EEE203 Lab

Continuous and Discrete Time Signals and Systems

Name: Sahand Sabour

Lab partner: Kai-Yu Lu

Student ID: 1614650

Submission Date: 2018/12/21

Abstract

This experiment is conducted to explore the theories behind filters, which have become a considerable part of modern electronics, and analyze different kinds of filters in order to further understand their functionality and characteristics. In this experiment, low-pass and band-pass filters, which are both types of active filters, are assembled, tested against several different input signal frequencies and the obtained results are recorded accordingly. Based on the obtained results, it can be concluded that the experiment was conducted successfully.

Introduction

Filters have become an essential part of modern electronics. This is due to the use of filters in order to remove unwanted signals and noise when an input signal is presented to the system. Essentially, filters can be categorized into two main categories: active and passive. Furthermore, the mentioned classification can be divided into a number of more detailed sections, which relates to the range of signals that can be used as input for the filter. In this experiment, low-pass and band-pass active filters were assembled, provided with several various input signals, and the obtained corresponding outputs were recorded and analyzed. Furthermore, a number of exercises on theories regarding filters are provided and explained in order to further explore the field of signal filters.

Experiment procedure

A) Exercises on the theory

1) *Express the following values of voltage gain in dB:*

   The following formula can be used in order to obtain the required values:

$$|T|_{dB} = 20log_{10}|A|$$

Gain	1	1/2	$\mathbf{1/\sqrt{2}}$	$\sqrt{\mathbf{2}}$	2	10	100
dB	0	-6.021	-3.01	3.01	6.021	20	40

2) *Give formulae for (a) the transfer function, (b) the frequency response, (c) the gain response and (d) the phase response of a second order Butterworth low-pass approximation with cut-off frequency 4 rad/sec.*

   Transfer function:

$$H(s) = \frac{1}{1 + 1.414\left(\frac{s}{4}\right) + (\frac{s}{4})^2}$$

Frequency response:

$$H(j\omega) = \frac{1}{1 + 1.414\left(\frac{j\omega}{4}\right) + (\frac{j\omega}{4})^2}$$

Gain response:

$$G(\omega) = \frac{1}{\sqrt{(1 + (\frac{\omega}{4})^4}}$$

Phase response:

$$\emptyset(\omega) = \arg\big(H(j\omega)\big) = \arctan\left(\frac{Im\big(H(j\omega)\big)}{Re\big(H(j\omega)\big)}\right) + (\pi, if\ RE(j\omega) < 0)$$

If 0 < ω < 4, we have:

$$\emptyset(\omega) = \arctan\left(\frac{4\sqrt{2}\,\omega}{\omega^2 - 16}\right)$$

If 4 < ω, we have:

$$\emptyset(\omega) = \arctan\left(\frac{4\sqrt{2}\,\omega}{\omega^2 - 16}\right) + \pi$$

3) *Complete the missing entries in the following table for the 4 rad/sec cut-off low-pass filter referred to above:*

ω (rad/sec)	0	0.5	1.0	1.5	4.0	10.0	20.0
Voltage gain	1	0.999	0.998	0.99	0.7	0.16	0.04
Gain in dB	0	-0.008	-0.017	-0.088	-3.09	-15.92	-27.95
Phase lead (deg)	0	-10.2	-20.7	-31.7	-90	-146	-163.6
Phase lag (rad)	0	0.18	0.36	0.56	1.6	2.55	2.86
Phase delay (sec)	0	0.361	0.361	0.361	0.41	0.26	0.14

4) *Referring to the table above, at frequencies greater than 4 rad/sec what is the reduction in gain in dB per octave (i.e. per doubling of frequency)?*

Reduction in gain = –27.95 – (–15.92) = –12.03 dB

5) *At frequencies greater than 4 rad/sec what is the reduction in gain in dB per decade (i.e. increasing the frequency 10 times)?*

The gain for when ω = 100 radians per second was calculated to be approximately –55.9 dB. Therefore, the reduction in gain = –55.9 – (–16.92) = –38.98

6) *A 1 radian/second sine wave will be delayed by about 0.8 seconds. TRUE/FALSE?*

False, as the delay for the mentioned wave would be 0.361 seconds.

7) *Is the frequency response linear phase? YES/NO?*

No, as this is a low-pass filter, it would only have linear phase for its frequency in low frequencies. However, according to the formula given in the second question, the response would tend to be approximately zero in high frequencies.

8) *Why is linear phase response desirable?*

In an ideal filter, no distortions would occur in the output signal. In order for no distortions to occur, the phase delay for each of the frequency signals, which collectively compose the input signal, should be equal. As equal phase delay for all the frequency signals requires linear phase response, for an ideal filter linear phase response is desirable.

9) *Butterworth low-pass filters of all orders have -3dB gain (relative to their gain at 0Hz) at $\omega = \omega_C$. TRUE or FALSE?*

True, since for $\omega = \omega_C$ we have

$$G(\omega) = \frac{1}{\sqrt{(1+(\frac{1}{1})^{2n}}} = \frac{1}{\sqrt{2}} rad/sec = -3.01dB$$

10) *Cascading three identical 2^{nd} order Butterworth high-pass filters produces a 6^{th} order Butterworth high-pass filter. TRUE or FALSE?*

False, since for a 2^{nd} order Butterworth high-pass filter we have

$$H(s) = \frac{1}{1 + 1.414\ s\omega_C + (\omega_C)^2}$$

Then for three identical 2^{nd} order Butterworth high-pass filters we get

$$H_1(s) = (\frac{1}{1 + 1.414\ s\omega_C + (\omega_C)^2})(\frac{1}{1 + 1.414\ s\omega_C + (\omega_C)^2})\ (\frac{1}{1 + 1.414\ s\omega_C + (\omega_C)^2})$$

Whereas, for a 6th order Butterworth high-pass filter we have

$$H_2(s) = (\frac{1}{1 + 0.518\ s\omega_C + (\omega_C)^2})(\frac{1}{1 + 1.414\ s\omega_C + (s\omega_C)^2})\ (\frac{1}{1 + 1.932\ s\omega_C + (s\omega_C)^2})$$

Giving $H_1(s) \neq H_2(s)$. Therefore, the mentioned statement is false.

11) *If a medium wave radio receiver were to be design using a band-pass ceramic filter whose pass-band is centered on 455kHz. It could be turned to different stations by multiplying the incoming signal by a sine wave of correctly chosen frequency. The effect of this multiplication is to change the frequency of the input signal to make it pass through the filter. The filter will then remove other radio stations, leaving only the one passed by the filter. We can understand this by remembering the formula:*

$$2\,cos(\omega_1 t)\,cos(\omega_2 t) = cos(\omega_1 + \omega_2)t + cos(\omega_1 - \omega_2)t$$

Assuming for simplicity that the incoming signal is a 750kHz sine-wave, what must be the frequency of the multiplying sine wave to make the frequency-changed signal pass through the ceramic filter?

Based on the problem description, since $\omega_1 - \omega_2 = 455\ kHZ$, we get that

if $\omega_1 = 750\ kHz$, then $\omega_2 = 750 - 455 = 295\ kHz$

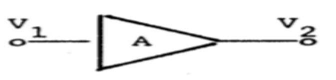
and

if $\omega_2 = 750\ kHz$, then $\omega_1 = 750 + 455 = 1205\ kHz$.

B) Laboratory exercises

Exercise 1

As the first section of the laboratory exercises, a circuit containing an operational amplifier was to be designed to provide a non-inverting amplifier with gain of 1.59. The figure below displays the symbol used for an operational amplifier in a circuit (Figure 1).



(b) Circuit symbol

Figure 1: op-amp symbol

The operational amplifier was used in a circuit design to provide the circuit shown in the following figure (Figure 2), which displays the required non-inverting amplifier design.

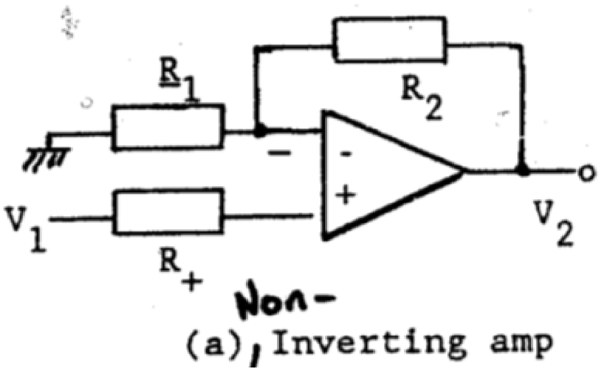


Figure 2: non-inverting amplifier

In order to achieve the required gain (gain = 1.59), as shown in the above figure, corresponding values of R_1, R_2 and R_+ were to be calculated. The necessary values were calculated as follows:

$$A_V = \frac{V_{out}}{V_{in}}$$

Where

$$V_{in} = IR_1$$

And

$$V_{out} = I(R_1 + R_2)$$

Which gives

$$A_V = \frac{I(R_1 + R_2)}{IR_1} = \frac{R_1 + R_2}{R_1} = 1.59$$

Therefore, by choosing R1 to be 10kΩ, the value of R2 can be obtained to be 5.9kΩ. However, due to the lack of 5.9kΩ in the laboratory session, 5.6kΩ were implemented in the design instead. After assembling the mentioned circuit (Figure 2), the circuit was tested with several sinusoidal inputs, which varied from 100Hz to 1MHz. The following graphs (Figures 3-7) display the output corresponding to each input frequency respectively, where the input signal is displayed in yellow while the output signal is displayed in blue.

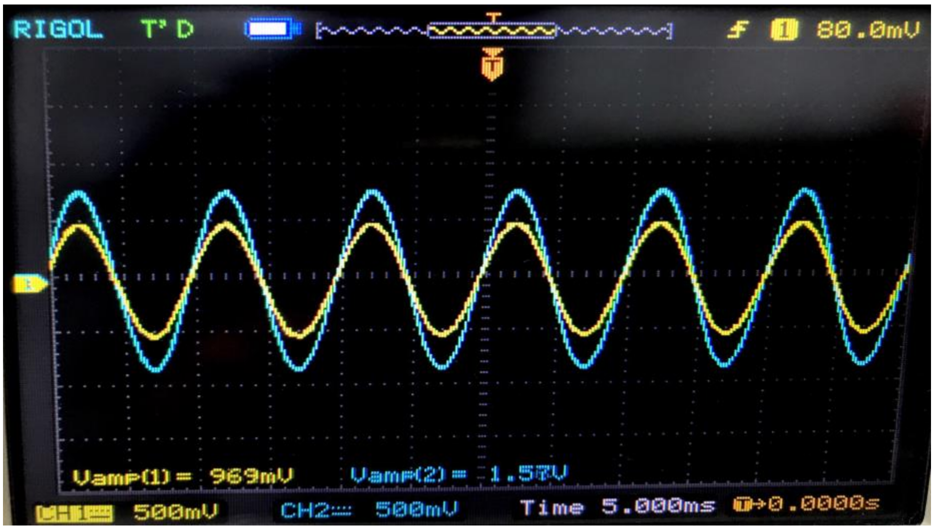


Figure 3: input and output signals' graph for 100Hz frequency

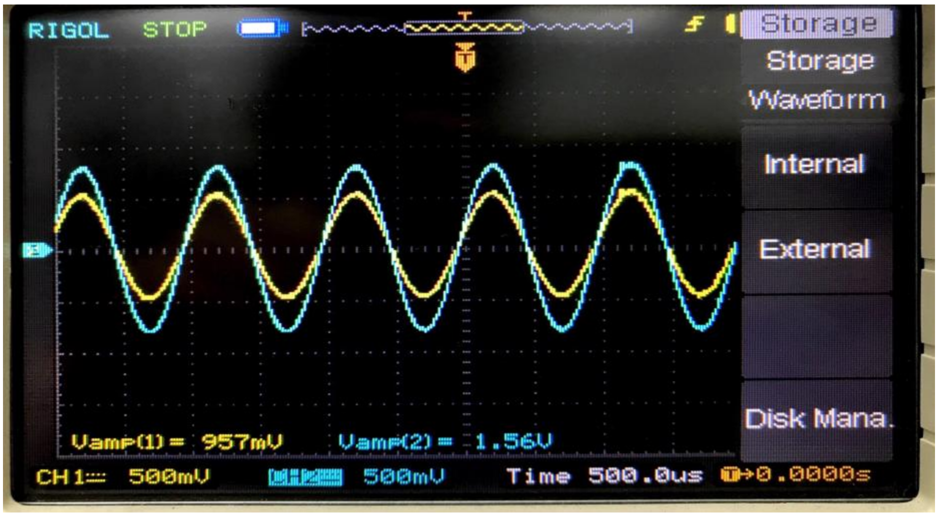


Figure 4: input and output signals' graph for 1kHz frequency

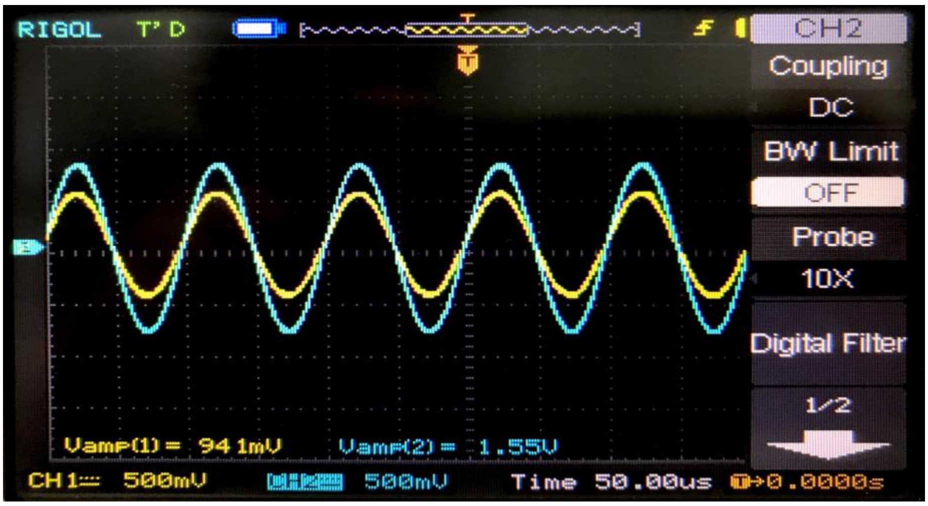


Figure 5: input and output signals' graph for 10kHz frequency

Figure 6: input and output signals' graph for 100kHz frequency

Figure 7: input and output signals' graph for 1MHz frequency

Moreover, the voltage gain for each of the mentioned input signals was recorded and is provided in the following table.

Frequency	100Hz	1kHz	10kHz	100kHz	1MHz
Voltage gain	1.62	1.63	1.64	1.7	0.32
Gain in dB	4.19	4.24	4.29	4.6	-9.89

Exercise 2

Subsequently, an active 2nd order Butterworth filter with cut-off frequency of 1592Hz was to be implemented and analyzed with a range of input frequencies. Figure 8 indicates the design to be assembled in order to obtain a 2nd order Butterworth filter.

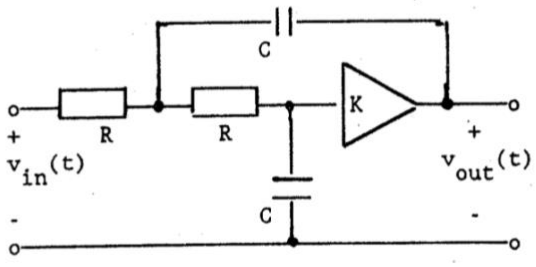


Figure 8: circuit design for 2nd order Butterworth filter

It should be mentioned that k in the above figure is the non-inverting amplifier that was assembled in the previous section. Moreover, as shown in the figure, values of R and C are to be determined in order to achieve a cut-off frequency of 1592Hz, as it is specified in the requirements of this circuit.

As for the transfer function of the 2nd order Butterworth filter we have

$$H_1(s) = \frac{1}{1 + 1.414\left(\frac{s}{\omega_c}\right) + (\frac{s}{\omega_c})^2}$$

Where ω_c is the cut-off frequency. Furthermore, based on the figure, the transfer function of the circuit can be obtained as the following:

$$H_2(s) = \frac{K}{1 + (3 - K)(RCs) + (RCs)^2}$$

Therefore, since $\omega_c = \frac{1}{RC}$ and $H_1(s) = H_2(s)$, we can conclude that

$$1.414 = 3 - K ==> K = 1.586 \;\cong 1.59$$

Additionally, as mentioned in the design specifications, cut-off frequency is required to be 1592Hz, which was calculated to be approximately 10000 radians per second (using $\omega = 2\pi f$). Therefore, by choosing R to be 10kΩ, C would be obtained as 10nF (using $\omega_c = \frac{1}{RC}$).

After assembling the circuit based on the obtained values, a number of input frequencies were provided to the circuit and the output was recorded as a result. The following graphs (Figures 9-14) display the gathered results, where the input signal is indicated by yellow and the output signal is displayed in blue.

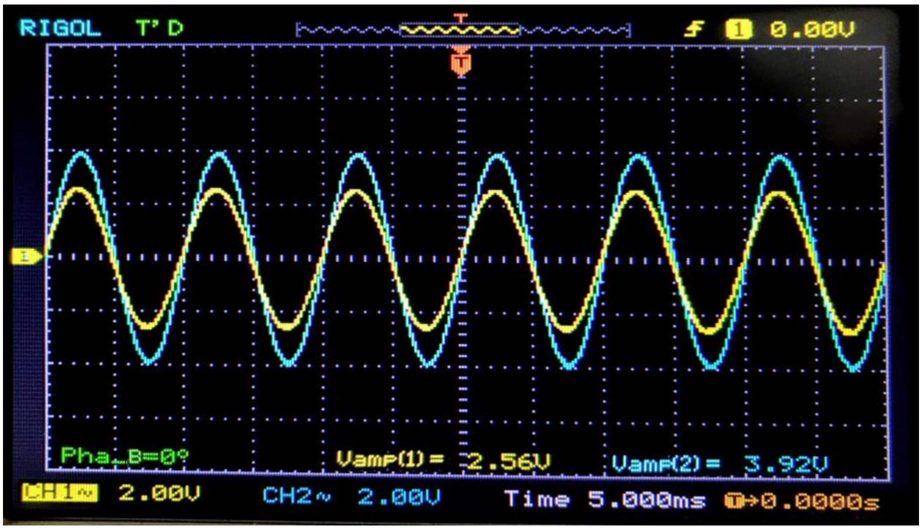


Figure 9: input and output signals' graph for 100Hz frequency

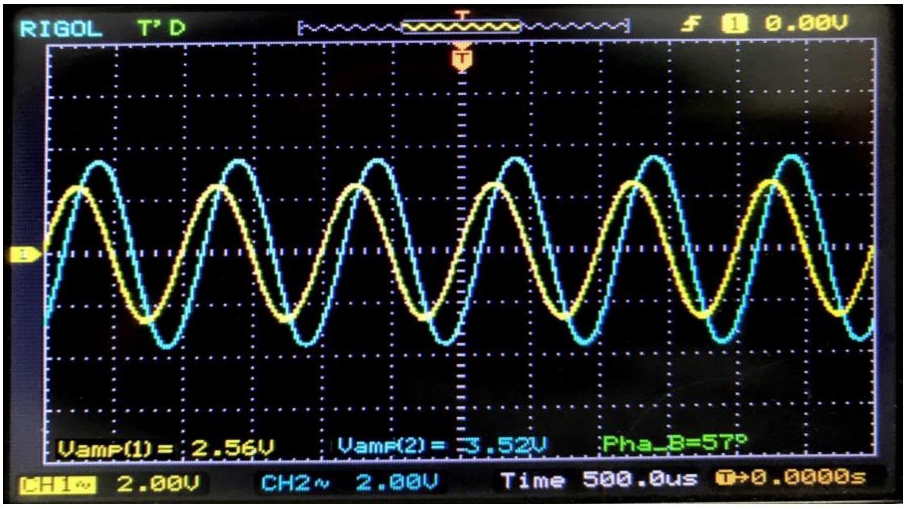


Figure 10: input and output signals' graph for 1kHz frequency

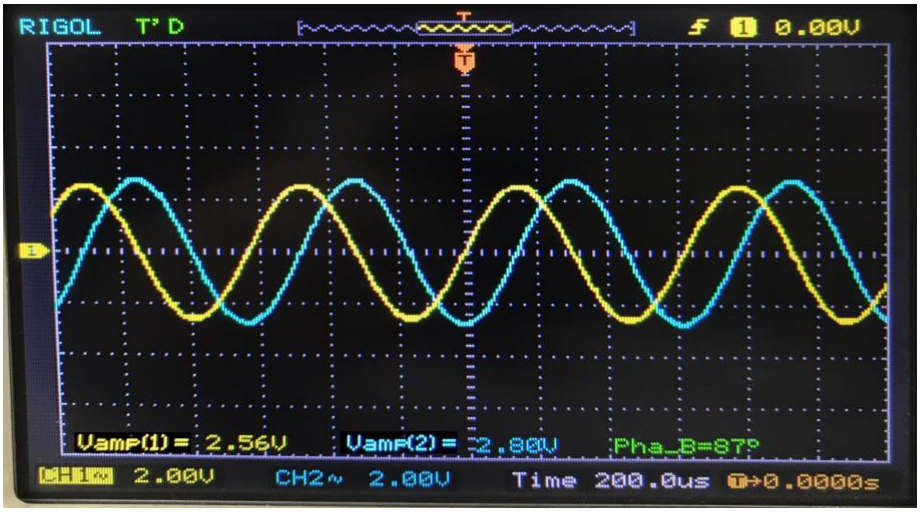


Figure 11: input and output signals' graph for 1600Hz frequency

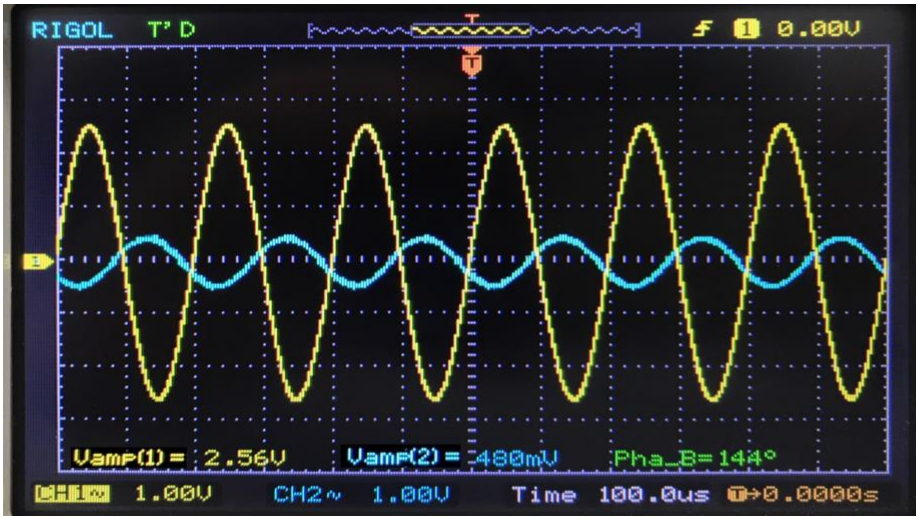


Figure 12: input and output signals' graph for 5kHz frequency

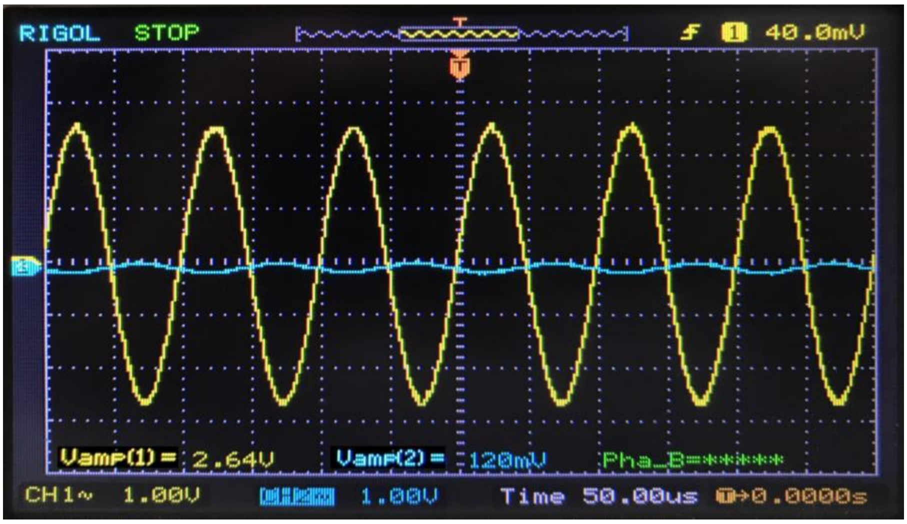


Figure 13: input and output signals' graph for 10kHz frequency

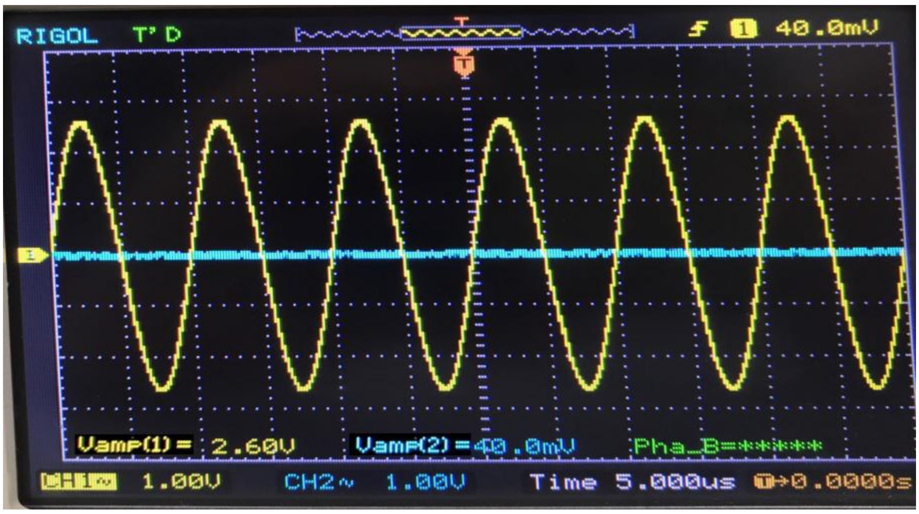


Figure 14: input and output signals' graph for 100KHz frequency

Moreover, the voltage gains for the circuit corresponding to each input frequency signal were recorded and are provided respectively in the table below:

Frequency	100Hz	1kHz	1600Hz	5kHz	10kHz	100kHz
Voltage gain	1.53	1.38	1.09	0.19	0.045	0.016
Gain in dB	3.69	2.79	0.75	-14.42	-26.9	-35.9
Phase lead (deg)	0	57°	87°	144°	N/A	N/A

Additionally, based on values that are provided in both the above and the below table, graphs of gain against log scale frequency and phase against linear scale frequency were plotted and are provided respectively below (Figure 15 and Figure 16).

Frequency (kHz)	0.5	1	1.5	2	2.5	3	3.5	4
Voltage gain	1.69	1.38	1.19	0.87	0.63	0.45	0.35	0.27
Gain in dB	4.5	2.79	1.51	-1.2	-4.01	-6.93	-9.12	-11.3
Phase lead (deg)	21°	32°	81°	100°	122°	132°	141°	153°

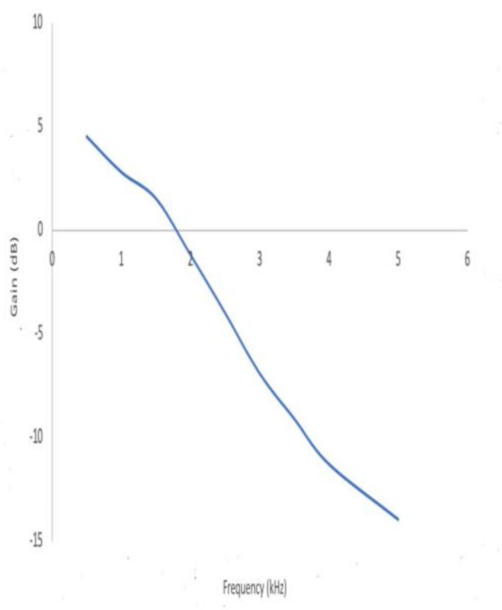


Figure 15: Gain response

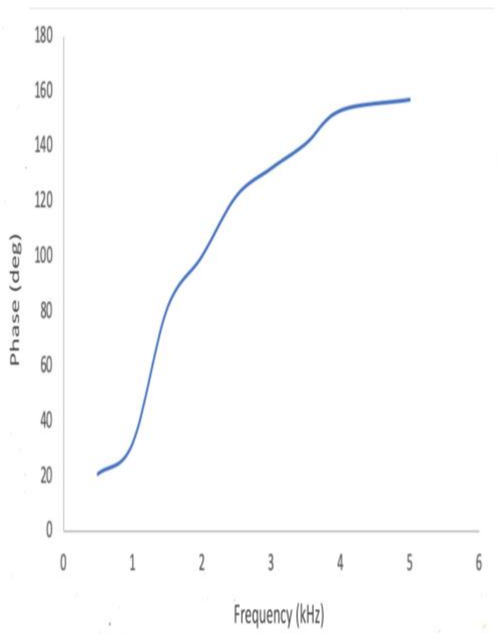


Figure 16: Phase response

Exercise 3

As the last section of this experiment, an active 2^{nd} order band-pass filter, with band pass range of 318.31Hz to 3.18kHz, was to be designed, assembled and analyzed. It was explained that the mentioned type of filter is regularly implemented for telephone applications.

In order to design the required active 2^{nd} order band-pass filter, the design of the previous sections was modified in order to achieve the desired results. The modification consisted of changing the place of capacitors and resistors with one another. The following figure (Figure 17) shows the implemented circuit design.

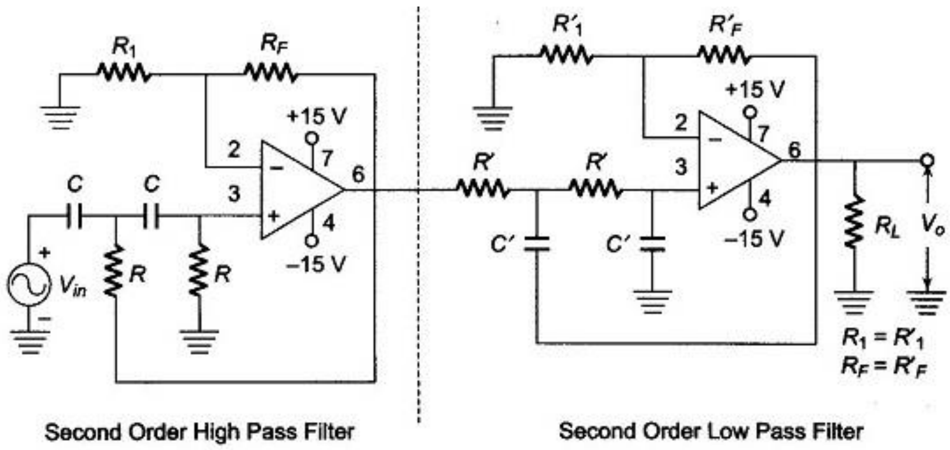


Figure 17: the 2^{nd} order band pass filter design [1]

The above figure indicates that the circuit contains a high-pass filter followed by a low-pass filter. As the band pass range is mentioned to be between 318.31Hz to 3.18kHz, it can be concluded that the cut-off frequency of 3.18kHz (20000 radians per second) should be considered for the low pass section while the cut-off frequency for the high-pass section would set to be 319.31Hz (2000 radians per second).

Accordingly, the corresponding resistor R and capacitor C values can be calculated using $\omega_c = \frac{1}{RC}$ as follows: for the low-pass section, by choosing R to be 10kΩ, C would be obtained as 5nF. Additionally, for the high-pass section, by choosing R to be 10kΩ, C would be obtained as 50nF.

By completing the design for the band pass filter, the output of the circuit was analyzed and recorded for a number of different frequencies. The following graphs (Figures 18-23) display the obtained results of each input frequency signal, where the input signal is indicated in yellow and the output signal is indicated in blue.

Figure 18: input and output signals' graph for 100Hz frequency

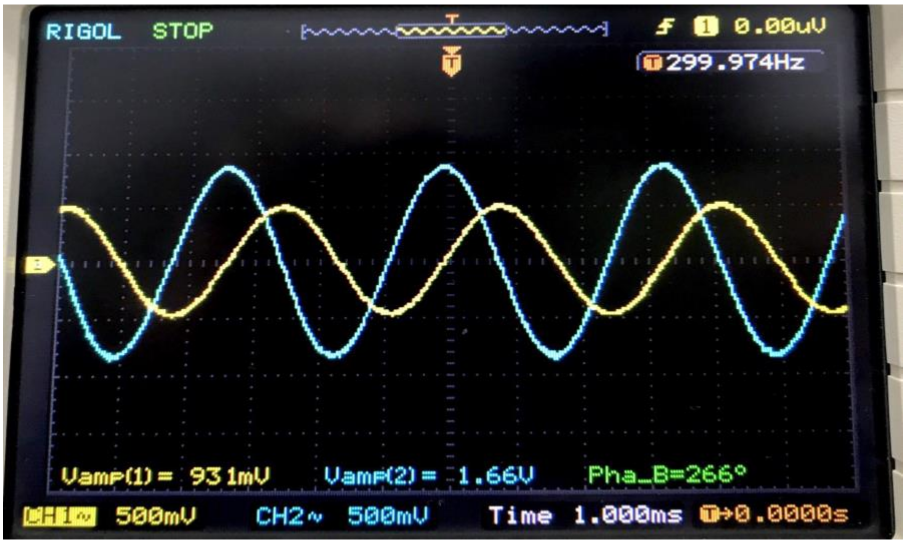


Figure 19: input and output signals' graph for 300Hz frequency

Figure 20: input and output signals' graph for 1600Hz frequency

Figure 21: input and output signals' graph for 3kHz frequency

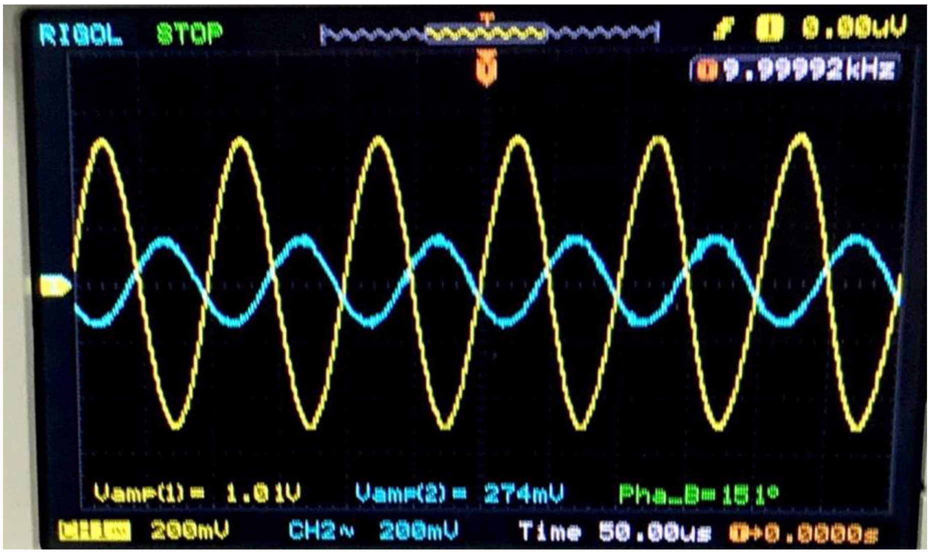


Figure 22: input and output signals' graph for 10kHz frequency

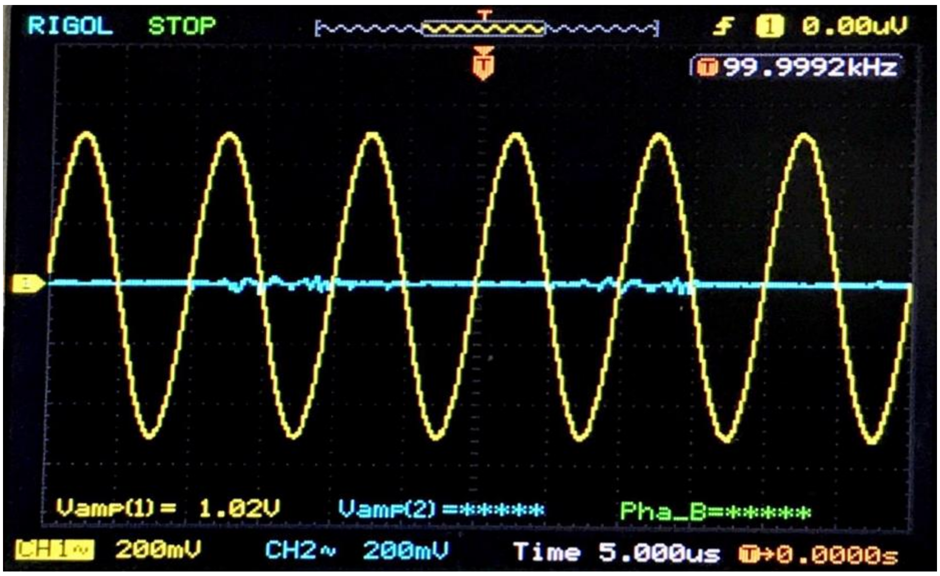


Figure 23: input and output signals' graph for 100kHz frequency

Moreover, the voltage gains for the circuit corresponding to each input frequency signal were recorded and are provided respectively in the table below:

Frequency	100Hz	300Hz	1600Hz	3kHz	10kHz	100kHz
Voltage gain	0.22	1.78	2.91	2.51	0.27	N/A
Gain in dB	-13.16	5	9.27	7.99	-11.37	N/A
Phase lead (deg)	-155°	-134°	20°	72°	151°	N/A

Additionally, based on values that are provided in both the above and the below table, graphs of gain against log scale frequency and phase against linear scale frequency were plotted and are provided respectively below (Figure 24 and Figure 25).

Frequency	0.1	0.2	0.3	0.35	0.4	0.45	0.5	3.1	3.5	4
Voltage gain	0.2	1.45	1.78	2.06	2.38	2.5	2.76	2.47	2.2	1.76
Gain in dB	-13.85	3.27	4.99	6.32	7.52	7.95	8.8	7.8	6.8	4.9
Phase lead (deg)	-152°	-105°	-92°	-78°	-62°	-57°	-44°	80°	88°	96°

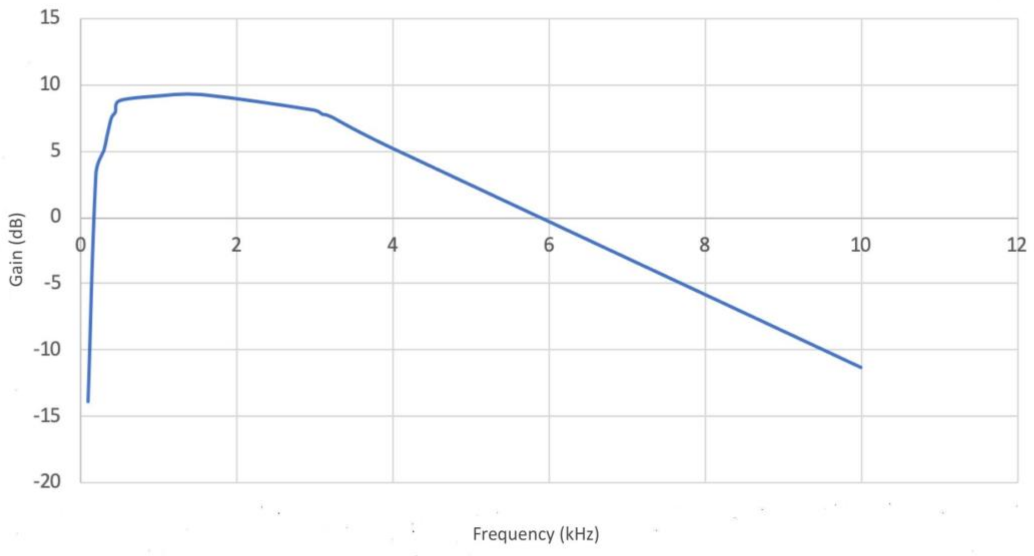


Figure 24: Gain response

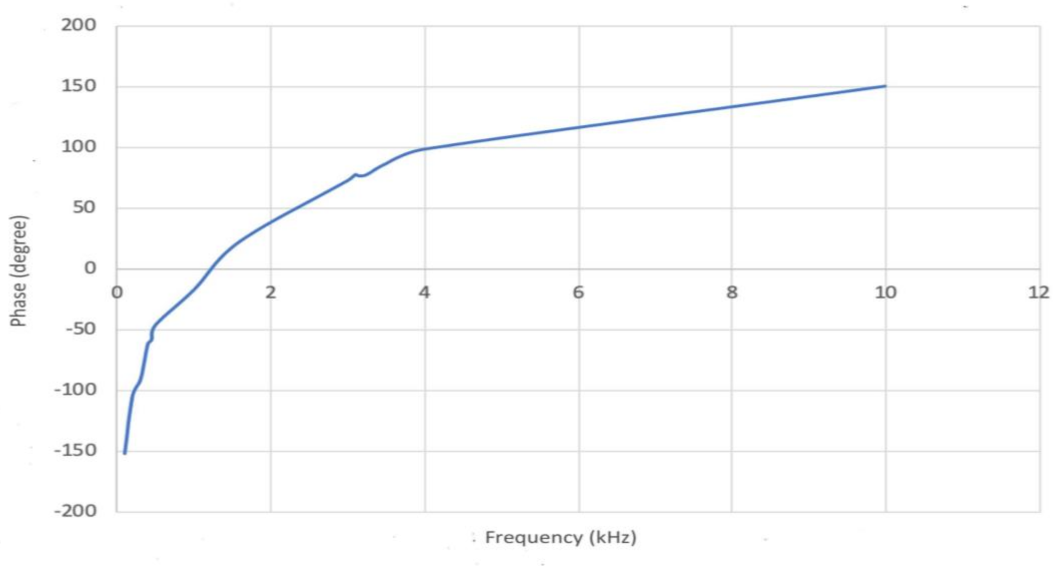


Figure 25: Phase response

Conclusion

In this experiment, operational amplifiers, low-pass, high-pass and band-pass filters were assembled and analyzed. It is believed that after conducting this experiment, valuable knowledge regarding both active filters and the lecture materials were obtained. Moreover, this experiment was believed to be a considerably great experience as it introduced filters with greater detail, compared to the EEE203 lectures, which is rather necessary in the field of electric and electronic engineering. It can be concluded that the experiment was conducted successfully and the obtained values are claimed to be accurate with small errors, due to existing laboratory and apparatus conditions.

Resources

[1] (n.d.) *Band Pass Filter Circuit* [Online]. Available: http://www.eeeguide.com/band-pass-filter-circuit/