Combinatorics HW 2.1

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1. **How many different permutations for word"Combinatorics"? (Case sensitive)**

   The word "Combinatorics" consists of 13 letters. Accordingly, the total number of different permutations, considering each letter as distinct, would be 13!. However, letters "i" and "o" have been repeated twice. It should be noted that letters "C" and "c" are not considered a repetition as the problem statement is looking for case sensitive permutations. Therefore, the total number of permutations should be divided by 2!2!. Hence, there exists $\frac{13!}{2!2!} = \underline{1556755200}$ different permutations for the word "Combinatorics".

2. **The coefficient number of $a^2b^2c^2$ in the expanded equation of $(2a+b+c)^6$ is ____。 [Please calculate the exact number]**

   Let's assume d = 2a. Since the power of the equation is 6, then we can think of each element of the expanded equation as $------$, where each empty location could be d, b or c and the final product would be the multiplication of these elements. For instance, ddbbcd gives d^3b^2c. Hence, the number of total permutations, assuming that each occurrence of these variables is distinct, would be 6!. In addition, in order to have $d^2b^2c^2$, we have to choose two locations from the six available locations for each variable d, b, and c; meaning that each of these variables would be repeated twice. Hence, the total number of permutations for the term $d^2b^2c^2$ would be $\frac{6!}{2!2!2!} = 90.$ This values represents the coefficient of this term as it records the total number of its permutations in this equation. Replacing d with 2a in $90d^2b^2c^2$ would give

$$90(2a)^2b^2c^2 = 360a^2b^2c^2$$

Therefore, the coefficient number of $a^2b^2c^2$ is $\underline{360}$.

3. **For the case of giving fruits to 3 kids, in total there are 12 identical apples, each child may at least have one apple, how many different ways to give apples to 3 kids?**

Assuming that the 3 kids are represented by variables x_1, x_2, and x_3 gives

$$x_1 + x_2 + x_3 = 12$$

Where $x_1 \geq 1, x_2 \geq 1, and\ x_3 \geq 1$. Hence, we can rewrite the equation by setting $y_i = x_i - 1$ as below

$$y_1 + y_2 + y_3 = 12 - 1 - 1 - 1 = 9$$

Where y_i is non-negative and $x_i \geq 1$. The number of solutions is

$$\mathrm{C(r + k - 1, r)} = \binom{9 + 3 - 1}{9} = \binom{11}{9} = \frac{11!}{9!\,2!} = 55$$

Therefore, there are 55 different ways to distribute 12 identical apples between 3 kids, in which each kid gets at least one apple.

4. **What is the number of integral solutions of the equation $x_1+x_2+x_3=30$, in which $x_1 \geqslant 5$, $x_2 \geqslant -8$, $x_3 \geqslant 5$.**

Similar to the approach in the previous question, we can have

$$y_1 = x_1 - 5$$

$$y_2 = x_2 + 8$$

$$y_3 = x_3 - 5$$

Where y_i is non-negative and x_i meets the required constraints. By replacing x in the original equation with y, we get

$$y_1 + y_2 + y_3 = 30 - 5 + 8 - 5 = 28$$

The number of integral solutions would be

$$\mathrm{C(r + k - 1, r)} = \binom{28 + 3 - 1}{28} = \binom{30}{28} = \frac{30!}{28!\,2!} = 435$$

Hence, there are 435 integral solutions for the equation with its constraints.