University of Liverpool
Department of Electrical Engineering & Electronics

Experiment 1

The Strain Gauge and Its Applications

Name: ____________________

Date: ____________________

Instructions

- Read this script before coming to the laboratory.
- The tables should be filled with the results and the questions should be answered while carrying out the experimental procedure.
- As you complete each part of the experiment, ensure that your results and calculations are viewed by demonstrator before proceeding.
- You should submit a word-processed report that is self-contained and independent of the labscript. It should also include results, graphs and answers to the questions.

Objectives

- Describe the basis of operation and the measurement of strain, tensile and compressive, using strain gauges.
- Apply strain gauge techniques in the investigation of a simple loaded beam structure.

Apparatus

- Two brass beam units; $beam_1$ is uniform and $beam_2$ with a hole (diameter =12mm), Dimensions: width (b)=25.4mm, depth (h)=6.27mm.
  Span between supports (ℓ) =20cm, overhang(X) =19.5cm.
- Strain gauges adhered on the beams as shown in Figure 1,

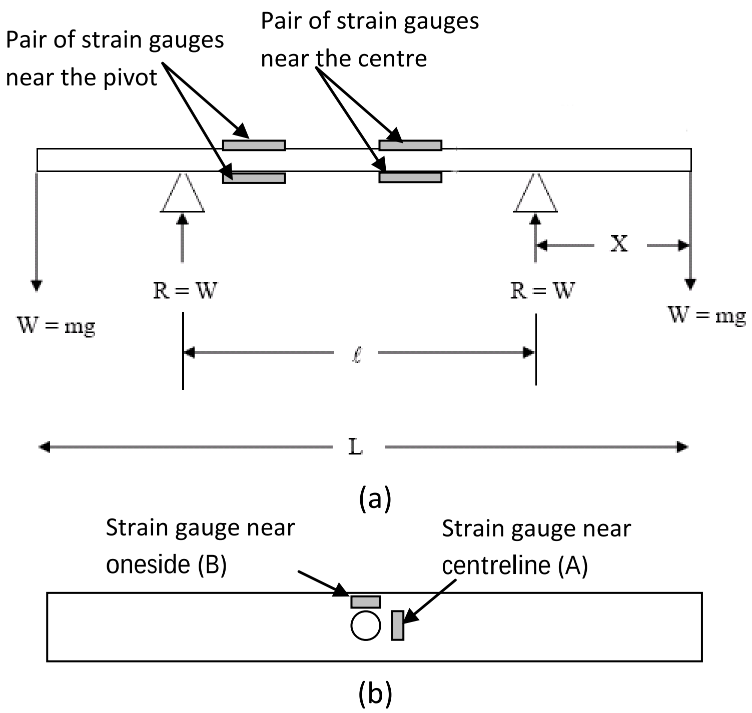


Figure 1: Schematic drawing for (a) the uniform beam loading, (b) the beam with a hole.

- Weights(g),
- Bridge box, see Figure 2,
- Power supply (5V),

- Digital multimeter,
- Displacement measuring device (dial gauge).

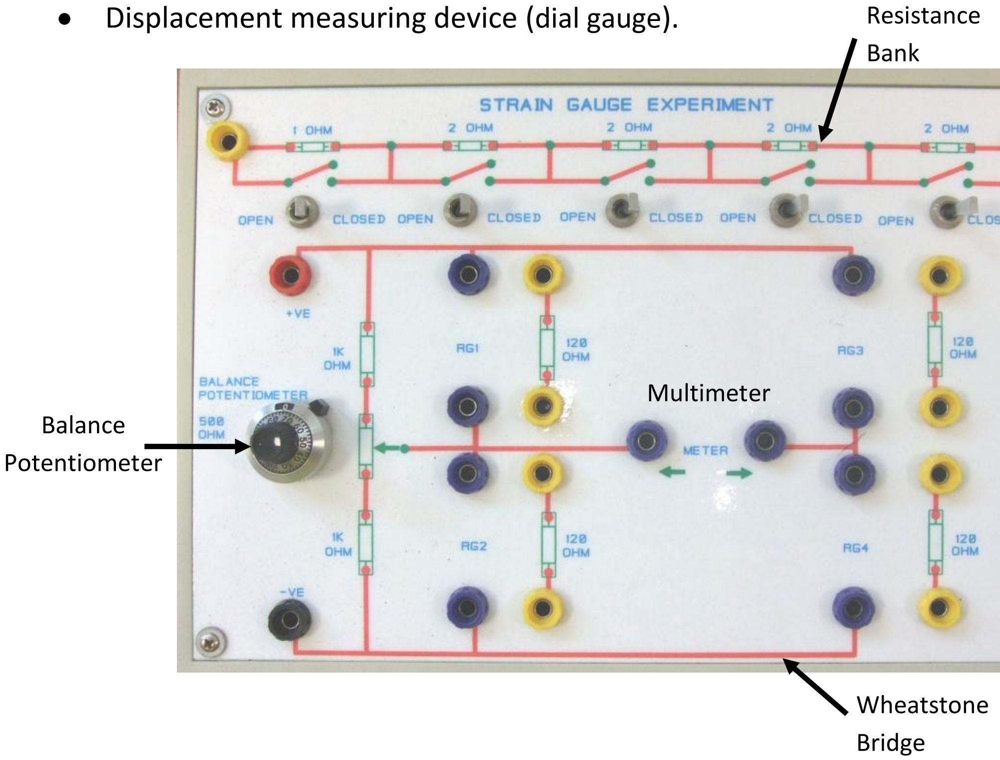


Figure 2: Bridge Box

Introduction

(a) Strain Definition:

Strain is the amount of deformation of a body due to an applied force. More specifically, strain (ε) is defined as the fractional change in length, as shown below.

$$\text{Strain }(\varepsilon) = \frac{change\ in\ length}{original\ length} = \frac{\Delta L}{\text{L}} \qquad (1)$$

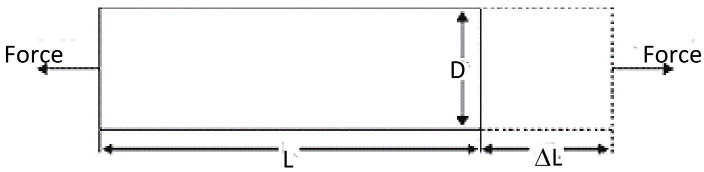


Figure 3

Strain can be positive (tensile) or negative (compressive). Although dimensionless, strain is sometimes expressed in units such as in/in or mm/mm. In practice, the magnitude of measured strain is very small. Therefore, strain is not expected to exceed 10^{-3}, thus the change in resistance will be less than 0.3Ω.

(b) Strain Gauge :

If a metal conductor is stretched or compressed, its resistance changes on account of the fact that both the length and diameter of the conductor change. There also a change in the value of resistivity of the conductor when it is strained and this property is called piezoresistive effect. This is the principle of strain gauge. Strain gauge is a device of which the electrical resistance varies in proportion to the amount of strain in the device. The most widely used gauge is the bonded metallic strain gauge.

A strain gauge of length L, area A, and diameter D when unstrained has resistance

$$R = (\rho L)/ A \qquad (2)$$

When a gauge is subjected to positive strain, its length increases while its area of cross section decreases, resistance of gauge increases with positive strain.

Bonded metallic (Foil type) strain gauges are commonly used due to their advantages over other strain gauges thus it is discussed in detail below.

The metallic strain gauge, shown in Figure 4, consists of a very fine wire or, more commonly, metallic foil arranged in a grid pattern. The grid pattern maximizes the amount of metallic wire or foil subject to strain in the parallel direction as shown Figure 4. The cross sectional area of the grid is minimized to reduce the effect of shear strain and Poisson strain.

The grid is bonded to a thin backing, called the carrier, which is attached directly to the test specimen. Therefore, the strain experienced by the test specimen is transferred directly to the strain gauge, which responds with a linear change in electrical resistance. Strain gauges are available commercially with nominal resistance values from 30 to 3000Ω, with 120, 350 and 1000Ω being the most common values.

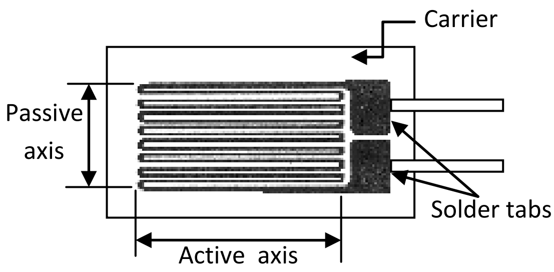


Figure 4: Bonded Metallic Strain Gauge.

It is very important that the strain gauge be properly mounted on to the test specimen so that the strain is accurately transferred from the test specimen, through the adhesive and strain gauge backing, to the foil itself. A fundamental parameter of the strain gauge is its sensitivity to strain, expressed quantitatively as the gauge factor (GF).

Gauge factor is defined as the ratio of fractional change in electrical resistance to the fractional change in length.

$$GF = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\varepsilon} \quad (3)$$

The Gauge Factor for metallic strain gauges is typically around 2.

(c) Strain Measurement Techniques:

Measurement of the fractional change in resistance

In practice, the strain measurements rarely involve quantities larger than a few millistrain ($\varepsilon \times 10^{-3}$). Therefore, to measure the strain requires accurate measurement of very small changes in resistance. To measure such small changes in resistance, strain gauges are almost always used in a bridge configuration with a voltage excitation source, where the change in the resistance is converted into electrical signal as shown in Figure 5-a.

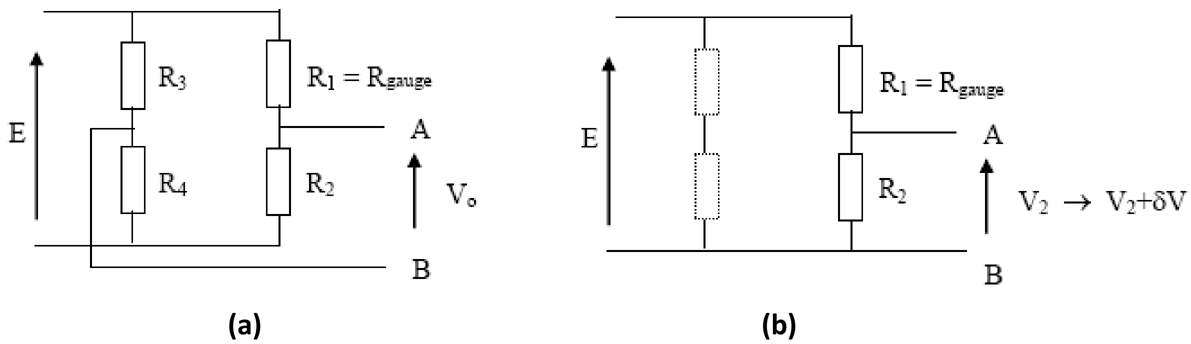


Figure 5: (a) The Wheatstone bridge, (b) Conditions appropriate to the selection of R_2.

Let R_1 be the arm of the bridge containing the strain gauge. Given the nominal resistance 120Ω, how should the values of resistors in the other arms of the bridge be set? Firstly take in isolation one half of the bridge i.e. the resistors R_1 (representing the gauge) and R_2, with the d.c. supply E and with voltage being measured across R_2 as shown in Figure 5-b. The problem is to maximise the change in voltage signal V_2 across R_2 caused by a small change in resistance $R_1 \rightarrow R_1 + \delta R$.

The change in voltage signal is

$$\delta V = \frac{E R_2}{R_1 + R_2 + \delta R} - \frac{E\, R_2}{R_1 + R_2} \quad (4a)$$

which upon rearranging the right hand side over the common denominator and noting $\delta R \ll R_1$, R_2 gives

$$\delta V = -E\left[\frac{R_2}{(R_1 + R_2)^2}\right] \cdot \delta R \quad (4b)$$

The bracketed term is a maximum for $R_1 = R_2$, hence R_2 should equal the nominal gauge resistance for the largest indicated change of voltage. Then with $R_1 = R_2$ and $\delta R \ll R_1$,

$$|\delta V| = \left[\frac{E}{4}\right] \frac{\delta R}{R_1} \tag{5}$$

If the second half of the bridge is incorporated as shown in Figure 5-a, the voltmeter at the output terminals measures the differential voltage V_2-V_4. By selecting R_3=R_4 and with R_2 set equal to the gauge resistance, the output voltage is zero when the gauge is unstrained, but will depart from zero when strain occurs, i.e. δV will directly constitute the output voltage V_o of the bridge. The change in resistance can thus be monitored with sensitivity and this is the obvious advantage of a bridge/null technique.

Temperature compensation circuits

Ideally, it is required that the resistance of the strain gauge to change only in response to applied strain. However, strain gauge material, as well as the specimen material to which the gauge is applied, will also respond to changes in temperature. Strain gauge manufacturers attempt to minimize sensitivity to temperature by processing the gauge material to compensate for the thermal expansion of the specimen material for which the gauge is intended. While compensated gauges reduce the thermal sensitivity, they do not totally remove it.

Changes in resistance associated with strain gauge operation are relatively small and could easily be masked by changes in gauge temperature. This influence can be countered by using a second gauge for compensation. This should be as nearly identical to the first as possible and included as R_2 in the Wheatstone bridge (Figure 5-a). The unstressed gauge should be placed as near to the active gauge as possible so that the temperature of the two will be the same. Let the fractional change in the resistance of the active gauge due to strain be x (i.e. x=δR/R) and the fractional change in the resistance of both gauges due to the temperature effects be y. Thus due to simultaneous influence of strain and temperature R_1 would be:

$$R_g = R_1(1+x)(1+y)$$

whilst for R_2:

$$R_g' = R_1(1+y)$$

The output signal V_0 of the bridge (with R_3 and R_4 fixed at R_1) follows as:

$$V_0 = \frac{R_g E}{R_g + R_g'} - \frac{E}{2} = \left[\frac{R_1(1+x)(1+y)}{R_1(1+x)(1+y) + R_1(1+y)} - \frac{1}{2}\right] E$$

This leads to

$$V_0 = \frac{x}{2(2+x)} E \tag{6}$$

The effect of temperature does not appear in the bridge output.

Strains are not expected to exceed 10^{-3}, as mentioned earlier, so to an accuracy of at least one percent the above equation may be written as:

$$V_0 = \frac{x}{4} E \tag{7}$$

which indicates a direct proportionally between V_0 and the fraction change in resistance (x) with a constant of proportionality (E/4) as earlier predicted in equation 6. Equation 6 may further be expressed in terms of strain using equation 3,

$$V_0 = \left[\frac{E}{4}\right] \cdot G \cdot \varepsilon \tag{8}$$

The bracketed term identifies the sensitivity of the measuring bridge (V). Thus, the output may be seen to be the product of:

Output voltage = Sensitivity • Gauge factor • Strain

Multi-gauge bridge circuits

The sensitivity of the bridge to strain can be doubled by making both gauges active in a half-bridge configuration. It can be further increased by making all four resistances of the arms of the bridge by active strain gauges in a full-bridge configuration.

Figure 6 illustrates the bending of a beam member. Consider two strain gauges attached to the member, one above and the other below its neutral axis and having their active axes along the length of the member. If the gauges are equidistant from the neutral axis, the tensile strain imposed on the gauge situated along AA' is equal to the compressive strain imposed on the second gauge situated along BB'. Both gauges will change in resistance by the same amount, but one increases in resistance while the other decreases.

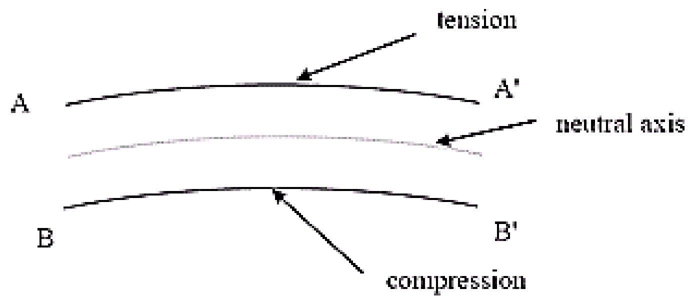


Figure 6

For the first gauge, at position R_1 of Figure 5-a,

$$R_{g1} = R_1(1 + x)$$

while for the second gauge, at position R_2,

$$R_{g2} = R_1(1 - x)$$

The voltage drop V_2 across R_2 will be

$$\frac{R_1(1+x)E}{R_1(1+x)+R_1(1-x)} = \frac{E}{2}(1+x)$$

The bridge output voltage will be

$$V_0 = \frac{E}{2}x = \left[\frac{E}{2}\right] \cdot G \cdot \varepsilon \qquad (9)$$

It can be seen that the output voltage will not be affected by a temperature change which affects both gauges in the same way.

Application to a two-point loaded beam

The extension of the top surface in Figure 7 by ΔL leads to the identification of a surface strain which is given by,

$$\varepsilon = \frac{\Delta l}{l}$$

where ℓ is the distance between supports.

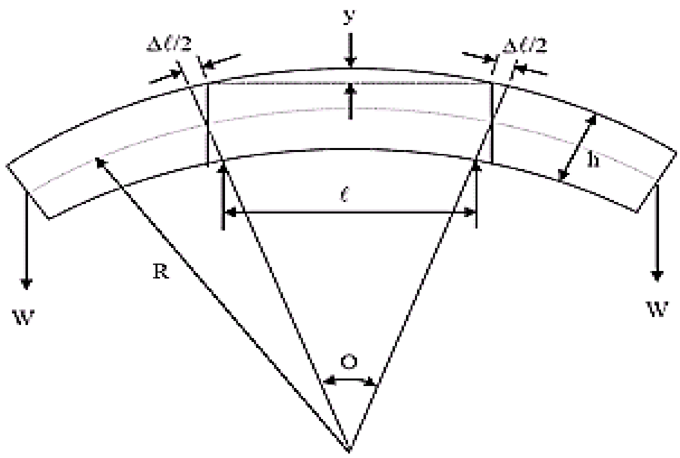


Figure 7: The geometry of the deformation of the beam (exaggerated).

With the symmetrical two-point loading arrangement as considered here, the bending moment and hence the strain is constant in the region between supports. This constant bending moment is given by,

$$M = \frac{W(L-l)}{2} \qquad (10)$$

where L is the distance between the loading points at the extremity of the beam, and W the load applied at each point.

The top most point of the beam under loading is displaced through a distance y, and the strain is given in terms of the displacement and the geometry of the loading arrangement as,

$$\varepsilon = \frac{4hy}{l^2} \tag{11}$$

where h is the height of the beam perpendicular to the bending axis as shown in Figure 7.

Strain/stress relationship

Following Hooke's Law the strain is linked to the stress according to

$$\sigma = Y\varepsilon \tag{12}$$

where Y represents the constant known as Young's modulus.

The stress can be also evaluated directly from the loading conditions and the geometry of the beam. In this manner, the stress follows as,

$$\sigma = \frac{3W(L-l)}{h^2 b} \tag{13}$$

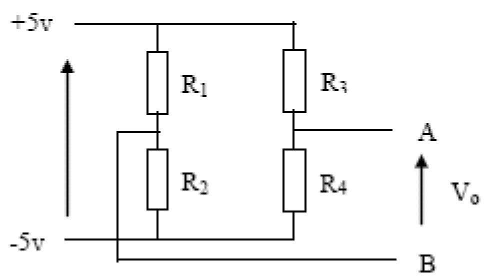
Measurements and Analysis

Experiment 1

Objective:
Determine the sensitivity of a Wheatstone bridge arrangement and compare with theoretical expressions for the sensitivity.

Procedure:

1. Construct the circuit shown in the figure below using the fixed 120Ω resistors provided on the strain gauge circuit box for each arm of the bridge, shown in Figure 2.



2. Connect R_3 in series with the switched resistor bank located across the top of the strain gauge circuit box.
3. Derive a 10V bridge supply using the 5V and –5V terminals of the power supply.
4. Connect a digital voltmeter across the bridge and set the voltmeter to '*millivolts*'.
5. Set all five switches in the resistor bank to their closed position.
6. Balance the bridge using the variable resistor on the LHS of the box (0.1mV is sufficient precision).

7. Using the switched resistor bank, increase the total value of R_3 in steps of 1Ω to 9Ω and measure the out of balance voltage δV.

δR	$\delta R/R_3$	δV
1		
2		
3		
4		
5		
6		
7		
8		
9		

8. Plot the output voltage (δV) against the fractional change in resistance ($\delta R/R_3$), where δR is the change in resistance of R_3.

Ask the demonstrator to check your graph before proceeding.

Questions:

1. Determine the slope of the graph, representing the sensitivity of the bridge.
   Experimental Sensitivity= ______________________
2. Calculate the theoretical sensitivity of the bridge (Equation 6).
   Theoretical Sensitivity= ______________________

3. Comment on the results.
   ____
   ____
   ____

Experiment 2

Objective:

Investigate the relative sensitivity of a single and a double-active gauge bridge, and determine the gauge factor G.

Part A:

Procedure:

1. Switch off the power supply.
2. Disconnect R_3 and connect the strain gauge, located towards the centre of the uniform beam (and on upper side), in its place.
3. Link the temperature compensation gauge in place of R_4.
4. Switch on the power supply and balance the bridge using the potentiometer. Setting the balance in this way effectively eliminates the weight of the beam. When balanced the meter should read less than 1mV.
5. Setup the gauge dial at the centre of the beam.
6. Increase the load on the beam by adding, symmetrically, 0.5kg weights up to at least 3kg on each side.
7. For each load note the bridge output voltage (δV) and the displacement (y) of the centre of the beam (see Figure 7).

Mass(kg)	δV	y
0.0		
0.5		
1.0		
1.5		
2.0		
2.5		
3.0		

Questions:

1. Why the temperature compensation strain gauge is used?
   ____
   ____

2. Is there any variation in temperature? And why?

Part B:

Procedure:

1. Replace the temperature compensation gauge representing R_4 by the strain gauge found on the underside of the beam (the partner to the strain gauge already in use).
2. Proceed as in part A to measure the bridge output (δV) as a function of the displacement (y) over a range of loading.
3. Plot the findings of bridge output against displacement for the two parts A and B on the same graph.

Mass(kg)	δV	y
0.0		
0.5		
1.0		
1.5		
2.0		
2.5		
3.0		

Ask the demonstrator to check your graph before proceeding.

Questions:

1. Compare the findings of bridge output against displacement for the two parts A and B. And the relative outputs with theory.

2. For your best set of measurements :
   (a) Calculate the strain using Equation 11 (show calculations).
   Strain (ε) =________________________________

   (b) Calculate the gauge factor (G). Hints, take the appropriate value for sensitivity of your measuring arrangement and recall that the bridge output is the product of sensitivity, the gauge factor, and the strain (Equations 8 and 9).

   (c) What is the effect of using multiple strain gauges on sensitivity?

Experiment 4

Objective:

Investigate the distribution of strain supports with a two-point loaded beam, further, determine Young's modulus by computing stresses from the known loading conditions.

Procedure:

1. Replace the pair of strain gauges located towards the centre of the beam by the pair of gauges fixed close to the pivots.
2. Investigate the bridge output with loading, again up to at least 3kg.

Mass(kg)	δV	ε	σ
0.0			
0.5			
1.0			
1.5			
2.0			
2.5			
3.0			

Questions:

1. Calculate the strain for the corresponding loads and show sample calculation. Hint, since the gauge factor G and the bridge sensitivity are known, it is not necessary to measure the displacement of the beam – rather the bridge output in conjunction with the sensitivity and the gauge factor can be taken directly as an indication of strain. The dial gauge and its support bracket may therefore be removed from on top of the beam.

2. Compute stresses by using Equation (13) from the known loading conditions. Show a sample calculation.

3. Plot a graph of stress versus strain.

Ask the demonstrator to check your graph before proceeding.

4. Determine the value of Young’s modulus for the beam material and compare your result with the $Y = 1.05\times10^{11}\,Nm^{-2}$ for brass.

5. How does the strain measured near the pivots compare with that found for the same loading near the centre in Experiment 2-B?

Experiment 5

Objective:

Investigate the modification to the stress pattern with the introduction of a hole in the beam.

Procedure:

1. Replace the uniform beam with a beam in which a central hole has been cut. Apart from the hole this beam is identical in material and size to that you have already used. There are two gauges mounted on this beam in different positions near to the edge of the hole.
2. Measure the longitudinal strain in the region of the hole (again taking the bridge output as an indication of strain) first using the gauge near to the centreline of the beam (position A in Figure 1-b) and secondly using that located near one side (position B in Figure 1-b).

	δV_{max}	ε_{max}	σ_{max}
Beam with hole			

Hints: Carry out your investigation with near maximum loading, so as to enhance the signal level, and incorporate temperature compensation (using the temperature compensation gauge on the support frame). Hence, as in Experiment 3, you may dispense with the dial gauge and use the output voltage of the bridge to measure the strain.

Questions:

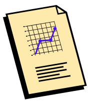
a) Compare between:
   1. the two strains at those two positions and explain the reason,

2. the strain under these conditions with that at the uniform beam at the same loading.

b) Find the stress concentration factor K at the centre of the beam (show calculations), defined as

$$K = \frac{\sigma_{max}(\text{beam with hole})}{\sigma_{max}(\text{beam without hole})}$$

c) Does the presence of the hole affect stresses and strains at points remote from the hole? And Why?



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Prepared by: Dr. K. I. Nuttall, February 2003
Modified by: Lina Momani, September 2008