

Combinatorics HW 3.2: Pigeon Hole Principle

Student ID: 2020280401

Name: Sahand Sabour

Score:

1. A basket of fruit is being arranged out of apples, bananas, and oranges.

What is the smallest number of pieces of fruits that should be put in the basket in order to guarantee that either there are at least 8 apples or at least 6 bananas or at least 9 oranges?

In the worst case scenario, 7 apples, 5 bananas, and 8 oranges would be initially put in the basket. Accordingly, the next fruit that is put in the basket, regardless of its type, would satisfy one of the given conditions. Hence, the smallest number of pieces of fruit that satisfy one of the above conditions would be $8+7+5+1 = \underline{21}$.

2. Show that for any given 52 integers there exists two of them whose sum, or else whose difference, is divisible by 100.

Initially, a pigeon hole could be dedicated to each possible remainder of division by 100 (% 100); that is, 100 holes with values 0, 1, 2, ..., 99. In this case, by the pigeon hole principle, we have that if we choose 101 integers, two of them would be put in the same hole. Hence, with subtraction, they cancel out each others' remainders and therefore, their difference is divisible by 100. For instance, if integers 199 and 299 are put in the '99' hole, then their difference would be 100, which is divisible by 100.

However, in this problem, only 52 integers are to be chosen. It can be observed that the the holes could be grouped together on the basis that the sum of their remainders in equal to 100. That is hole '1' could be grouped with '99', '2' with '98', etc. This results in 49 new holes in the form of ['1','99'],

['2', '98'], and so on. However, two holes from the previous step still remain as they could not be grouped with any other hole to form a sum of 100: '0' and '50'. Hence, there would be 51 holes in total. According to the pigeon principle, by having 52 integers and 51 holes, there's at least one hole with two integers. If the two integers are in one of the 49 holes, then their sum would have a remainder of 100, which makes them divisible by 100. In addition, if the two integers are in either '0' or '50', their sum would still give a remainder of 0 or 100, which both show that the sum is divisible by 100. **Hence, it can be concluded that for any given 52 integers, there are at least two integers whose sum or difference is divisible by 100.**