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In[1]:= Simplify[
  D[InterpolatingPolynomial[Table[{xi + k  $\frac{h}{2}$ , f[xi +  $\frac{k}{2}$ ]}], {k, -3, 3, 2}], z], {z, 1}] /.
  z → xi]
Simplify[D[InterpolatingPolynomial[Table[{xi + k  $\frac{h}{2}$ , f[xi +  $\frac{k}{2}$ ]}], {k, -5, 5, 2}], z],
  {z, 2}] /. z → xi]
Out[1]= 
$$\frac{f\left[x_{-\frac{3}{2}+i}\right] - 27 f\left[x_{-\frac{1}{2}+i}\right] + 27 f\left[x_{\frac{1}{2}+i}\right] - f\left[x_{\frac{3}{2}+i}\right]}{24 h}$$

Out[2]= 
$$-\frac{5 f\left[x_{-\frac{5}{2}+i}\right] - 39 f\left[x_{-\frac{3}{2}+i}\right] + 34 f\left[x_{-\frac{1}{2}+i}\right] + 34 f\left[x_{\frac{1}{2}+i}\right] - 39 f\left[x_{\frac{3}{2}+i}\right] + 5 f\left[x_{\frac{5}{2}+i}\right]}{48 h^2}$$

In[3]:= UFDWeights[m_, n_, s_] :=
  CoefficientList[Normal[Series[x^s Log[x]^m, {x, 1, n}]] / h^m], x]
In[4]:= UFDWeights[1, 3, 3/2]
Out[4]= 
$$\left\{\frac{1}{24 h}, -\frac{9}{8 h}, \frac{9}{8 h}, -\frac{1}{24 h}\right\}$$

In[5]:= UFDWeights[2, 5, 5/2]
Out[5]= 
$$\left\{-\frac{5}{48 h^2}, \frac{13}{16 h^2}, -\frac{17}{24 h^2}, -\frac{17}{24 h^2}, \frac{13}{16 h^2}, -\frac{5}{48 h^2}\right\}$$

In[6]:= StaggerdDiff[m_, n_] :=
  Simplify[D[InterpolatingPolynomial[Table[{xi + k  $\frac{h}{2}$ , f[xi +  $\frac{k}{2}$ ]}], {k, -n, n, 2}], z],
  {z, m}] /. z → xi]
In[7]:= StaggerdDiff[1, 1]
Out[7]= 
$$\frac{-f\left[x_{-\frac{1}{2}+i}\right] + f\left[x_{\frac{1}{2}+i}\right]}{h}$$

In[8]:= StaggerdDiff[1, 3]
Out[8]= 
$$\frac{f\left[x_{-\frac{3}{2}+i}\right] - 27 f\left[x_{-\frac{1}{2}+i}\right] + 27 f\left[x_{\frac{1}{2}+i}\right] - f\left[x_{\frac{3}{2}+i}\right]}{24 h}$$

In[10]:= StaggerdDiff[2, 3]
Out[10]= 
$$\frac{f\left[x_{-\frac{3}{2}+i}\right] - f\left[x_{-\frac{1}{2}+i}\right] - f\left[x_{\frac{1}{2}+i}\right] + f\left[x_{\frac{3}{2}+i}\right]}{2 h^2}$$

In[16]:= StaggerdDiff[2, 5]
Out[16]= 
$$-\frac{5 f\left[x_{-\frac{5}{2}+i}\right] - 39 f\left[x_{-\frac{3}{2}+i}\right] + 34 f\left[x_{-\frac{1}{2}+i}\right] + 34 f\left[x_{\frac{1}{2}+i}\right] - 39 f\left[x_{\frac{3}{2}+i}\right] + 5 f\left[x_{\frac{5}{2}+i}\right]}{48 h^2}$$

In[17]:= StaggerdDiff[3, 3]
Out[17]= 
$$\frac{-f\left[x_{-\frac{3}{2}+i}\right] + 3 f\left[x_{-\frac{1}{2}+i}\right] - 3 f\left[x_{\frac{1}{2}+i}\right] + f\left[x_{\frac{3}{2}+i}\right]}{h^3}$$


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In[18]:= **StaggerdDiff**[3, 5]

$$\text{Out}[18]= \frac{f\left[x_{-\frac{5}{2}+i}\right]-13 f\left[x_{-\frac{3}{2}+i}\right]+34 f\left[x_{-\frac{1}{2}+i}\right]-34 f\left[x_{\frac{1}{2}+i}\right]+13 f\left[x_{\frac{3}{2}+i}\right]-f\left[x_{\frac{5}{2}+i}\right]}{8 h^3}$$

In[20]:= **StaggerdDiff**[4, 5]

$$\text{Out}[20]= \frac{f\left[x_{-\frac{5}{2}+i}\right]-3 f\left[x_{-\frac{3}{2}+i}\right]+2 f\left[x_{-\frac{1}{2}+i}\right]+2 f\left[x_{\frac{1}{2}+i}\right]-3 f\left[x_{\frac{3}{2}+i}\right]+f\left[x_{\frac{5}{2}+i}\right]}{2 h^4}$$

In[21]:= **StaggerdDiff**[4, 7]

$$\text{Out}[21]= \frac{1}{48 h^4} \left(-7 f\left[x_{-\frac{7}{2}+i}\right]+59 f\left[x_{-\frac{5}{2}+i}\right]-135 f\left[x_{-\frac{3}{2}+i}\right]+83 f\left[x_{-\frac{1}{2}+i}\right]+83 f\left[x_{\frac{1}{2}+i}\right]-135 f\left[x_{\frac{3}{2}+i}\right]+59 f\left[x_{\frac{5}{2}+i}\right]-7 f\left[x_{\frac{7}{2}+i}\right] \right)$$