Combinatorics and Algorithms Design

C & A

Yuchun Ma(马昱春)

Ying Zhao (赵颖)

Information

- Prof. Yuchun Ma

  Email: myc@mail.tsinghua.edu.cn

  Office: East main building 8-424

- TA：
  - Zijian Lv

  (Email: lvzj19@mails.tsinghua.edu.cn)
  - Kai Su

  (Email: suk19@mails.tsinghua.edu.cn)

Introduce yourself!

WeChat group

Your ID to be
Student ID+Name(English/Chinese)

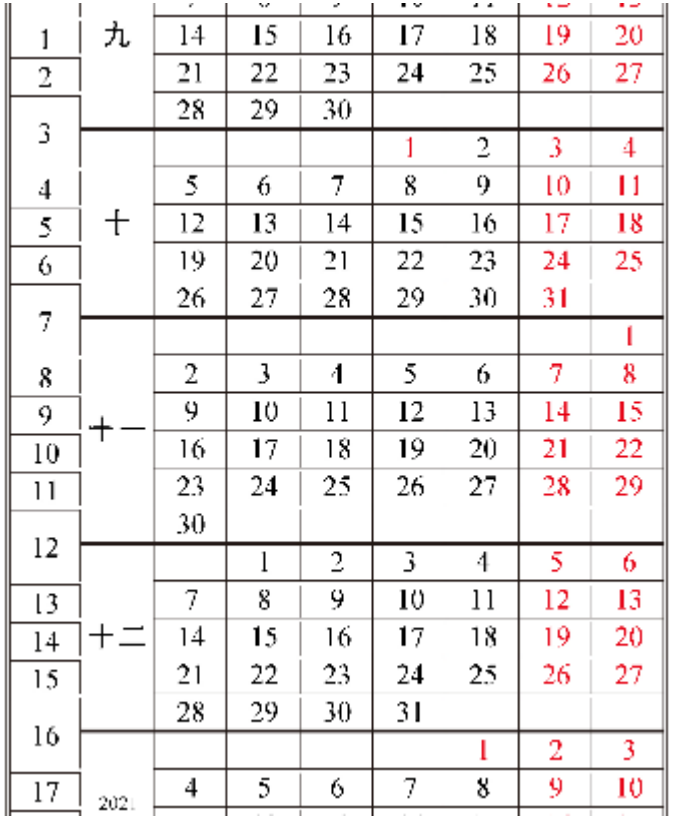
Introduce the course!

C & A

Textbook:

[1] Richard Brualdi, *Introductory Combinatorics*, fifth edition
https://item.jd.com/10059101.html

Office Hour?

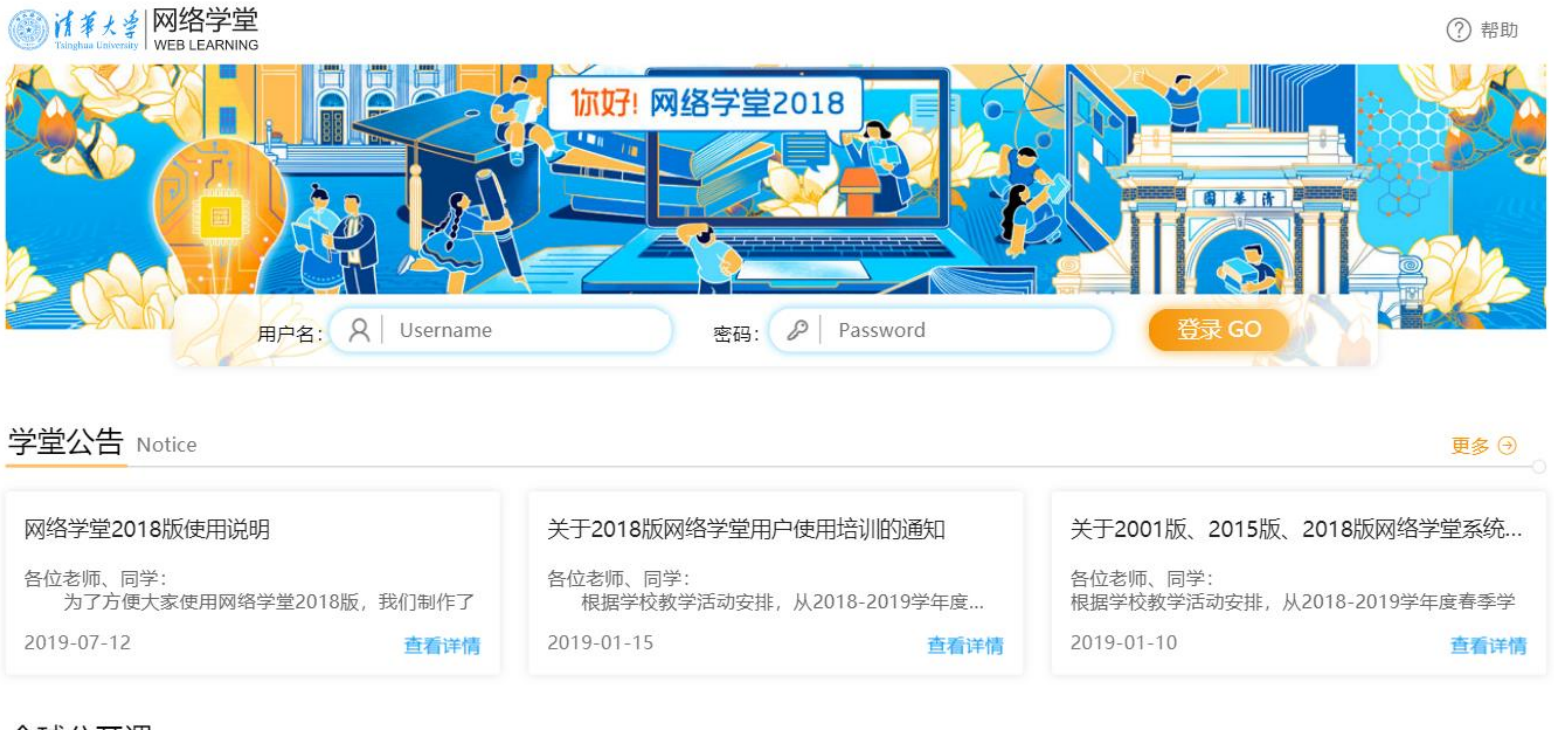


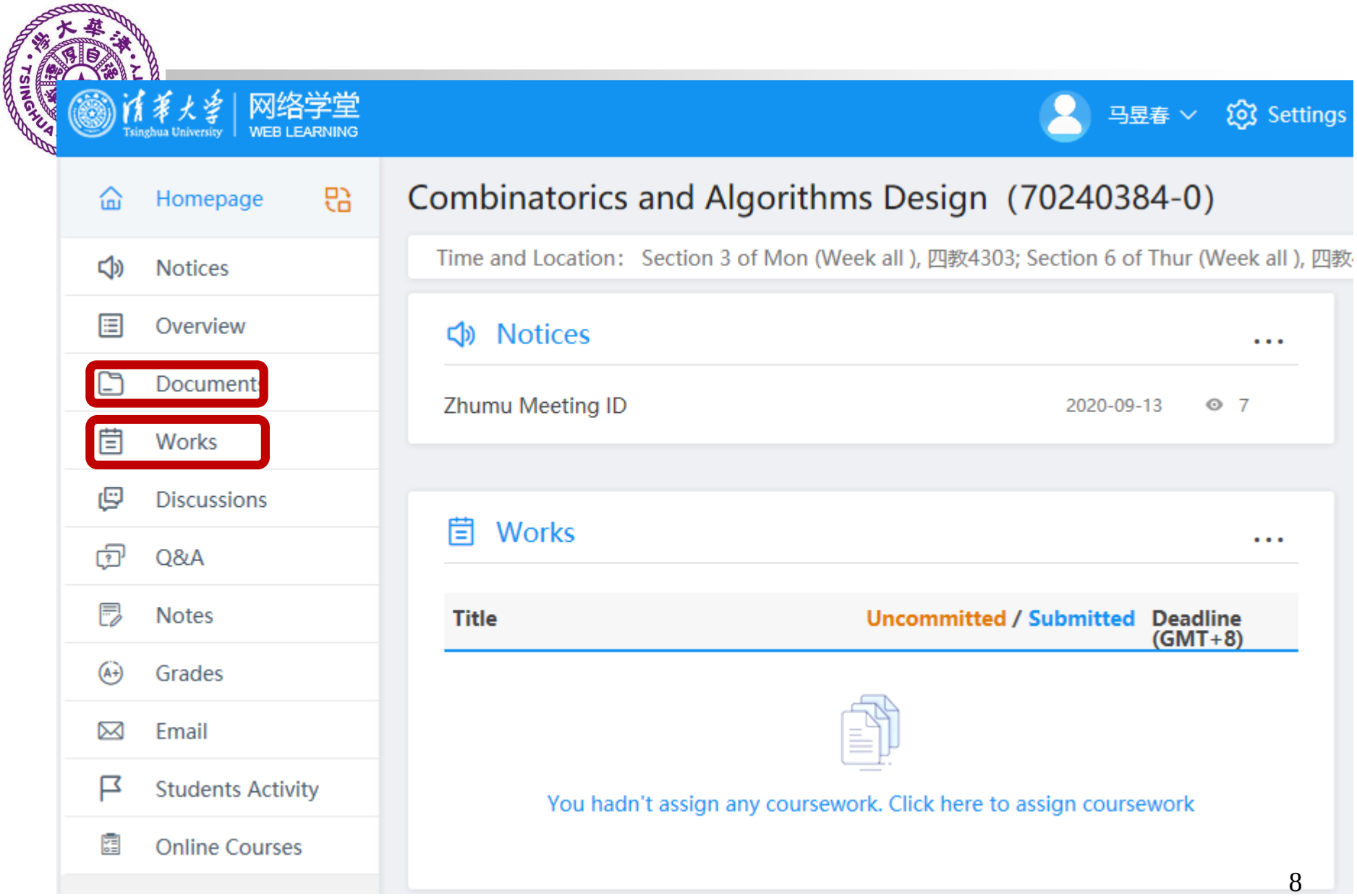
1	九	14	15	16	17	18	19	20
2		21	22	23	24	25	26	27
3		28	29	30				
					1	2	3	4
4		5	6	7	8	9	10	11
5	十	12	13	14	15	16	17	18
6		19	20	21	22	23	24	25
7		26	27	28	29	30	31	
								1
8		2	3	4	5	6	7	8
9		9	10	11	12	13	14	15
10	十一	16	17	18	19	20	21	22
11		23	24	25	26	27	28	29
12		30						
			1	2	3	4	5	6
13		7	8	9	10	11	12	13
14	十二	14	15	16	17	18	19	20
15		21	22	23	24	25	26	27
16		28	29	30	31			
						1	2	3
17	202	4	5	6	7	8	9	10

- 9.14 Introduction
- 9.17 Permutation and Combination
- 9.21 Permutation and Combination
- 9.24 Generating permutations
- 9.28 IEP
- 10.5 pigeon-hole
- 10.8 Summary and Practice
- 10.10 Generating Function
- 10.12 Generating Function
- 10.15 Magic sequences
- 10.19 Summary and Practice
- 10.22 Linear Programming
- 10.26 Linear programming
- 10.29 Summary
- Week 8 Final Exam

Course Materials

http://learn.tsinghua.edu.cn/




清華大學 Tsinghua University
网络学堂
WEB LEARNING
马昱春
Settings
Homepage
Notices
Overview
Document
Works
Discussions
Q&A
Notes
Grades
Email
Students Activity
Online Courses
Combinatorics and Algorithms Design（70240384-0）
Time and Location： Section 3 of Mon (Week all), 四教4303; Section 6 of Thur (Week all), 四教
Notices
Zhumu Meeting ID
2020-09-13
7
Works
Title
Uncommitted / Submitted
Deadline (GMT+8)
You hadn't assign any coursework. Click here to assign coursework

Homework

- Announced on learning.tsinghua.edu.cn
  - Word file with questions provided.
  - Online Submission: pdf file only.
  - Detailed explanation is needed.
  - Handwritten or typed. (Recommend you to print the HW sheet out and handwrite all the answers and then scan your answer sheet to pdf file.)
  - Due time: Monday 9:00am
  - Late submission is not accepted. The maximal extension would be on Thursday 10am by email, but each people is allowed only once. Otherwise, 0 point.

Supporting materials

edX Courses Programs Schools & Partners About Search:

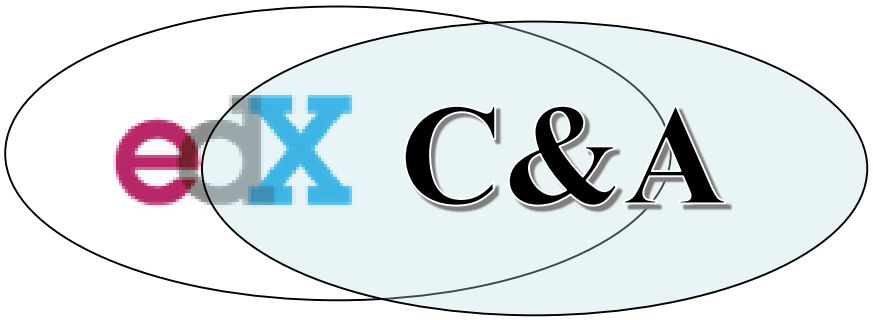
Home > All Subjects > Math > Combinatorial Mathematics 组合数学

Combinatorial Mathematics 组合数学

Discover how to apply counting principles and combinatorics to solve problems in computer science, financial analysis, and your daily life.

清華大學 Tsinghua University

https://www.edx.org/course/combinatorial-mathematics-zu-he-shu-xue-tsinghuax-60240013x-2



投票 最多可选1项

设置

Are you a MOOCer?

A I never heard MOOC before.

B I heard MOOC before, but never learn a course.

C I have taken 1-3 MOOC courses.

D I have taken several MOOC courses and finished some of them.

提交

What is Combinatorics

What are those videos?

- Around 80% course knowledges could be found in the videos.
  - Pre-class video: learn by your self
    - Online-quiz
    - In-class Quiz
  - Post-class video: Help you to understand the lecturing, self-learning
  - Supporting video: Related with your homeworks/projects

Learning process in this course

- Pre-class
  - RainClassroom
    - Short videos and Online quizzes **(5 points)**
- In-class
  - Quizzes **(2 points)**
  - Lecturing
  - Interactive quizzes
- After-class
  - Links of Supporting videos
  - Playback or video links
  - Homework **(10 points)**
  - Programming tasks **4 tasks from 5 (8 points)**

Final Exam 25 points

Substitute 2 points

Extra open projects

Contribution

Evaluation

Grades: 50 points.

- Homework : 10 points; paper work;
- Online-quiz: 5 points
- In-class quiz or Substitute: 2 points
- Online Programming tasks: 8 points
- Final Exam : 25 points
  - If you are going to be absent on the day of an examination, you must provide a University-approved excuse for your absence before the day of the examination.

Open Projects

- Besides OJ tasks, you may implement your own projects about combinatorics.
  - Any topics related to combonarotics are welcome.
  - Novelty and Interesting
  - Write a report or paper about your projects.
  - Submit the package on learn.Tsinghua.edu.cn
    - Code C/C++/Python
    - Readme file
    - Report/paper

Rules in forum/weChat group

- No answers, only ideas!
- Anything related with combinatorics
- Any comments
- Any suggestions

Policies

Policy on Cheating: If you copy a classmate's assignment or permit a classmate to copy your assignment, you are cheating. If you have received help with an assignment problem, we expect that you will write out a solution in your own words. If you are cheating, your mark will be 0.

Policy on Lectures and Assignments: Students are expected to attend all lectures and to submit all assignments for grading.

Survey

Chapter 1
What is Combinatorics

Combinatorics is a branch of pure **mathematics** concerning the study of **discrete** (and usually finite) objects. It is related to many other areas of mathematics, such as algebra, probability theory, ergodic theory and geometry, as well as to applied subjects in **computer science** and statistical physics.

http://en.wikipedia.org/wiki/Combinatorics

Combinatorics is concerned with the

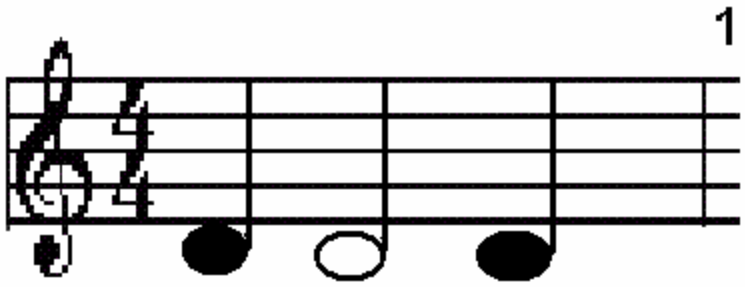
Existence, Enumeration, Analysis, and Optimization

of discrete structure.

Discrete Structure

- Discrete Structure
  - Fundamentally discrete rather than continuous
  - Discrete objects can often be enumerated by integers
  - Dealing with countable sets

Red **Green** **Blue**



Arrangement of Discrete Structure

Given three integers {1,2,3}, how to arrange them in a sequence in which each integer appear once and only once?

1 2 3 1 3 2 2 1 3 2 3 1 3 1 2 3 2 1

- Arrangement
  - A set of objects are arranged such that certain requirements are satisfied.
    - Is the arrangement always possible?

    Existence
  - A specified arrangement is possible and there are several ways of achieving it.
    - Count their number
    - Classify them into types

    Enumeration
  - Investigate the properties and structures.
    - Whether the structure has implications for the classification problem
    - Whether it has potential applications?

    Analysis
  - More than one arrangement is possible.
    - Find a best or optimal arrangement.

    Optimization

Arrangement of Discrete Structure

Combinatorics is concerned with the

Existence, Enumeration, Analysis, and Optimization

of discrete structure.

- Arrangement
  - A set of objects are arranged such that certain requirements are satisfied.
    - Is the arrangement always possible?

    Existence
  - A specified arrangement is possible and there are several ways of achieving it.
    - Count their number
    - Classify them into types

    Enumeration
  - Investigate the properties and structures.
    - Whether the structure has implications for the classification problem
    - Whether it has potential applications?

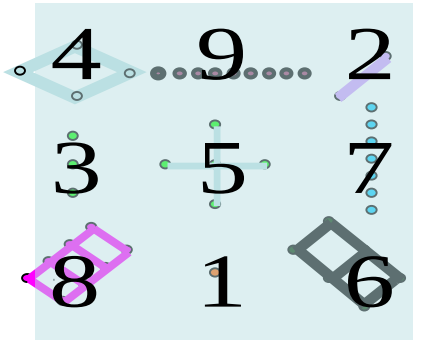
    Analysis
  - More than one arrangement is possible.
    - Find a best or optimal arrangement.

    Optimization

History of Combinatorics

- Combinatorics is an old but also young branch of mathematics
  - As old as 4200 years
  - Legend of Lo Shu(洛书) "scroll of the river Lo”:

The great king Yu
(2205BC -2105BC)



Magic Square

4	9	2
3	5	7
8	1	6

A magic square: a square array of numbers in which the sum of all rows, all columns and both diagonals is the same.

- a **magic square** of order n is an arrangement of n^2 numbers, usually distinct integers, in a square, such that the n numbers in all rows, all columns, and both diagonals sum to the same constant.
- A **normal** magic square contains the integers from 1 to n^2.
- The constant sum in every row, column and diagonal is called the magic constant or magic sum, M.

$$Sum = 1+2+3...+n^2 = \frac{n^2(n^2+1)}{2} \quad \xrightarrow[n \text{ rows}]{n*M = Sum} \quad M = \frac{n\ (n^2+1)}{2}$$

单选题 1分

设置

If I want to put 2,4,6….$2*4^2$ into a 4*4 grids to construct a magic square, how much is the magic sum?

A 15

B 34

C 68

D 272

提交

Existence?

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

投票 最多可选1项

设置

Do you think the magic square in 2*2 grids exists?

A Yes!

B No!

提交

Existence

- Normal magic squares exist for all orders $n \geq 1$ except $n = 2$,
  - $n = 1$ is trivial, consisting of a single cell containing the number 1.

  - n=2 magic sum: $M = \frac{n\ (n^2+1)}{2} = \frac{2\ (2^2+1)}{2} = 5$
    - {1,2,3,4} $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ a,b,c,d are different integers from{1,2,3,4}

$a + b = 5$

$c + d = 5$

$a + d = 5$

$c + b = 5$

$b - d = 0$; $b = d$ 。 **CONFLICT**

Magic square of order 2 does not exist!

HOW TO CONSTRUCT A MAGIC SQUARE?

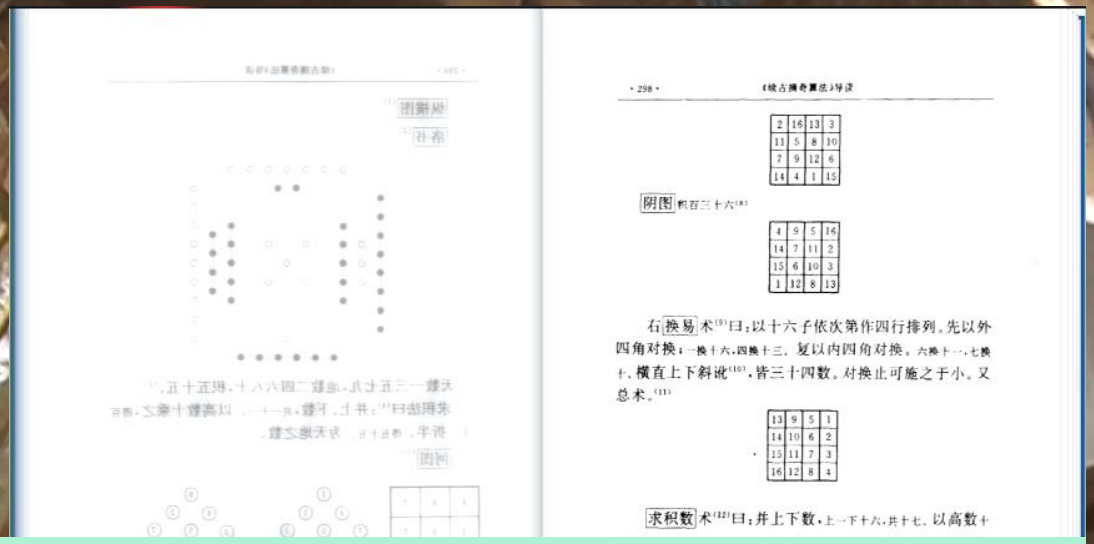
Construction

8	1	6
3	5	7
4	9	2

Given {1,2,3…9},
how to construct a magic square?

- A method for constructing magic squares of odd order was published by the French diplomat de la Loubère in his book. (Du Royaume de Siam, 1693),
  - Starting from the central column of the first row with the number 1,
  - the fundamental movement for filling the squares is diagonally up and right, one step at a time.
  - If a filled square is encountered, one moves vertically down one square instead, then continuing as before.
  - When a move would leave the square, it is wrapped around to the last row or first column, respectively.


Next we'll use this device to introduce to everyone



· 298 · 《续古摘奇算法》导读

2	16	13	3
11	5	8	10
7	9	12	6
14	4	1	15

附图 幔百三十六[8]

4	9	5	16
14	7	11	2
15	6	10	3
1	12	8	13

右换易术[9]曰：以十六子依次第作四行排列。先以外四角对换：一换十六，四换十三。复以内四角对换。六换十一，七换十。横直上下斜说[10]，皆三十四数。对换止可施之于小。又总术。[11]

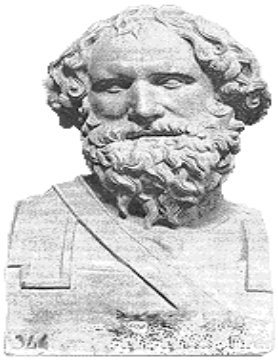
13	9	5	1
14	10	6	2
15	11	7	3
16	12	8	4

求积数术[12]曰：并上下数，上一下十六，共十七。以高数十

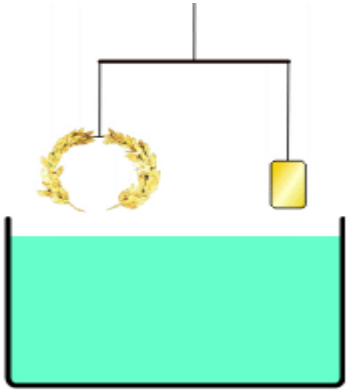
16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

So it can be seen that the secrets behind the ordering in magic squares

Archimedes



- **Archimedes (Greek** 287 BC – 212 BC**)**
  - The greatest mathematician of antiquity and one of the greatest of all time;
  - **Founder of the modern science**
  - **The Golden Crown**
    - Assess the purity of an irregular golden votive crown.
    - Balance the crown against pure gold in air, and then submerge the scale with crown and gold in water to see if they still balance.
  - **Law of lever**



Give me a place to stand on, and I will move the Earth.

http://en.wikipedia.org/wiki/Archimedes

Archimedes's scrolls

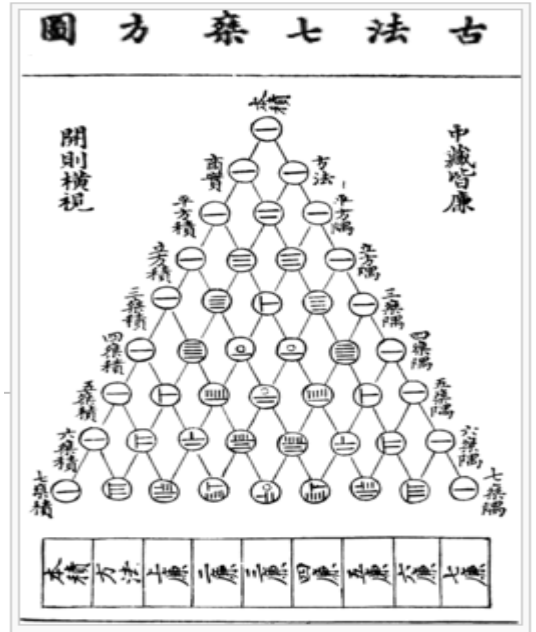

After imaging a page from the palimpsest, the original Archimedes text is now seen clearly

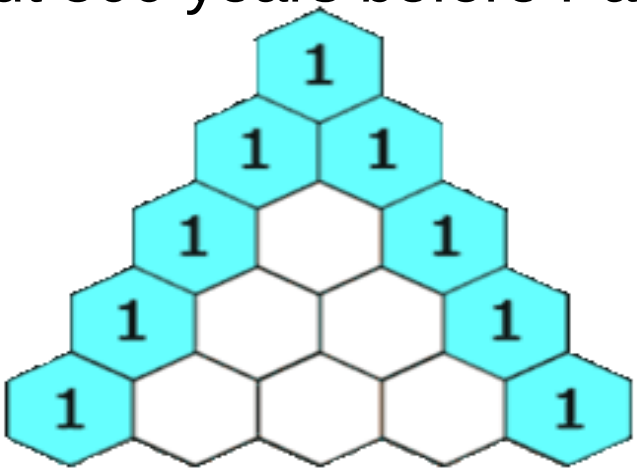
- **Archimedes Palimpsest ['pælɪm(p)sest]**
  - Copy of Archimedes work
  - Made in the 10th century
  - Original Archimedes codex was unbound, scraped and washed, along with at least six other parchment manuscripts
  - **Stomachion (Tiling)** a 14-piece dissection puzzle forming a square
    - Archimedes discussed the *number of ways* to put the pieces back in their box.
    - 17,152 if allow them to be turned over;
    - 64 – if not allow them to be turned over

http://en.wikipedia.org/wiki/Archimedes_Palimpsest

Jia Xian Triangle

- **Jia Xian** (贾宪) (approximately 1010-1070 AD)
  - Jia Xian Triangle / Pascal Triangle around the first half of 11th century, about 500 years before Pascal





Each number in the triangle is the sum of the two directly above it.

http://en.wikipedia.org/wiki/Pascal%27s_triangle

Combinatorics

Leibniz invented the name of Combinatorics in his essay *Dissertatio de arte combinatoria* (1666)

- Applications
  - Physical science (with the application of traditional mathematics)
  - Social science
  - Biological sciences
  - Information theory
  - etc.

CHINA OPEN
中国网球公开赛

01	Novak DJOKOVIC	Serbia	Singles Ranking 1
02	Rafael NADAL	Spain	Singles Ranking 2
03	Tomas BERDYCH	Czech Republic	Singles Ranking 7
04	Grigor DIMITROV	Bulgaria	Singles Ranking 8
05	Ernests GULBIS	Latvia	Singles Ranking 12
06	Richard GASQUET	France	Singles Ranking 14
07	John ISNER	United States	Singles Ranking 15
08	Marin CILIC	Croatia	Singles Ranking 16
09	Fabio FOGNINI	Italy	Singles Ranking 17
10	Tommy ROBREDO	Spain	Singles Ranking 18
11	Mikhail YOUZHNY	Russia	Singles Ranking 24
12	Leonardo MAYER	Argentina	Singles Ranking 26
13	Julien BENNETEAU	France	Singles Ranking 27
14	Ivo KARLOVIC	Croatia	Singles Ranking 29
15	Santiago GIRALDO	Colombia	Singles Ranking 31

32 Male players

61 Female players

How many matches?

2013年中国网球公开赛男单签表					
第一轮	第二轮	1/4决赛	半决赛	决赛	冠军
(1)德约科维奇 (塞尔维亚)					
罗索尔 (捷克)					
沃[illegible]斯科[illegible] (西班牙)					
[illegible]赛球员					
奎雷伊 (美国)					
[illegible] (俄罗斯)					

2013年中国网球公开赛女单下半区签表					
第一轮	第二轮	第三轮	1/4决赛	半决赛	决赛
(5)埃拉尼 (意大利)					
菲利普肯斯 (比利时)					
资格赛球员					
资格赛球员					
勒普琴科 (美国)					
沃特森 (英国)					
首轮轮空	(9)科[illegible]托娃 ([illegible]克)				
(9)科维托娃 (捷克)					
(13)利斯基 (德国)					
资格赛球员					
首轮轮空	大威廉姆斯 (美国)				
大威廉姆斯 (美国)					
雅瓦诺夫斯基 (塞尔维亚)					
科斯蒂亚 (罗马尼亚)					
汉图楚娃 (斯洛伐克)					
(4)李娜 (中国)					
(8)扬科维奇 (塞尔[illegible])					
帕芙柳琴科娃 (俄罗[illegible])					
资格赛球员					
资格赛球员					
巴瑟尔 (德国)					
资格赛球员					
梅拉德诺维奇 (法国)					
(12)纳瓦罗 (西班牙)					
(15)斯托瑟 (奥大利亚)					
萨法洛娃 (捷克)					
卡内皮 (爱沙尼亚)					
格尔格斯 (德国)					
谢淑薇 (中华台北)					
库兹涅佐娃 (俄罗斯)					

第一轮	第二轮	第三轮	1/4决赛	半决赛	决赛
(1)小威廉姆斯 (美国)					
维斯妮娜 (俄罗斯)					
斯齐亚沃尼 (意大利)					
科内特 (法国)					
基里连科 (俄罗斯)					
郑洁 (中国)					
U-拉德万斯卡 (波兰)					
(16)[illegible]勒普 (罗马尼亚)					
(1[illegible]斯[illegible]娃 (美[illegible])					
[illegible]兹尼亚克 (加拿大)					
资格赛球员					
莱巴里科娃 (斯洛伐克)					
贝克 (德国)					
尼库莱斯库 (罗马尼亚)					
首轮轮空	(6)沃兹尼亚奇 (丹麦)				
(6)沃兹尼亚奇 (丹麦)					
(3)A-拉德[illegible]斯卡 (波[illegible])					
[illegible]格 ([illegible]士)					
凯斯 (美国)					
齐布尔科娃 (斯洛伐克)					
资格赛球员					
普依格 (波多黎各)					
佩内塔 (意大利)					
(14)伊万诺维奇 (塞尔维亚)					
(10)文奇 (意大利)					
斯维托里娜 (乌克兰)					
张帅 (中国)					
彭帅 (中国)					
扎科帕洛娃 (捷克)					

Model Transform

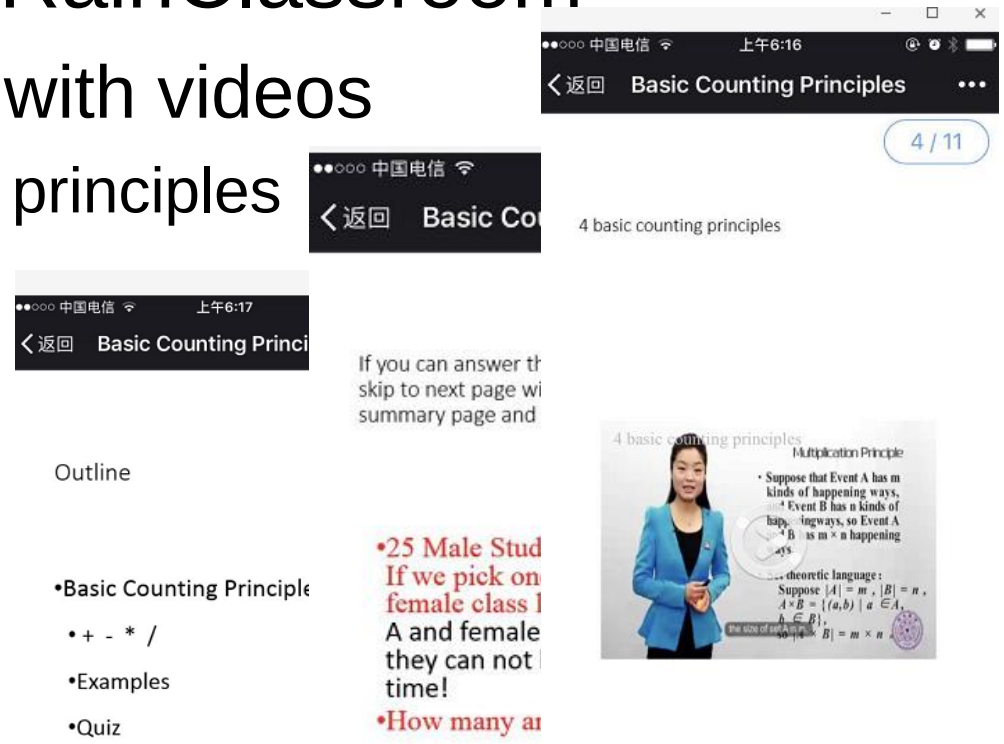
- **Q:** A large tournament has 100 entrants in total. If it is a single elimination tournament, how many matches have to be played out before the champion can be decided?
- **A:** There is a one to one correspondence between the number of matches and the number of player eliminated.
- Totally, 99 matches。

What are you expected to do?

- Get familiar with learn.Tsinghua.edu.cn
  - After-class materials
    - Slides
    - Playback or Video links posted on web
  - HW (Due on Sep.21)

Before next class(Thursday)

- Get familiar with RainClassroom
  - Pre-class Slides with videos
    - 4 basic counting principles
  - Online quizzes



Summary

Combinatorics is concerned with the

Existence, Enumeration, Analysis, and Optimization

of discrete structure.

Learn, Practice and Use Math!

Keep up with course paces

L-PUM