

$$\bar{n}_1 = (2x_1 + 1)(2u_1 + 1) - 2\Delta_1$$

$$\bar{n}_1' = (2x_1' + 1)(2u_1' + 1) - 2\Delta_1' \Delta x_{12}$$

$$\begin{cases} x_1 = x_1' + \Delta x_1' \\ u_1 = u_1' + \Delta u_1' \end{cases}$$

$$\begin{matrix} \downarrow \\ \varepsilon + \frac{1}{\Delta x_{12}} \Delta_1 \end{matrix}$$

$$\rightarrow -2\Delta_1' \Delta x_{12}$$

$$= -2\left(\varepsilon + \frac{1}{\Delta x_{12}} \Delta_1\right) \Delta x_{12}$$

$$= -2\varepsilon - 2(\varepsilon + \Delta_1)$$

$$\Rightarrow (2x_1' + 1)$$

$$(2(x_1' + \Delta x_1') + 1)(2(u_1' + \Delta u_1') + 1) - 2\Delta_1$$

$$= (2x_1' + 1)(2u_1' + 1) - 2\Delta x_{12} \Delta_1'$$

$$(2x_1' + 1)(2(u_1' + \Delta u_1') + 1) + 2\Delta x_1'(2(u_1' + \Delta u_1') + 1) - 2\Delta_1$$

$$= (2x_1' + 1)(2u_1' + 1) - 2\Delta x_{12} \Delta_1'$$

$$(2x_1' + 1)(2u_1' + 1) + (2x_1' + 1)(2\Delta u_1') + 2\Delta x_1'(2u_1' + 1)$$

$$+ 2\Delta x_1' \cdot (2\Delta u_1') - 2\Delta_1$$

$$= (2x_1' + 1)(2u_1' + 1) - 2\Delta x_{12} \Delta_1'$$

$$(2x_1' + 1)(2\Delta u_1') + 2\Delta x_1'(2(u_1' + \Delta u_1') + 1) - 2\Delta_1$$

$$= -2\Delta x_{12} \Delta_1'$$

$$\Delta_1' = \frac{(2x_1' + 1)(2\Delta u_1') + 2\Delta x_1'(2(u_1' + \Delta u_1') + 1) - 2\Delta_1}{2\Delta x_{12}}$$

$$= \frac{(2x_1' + 1)(\Delta u_1') + \Delta x_1'(2(u_1' + \Delta u_1') + 1) - \Delta_1}{\Delta x_{12}}$$

$$= \Delta_1 \cdot \frac{(2x_1' + 1)(\Delta u_{1\beta}') + \Delta x_{1\beta}'(2(u_1' + \Delta u_{1\beta}') + 1) - 1}{\Delta x_{12}}$$

$$= \Delta_1 \cdot \frac{(2x_1' + 1)(\Delta u_{1\beta}') + \Delta x_{1\beta}'(2u_1' + 1) + \Delta x_{1\beta}' \Delta u_{1\beta}' - 1}{\Delta x_{12}}$$

$$= \Delta_1 \cdot \frac{(2x_1' + 1)(u_{1\beta} \Delta x_{12}) + (u_{1\beta} \Delta x_{12})(2u_1' + 1) + u_{1\beta} \Delta x_{12} \cdot u_{1\beta} \Delta x_{12} - 1}{\Delta x_{12}}$$

$$= \varepsilon$$

$$\Delta_1' = \Delta_1 \left[(2x_1' + 1) u_{1\beta} + u_{1\beta} (2u_1' + 1) + u_{1\beta} u_{1\beta} \Delta x_{12} \right] - \frac{1}{\Delta x_{12}} \cdot \Delta_1 \quad \text{if } \Delta x_{12} > 1$$

$$\Rightarrow \lim_{\Delta x_{12} \rightarrow \infty} \textcircled{1} \rightarrow 0$$

$$\Delta x_{1\beta}' = \frac{u_{1\beta} \Delta x_{12} + 1}{\Delta u_{1\beta}'}$$

$$\text{Be } \Delta u_{1\beta}' = 1$$

$$\Rightarrow \Delta x_{1\beta}' = u_{1\beta} \Delta x_{12} + 1$$

$$\begin{aligned} x_1' &= x_1 - \Delta x_1' \\ \Rightarrow x_1' &= x_1 - \Delta_1 u_{1\beta} \Delta x_{12} \end{aligned}$$

$$\begin{aligned} u_1' &= u_1 - \Delta u_1' \\ &= u_1 - \Delta_1 u_{1\beta} \Delta x_{12} \end{aligned}$$

$$\begin{aligned} \Delta u_1' &= \Delta_1 \cdot \Delta u_{1\beta}' \\ &= \Delta_1 \cdot u_{1\beta} \Delta x_{12} \end{aligned}$$

$$\begin{aligned} \Delta x_1' &= \Delta_1 \cdot \Delta x_{1\beta}' \\ &= \Delta_1 \cdot u_{1\beta} \Delta x_{12} \end{aligned}$$

$$\begin{aligned} \text{Be} \\ x_{1\beta}' &= \beta \cdot x_1 \end{aligned}$$

$$\begin{aligned}
 \Delta_1' &= \Delta_1 \left[(2x_1' + 1)u_u + u_x(2u_1' + 1) + u_u u_x \Delta x_{1,2} \right] - \Delta_1 \frac{1}{\Delta x_{1,2}} \\
 &= \Delta_1 \left[(2(x_1 - \Delta_1 u_x \Delta x_{1,2}) + 1)u_u + \right. \\
 &\quad \left. + u_x(2(u_1 - \Delta_1 u_u \Delta x_{1,2}) + 1) + u_u u_x \Delta x_{1,2} \right] - \Delta_1 \frac{1}{\Delta x_{1,2}} \\
 &= \Delta_1 \left[(2x_1 + 1)u_u - 2\Delta_1 u_x u_u \Delta x_{1,2} \right. \\
 &\quad \left. + (2u_1 + 1)u_x - 2\Delta_1 u_x u_u \Delta x_{1,2} + u_u u_x \Delta x_{1,2} \right] - \Delta_1 \frac{1}{\Delta x_{1,2}}
 \end{aligned}$$

$$\Delta_1' = \Delta_1 \left[(2x_1 + 1)u_u + (2u_1 + 1)u_x - 4\Delta_1 u_x u_u \Delta x_{1,2} + u_u u_x \Delta x_{1,2} \right] - \Delta_1 \frac{1}{\Delta x_{1,2}}$$

$$\bar{n}_1' = (2x_1' + 1)(2u_1' + 1) - 2\Delta_1' \Delta x_{1,2}$$

(with $u_x = 1 = u_u$)

$$\begin{aligned}
 \bar{n}_1' &= (2x_1' + 1)(2u_1' + 1) - 2 \left\{ \Delta_1 \left[(2x_1' + 1)u_u + (2u_1' + 1)u_x + u_u u_x \Delta x_{1,2} \right] - \frac{\Delta_1}{\Delta x_{1,2}} \right\} \Delta x_{1,2} \\
 &= (2x_1' + 1)(2u_1' + 1) - \left\{ 2\Delta_1 \left[(2x_1' + 1)u_u + (2u_1' + 1)u_x + u_u u_x \Delta x_{1,2} \right] \right\} \Delta x_{1,2} \\
 &\quad - \left\{ 2 \cdot \frac{\Delta_1}{\Delta x_{1,2}} \right\} \Delta x_{1,2}
 \end{aligned}$$

$$\begin{aligned}
 \bar{n}_1' &= (2x_1' + 1)(2u_1' + 1) - 2\Delta_1 \Delta x_{1,2} \left[(2x_1' + 1)u_u + (2u_1' + 1)u_x + u_u u_x \Delta x_{1,2} \right] \\
 &\quad - 2\Delta_1
 \end{aligned}$$

$$\begin{aligned}
 (2x_1' + 1)(2u_1' + 1) &= (2(x_1 - \Delta_1 u_x \Delta x_{1,2}) + 1)(2(u_1 - \Delta_1 u_u \Delta x_{1,2}) + 1) \\
 &= (2x_1 + 1)(2(u_1 - \Delta_1 u_u \Delta x_{1,2}) + 1) \\
 &\quad - (2\Delta_1 u_x \Delta x_{1,2})(2(u_1 - \Delta_1 u_u \Delta x_{1,2}) + 1) \\
 &= (2x_1 + 1)(2u_1 + 1) - (2x_1 + 1)(2\Delta_1 u_u \Delta x_{1,2}) \\
 &\quad - (2\Delta_1 u_x \Delta x_{1,2})(2u_1 + 1) \\
 &\quad + (2\Delta_1 u_x \Delta x_{1,2})(2\Delta_1 u_u \Delta x_{1,2})
 \end{aligned}$$

$$\overline{n}_1' = (2x_1 + 1)(2u_1 + 1)$$

$$- (2x_1 + 1)(2\Delta_1 u_k \Delta x_{112})$$

$$- (2\Delta_1 u_x \Delta x_{112})(2(u_1 - \Delta_1 u_k \Delta x_{112}) + 1)$$

$$- 2\Delta_1 \Delta x_{112} [(2(x_1 - \Delta_1 u_x \Delta x_{112}) + 1) u_k + (2(u_1 - \Delta_1 u_k \Delta x_{112}) + 1) + u_k u_x \Delta x_{112}]$$

$$- 2\Delta_1$$

$$\overline{n}_1' = (2x_1 + 1)(2u_1 + 1)$$

$$- (2x_1 + 1)(2\Delta_1 u_k \Delta x_{112}) \quad \checkmark$$

$$- (2\Delta_1 u_x \Delta x_{112})(2(u_1 - \Delta_1 u_k \Delta x_{112}) + 1) \quad \checkmark$$

$$- (2\Delta_1 \Delta x_{112})(2x_1 + 1) u_k \quad \checkmark$$

$$+ (2\Delta_1 \Delta x_{112})(2\Delta_1 u_x \Delta x_{112} u_k) \quad \checkmark$$

$$- (2\Delta_1 \Delta x_{112})(2(u_1 - \Delta_1 u_k \Delta x_{112}) + 1) \quad \checkmark$$

$$- (2\Delta_1 \Delta x_{112})(u_k u_x \Delta x_{112})$$

$$- 2\Delta_1$$

$$\overline{n}_1' = (2x_1 + 1)(2u_1 + 1)$$

$$- (2x_1 + 1)(2\Delta_1 \Delta x_{112})(\cancel{u_k u_k}) (2u_k)$$

$$- (2\Delta_1 u_x \Delta x_{112})(2u_1 + 1)$$

$$+ (2\Delta_1 u_x \Delta x_{112})(2\Delta_1 u_k \Delta x_{112})$$

$$+ (2\Delta_1 \Delta x_{112})(2\Delta_1 u_x \Delta x_{112} u_k)$$

$$- (2\Delta_1 \Delta x_{112})(2u_1 + 1)$$

$$+ (2\Delta_1 \Delta x_{112})(2\Delta_1 u_k \Delta x_{112})$$

$$- (2\Delta_1 \Delta x_{112})(u_k u_x \Delta x_{112})$$

$$\cancel{u_k u_k} - 2\Delta_1$$

$$u_x, u_k \in \mathbb{Z}$$