EEE203 Assignment 1 Solution:

Q1

We are given Figure S3.4-1.

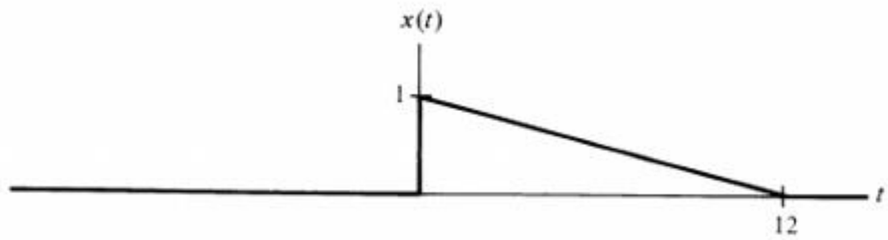

Figure S3.4-1

$x(-t)$ and $x(1-t)$ are as shown in Figures S3.4-2 and S3.4-3.

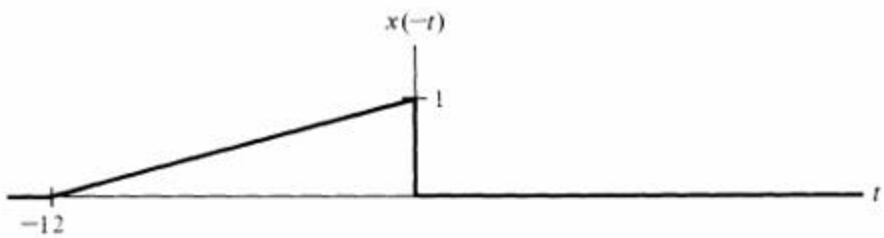

Figure S3.4-2

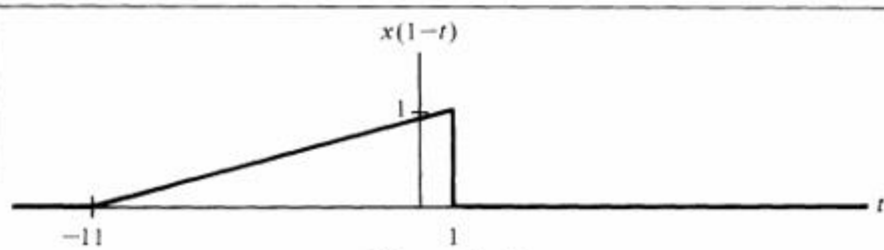

Figure S3.4-3

(a) $u(t+1) - u(t-2)$ is as shown in Figure S3.4-4.

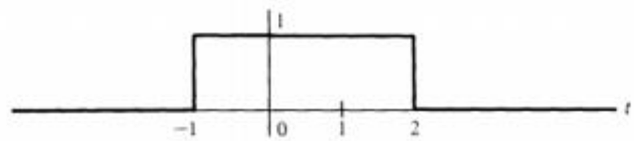

Figure S3.4-4

Hence, $x(1-t)[u(t+1) - u(t-2)]$ looks as in Figure S3.4-5.

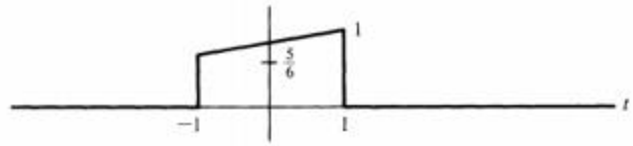

Figure S3.4-5

(b) $-u(2-3t)$ looks as in Figure S3.4-6.

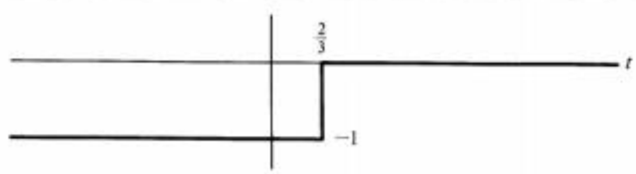

Figure S3.4-6

Hence, $u(t+1) - u(2-3t)$ is given as in Figure S3.4-7.

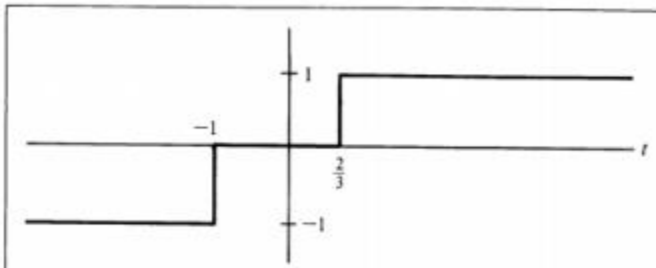

Figure S3.4-7

So $x(1-t)[u(t+1) - u(2-3t)]$ is given as in Figure S3.4-8.

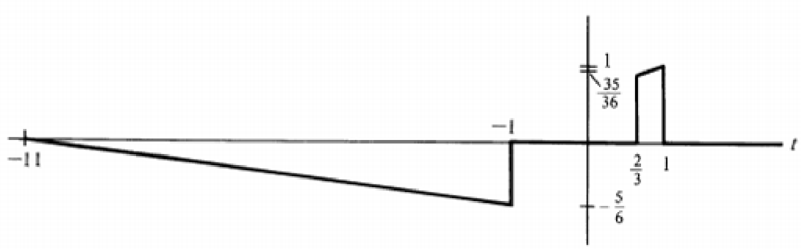

Figure S3.4-8

Q2

2. (a) $y(t) = x(t)u(t)$

$y_1(t) = x_1(t)u(t)$ $\quad y_2(t) = x_2(t)u(t)$

↳ $y(t) = u(t)[ax_1(t) + bx_2(t)] = ax_1(t)u(t) + bx_2(t)u(t) = ay_1(t) + by_2(t)$

∴ Linear.

$y(t-\tau) = x(t-\tau)u(t-\tau) \neq x(t-\tau)u(t)$ ∴ not Time-invariant

∵ t ∴ memoryless, depends on current ∴ causal

(b) $y(t) = x(1-t)$

$y_1(t) = x_1(1-t)$ $\quad y_2(t) = x_2(1-t)$

$y(t) = ax_1(1-t) + bx_2(1-t) = ay_1(t) + by_2(t)$ ∴ Linear

$y(t-\tau) = x(1-t+\tau) \neq x(1-t-\tau)$ ∴ not Time-invariant

eg. $y(-1) = x(2)$ ∴ depend on the future ∴ not causal

(c) $y(t) = x(2t)$

$y_1(t) = x_1(2t)$ $\quad y_2(t) = x_2(2t)$

∴ $y(t) = ax_1(2t) + bx_2(2t) = ay_1(t) + by_2(t)$ ∴ Linear

$y(t-\tau) = x(2t-2\tau) \neq x(2t-\tau)$ ∴ not Time invariant

eg. $y(\frac{1}{2}) = x(1)$ ∴ depend on future ∴ not causal

(d) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

$y_1(t) = \int_{-\infty}^{t} x_1(\tau)\,d\tau$ $\quad y_2(t) = \int_{-\infty}^{t} x_2(\tau)\,d\tau$

∴ $y(t) = a\int_{-\infty}^{t} x_1(\tau)\,d\tau + b\int_{-\infty}^{t} x_2(\tau)\,d\tau = \int_{-\infty}^{t}[ax_1(\tau) + bx_2(\tau)]\,d\tau = ay_1(t) + by_2(t)$

∴ Linear

$y(t-\tau_1) = \int_{-\infty}^{t-\tau_1} x(\tau)\,d\tau = \int_{-\infty}^{t-\tau_1} x(\tau)\,d\tau$ ∴ Time-invariant

∵ depend on the current and previous ∴ causal

Q3

(a)

The convolution can be evaluated by using the convolution formula. The limits can be verified by graphically visualizing the convolution.

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(\tau)h\,(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} e^{-(\tau-1)}u(\tau-1)u(t-\tau+1)d\tau \\
&= \begin{cases} \int_{1}^{t+1} e^{-(\tau-1)}d\tau, & t>0, \\ 0, & t<0, \end{cases}
\end{aligned}
$$

Let $\tau' = \tau - 1$. Then

$$
y(t) = \begin{cases} \int_{0}^{t} e^{-\tau'}d\tau' \\ 0 \end{cases} = \begin{cases} 1-e^{-t}, & t>0, \\ 0, & t<0 \end{cases}
$$

(b)

The convolution can be evaluated graphically or by using the convolution formula.

$$
y(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau-2)\,d\tau = x(t-2)
$$

So $y(t)$ is a shifted version of $x(t)$.

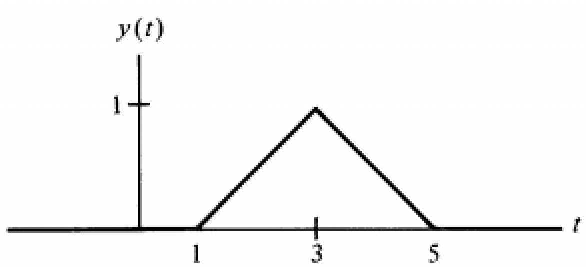

Figure S4.3-2

Q4

$$\mathbf{r}(\mathbf{t}) = \mathbf{e}(\mathbf{t}) * \mathbf{h}_1(\mathbf{t}) + \mathbf{e}(\mathbf{t}) * \mathbf{h}_2(\mathbf{t}) * \mathbf{h}_1(\mathbf{t}) * \mathbf{h}_3(\mathbf{t})$$

$$= \mathbf{e}(\mathbf{t}) * \left[\mathbf{h}_1(\mathbf{t}) + \mathbf{h}_2(\mathbf{t}) * \mathbf{h}_1(\mathbf{t}) * \mathbf{h}_3(\mathbf{t}) \right]$$

$$= \mathbf{e}(\mathbf{t}) * \mathbf{h}(\mathbf{t})$$

$$\therefore \mathbf{h}(\mathbf{t}) = \mathbf{h}_1(\mathbf{t}) + \mathbf{h}_2(\mathbf{t}) * \mathbf{h}_1(\mathbf{t}) * \mathbf{h}_3(\mathbf{t})$$

$$\mathbf{h}_1(\mathbf{t}) = \mathbf{u}(\mathbf{t}),\quad \mathbf{h}_2(\mathbf{t}) * \mathbf{h}_1(\mathbf{t}) * \mathbf{h}_3(\mathbf{t}) = \delta(\mathbf{t}-1) * \mathbf{u}(\mathbf{t}) * \left[-\delta(\mathbf{t}) \right] = -\mathbf{u}(\mathbf{t}-1)$$

$$\therefore \mathbf{h}(\mathbf{t}) = \mathbf{u}(\mathbf{t}) - \mathbf{u}(\mathbf{t}-1)$$

Q5

(a)

Note that the period is $T_0 = 6$. Fourier coefficients are given by

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

We take $\omega_0 = 2\pi/T_0 = \pi/3$. Choosing the period of integration as -3 to 3, we have

$$\begin{aligned}
a_k &= \frac{1}{6}\int_{-2}^{-1} e^{-jk(\pi/3)t}\,dt - \frac{1}{6}\int_{1}^{2} e^{-jk(\pi/3)t}\,dt \\
&= \frac{1}{6}\frac{1}{-jk(\pi/3)}e^{-jk(\pi/3)t}\Big|_{-2}^{-1} - \frac{1}{6}\frac{1}{-jk(\pi/3)}e^{-jk(\pi/3)t}\Big|_{1}^{2} \\
&= \frac{1}{-j2\pi k}[e^{+j(\pi/3)k} - e^{+j(2\pi/3)k} - e^{-j(2\pi/3)k} + e^{-j(\pi/3)k}] \\
&= \frac{\cos(2\pi/3)k}{j\pi k} - \frac{\cos(\pi/3)k}{j\pi k}
\end{aligned}$$

Therefore,

$$x(t) = \sum_k a_k e^{jk\omega_0 t}, \qquad \omega_0 = \frac{\pi}{3}$$

and

$$a_k = \frac{\cos(2\pi/3)k - \cos(\pi/3)k}{j\pi k}$$

Note that $a_0 = 0$, as can be determined either by applying L'Hôpital's rule or by noting that

$$a_0 = (1/T_0)\int_{T_0} x(t)dt.$$

(b)

The period is $T_0 = 2$, with $\omega_0 = 2\pi/2 = \pi$. The Fourier coefficients are

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

Choosing the period of integration as $-\frac{1}{2}$ to $\frac{3}{2}$, we have

$$\begin{aligned} a_k &= \frac{1}{2}\int_{-1/2}^{3/2} x(t)e^{-jk\omega_0 t}\,dt \\ &= \frac{1}{2}\int_{-1/2}^{3/2} [\delta(t) - 2\delta(t-1)]e^{-jk\omega_0 t}\,dt \\ &= \frac{1}{2} - e^{-jk\omega_0} = \frac{1}{2} - (e^{-j\pi})^k \end{aligned}$$

Therefore,

$$a_0 = -\tfrac{1}{2}, \qquad a_k = \tfrac{1}{2} - (-1)^k$$