Lista 5 - CÁLCULO NUMÉRICO

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1. Seja f uma função. Suponha que f é definida no intervalo $\Big[a, b\Big]$ - para alguns $a, b \in R$. Além disso, suponha que esse intervalo é subdividido em quatro pontos x_0, x_1, x_2 e x_3 separados por uma distância h. Diante disso, temos que:

$$\int_{x_0}^{x_3} f(x)dx \;\approx\; \int_{x_0}^{x_3} P_3(x)dx$$

onde $P_3(x)$ é o polinômio de Gregor-Newton de grau **3**. Ou, reescrevendo $P_3(x)$:

$$P_3(x) = f(x_0) + (x - x_0)\cdot\frac{\Delta^1 f(x_0)}{1!h^1} + (x-x_0)(x-x_1)\cdot\frac{\Delta^2 f(x_0)}{2!h^2} + (x-x_0)(x-x_1)(x-x_2)\cdot\frac{\Delta^3 f(x_0)}{3!h^3}$$

Agora, fazendo substituição de variáveis, teremos que:

$$z = \frac{x - x_0}{h} \Longrightarrow x - x_0 = z\cdot h$$

Em seguida, temos também que:

$$x_1 - x_0 = h \Longrightarrow x_1 = x_0 + h$$

Consequentemente,

$$\begin{aligned} x - x_1 &= x - (x_0 + h) \\ &= (x - x_0) + h \\ &= (z\cdot h) + h \\ &= (z+1)\cdot h \end{aligned}$$

E,

$$x_2 - x_0 = 2\cdot h \Longrightarrow x_2 = x_0 + 2\cdot h$$

Consequentemente,

$$\begin{aligned} x - x_2 &= x - (x_0 + 2\cdot h) \\ &= (x - x_0) + 2\cdot h \\ &= (z\cdot h) + 2\cdot h \\ &= (z+2)\cdot h \end{aligned}$$

Em seguida, reescrevendo o polinômio com a troca de variáveis

$$\begin{aligned} P_3(x) &= f(x_0) + zh\cdot\frac{\Delta^1 f(x_0)}{1!h^1} + zh(z-1)h\cdot\frac{\Delta^2 f(x_0)}{2!h^2} + zh(z-1)h(z-2)h\cdot\frac{\Delta^3 f(x_0)}{3!h^3} \\ &= f(x_0) + z\cdot\Delta^1 f(x_0) + z(z-1)\cdot\frac{\Delta^2 f(x_0)}{2} + z(z-1)(z-2)\cdot\frac{\Delta^3 f(x_0)}{6} \end{aligned}$$

E, montando a integral

$$\begin{aligned} \int_{x_0}^{x_3} P_3(x)dx &= \int_0^3 P_3(z)hdz \\ &= h\cdot\int_0^3 P_3(z)dz \\ &= h\cdot\int_0^3 \Big[f(x_0) + z\cdot\Delta^1 f(x_0) + z(z-1)\cdot\frac{\Delta^2 f(x_0)}{2} + z(z-1)(z-2)\cdot\frac{\Delta^3 f(x_0)}{6}\Big]dz \\ &= h\cdot f(x_0)\cdot\int_0^3 dz + h\cdot\Delta^1 f(x_0)\int_0^3 zdz + h\cdot\frac{\Delta^2 f(x_0)}{2}\cdot\int_0^3 (z^2 - z)dz \\ &\quad + h\cdot\frac{\Delta^3 f(x_0)}{6}\cdot\int_0^3 (z^3 - 3z^2 + 2z)dz \end{aligned}$$

onde

$\Delta^1 f(x_0) = f(x_1) - f(x_0)$
$\Delta^2 f(x_0) = f(x_2) - 2f(x1) + f(x_0)$
$\Delta^3 f(x_0) = f(x_3) - 3f(x2) + 3f(x_1) - f(x_0)$

Logo, temos que:

$$
\begin{aligned}
h \cdot \int_0^3 P_3(z)dz \;&=\; h \cdot f(x_0) \cdot \int_0^3 dz + h \cdot \Delta^1 f(x_0) \int_0^3 z dz + h \cdot \frac{\Delta^2 f(x_0)}{2} \cdot \int_0^3 (z^2 - z)dz + \\
&\quad h \cdot \frac{\Delta^3 f(x_0)}{6} \cdot \int_0^3 (z^3 - 3z^2 + 2z)dz \\
&= h \cdot f(x_0) \cdot \int_0^3 dz + h \cdot \Big[f(x_1) - f(x_0)\Big] \cdot \int_0^3 z dz \\
&\quad + h \cdot \Big[f(x_2) - 2 \cdot f(x_1) + f(x_0)\Big] \cdot \int_0^3 (z^2 - z)dz \\
&\quad + h \cdot \Big[f(x_3) - 3 \cdot f(x_2) + 3 \cdot f(x_1) + f(x_0)\Big] \int_0^3 (z^3 - 3z^2 + 2z)dz \\
&= h \cdot f(x_0) \cdot \Big[\, z \,\Big|_0^3 \,\Big] + h \cdot \Big(\, f(x_1) - f(x_0) \,\Big) \cdot \left[\, \frac{z^2}{2} \,\Bigg|_0^3 \,\right] \\
&\quad + \frac{h}{2} \cdot \Big(\, f(x_2) - 2 \cdot f(x_1) + f(x_0) \,\Big) \cdot \left[\, \left(\frac{z^3}{3} - \frac{z^2}{2}\right) \,\Bigg|_0^3 \,\right] \\
&\quad + \frac{h}{6} \cdot \Big(\, f(x_3) - 3 \cdot f(x_2) + 3 \cdot f(x_1) - f(x_0) \,\Big) \cdot \left[\, \left(\frac{z^4}{4} - z^3 + z^2\right) \,\Bigg|_0^3 \,\right] \\
&= h \cdot f(x_0) \cdot \Big[\, 3 - 0 \,\Big] + h \cdot \Big(\, f(x_1) - f(x_0) \,\Big) \cdot \left[\, \frac{3^2}{2} \,-\, \frac{0^2}{2} \,\right] \\
&\quad + \frac{h}{2} \cdot \Big(\, f(x_2) - 2 \cdot f(x_1) + f(x_0) \,\Big) \cdot \left[\, \left(\frac{3^3}{3} - \frac{3^2}{2}\right) \,-\, \left(\frac{0^3}{3} - \frac{0^2}{2}\right) \,\right] \\
&\quad + \frac{h}{6} \cdot \Big(\, f(x_3) - 3 \cdot f(x_2) + 3 \cdot f(x_1) - f(x_0) \,\Big) \cdot \left[\, \left(\frac{3^4}{4} - 3^3 + 3^2\right) \,-\, \left(\frac{0^4}{4} - 0^3 + 0^2\right) \,\right] \\
&= h \cdot f(x_0) \cdot 3 + h \cdot \Big(\, f(x_1) - f(x_0) \,\Big) \cdot \frac{9}{2} \\
&\quad + \frac{h}{2} \cdot \Big(\, f(x_2) - 2 \cdot f(x_1) + f(x_0) \,\Big) \cdot \left[\, \left(9 - \frac{9}{2}\right) \,-\, 0 \,\right] \\
&\quad + \frac{h}{6} \cdot \Big(\, f(x_3) - 3 \cdot f(x_2) + 3 \cdot f(x_1) - f(x_0) \,\Big) \cdot \left[\, \left(\frac{81}{4} - 18\right) \,-\, 0 \,\right] \\
&= h \cdot f(x_0) \cdot 3 + h \cdot \Big(\, f(x_1) - f(x_0) \,\Big) \cdot \frac{9}{2} \,+\, \frac{h}{2} \cdot \Big(\, f(x_2) - 2 \cdot f(x_1) + f(x_0) \,\Big) \cdot \frac{9}{2} \\
&\quad + \frac{h}{6} \cdot \Big(\, f(x_3) - 3 \cdot f(x_2) + 3 \cdot f(x_1) - f(x_0) \,\Big) \cdot \frac{9}{4} \\
&= 3h \cdot f(x_0) + \frac{9h}{2} \cdot \Big(\, f(x_1) - f(x_0) \,\Big) \,+\, \frac{9h}{4} \cdot \Big(\, f(x_2) - 2 \cdot f(x_1) + f(x_0) \,\Big) \\
&\quad + \frac{3h}{8} \cdot \Big(\, f(x_3) - 3 \cdot f(x_2) + 3 \cdot f(x_1) - f(x_0) \,\Big) \\
&= 3h \cdot f(x_0) + \frac{9h}{2} \cdot f(x_1) \,-\, \frac{9h}{2} \cdot f(x_0) \,+\, \frac{9h}{4} \cdot f(x_2) \,-\, \frac{9h}{4} \cdot 2 \cdot f(x_1) \,+\, \frac{9h}{4} \cdot f(x_0) \\
&\quad + \frac{3h}{8} \cdot f(x_3) \,-\, \frac{3h}{8} \cdot 3 \cdot f(x_2) \,+\, \frac{3h}{8} \cdot 3 \cdot f(x_1) \,-\, \frac{3h}{8} \cdot f(x_0)
\end{aligned}
$$

$$
\begin{aligned}
&= f(x_0)\cdot\Big(3h-\frac{9h}{2}+\frac{9h}{4}-\frac{3h}{8}\Big) \;+\; f(x_1)\cdot\Big(\frac{9h}{2}-\frac{9h}{2}+\frac{9h}{8}\Big) \;+\; f(x_2)\cdot\Big(\frac{9h}{4}-\frac{9h}{8}\Big)\\
&\quad + f(x_3)\cdot\Big(\frac{3h}{8}\Big)\\
&= f(x_0)\cdot\Big(\frac{3h}{8}\Big) \;+\; f(x_1)\cdot\Big(\frac{9h}{8}\Big) \;+\; f(x_2)\cdot\Big(\frac{9h}{8}\Big) \;+\; f(x_3)\cdot\Big(\frac{3h}{8}\Big)\\
&= \Big(\frac{3h}{8}\Big)\cdot\Big(f(x_0) \;+\; 3\cdot f(x_1) \;+\; 3\cdot f(x_2) \;+\; f(x_3)\Big)
\end{aligned}
$$

2. (a) Calculamos,

$$
\begin{aligned}
\int_{-5}^{5} xsen(x)\,dx &= -xcos(x) - \int_{-5}^{5} -cos(x)\,dx\\
&= \Big[\lim_{x\to 5^-} -xcos(x)+sen(x)\Big] - \Big[\lim_{x\to -5^+} -xcos(x)+sen(x)\Big]\\
&= \big[-5\cos(5)+\sin(5)\big] - \big[5\cos(5)-\sin(5)\big]\\
&= -10\cos(5)+2\sin(5)
\end{aligned}
$$

Table 1: Tabela de resultado dos métodos númerico

k	Mét. do trapézio	1/3 Simpson	3/8 Simpson
1	-4.578	-4.463	
2	-4.703	-4.744	
3	-4.731	-4.752	
4	-4.741	-4.753	
5	-4.746	-4.754	
6	-4.748	-4.754	

(b)

3. Analisando o gráfico, obtemos os seguintes pontos: $(\,0,\, v\,)$ e $(\,c,\; 2v\,)$. Diante disso, teremos que:

$$
\begin{aligned}
y-y_0 = m\cdot(x-x_0) &\implies 2v-v = m\cdot(c-0)\\
&\implies v = m\cdot c\\
&\implies m = \frac{v}{c}
\end{aligned}
$$

Devido ao fato de ser uma reta teremos a seguinte integral:

$$
\int_0^c \Big(\frac{v}{c}\Big)\,t+v\;dt
$$

Em seguida, aplicando-se integração por regra do trapézio, calculamos:

$$
\begin{aligned}
\int_0^c \Big(\frac{v}{c}\Big)\,t+v\;dt &\approx \frac{c-0}{2}\cdot\Big(\Big(\frac{v}{c}\cdot c+v\Big)+\Big(\frac{v}{c}\cdot 0+v\Big)\Big)\\
&= \frac{c}{2}\cdot(2v+v))\\
&= \frac{3vc}{2}
\end{aligned}
$$