

# Graph - 2

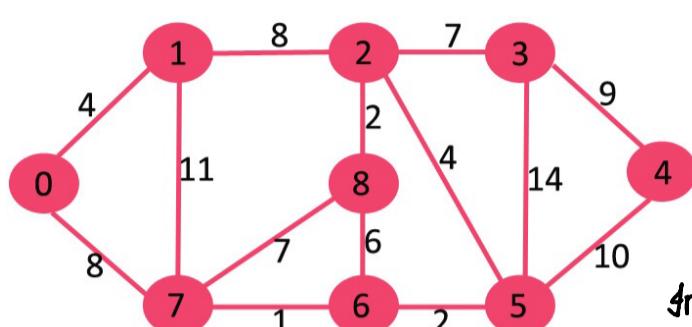
- Karun Karthik

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## 11) Dijkstra Algorithm → single source shortest path (only +ve weights)

→ Helps in finding the shortest path to every node from src node.



$n = 9$  (nodes from 0 to 8)

$src = 1$

dist away = min cost from src to every other vertex

initially cost = 

0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8

 vis = { } 3

→ As it is weighted graph, we'll use priority queue (PQ) instead of normal queue. An element pushed into it will be of form curr node, curr cost

→ PQ always pops element with least curr cost, always calculated from src to curr node.



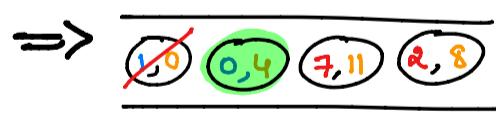
⇒ now neighbours of 1 = 0, 4 7, 11 2, 8 ∴ push



vis = {1}

cost[1] = 0

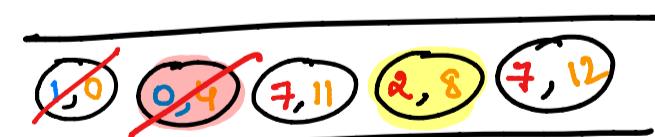
→ lowest cost among 4, 11, 8  
is 4 ∴ pop it & push its  
neighbours.



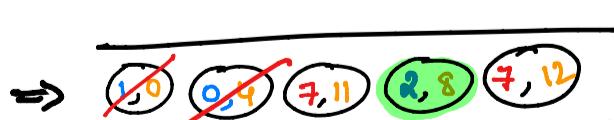
⇒ now neighbours of 0 = 1 (visited), 7, 12 ∴ push

vis = {1, 0}

cost[0] = 4



→ lowest cost is 8 ∴  
pop & push its neighbours



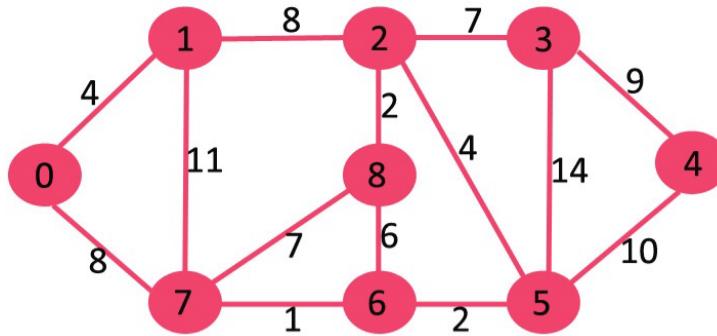
⇒ neighbours of 2 = 1 (visited), 8, 10, 3, 15, 5, 12 ∴ push

vis = {1, 0, 2}

cost[2] = 8



→ lowest cost is 10 ∴  
pop & push its neighbours



⇒



$$\text{vis} = \{1, 0, 2, 8\}$$

$$\text{cost}[8] = 10$$



↳ lowest cost = 11 ∴ pop & push its neighbours.

⇒



⇒ neighbours of 7 = 0, 1, 8 are visited.

& ~~6,12~~ ∴ push

$$\text{vis} = \{1, 0, 2, 8, 7\}$$

$$\text{cost}[7] = 11$$



↳ lowest cost = 12

∴ Anything among 5, 6 can be selected & pop & push its neighbours  
Not 7, because it is already visited & cost is < 12.

⇒



⇒ neighbours of 5 = ~~4,22~~, ~~3,26~~, ~~6,14~~ ∴ push

$$\text{vis} = \{1, 0, 2, 8, 7, 5\}$$

$$\text{cost}[5] = 12$$



→ lowest cost = 12  
∴ pop & push its neighbours

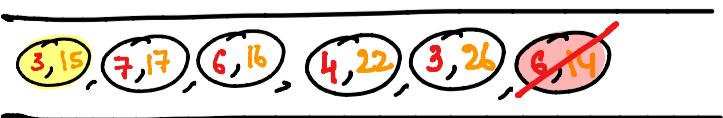
⇒



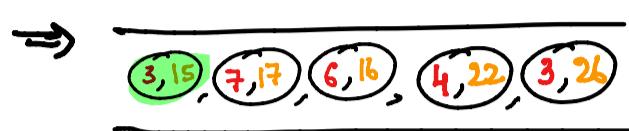
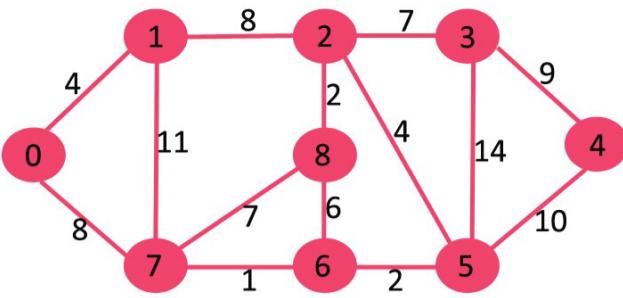
⇒ neighbours of 6 = 5, 7, 8 are visited .

∴ no push

→ next lowest is 14, but 6 is already visited .



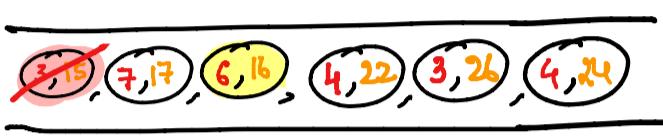
∴ Next lowest is 15, ∴ pop & push its neighbours



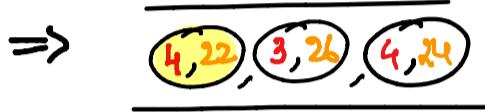
$\Rightarrow$  neighbours of 3 = 2, 5 (visited)  $(4, 24)$   $\therefore$  push

$$vis = \{1, 0, 2, 8, 7, 5, 6, 3\}$$

$$cost[3] = 15$$



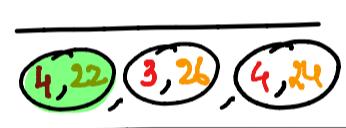
$\rightarrow$  next lowest cost = 16  
but 6 is already visited  $\therefore$  pop



$\rightarrow$  next lowest cost = 22

$\therefore$  pop q & push its neighbour.

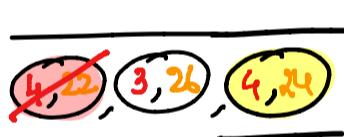
$\rightarrow$  next lowest cost = 17  
but 7 is already visited  $\therefore$  pop



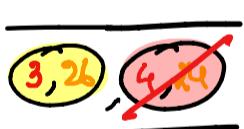
$\Rightarrow$  neighbours of 4 = 3, 5 (visited)  $\therefore$  no push

$$vis = \{1, 0, 2, 8, 7, 5, 6, 3, 4\}$$

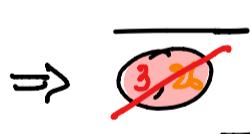
$$cost[4] = 22$$



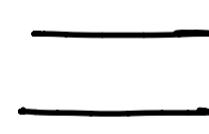
$\rightarrow$  next lowest cost = 24  
but 4 is already visited  
 $\therefore$  pop



$\rightarrow$  next lowest cost = 26  
but 3 is already visited  
 $\therefore$  pop



$\Rightarrow$



$\therefore$  empty PQ.

Answer  $\Rightarrow$

4	0	8	15	22	12	12	11	10
0	1	2	3	4	5	6	7	8

Dijkshaus = BFS + PQ

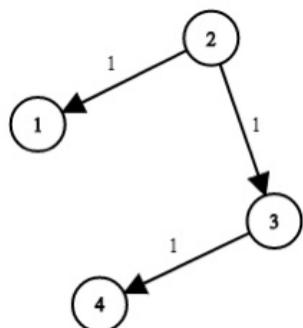
$T_C \rightarrow O(V + E \log V)$

$S_C \rightarrow O(V)$

## Code →

```
1 class Solution
2 {
3     public:
4     vector <int> dijkstra(int V, vector<vector<int>> adj[], int src)
5     {
6         vector<int>cost(V,0);
7         cost[src]=0;
8
9         vector<bool>vis(V, false);
10        priority_queue<pair<int,int>,vector<pair<int,int>>,greater<pair<int,int>>> pq;
11
12        pq.push({0,src}); // {cost, node}
13
14        while(!pq.empty())
15        {
16            pair<int,int>p = pq.top();
17            int currCost = p.first;
18            int currNode = p.second;
19            pq.pop();
20
21            if(vis[currNode]) continue;
22
23            vis[currNode] = true;
24            cost[currNode] = currCost;
25
26            for(int i=0;i<adj[currNode].size();i++)
27            {
28                int neighbourNode = adj[currNode][i][0];
29                int weight = adj[currNode][i][1];
30                // if already visited then skip
31                if(vis[neighbourNode]) continue;
32                // else push
33                pq.push({currCost + weight, neighbourNode});
34            }
35        }
36        return cost;
37    }
38 };
39
```

## 12 Network Delay Time



$\text{src} = 2$ .

You are given a network of  $n$  nodes, labeled from 1 to  $n$ . You are also given  $\text{times}$ , a list of travel times as directed edges  $\text{times}[i] = (u_i, v_i, w_i)$ , where  $u_i$  is the source node,  $v_i$  is the target node, and  $w_i$  is the time it takes for a signal to travel from source to target.

We will send a signal from a given node  $k$ . Return the time it takes for all the  $n$  nodes to receive the signal. If it is impossible for all the  $n$  nodes to receive the signal, return  $-1$ .

✓ Similar to Dijkstra's algo.  $\text{cost} = \boxed{0 \ 0 \ 0 \ 0 \ 0}$   $\text{vis} = \{2\}$   $\text{pq} = \underline{\quad \quad \quad}$

$\Rightarrow$  push  $(2, 0)$  to pq.  $\Rightarrow \underline{(2, 0) \quad \quad \quad}$

$\Rightarrow \underline{(2, 0)}$  neighbours =  $(1, 1), (3, 1)$   $\therefore$  push  
 $\text{vis} = \{2\}$   $\text{cost}[2] = 0$   $\rightarrow$  next lowest cost = 1  $\therefore$  choose 1 or 3  
 $\therefore$  pop & push their neighbour.

$\Rightarrow \underline{(1, 1), (3, 1)}$  no new neighbours  $\therefore$  pop  
 $\text{vis} = \{2, 1\}$   $\text{cost}[1] = 1$   $\rightarrow$  next lowest cost = 1  
 $\therefore$  pop & push their neighbour.

$\Rightarrow \underline{(3, 1)}$  neighbour =  $(4, 2)$   $\therefore$  push  
 $\text{vis} = \{2, 1, 3\}$   $\text{cost}[3] = 1$   $\rightarrow$  next lowest cost = 2  
 $\therefore$  pop & push neighbours.

$\Rightarrow \underline{(4, 2)}$  no new neighbours  $\therefore$  pop  
 $\text{vis} = \{2, 1, 3, 4\}$   $\text{cost}[4] = 2$   $\rightarrow$  pq is empty.

$\therefore \text{cost} = \boxed{0 \ 1 \ 0 \ 1 \ 2}$

$T_c \rightarrow O(V + E \log V)$   
 $S_c \rightarrow O(V)$

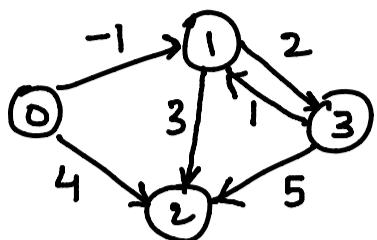
$\rightarrow$  check if all nodes are in visited,  
else return -1.  
 $\rightarrow$  Return max value in cost as

## Code →

```
1  class Solution {
2  public:
3
4      int networkDelayTime(vector<vector<int>>& times, int n, int k) {
5          vector<vector<vector<int>>> graph = createGraph(times,n);
6          return minTime(graph,n,k);
7      }
8
9      vector<vector<vector<int>>> createGraph(vector<vector<int>>& edges,int n) {
10
11         vector<vector<vector<int>>> graph(n+1);
12
13         for(int i=0;i<=n;i++) {
14             graph.push_back({{}});
15         }
16         // add every edge to the graph
17         for(vector<int> edge:edges) {
18             int source = edge[0];
19             int dest = edge[1];
20             int cost = edge[2];
21             graph[source].push_back({dest,cost});
22         }
23         return graph;
24     }
25
26     int minTime(vector<vector<vector<int>>> &graph,int n,int src) {
27
28         vector<int> cost(n+1,0);
29         cost[src] = 0;
30         vector<bool>vis(n+1, false);
31
32         priority_queue<pair<int,int>,vector<pair<int,int>>,greater<pair<int,int>>>pq;
33         pq.push({0,src}); // {cost, node}
34
35         while(!pq.empty()) {
36             pair<int,int>p = pq.top();
37             int currNode = p.second;
38             int currCost = p.first;
39             pq.pop();
40             // if already visited then skip
41             if(vis[currNode])    continue;
42
43             vis[currNode] = true;
44             cost[currNode] = currCost;
45
46             for(int i=0;i<graph[currNode].size();i++)
47             {
48                 int neighbourNode = graph[currNode][i][0];
49                 int weight = graph[currNode][i][1];
50                 // if already visited then skip
51                 if(vis[neighbourNode])  continue;
52                 // else push into pq
53                 pq.push({currCost + weight, neighbourNode});
54             }
55         }
56
57         for(int i=1; i<=n; i++)
58             if(vis[i]==0)    return -1;
59
60         int ans = 0;
61         for(int x:cost)    ans = max(ans,x);
62         return ans;
63     }
64 }
```

⑬ Bellman Ford Algorithm → useful when weights  $< 0$  (Dijkstra fails)  
 ↳ dp algo → useful when finding negative weight cycle.  
 $[src, dest, wt]$

Eg  $n = 4$  edges =  $\{[0, 1, -1], [0, 2, 4], [1, 2, 3], [1, 3, 2], [3, 1, 1], [3, 2, 5]\}$



initially dist 

inf	inf	inf	inf
0	1	2	3

$\Rightarrow \text{dist}[src] = 0$  &

$\Rightarrow$  relax every edge  $n-1$  time is run for loop & perform the following operation

$$\text{dist}[dest] = \min(\text{dist}[src] + \text{weight}, \text{dist}[dest])$$

$\Rightarrow$  finally relax one more time &

if  $\text{dist}[dest] > \text{dist}[src] + \text{wt} \Rightarrow$  -ve weight cycle present

$\Rightarrow$  we should relax 3 times &  $src=0 \Rightarrow \text{dist}[src]=0$  dist 

0	inf	inf	inf
0	1	2	3

$\rightarrow$  for edge  $[0, 1, -1]$ ,  $\text{dist}[1] = \min(0 + (-1), \text{inf}) = -1$

$[0, 2, 4]$ ,  $\text{dist}[2] = \min(0 + 4, \text{inf}) = 4$

$[1, 2, 3]$ ,  $\text{dist}[2] = \min(-1 + 3, 4) = 2$

$[1, 3, 2]$ ,  $\text{dist}[3] = \min(-1 + 2, \text{inf}) = 1$

$[3, 1, 1]$ ,  $\text{dist}[1] = \min(1 + 1, -1) = -1$

$[3, 2, 5]$ ,  $\text{dist}[2] = \min(1 + 5, 2) = 2$ .

$$\therefore \text{dist} = \begin{array}{|c|c|c|c|} \hline 0 & -1 & 2 & 1 \\ \hline 0 & 1 & 2 & 3 \\ \hline \end{array}$$

$\rightarrow$  now use the above dist & perform same operation twice, in this case dist remains same.

$\rightarrow$  during final relaxation, -ve weight cycle condition is not met.

Answer  $\Rightarrow$  dist = 

0	-1	2	1
0	1	2	3

$$TC \rightarrow O(V * E)$$

$$SC \rightarrow O(V)$$

# ⑯ Negative weight cycle → Bellman Ford Algorithm.

→ To check the presence of negative weight cycle using Bellman Ford Algorithm.

$$TC \rightarrow O(V * E)$$
$$SC \rightarrow O(V)$$

Code →

```
● ● ●  
1 class Solution {  
2 public:  
3     int isNegativeWeightCycle(int n, vector<vector<int>>edges){  
4         vector<int>dis(n, INT_MAX);  
5         // initially, dist to src is 0  
6         dis[0] = 0;  
7         // relax n-1 times  
8         for(int i=0;i<n-1;i++)  
9         {  
10             for(auto edge:edges)  
11             {  
12                 int src = edge[0];  
13                 int dest = edge[1];  
14                 int wt = edge[2];  
15                 if(dis[src]!=INT_MAX) // to avoid integer overflow  
16                     dis[dest] = min(dis[dest],dis[src]+wt);  
17             }  
18         }  
19         // final relaxation  
20         for(auto edge:edges)  
21         {  
22             int src = edge[0];  
23             int dest = edge[1];  
24             int wt = edge[2];  
25             if(dis[src]!=INT_MAX && dis[dest]>dis[src]+wt)  
26                 return 1;  
27         }  
28         return 0;  
29     }  
30 };
```

## 15) Floyd Warshall Algorithm

- All source shortest path & -ve edges allowed.
- Since its all source shortest path we need to run the loop for all nodes, considering it as intermediary vertex.
- $\text{cost}[i][j] = \min(\text{cost}[i][j], \text{cost}[i][k] + \text{cost}[k][j])$

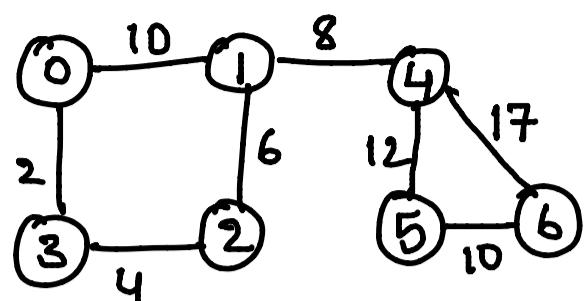
$$TC \rightarrow O(N^3) \quad SC \rightarrow O(N^2)$$

Code →

```
● ● ●  
1 class Solution {  
2     public:  
3         void shortest_distance(vector<vector<int>>&matrix){  
4             int V = matrix.size();  
5             vector<vector<int>> costs(matrix.size(), vector<int>(matrix.size()));  
6  
7             for(int i=0;i<V;i++)  
8                 for(int j=0;j<V;j++)  
9                     costs[i][j] = matrix[i][j];  
10  
11            for(int k=0;k<V;k++)  
12                for(int i=0;i<V;i++)  
13                    for(int j=0;j<V;j++){  
14                        // if intermediate is not -1 then  
15                        if(costs[i][k]!=-1 && costs[k][j]!=-1){  
16                            if(costs[i][j]==-1)  
17                                costs[i][j] = costs[i][k]+costs[k][j];  
18                            else  
19                                costs[i][j] = min(costs[i][j], costs[i][k]+costs[k][j]);  
20                        }  
21                    }  
22  
23            for(int i=0;i<V;i++)  
24                for(int j=0;j<V;j++)  
25                    matrix[i][j] = costs[i][j];  
26  
27        }  
28    };
```

16 Prim's Algorithm → Minimum Spanning Tree (MST)

Eg

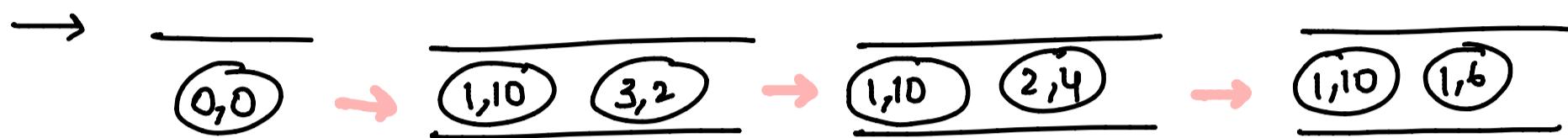


$$Vis = \{ \}$$

PQ (node, weight)

↑ returns node with lowest cost/weight.

\* To find MST, just push node along with its weight.

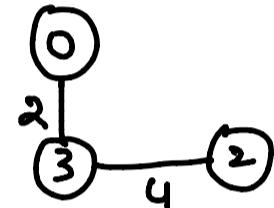


$$Vis = \{ \}$$

$$Vis = \{ 0 \}$$

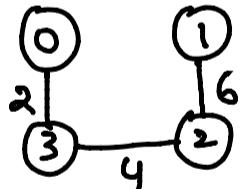
$$Vis = \{ 0, 3 \}$$

$$Vis = \{ 0, 3, 2 \}$$



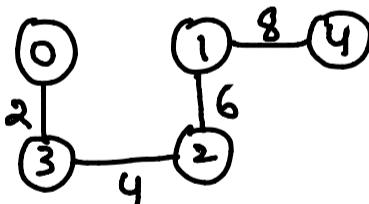
$$\begin{array}{c} (1, 10) \\ (4, 8) \end{array}$$

$$Vis = \{ 0, 3, 2, 1 \}$$



$$\begin{array}{c} (1, 10) \\ (5, 12) \\ (6, 17) \end{array}$$

$$Vis = \{ 0, 3, 2, 1, 4 \}$$

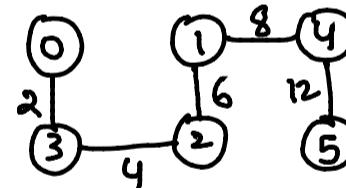


$$\begin{array}{c} (5, 12) \\ (6, 17) \end{array}$$

$$Vis = \{ 0, 3, 2, 1, 4 \}$$

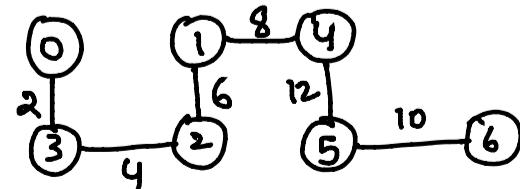
$$\begin{array}{c} (6, 17) \\ (6, 10) \end{array}$$

$$Vis = \{ 0, 3, 2, 1, 4, 5 \}$$



$$(6, 17)$$

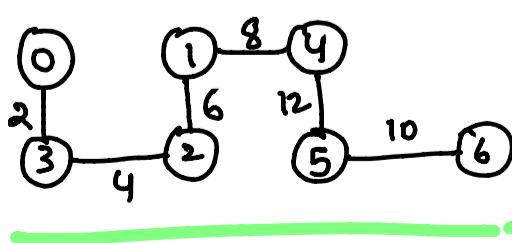
$$Vis = \{ 0, 3, 2, 1, 4, 5, 6 \}$$



$$\underline{\hspace{10cm}}$$

$$Vis = \{ 0, 3, 2, 1, 4, 5, 6 \}$$

$$TC \rightarrow O(V + E \log V)$$



$$SC \rightarrow O(V)$$

Code →

```
1 class Solution
2 {
3     public:
4     //Function to find sum of weights of edges of the Minimum Spanning Tree.
5     int spanningTree(int V, vector<vector<int>> adj[])
6     {
7         int minCost = 0;
8         vector<int> costs(V, INT_MAX);
9         costs[0] = 0;
10        vector<bool> vis(V, false);
11        priority_queue<pair<int,int>, vector<pair<int,int>>, greater<pair<int,int>>>pq;
12        pq.push({0,0}); // {cost, Node}
13
14        while(!pq.empty())
15        {
16            pair<int,int> p = pq.top();
17            int currNode = p.second;
18            int currCost = p.first;
19            pq.pop();
20
21            if(vis[currNode])    continue;
22
23            minCost += currCost;
24
25            vis[currNode] = true;
26            costs[currNode] = currCost;
27
28            for(int i=0;i<adj[currNode].size();i++)
29            {
30                int neighbourNode = adj[currNode][i][0];
31                int neighbourNodeCost = adj[currNode][i][1];
32                if(vis[neighbourNode])  continue;
33                pq.push({neighbourNodeCost, neighbourNode});
34            }
35        }
36        return minCost;
37    }
38 };
39
```

# 17 Min Cost to Connect all points

→ Create graph with each node containing  $Wt \triangleq$  Node value

$$Wt = \text{abs}(X_i - X) + \text{abs}(Y_i - Y)$$

→ Perform Prims algo.

$$TC \rightarrow O(V + E \log V)$$

$$SC \rightarrow O(V)$$

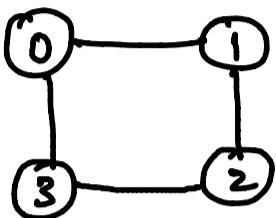
Code →

```
1
2 class Solution {
3 public:
4     int minCostConnectPoints(vector<vector<int>>& points) {
5
6         int n = points.size();
7         vector<vector<pair<int, int>>> graph(n);
8
9         for (int i = 0; i < n; i++) {
10            for (int j = 0; j < n; j++) {
11                if (i == j) continue;
12                graph[i].push_back({abs(points[i][0] - points[j][0]) + abs(points[i][1] - points[j][1]), j});
13            }
14        }
15
16        priority_queue<pair<int,int>,vector<pair<int,int>>,greater<pair<int,int>>>pq;
17        vector<bool> vis(n, false);
18        pq.push({0, 0}); // {cost, Node}
19
20        int ans = 0;
21        while (!pq.empty())
22        {
23            pair<int,int> p = pq.top();
24            int currNode = p.second;
25            int currCost = p.first;
26            pq.pop();
27
28            if (vis[currNode]) continue;
29            ans += currCost;
30            vis[currNode] = true;
31
32            for(int i=0;i<graph[currNode].size();i++)
33            {
34                int neighbourNode = graph[currNode][i].second;
35                int neighbourNodeCost = graph[currNode][i].first;
36                if(vis[neighbourNode]) continue;
37                pq.push({neighbourNodeCost, neighbourNode});
38            }
39        }
40        return ans;
41    }
42 }
43 };
44 }
```

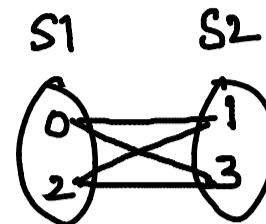
### (18) Is graph Bipartite

Bipartite graph is undirected graph, such that all vertices can be divided into 2 sets,  $S_1 \& S_2$  and no two vertices present in same set share an edge.

Eg  $n = 4$



then



$\therefore$  the graph is bipartite.

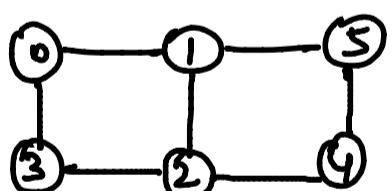
$\Rightarrow$  for graph to be bipartite,

- it needs to be undirected acyclic graph (or)
- it needs to be even length cyclic graph

$\rightarrow$  we generally denote sets by coloring it, color = 0, 1.

$$\begin{matrix} \downarrow & \downarrow \\ S_1 & S_2 \end{matrix}$$

Eg  $n = 6$



$$vis = \{3\} \quad S_1 = \{3\} \quad S_2 = \{3\}$$

initially color

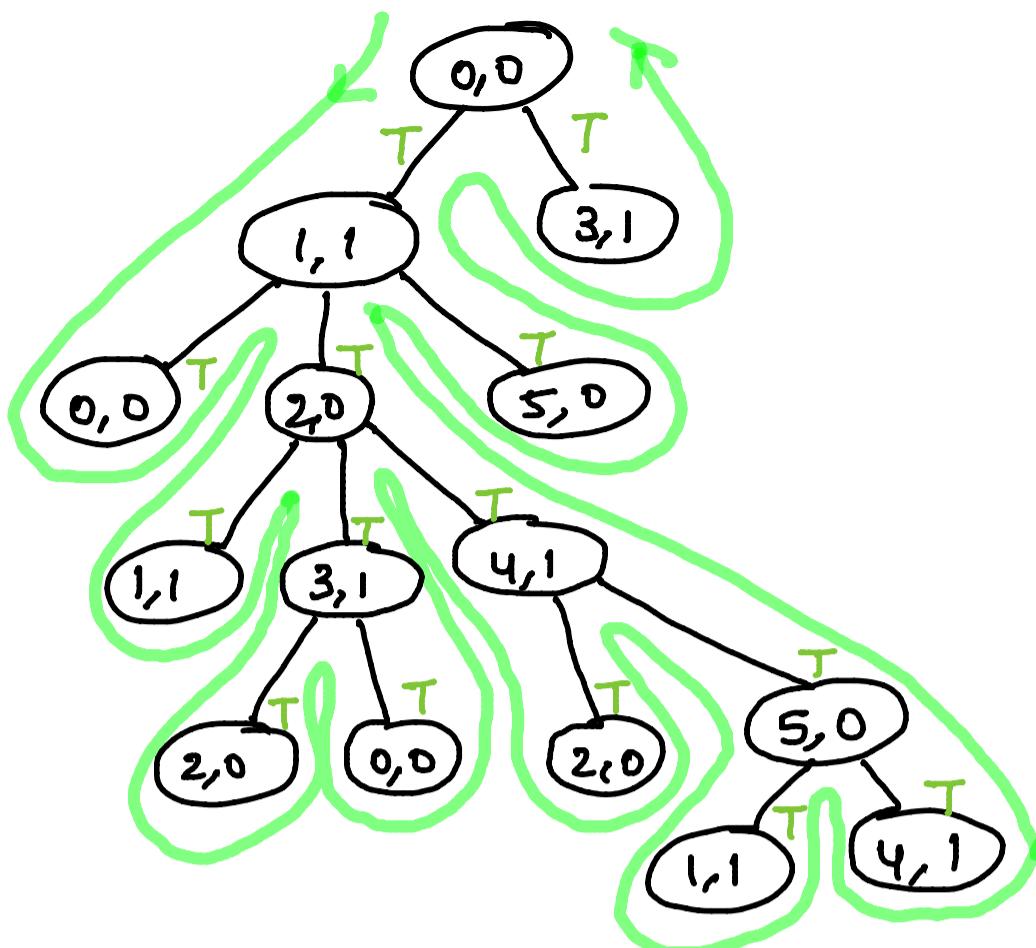
-1	-1	-1	-1	-1	-1
0	1	2	3	4	5

$\rightarrow$  at each vertex, check if it visited or not.

$\rightarrow$  if visited then check if it's present in the intended set or not.

$\rightarrow$  if yes then return true, else false

$\rightarrow$  return AND of all the boolean values.



Code →

```
● ● ●  
1 class Solution {  
2 public:  
3  
4     bool isBipartite(vector<vector<int>>& graph) {  
5  
6         int n= graph.size();  
7         vector<int>colors(n,-1);  
8  
9         for(int curr=0; curr<n ; curr++){  
10             // if already colored then skip  
11             if(colors[curr]!=-1)  continue;  
12             // check for even length cycle  
13             if(hasEvenLengthCycle(graph, curr, 0, colors)==false)  return false;  
14         }  
15         return true;  
16     }  
17  
18     bool hasEvenLengthCycle(vector<vector<int>>& graph,int curr,int color,vector<int>&colors)  
19     {  
20         if(colors[curr]!=-1)  
21             return colors[curr]==color;  
22  
23         // if not colored then color it  
24         colors[curr] = color;  
25  
26         // check for neighbours  
27         for(int neigh: graph[curr])  
28         {  
29             if(hasEvenLengthCycle(graph, neigh, 1-color, colors)==false)  
30                 // 1- color will handle both changing colors 0 to 1 and 1 to 0  
31                 return false;  
32         }  
33         return true;  
34     }  
35  
36 };
```

# 19 Possible Bipartition →

- Create a graph using dislikes array.
- use previous problem's approach to solve it.

Code →

TC → O(V+E) SC → O(V+E)

```
● ○ ●  
1 class Solution {  
2 public:  
3  
4     bool dfs(vector<int> graph[], int curr, vector<int>& color){  
5  
6         // if not colored then color  
7         if(color[curr] == -1)  
8             color[curr] = 1;  
9  
10        // process the neighbours and check their colors  
11        for(auto neigh : graph[curr])  
12        {  
13            if(color[neigh] == -1)  
14            {  
15                color[neigh] = 1 - color[curr];  
16                if(dfs(graph, neigh, color)==false) return false;  
17            }  
18            else if(color[neigh] == color[curr]) return false;  
19        }  
20        return true;  
21    }  
22  
23    bool possibleBipartition(int n, vector<vector<int>>& dislikes) {  
24        vector<int> color(n+1, -1);  
25        vector<int> graph[n+1];  
26  
27        // populating the graph  
28        for(auto edge : dislikes){  
29            graph[edge[0]].push_back(edge[1]);  
30            graph[edge[1]].push_back(edge[0]);  
31        }  
32  
33        for(int i=1; i<=n; i++){  
34            if(color[i] == -1)  
35                if(!dfs(graph, i, color)) return false;  
36        }  
37  
38        return true;  
39    }  
40};
```

20 Disjoint Set  $\rightarrow$  UNION & FIND./getParent

$\hookdownarrow$  helps in finding parent of component  
helps in UNION of components/vertices.

Eg  $0 \ 1 \Rightarrow \text{UNION}(0, 1) \rightarrow$  

Eg  $n=7$  initially every component is parent of itself



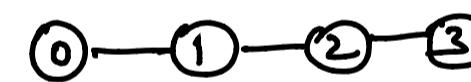
parent =	<table border="1" data-bbox="696 819 1466 983"><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr></table>	0	1	2	3	4	5	6	0	1	2	3	4	5	6
0	1	2	3	4	5	6									
0	1	2	3	4	5	6									

now  $\text{getParent}(2) = 2$ ,  $\text{getParent}(3) = 3$ .

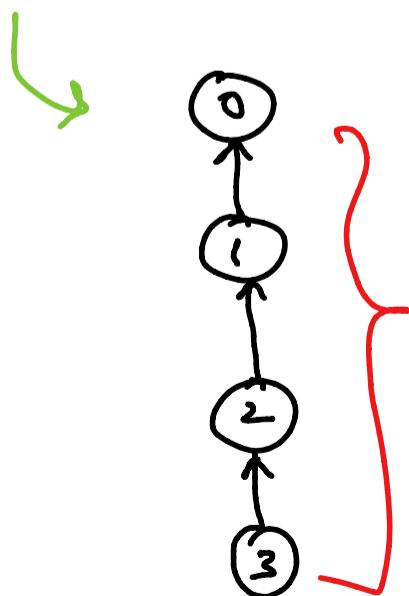
& if  $\text{UNION}(0, 1) \Rightarrow$   &  $\text{parent}[1] = 0$

now  $\text{getParent}(1) = 0$

&  $\text{UNION}(1, 2) \Rightarrow$  

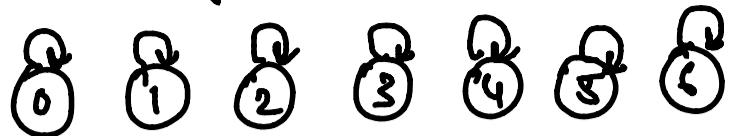
$\text{UNION}(2, 3) \Rightarrow$  

&  $\text{getParent}(3) = 0$



This increases the recursive calls  
and the tree is unbalanced  
so we'll use rank array to  
store min. height tree for node.

$n=7$  initially every component is parent of itself



parent =

0	1	2	3	4	5	6
0	1	2	3	4	5	6

rank =

0	0	0	0	0	0	0
0	1	2	3	4	5	6

$\Rightarrow \text{UNION}(0,1) \Rightarrow$  then  $\text{find}(0) \neq \text{find}(1) \neq 0 \neq 1 \therefore$  diff components.  
as they are diff components find rank &  $\text{rank}[0] = \text{rank}[1] = 0$

$\therefore$  select either 0 or 1 & make it as root & inc the rank by 1



parent =

0	0	2	3	4	5	6
0	1	2	3	4	5	6

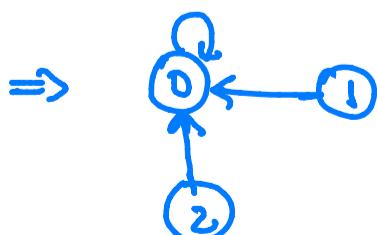
rank =

1	0	0	0	0	0	0
0	1	2	3	4	5	6

$\Rightarrow \text{UNION}(1,2) \Rightarrow \text{parent}(1)=0 \neq \text{parent}(2)=2$   
now  $\text{rank}[0]=1 \neq \text{rank}[2]=0$

as  $\text{rank}[0] > \text{rank}[2]$ ,

vertex 0 should be the parent  
& do not update rank if they are unequal.



parent =

0	0	0	3	4	5	6
0	1	2	3	4	5	6

rank =

1	0	0	0	0	0	0
0	1	2	3	4	5	6

# Code

```
1  class DisjSet {
2      int *rank, *parent, n;
3
4      public:
5      DisjSet(int n)
6      {
7          rank = new int[n];
8          parent = new int[n];
9          this->n = n;
10         makeSet();
11     }
12
13     void makeSet()
14     {
15         for (int i = 0; i < n; i++) {
16             parent[i] = i;
17         }
18     }
19
20     int find(int x)
21     {
22         // if x is not parent of itself then
23         // find parent recursively
24         if (parent[x] != x) {
25             parent[x] = find(parent[x]);
26         }
27         return parent[x];
28     }
29
30     void Union(int x, int y)
31     {
32         int xset = find(x);
33         int yset = find(y);
34
35         // if set of x and y are same then return
36         if (xset == yset)    return;
37
38         // place the elements in small rank
39         if (rank[xset] < rank[yset]) {
40             parent[xset] = yset;
41         }
42         else if (rank[xset] > rank[yset]) {
43             parent[yset] = xset;
44         }
45         // if same rank then increment it
46         else {
47             parent[yset] = xset;
48             rank[xset] = rank[xset] + 1;
49         }
50     }
51 };
52 }
```

(21)

## Kruskal's Algorithm →

- This is used to find minimum spanning tree.
- can be implemented using Disjoint set.
- sort all the edges in ↑ order of weight.
- pick smallest edge & check if it contributes to cycle in graph
- if yes then discard else include.

Code →

```

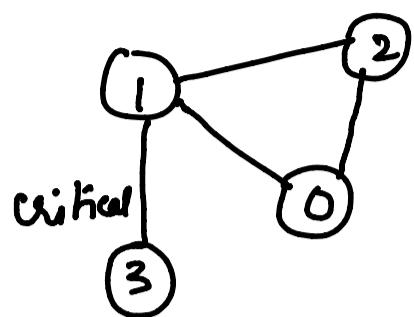
● ● ●

1 class Graph {
2     vector<vector<int>> edgelist;
3     int V;
4
5 public:
6     Graph(int V) { this->V = V; }
7
8     void addEdge(int x, int y, int w)
9     {
10         edgelist.push_back({ w, x, y });
11     }
12
13     void kruskals_mst()
14     {
15         // 1. Sort all edges
16         sort(edgelist.begin(), edgelist.end());
17
18         // Initialize the DSU - DisjointSet
19         DSU s(V);
20         int ans = 0;
21         for (auto edge : edgelist) {
22             int w = edge[0];
23             int x = edge[1];
24             int y = edge[2];
25             // take that edge in MST if it does form a cycle
26             if (s.find(x) != s.find(y)) {
27                 s.union(x, y);
28                 ans += w;
29                 cout << x << " -- " << y << " == " << w
30                             << endl;
31             }
32         }
33         cout << "Minimum Cost Spanning Tree: " << ans;
34     }
35 };

```

## 22 Critical Connection in a Network

Eg  $n=4$  edges =  $[[0, 1], [1, 2], [2, 0], [1, 3]]$



→ Critical connection is a connection, when removed from graph, would result in breaking graph into different components.

Here if  $[1, 3]$  is removed then graph becomes disconnected..

### Approach 1

- Remove one edge each time
- Perform dfs
- If all vertices are not visited then
- Removed edge is a critical connection.

### Approach 2

- Initialise distime array & mintime array with -1.
- discovery time for vertex → min time for vertex to be discovered.
- perform dfs from one node
  - if  $\text{neighbours} == \text{parent}$  then continue
  - else if neighbour is already visited then  
 $\text{mintime}[\text{curr}] = \min(\text{mintime}[\text{curr}], \text{distime}[\text{neigh}])$
  - while returning  $\text{mintime}[\text{curr}] = \min(\text{mintime}[\text{curr}], \text{mintime}[\text{neigh}])$   
if at any point if  $\text{distime}[\text{curr}] < \text{mintime}[\text{neigh}]$   
This indicates critical connection

Code →

```
● ● ●
1 class Solution {
2 public:
3
4     vector<vector<int>> criticalConnections(int n, vector<vector<int>& connections) {
5         vector<int> graph[n];
6         for(vector<int> edge: connections){
7             int u = edge[0];
8             int v = edge[1];
9             graph[u].push_back(v);
10            graph[v].push_back(u);
11        }
12        return findCriticalConnections(n, graph);
13    }
14
15    vector<vector<int>> findCriticalConnections(int n, vector<int> graph[]){
16        vector<int> disTime(n,-1);
17        vector<int> lowTime(n,-1);
18        int time = 0;
19        vector<vector<int>> answer;
20        tarjansDFS(graph, 0, -1, disTime, lowTime, time, answer);
21        return answer;
22    }
23
24    void tarjansDFS(vector<int> graph[], int curr, int parent, vector<int>&disTime,
25    vector<int> &lowTime, int &time, vector<vector<int>> &answer){
26
27        disTime[curr] = time;
28        lowTime[curr] = time;
29        time += 1;
30
31        for(int neigh: graph[curr]){
32            if(neigh == parent) continue;
33
34            if(disTime[neigh]!=-1){
35                lowTime[curr] = min(lowTime[curr], disTime[neigh]);
36                continue;
37            }
38
39            tarjansDFS(graph, neigh, curr, disTime, lowTime, time, answer);
40            lowTime[curr] = min(lowTime[curr], lowTime[neigh]);
41
42            if(disTime[curr] < lowTime[neigh]){
43                vector<int> temp;
44                temp.push_back(curr);
45                temp.push_back(neigh);
46                answer.push_back(temp);
47            }
48        }
49        return;
50    }
51
52};
```

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