Experiment 1

The strain gauge and its applications

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Experiment 1

The aim of this experiment is to simulate the gauge operation under the changes of the strain. The following figure (Figure 1) was provided in the lab manual, in which resistors R_1, R_2, R_3, and R_4 are 120Ω each and R_3 is connected to the switched resistor bank of the given strain gauge circuit box. This configuration is known as the Wheatstone bridge. It should be mentioned that the switched resistor bank can be introduced as a part of the circuit which consists of switches that could be used to either connect or disconnect a number of resistors in series. This would allow modification of the overall resistance R_{TH}, which is in series with R_3.

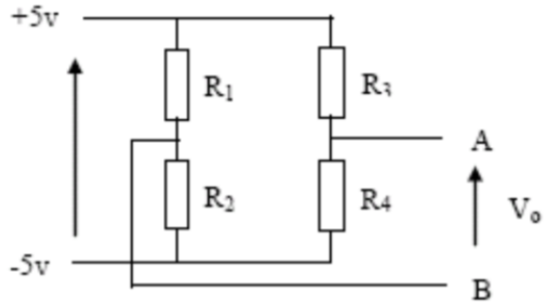

Figure 1: The Wheatstone bridge configuration

According to the figure above, since R_3 and R_4 are assembled in series, as the resistance of R3 is increased via the resistor bank, the voltage drop across R_4 decreases. Accordingly, as the output voltage V_o is the potential difference between voltages across R_2 and R_4, increasing the resistance of R_3 would decrease the output voltage likewise. Hence, the following can be derived:

$$\Delta R = E(\frac{R_2}{R_1 + R_2 + \Delta R} - \frac{R_4}{R_3 + R_4}) \tag{1}$$

Where E is the voltage across the circuit.

Since $R_1 = R_2 = R_3 = R_4 = R = 120\Omega$, we have

$$\Delta R = E(\frac{R}{2R + \Delta R} - \frac{R}{2R}) = \frac{-E\Delta R}{4R + 2\Delta R} \tag{2}$$

Assuming that $\Delta R << 2R$, the above equation can be simplified as follows

$$\Delta R = \left[\frac{E}{4}\right]\frac{\Delta R}{R} \tag{3}$$

Where $\left[\frac{E}{4}\right]$ is referred to as the sensitivity.

Questions

1. **Determine the slope of the graph, representing the sensitivity of the bridge.**

   The measured values, which were obtained from this experiment, are provided respectively in the table below:

$\Delta R(\Omega)$	$\frac{\Delta R}{R_3}$	$\Delta V(mV)$
1	0.0083	20.7
2	0.0167	41.5
3	0.0250	62.6
4	0.0334	82.4
5	0.0417	101
6	0.0500	122.9
7	0.0584	142.3
8	0.0667	162.8
9	0.0750	181.9

Table 1: Experimental values for Experiment 1

Based on the values in the above table, the graph of the output voltage against the fractional change in resistance was plotted (Figure 2).

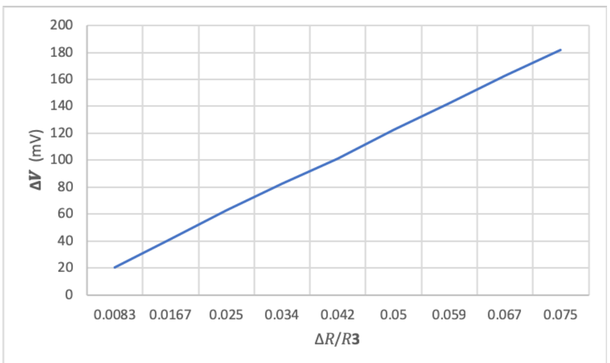

Figure 2: graph of ΔV against $\frac{\Delta R}{R_3}$

Therefore, the experimental sensitivity can be obtained by calculating the slope of the above graph. Consequently, since the graph is linear, two arbitrary points on the graph can be used in order to find the slope.

Let Point A = (0.0083, 20.7) and Point B= (0.075,181.9), then

$$Slope = \frac{(181.9 - 20.7) \times 10^{-3}}{0.075 - 0.0083} = 2.417 \tag{4}$$

Where m is the slope of the graph. Accordingly, the calculations of this section can be concluded as follows

$$\boldsymbol{ExperimentalSensitivity = 2.417}$$

2. **Calculate the theoretical sensitivity of the bridge.**

   As stated in equation (3), $\left[\frac{E}{4}\right]$ is the theoretical sensitivity of the bridge. Since the voltage across the circuit in this experiment is 10V ($E = +5V - (-5V) = 10V$), the value of the theoretical sensitivity can be calculated as the following:

$$\boldsymbol{TheoreticalSensitivity = \frac{10}{4} = 2.5}$$

3. **Comment on the result.**

   Due to the obtained value regarding the experimental sensitivity being fairly close to the theoretical value of sensitivity, it can be claimed that the experiment was rather successful. The small difference between the experimental and theoretical values is believed to be related to both the lab conditions not being standard and the assumption that was made regarding equation (3). Hence, the mentioned error can be ignored.

Experiment 2

Part A

The gauge resistance is known to change in according to changes in the strain. However, it is also known to be affected by change of temperature. This experiment aims to remove the error caused by the change in temperature. In order to achieve the mentioned condition, an additional gauge, also known as the temperature compensation gauge, is added to the configuration. Consequently, the following equations can be derived:

$$R_g = R_3(1+x)(1+y) \tag{5}$$

$$R'_g = R_3(1+y) \tag{6}$$

Where R_g and R'_g are the gauge resistances on R_3 and R_4 (check Figure 1) respectively. Moreover, x refers to the fractional change in the strain while y refers to the fractional change in temperature. Hence, the output voltage of the bridge is obtained as follows:

$$V_o = \frac{E.R_g}{R_g + R'_g} - \frac{E}{2} \tag{7}$$

$$V_o = E\Big[\frac{R_3(1+x)(1+y)}{R_3(1+x)(1+y) + R_3(1+y)} - \frac{1}{2}\Big] \tag{8}$$

Therefore, it can be noticed that

$$V_o = \frac{E.x}{2(2+x)} \tag{9}$$

According to equation 9, it is demonstrated that y, which indicates the change in the temperature, does not affect the output voltage of the bridge. Furthermore, if an accuracy of 0.001 is assumed, the following equation can be obtained:

$$V_o = \frac{E.x}{4} = \frac{E.G.\varepsilon}{4} \tag{10}$$

Questions

1. **Why is the temperature compensation used?**

   As previously mentioned and concluded by equation 9, the presence of a temperature compensation gauge would remove the effect of temperature on the change in the gauge output. Consequently, the measured output values would not suffer from any errors regarding temperature and would merely depend on the changes in the strain.

2. **Is there any variation in the temperature? And why?**

   There are indeed variations in the temperature. The reason is believed to be due to the difference between the temperature in the university's laboratory and the place in which the gauge was produced.

Part B

In this section, the temperature compensation gauge from Part A is replaced by the strain gauge on the bottom side of the beam. With this configuration, both sides of the beam would be operating. Hence, the following equations can be derived:

$$R_{g1} = R_3(1 + x) \tag{11}$$

$$R_{g2} = R_3(1 - x) \tag{12}$$

Where R_{g1} and R_{g2} are the gauge resistances of the upper and lower sides of the beam respectively. R_3 is the resistor displayed in Figure 1 and x indicates the fractional change in the strain.

$$V_o = \frac{E.R_{g1}}{R_{g1} + R_{g2}} \tag{13}$$

$$V_o = \frac{E.R_3(1 + x)}{R_3(1 + x) + R_3(1 - x)} = \frac{E}{2}(1 + x) \tag{14}$$

Hence, the bridge's output voltage would be obtained using the following equation:

$$V_o = \frac{E.x}{2} = \frac{E.G.\varepsilon}{2} \tag{15}$$

It can be noticed that, in comparison with the first section, the output voltage has doubled in value (compare equations 10 and 15).

Questions

1. **Compare the findings of the bridge output against displacements for the two parts A and B. And the relative outputs with theory.**

   The following values were obtained as a result of conducting the first section of this experiment (Table 2).

$\Delta R(\Omega)$	$\Delta V(mV)$	$y(\mu m)$
0.0	0.046	0
0.5	0.217	76
1.0	0.486	163
1.5	0.768	236
2.0	1.042	306
2.5	1.304	391
3.0	1.568	443

Table 2: Experimental values for Experiment 2 - Part A

Furthermore, the obtained values from the second section of the experiment were recorded in the following table (Table 3).

$\Delta R(\Omega)$	$\Delta V(mV)$	$y(\mu m)$
0	0.068	0
0.5	0.477	74
1	1.032	159
1.5	1.562	236
2	2.102	312
2.5	2.643	396
3	3.179	467

Table 3: Experimental values for Experiment 2 - Part B

Consequently, the above values were used in order to plot the graph of the output voltage against the displacement (Figure 3).

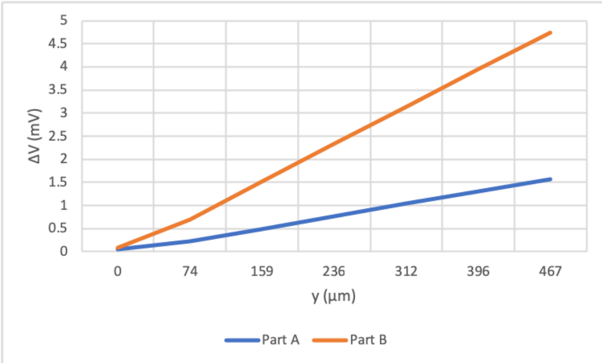

Figure 3: graph of ΔV versus the displacement y

It can be concluded that based on the obtained results, the experimental values suggest that with similar displacement, the output voltage has approximately doubled in the latter case.

From the theoretical point of view, equations 4 and 5 can be used to demonstrate the mentioned increase.

$$V_{o(PartA)} = \frac{E.x}{4} = \frac{E.G.\varepsilon}{4} \tag{16}$$

$$V_{o(PartB)} = \frac{E.x}{2} = \frac{E.G.\varepsilon}{2} \tag{17}$$

Giving

$$V_{o(PartB)} = 2V_{o(PartA)} \tag{18}$$

It can be seen that both the experimental and theoretical approaches suggest that the output voltage would be doubled when both sides of the beam are used.

2. **For your best set of measurements:**

   (a) **Calculate the strain**

   The strain can be calculated via the following equation

$$\varepsilon = \frac{4hy}{l^2} \tag{19}$$

Where h is the height of the beam perpendicular to the bending axis, y is the displacement, and l is the distance between the loading points of the beam. An arbitrary mass (3Kg) was chosen for which the strain would be calculated. As the displacement for the mentioned mass had small difference in sections A and B, the average displacement of these sections is used to calculate the strain.

$$\varepsilon = \frac{4(6.27mm)(455\mu m)}{(20cm)^2} \tag{20}$$

$$\boldsymbol{\varepsilon = 2.85 \times 10^{-4}} \tag{21}$$

(b) **Calculate the gauge factor (G)** Since we have that

$$V_{o(PartA)} = \frac{E.G.\varepsilon}{4} \tag{22}$$

and

$$V_{o(PartB)} = \frac{E.G.\varepsilon}{2} \tag{23}$$

Similar to previous question, an arbitrary mass (3kg) can be used in order to calculate the gauge factor.

$$G = \frac{4 \times V_{o(PartA)}}{E.\varepsilon} = \frac{2 \times V_{o(PartB)}}{E.\varepsilon} \tag{24}$$

$$G = \frac{4 \times (1.568mV)}{(10)(2.85 \times 10^-4)} = \frac{2 \times (3.179mV)}{(10)(2.85 \times 10^-4)} \tag{25}$$

$$\boldsymbol{G = 2.23} \tag{26}$$

(c) **What is the effect of using multiple strain gauges on sensitivity?**

As the slope of the graph for output voltage against displacement is the sensitivity of the bridge, Figure 3 can be used to calculate the required sensitivity. Hence, two arbitrary points on each graph are chosen in order to obtain the slope of each graph respectively. Let Point A = (0,0.046) Point B = (443,1.568), then

$$Slope_{PartA} = \frac{(1.568 - 0.046) \times 10^{-3}}{(443 - 0) \times 10^{-6}} = 3.43 \tag{27}$$

Let Point C = (0,0.045) and Point D = (467, 3.179), then

$$Slope_{PartB} = \frac{(3.179 - 0.045) \times 10^{-3}}{(467 - 0) \times 10^{-6}} = 6.71 \tag{28}$$

Sensitivity for Part A=3.43

Sensitivity for Part B=6.71

It can be seen that the use of multiple strain gauges has increased the sensitivity (approximately doubled).

Experiment 4

This experiment intends to investigate the distribution of the strain supports with a two-point loaded beam and determine the Young's modulus. In this section, the same configuration as the previous section is used, with the slight difference that a pair of gauges would be located near the pivots rather than a single gauge in the middle of the beam. In order to obtain an insight on what the Young's modulus is, Hooke's law must first be investigated. According to Hooke's law, we have that

$$\sigma = Y.\varepsilon \tag{29}$$

Where σ, ε, and Y refer to strain, stress, and Young's modulus respectively. Furthermore, stress can also be obtained, using the loading conditions and geometry of the beam, via the following equation:

$$\sigma = \frac{3W(L - l)}{h^2 b} \tag{30}$$

where W is the weight of the load ($= mg$), L is the distance between pivots, l is the distance between supports, h is the height of the beam perpendicular to the bending axis, and b is the width of the beam.

Questions

1. **Calculate the strain for the corresponding load and show sample calculation.**

   In order to calculate the strain for each load, the following equation can be used.

$$\varepsilon = \frac{2V_o}{G.E} \tag{31}$$

Since the same gauge as previous sections is employed in this experiment, the gauge factor of 2.23, which was calculated in the previous experiment, can be used. Hence, by measuring the output voltage corresponding to each load, the strain corresponding to each load can be obtained likewise. Conclusively, the obtained values were recorded and provided respectively in the table below:

Mass	$\Delta V(mV)$	ε
0.0	0.0	0
0.5	0.5	0.44×10^{-4}
1.0	1.2	1.07×10^{-4}
1.5	1.6	1.43×10^{-4}
2.0	2.2	1.97×10^{-4}
2.5	2.7	2.42×10^{-4}
3.0	3.27	2.93×10^{-4}

Table 4: The strain corresponding to each load

A sample calculation for when mass is chosen as 3Kg is as follows:

$$\varepsilon = \frac{2 \times (3.27mV)}{(2.23) \times (10V)} = 2.93 \times 10^{-4} \quad (32)$$

2. **Compute stresses by using Equation (13) from the known loading conditions. Show a sample calculation.**

   Equation 30 can be used to calculate the stress. The calculated values of stress corresponding to each load is provided respectively in the table below:

Mass	$\Delta V(mV)$	σ
0	0	0
0.5	0.5	0.67×10^{7}
1	1.2	1.14×10^{7}
1.5	1.6	1.72×10^{7}
2	2.2	2.29×10^{7}
2.5	2.7	2.87×10^{7}
3	3.27	3.44×10^{7}

Table 5: The stress corresponding to each load

A sample calculation for when mass is chosen as 3Kg is provided below:

$$\sigma = \frac{3 \times (3Kg) \times 9.8 \times (59cm - 20cm)}{(6.27mm)^2 \times (25.4cm)} = 3.44 \times 10^7 \tag{33}$$

3. **Plot a graph of stress versus strain.**

   According to the values obtained from the experiment and provided in the previous two questions, the following graph (Figure 4) was plotted.

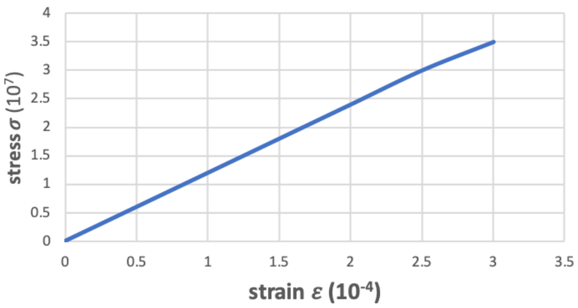

Figure 4: The Wheatstone bridge configuration

4. **Determine the value of Young's modulus for the beam material and compare your result with the $\boldsymbol{Y = 1.05 \times 10^{11} Nm^2}$ for brass.**

   As previously mentioned (check equation 29), Young's modulus is obtained by $Y = \varepsilon/\sigma$, which is equal to the slope of the plotted graph (Figure 4). Hence, two arbitrary points on the graph can be used to calculate the slope.

   Let Point A = (0,0) and Point B = (2.93, 3.44), then

$$Y = \frac{(3.44 - 0) \times 10^7}{(2.93 - 0) \times 10^{-4}} = 1.174 \times 10^{11} \tag{34}$$

   Therefore, the Young's modulus is calculated to be $1.174 \times 10^{11} Nm^2$, which is considerably close to the provided value for the Young's modulus of brass ($1.05 \times 10^{11} Nm^2$).

5. **How does the strain measured near the pivots compare with that found for the same loading near the center in Experiment 2-Part B?**

Using the same procedure as in the first question, the strain for Experiment 2-Part B can be calculated. The required values are calculated and provided respectively below.

Mass	ε near the center	ε near the pivots
0	0	0
0.5	0.46×10^{-4}	0.44×10^{-4}
1.0	1.12×10^{-4}	1.07×10^{-4}
1.5	1.52×10^{-4}	1.43×10^{-4}
2.0	2.03×10^{-4}	1.97×10^{-4}
2.5	2.46×10^{-4}	2.42×10^{-4}
3.0	3.01×10^{-4}	2.93×10^{-4}

Table 6: Values of the strain for Experiments 2-B and 4

The obtained results seem to suggest that the strain is rather greater near the center of the beam compared to strain measured near the pivots. Hence, it is demonstrated that the changes in strain occur more intensely towards the center.

Experiment 5

This experiment investigates the change in the strain pattern in a beam with a hole compared to the previously employed beam (without hole).

Questions

1. **Compare between:**

   (a) **The two strains at those two positions and explain the reason.**
   Since the same beam as previous sections is used in this experiment, the same Gauge factor, which was calculated to be 2.23, can be used to calculate strain at two positions.
   By equation 10 we have that

$$\varepsilon = \frac{4 \times V_o}{E.G} \tag{35}$$

   Where $V_{o(A)}$ and $V_{o(B)}$ with load of 3Kg were measured to be 1.64mV and 3.67mV respectively. Therefore,

$$\varepsilon_A = \frac{4 \times 1.64mV}{10V \times 2.23} = 2.94 \times 10^{-4} \tag{36}$$

$$\varepsilon_B = \frac{4 \times 3.67mV}{10V \times 2.23} = 6.58 \times 10^{-4} \tag{37}$$

Hence, it is demonstrated that strain is considerably greater when the weights are placed near the pivots.

(b) **Strain under these conditions with that at the uniform beam at the same loading.**

Based on the previous results (check Experiment 2-B and previous question), the values of strain for beams with and without holes were as follows:

$$\varepsilon_{\text{without hole}} = \mathbf{2.85 \times 10^{-4}} \tag{38}$$

$$\varepsilon_{\text{with hole}} = \mathbf{6.58 \times 10^{-4}} \tag{39}$$

It is shown that strain for when a hole is present in the beam is considerably higher compared to the beam without a hole.

2. **Find the stress concentration factor K at the center of the beam (show calculations), defined as**

$$K = \frac{\sigma_{max}(\text{beam with a hole})}{\sigma_{max}(\text{beam without a hole})} \tag{40}$$

According to the measured values (Equations 38 and 39), the stress concentration factor is

$$\boldsymbol{K} = \frac{\mathbf{6.28 \times 10^{-4}}}{\mathbf{2.85 \times 10^{-4}}} = \mathbf{2.2} \tag{41}$$

3. **Does the presence of the hole affect stresses and strains at points remote from the hole? And why?**

The presence of the hole does not affect stresses and strains at points remote from the hole.

Stress concentration can be defined as the location in which the stress is significantly larger than average[1]. In this experiment, it was demonstrated that the maximum stress occurs in the center of the beam.

Thus, the center of the beam is beam's stress concentration. Hence, the mentioned hole would only affect stresses and strains in a localized area near the center and not the whole of the beam.

References

[1] Safih (2012) *Stress Concentration Fundamentals* [Online]. Available from: https://www.engineersedge.com/material_science/stress_concentration_fundamentals_9902.htm