Combinatorics HW 1.1

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1. **A large tournament has 569 entrants in total. If it is a single elimination tournament, how many matches have to be played out before the champion can be decided? (Please calculate the precise value)**

   As mentioned in the problem statement, at the end of each match in this tournament, there would be a single winner and therefore, a single elimination; That is, a single player is eliminated after each match. Hence, it can be derived that there exists a linear relationship/correspondence between the number of matches to be played and the number of players to be eliminated: n matches have to be played in order to have n eliminations. Correspondingly, in order to have a single winner/champion from x players, x-1 players have to be eliminated, which requires x-1 matches to be played. Therefore, in this situation with 569 entrants, 569 - 1 = <u>568</u> matches have to be played out before the champion is decided.

2. **The figure below shows a partial 4X4 matrix，is there some way of filling up the rest of the omitted entries to produce a magic square of size 4？**

$$\begin{bmatrix} 2 & 3 & \blacksquare & \blacksquare \\ 4 & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare \end{bmatrix}$$

In a normal magic square, the numbers 1 to n^2 are used to fill in the square, where n is the size of the rows or columns of the square (n=4 in this case). Therefore, the sum of all the elements in the square can be calculated as

$$\text{Sum} = 1 + 2 + 3 + \ldots + \mathrm{n}^2 = \frac{\mathrm{n}^2(\mathrm{n}^2+1)}{2} = \frac{4^2(4^2+1)}{2}$$

Based on the definition of the magic square, the sum of numbers in each row, each column, and each diagonal would be the same value. This value is referred to as the magic sum and can be calculated as

$$\text{Magic Sum (M)} = \frac{\text{Sum}}{\text{n}} = \frac{\frac{4^2(4^2+1)}{2}}{4} = 34$$

Let $\blacksquare_{ij}$ denote an omitted entry in the i^{th} row and j^{th} column of the given matrix. By observing the top row and left-most column of the given square, the following two equations can be derived

$$2 + 3 + \blacksquare_{13} + \blacksquare_{14} = 34$$
$$2 + 4 + \blacksquare_{31} + \blacksquare_{41} = 34$$

Which gives

$$\blacksquare_{13} + \blacksquare_{14} = 29$$
$$\blacksquare_{31} + \blacksquare_{41} = 28$$

Regarding the top row, as numbers 1 to 16 should be used to fill in the square, there would be two combination of numbers to fill this row (13 + 16= 14 + 15 = 29). Considering the first option (13 and 16), there would be no possible way to complete the left-most column as the only way to have a sum of 28 in the missing two elements would be either 16+12 or 15+13, which both result in repeating an already used number. Hence, 14 and 15 would be used to complete the top row while 16 and 12 are used to fill in the left-most column, which gives 4 ways to update the square:

A) $\begin{bmatrix} 2\;3\;14 15 \\ 4 \\ 12 \\ 16 \end{bmatrix}$ B) $\begin{bmatrix} 2\;3\;14 15 \\ 4 \\ 16 \\ 12 \end{bmatrix}$ C) $\begin{bmatrix} 2\;3\;15 14 \\ 4 \\ 12 \\ 16 \end{bmatrix}$ D) $\begin{bmatrix} 2\;3\;15 14 \\ 4 \\ 16 \\ 12 \end{bmatrix}$

Correspondingly, we would analyze the diagonal between the top right corner and bottom left corner. With option A, we would have

$$15 + \blacksquare_{23} + \blacksquare_{32} + 16 = 34$$
$$\blacksquare_{23} + \blacksquare_{32} = 3$$

This is not possible as 3 = 2 + 1 and 2 has already been used. Option B gives

$$15 + \blacksquare_{23} + \blacksquare_{32} + 12 = 34$$

$$■_{23} + ■_{32} = 7$$

In this option, we are able to use 1 and 6 as the other sums are not possible (2+5 and 3+4). In option C, we have that

$$14 + ■_{23} + ■_{32} + 16 = 34$$

$$■_{23} + ■_{32} = 4$$

There are no possible solutions for this as 4 = 3 + 1 and 3 has already been used. Lastly, option D gives

$$14 + ■_{23} + ■_{32} + 12 = 34$$

$$■_{23} + ■_{32} = 8$$

Which is possible if 1 and 7 are used. Therefore the table could be updated in 4 ways:

A) $\begin{bmatrix} 2 & 3 & 14 & 15 \\ 4 & & 1 & \\ 16 & 6 & & \\ 12 & & & \end{bmatrix}$ B) $\begin{bmatrix} 2 & 3 & 14 & 15 \\ 4 & & 6 & \\ 16 & 1 & & \\ 12 & & & \end{bmatrix}$ C) $\begin{bmatrix} 2 & 3 & 15 & 14 \\ 4 & & 1 & \\ 16 & 7 & & \\ 12 & & & \end{bmatrix}$ D) $\begin{bmatrix} 2 & 3 & 15 & 14 \\ 4 & & 7 & \\ 16 & 1 & & \\ 12 & & & \end{bmatrix}$

Observing the second column from the left in options B and D would give

$$3 + ■_{22} + 1 + ■_{42} = 34$$

$$■_{22} + ■_{42} = 30$$

Which is impossible because the largest available number is 13: that is, 14, 15, and 16 are already used to fill in the square. Moreover, in options A and C, analyzing the second row from the top gives that

$$4 + ■_{22} + 1 + ■_{24} = 34$$

$$■_{22} + ■_{24} = 29$$

Which is again impossible as the largest available number is 13.

Therefore, there is <u>no way</u> of filling up the empty entries to create a 4x4 magic square.