Experiment No.-4: Response of a system described by difference equation

1. Overview

In this lab we will use MATLAB to solve difference-equations.

2. Difference equation of LTI system

The difference equation of an LTI system can be written as,

$$\sum_{k=0}^{K} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]$$

where $x[n]$ is the system input and $y[n]$ is the system output. If `x` is a MATLAB vector containing the input $x[n]$ on the interval $n_x \le n \le n_x + N_x - 1$ and the vectors `a` and `b` contain the coefficients a_k and b_m then `filter(b,a,x)` returns the output of the causal LTI system satisfying

$$\sum_{k=0}^{K} a[k+1]y[n-k] = \sum_{m=0}^{M} b[m+1]x[n-m]$$

Note that $a[k+1] = a_k$ and $b[m+1] = b_m$, since MATLAB requires all vectors to begin at one. For example, to specify the system described by the difference equation

$$y[n] + 2y[n-1] = x[n] - 3x[n-1],$$

you would define `a = [1 2]` and `b = [1 -3]`.
Also note that the output of filter contains samples of $y[n]$ in the same interval as the samples of `x`. Also, `filter` need samples in the interval $n_x - M \le n \le n_x - 1$ in order to compute the first output sample of $y[n]$. If they are not provided, `filter` assumes these samples are zero.
Define `a1` and `b1` to describe the causal LTI system shown below:

$$y[n] = 0.5x[n] + x[n-1] + 2x[n-2]$$

```
x = [1 2 3 4];
a1 = [1];
b1 = [0.5 1 2];
y1 = filter(b1,a1,x)
```

Use `filter` to compute response $y[n]$ to input $x[n] = nu[n]$: Try the following system:

$$y[n] = 0.8y[n-1] + 2x[n]$$

```
x = [1 2 3 4];
a1 = [1 -0.8];
b1 = [2];
y1 = filter(b1,a1,x)
```

3. Exercise

1. Write a program in MATLAB to,

(a) Define coefficient vectors `a` and `b` to describe the causal LTI system described by,

$$y[n] = 0.5x[n] + x[n-1] + 2x[n-2].$$

(b) Define coefficient vectors `a` and `b` to describe the causal LTI system described by,

$$y[n] = 0.8y[n-1] + 2x[n].$$

(c) Define coefficient vectors `a` and `b` to describe the causal LTI system described by,

$$y[n] - 0.8y[n-1] = 2x[n-1].$$

For each of these systems use the `filter` command to compute the response $y[n]$ for the input signal $x[n] = u[n], 0 \leq n \leq 3$. Also plot the response for each of the above cases.

Source: https://web.stanford.edu/~kairouzp/teaching/ece311/secure/lab3/lab3.pdf