HW - Week 14

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1. We have six points, each being denoted by p_i, where i is their index in the sequence (from 0 to 5). Accordingly we would have 5 As as follows:

$$A_1 = 5 \times 10$$

$$A_2 = 10 \times 3$$

$$A_1 = 3 \times 12$$

$$A_1 = 12 \times 5$$

$$A_1 = 5 \times 50$$

Since we have that

$$m[i,j] = \min\{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j\} \text{ where } i \le k < j$$

Giving m[i,j] = 0 if i=j and

$$m[1, 2] = \min(0 + 0 + 150) = 150$$

$$m[2, 3] = \min(0 + 0 + 360) = 360$$

$$m[3, 4] = \min(0 + 0 + 180) = 180$$

$$m[4, 5] = \min(0 + 0 + 3000) = 3000$$

$$m[1, 3] = \min(0 + 360 + 600, 150 + 0 + 180) = 330$$

$$m[2, 4] = \min(0 + 180 + 150, 330 + 0 + 600) = 330$$

$$m[3, 5] = \min(0 + 3000 + 1800, 180 + 0 + 750) = 930$$

$$m[1, 4] = \min(0 + 330 + 250, 150 + 180 + 75, 330 + 0 + 300) = 405$$

$$m[2, 5] = \min(0 + 930 + 1500, 360 + 3000 + 6000, 330 + 0 + 2500) = 2430$$

$$m[1, 5] = \min(0 + 2430 + 2500, 150 + 930 + 750, 330 + 3000 + 3000, 405 + 0 + 1250) = 1655$$

We can construct the m and s tables (i columns, j rows) as follows

m	1	2	3	4	5
1	0	-	-	-	-
2	150	0	-	-	-
3	330	360	0	-	-
4	405	330	180	0	-
5	1655	2430	930	3000	0

s	1	2	3	4	5
1	-	-	-	-	-
2	1	-	-	-	-
3	2	2	-	-	-
4	2	2	3	-	-
5	4	2	4	4	-

Giving the minimum number of required scalar multiplications to be m[1,5] = 1655. Accordingly the optimal solution would be $((A_1A_2)(A_3A_4))A_5$.

2. An optimal substructure can be defined as a situation in which an optimal solution to the problem contains the optimal solutions for its sub-problems. For this problem, the first step would be to split the problem into one or more subproblems, which is acheived by dividing the matrix chain $A_1A_2..A_n$ into two smaller sub-chains $A_1A_2...Am$ and $A_{k+1}A_{2k+2}...A_n$. Accordingly, we could define the optimal solution for each sub-chain to be the most expensive scalar multiplication in the chain. Therefore, the paranthesization in each sub-chain should be chosen in a way to produce the highest cost. This can be proven by the cut and paste method; Let us consider S as the optimal solution for the matrix chain $A_1A_2..A_n$. If the solution for either of the sub-chains is not optimal, S would not be optimal either, which is a contradiction. Hence, this problem exhibits optimal structure.

3. We need to prove that by taking away a_n, the remaining solution S - $\{a_n\}$ is optimal for the following sub-problem $KS_{n-1, w}$: finding the solution from a_1, a_2, ..., a_n with knapsack capacity w. Hence, we suppose that S' is a different solution from S - $\{a_n\}$ and it is optimal to $KS_{n-1, w}$. Accordingly, if we have that S' is a feasible solution and also a better solution than S, we would have a contradiction. Initially, we could check the feasiblity of S' by

$$\sum_{a_j \in S'} w_j \leq w$$

which makes S’ a feasible solution as the sum of its weights are lower than or equalt to the capacity w. Accordingly, we would have

$$\sum_{a_j \in S'} v_j > \sum_{a_j \in S} v_j$$

since S’ is a better solution than S, which shows a contradiction.