HW - Week 11

Name: Sahand Sabour   **Student ID:** 2020280401

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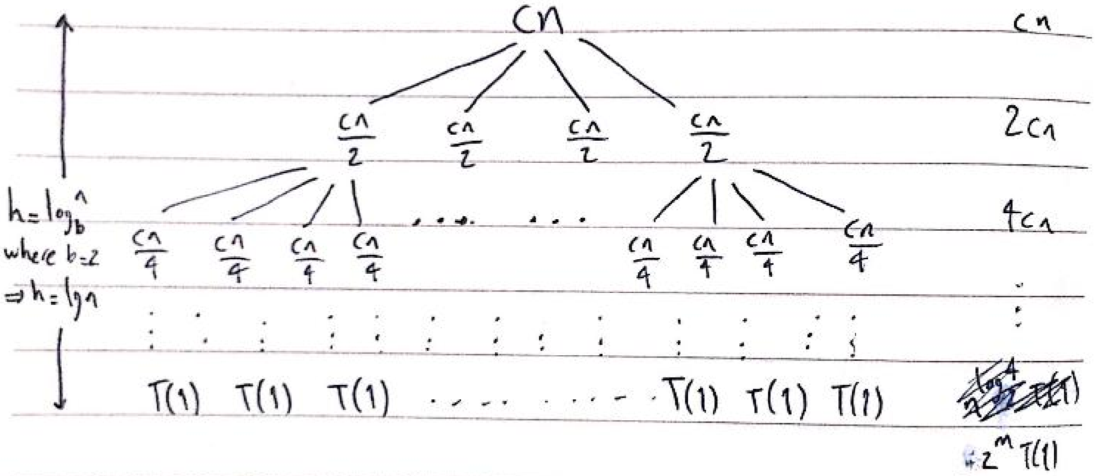

where m is the number of steps

So, we have that $h = \lg n$, there are $n^{\log_b a} = n^{\log_2 4} = n^2$ leaves and there are 2^m element at step m. Therefore, the time complexity for all the nodes in step m would be $T(n) = \sum_{m=0}^{\lg n - 1} 2^m cn + \Theta(n^2) = \frac{2^{\lg n} - 1}{2-1} cn + \Theta(n^2) = \Theta(n^2)$

Hence, given T(n) = Θ(n^2), we have that the upper bound is O(n^2) and the lower bound is Ω(n^2). Accordingly, we use substitution to provide the tightness of these asymptotic bounds (prove that this guess , which states T(n) = O(n^2) as the upper bound, is correct). Therefore, using given constants d and e>0, we get that

$$\mathrm{T(n)} \leq 4\mathrm{t}[\mathrm{n}/2] + \mathrm{cn}$$
$$\leq 4\mathrm{d}[\mathrm{n}/2]^2 + \mathrm{cn}$$
$$\leq 4\mathrm{d}(\mathrm{n}/2)^2 - 4(\mathrm{d'n}/2) + \mathrm{cn}$$
$$= \mathrm{dn}^2 - 2\mathrm{d'n} + \mathrm{cn}$$
$$\leq \mathrm{dn}^2 - \mathrm{d'n}$$

Where d’>c. Therefore, the upper bound is T(n) = O(n^2). Accordingly, for the lower bound, we want to prove that T(n) = $\Omega(n^2)$. Hence, for a given constant d>0, we have

$$\mathrm{T(n)} \geq 4\mathrm{t}[\mathrm{n}/2] + \mathrm{cn}$$
$$\geq 4\mathrm{d}((\mathrm{n}/2) - 1)^2 + \mathrm{cn}$$
$$= \mathrm{dn}^2 - 4\mathrm{dn} + \mathrm{cn} + 4\mathrm{d}$$

Where -4d+c≥4. This proves that the lower bound is T(n) = $\Omega(n^2)$.

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B) For this recurrence a = 1, b= 10/7, and f(n) = n. Therefore, we have that

$$n^{log_b^a} = n^{log_{10/7}^1} = n^0 = 1$$

Since f(n)=$\Theta(n^{log_{10/7}^{1+\epsilon}})$, where $\epsilon = \frac{3}{7}$ and $\mathrm{af(n/b)} = \mathrm{f(7n/10)}$ ≤ cf(n) where c= 7/10 < 1, by the case 3 of master theorem the solution is T(n) = Θ(f(n))= Θ(n).

C) For this recurrence a = 16, b= 4, and f(n) = n^2. Therefore, we have that

$$n^{log_4^{16}} = n^2$$

Since f(n)=$\Theta(n^{log_4^{16}})$, then by the case 2 of master theorem the solution is T(n) = $\Theta(n^{log_4^{16}} lgn)$ = $\Theta(n^2 lgn)$.

D) For this recurrence a = 7, b= 3, and f(n) = n^2. Therefore, we have that

$$n^{log_b^a} = n^{log_3^7}$$

Since f(n)=$\Theta(n^{log_3^{7+\epsilon}})$, where $\epsilon = 1$ and af(n/b) =7 f(n/3) ≤ cf(n) where c= 7/9 < 1, by the case 3 of master theorem the solution is T(n) = Θ(f(n))= $\Theta(n^2)$.

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B) The worst-case running times for each strategy is provided respectively below:

1. T(n) = 2T(n/2) + Θ(n) so a=2, b=2, and f(n) = Θ(n). Therefore, we have that

$$n^{log_b^a} = n^{log_2^2} = n$$

Since f(n) =Θ($n^{log_2^2}$), by case 2 of the master theorem the solution is $\mathrm{T(n)} =$ Θ($n^{log_2^2} lgn$) = Θ($nlgn$).

2. T(n) = 2T(n/2) + Θ(N) = 4T(n/4) + 2Θ(N) + Θ(N) = $\sum_{i=0}^{lgn} 2^i$ Θ(N) $=$ Θ(n^2).

3. T(n) = 2T(n/2) + Θ(n) so similar to part 1, by case 2 of the master theorem we have that $\mathrm{T(n)} =$ Θ($n^{log_2^2} lgn$) = Θ($nlgn$).