HW Magic Sequences

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1. **Using 5 numbers 1, 2, 3, 4, 5 to fill in 1x n grids, each grid is filled with one digit. If there are odd number of grids that have 1 written on them, and an even number of grids with 2, please write the corresponding exponential generating function and figure out how many arrangements there for 1*6 grids?____**

The corresponding exponential generating function would be as follows:

$$G_e(x) = (\frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots)(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots)(1 + x + \frac{x^2}{2!} + \ldots)^3$$

The above generating function can be written as

$$G_e(x) = (\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!})(\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!})(\sum_{n=0}^{\infty} \frac{x^n}{n!})^3$$

Since $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ and $e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!}$, we can get

$$e^x \pm e^{-x} = \sum_{n=0}^{\infty} \frac{x^n \pm (-x)^n}{n!}$$

Therefore, distinguishing the odd and even numbers for n, we would get

$$\frac{e^x + e^{-x}}{2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} \text{ and } \frac{e^x - e^{-x}}{2} = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

Accordingly, the generating function can be rewritten as:

$$G_e(x) = (\frac{e^x - e^{-x}}{2})(\frac{e^x + e^{-x}}{2})(e^x)^3 = \frac{e^{5x} - e^x}{4} = \sum_{n=0}^{\infty} (\frac{5^n - 1^n}{4})\frac{x^n}{n!}$$

Giving $\frac{5^6 - 1^6}{4} = \frac{15625-1}{4} = \underline{3906}$ different arrangements for 1*6 grids.

2. **There are six people in a library queuing up, three of them want to return the book “Interviewing Skills”, and 3 of them want to borrow the same book. If at the beginning, all the books of “Interviewing Skills” are out of stock in the library, how many ways can these people line up?____**

$$C_n = C(2n, n) - C(2n, n-1) = \frac{1}{n+1}\binom{2n}{n}$$

Therefore, for this problem, we would need to calculate C_3, since there are 3 people borrowing and 3 people returning.

$$C_3 = \frac{1}{3+1}\binom{6}{3} = 5$$

Accordingly, since the people giving back the book and people borrowing the book each can have 3! different arrangements between themselves, there would be a total of 5*3!*3! = 180 ways that these people could line up.