertices of degree less than  $k$ . If a non-empty  $k$ -core exists, then, clearly,  $G$  has degeneracy at least  $k$ , and the degeneracy of  $G$  is the largest  $k$  for which  $G$  has a  $k$ -core.

Here is an example:



You can read the wiki for more detailed information of k-core.

[https://en.wikipedia.org/wiki/Degeneracy\\_\(graph\\_theory\)](https://en.wikipedia.org/wiki/Degeneracy_(graph_theory)).

In this assignment, you can implement this algorithm in two ways:

1. Your code is used to calculate the coreness value of all vertices. And output the largest coreness value.
2. Your program need a parameter called  $k$ , and return or output vertices whose coreness values is greater than or equal to  $k$ .

The input graph of k-core algorithm need be undirected.