Chapter 3

Discrete-Time Fourier Transform

P3.1 MATLAB Function `[X] = dtft(x,n,w)`

```
function [X] = dtft(x,n,w)
% Computes Discrete-time Fourier Transform
% [X] = dtft(x,n,w)
%
% X = DTFT values computed at w frequencies
% x = finite duration sequence over n (row vector)
% n = sample position row vector
% w = frequency row vector
X = x*exp(-j*n'*w);
```

1. $x(n) = (0.6)^{|n|}\,[u(n+10) - u(n-11)]$.

```
% P0301a: DTFT of x1(n) = 0.6 ^ |n|*(u(n+10)-u(n-11))
clc; close all;
%
[x11,n11] = stepseq(-10,-11,11); [x12,n12] = stepseq(11,-11,11);
[x13,n13] = sigadd(x11,n11,-x12,n12); n1 = n13; x1 = (0.6 .^ abs(n1)).*x13;
w1 = linspace(-pi,pi,201); X1 = dtft(x1,n1,w1);
magX1 = abs(X1); phaX1 = angle(X1);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0301a');
subplot(2,1,1); plot(w1/pi,magX1,'LineWidth',1.5);
axis([-1 1 0 4.5]); wtick = [-1:0.2:1]; magtick = [0:0.5:4.5];
xlabel('\omega/\pi',FNTSZ,LFS);
ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick);
subplot(2,1,2); plot(w1/pi,phaX1*180/pi,'LineWidth',1.5);
axis([-1,1,-180,180]); phatick = [-180 0 180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0301a;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.1.

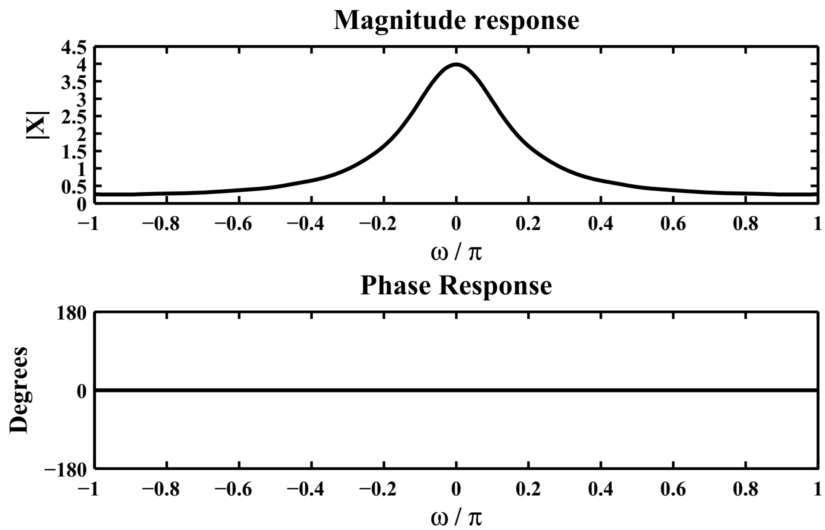

Figure 3.1: Problem P3.1.1 DTFT plots

2. $x(n) = n(0.9)^n [u(n) - u(n-21)]$.

```
% P0301b: % DTFT of x2(n) = n.*(0.9 ^ n) .*(u(n)-u(n-21))
clc; close all;
%
[x21,n21] = stepseq(0,0,22); [x22,n22] = stepseq(21,0,22);
[x23,n23] = sigadd(x21,n21,-x22,n22); n2 = n23; x2 = n2.*(0.9 .^ n2).*x23;
w2 = linspace(-pi,pi,201); X2 = dtft(x2,n2,w2);
magX2 = abs(X2); phaX2 = angle(X2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0301b');
subplot(2,1,1); plot(w2/pi,magX2,'LineWidth',1.5);
wtick = [-1:0.2:1]; magtick = [0:10:60];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick);
subplot(2,1,2); plot(w2/pi,phaX2*180/pi,'LineWidth',1.5);
axis([-1,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0301b;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.2.

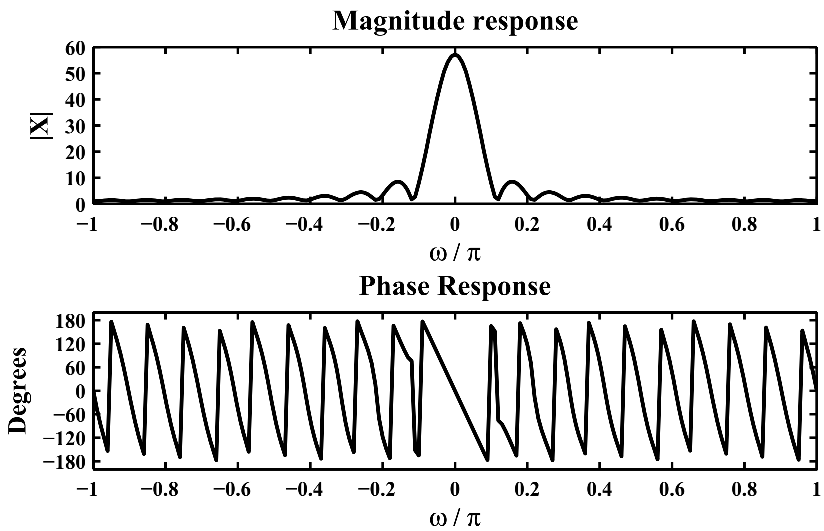

Figure 3.2: Problem P3.1.2 DTFT plots

3. $x(n) = [\cos(0.5\pi n) + j\sin(0.5\pi n)][u(n) - u(n-51)]$.

```
% P0301c: % DTFT of x3(n) = (cos(0.5*pi*n)+j*sin(0.5*pi*n)).*(u(n)-u(n-51))
clc; close all;
%
[x31,n31] = stepseq(0,0,52); [x32,n32] = stepseq(51,0,52);
[x33,n33] = sigadd(x31,n31,-x32,n32); n3 = n33;
x3 = (cos(0.5*pi*n3)+j*sin(0.5*pi*n3)).*x33;
w3 = linspace(-pi,pi,201); X3 = dtft(x3,n3,w3);
magX3 = abs(X3); phaX3 = angle(X3);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0301c');
subplot(2,1,1); plot(w3/pi,magX3,'LineWidth',1.5);
wtick = [-1:0.2:1]; magtick = [0:10:60];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick);
subplot(2,1,2,'LineWidth',1.5); plot(w3/pi,phaX3*180/pi);
axis([-1,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0301c;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.3.

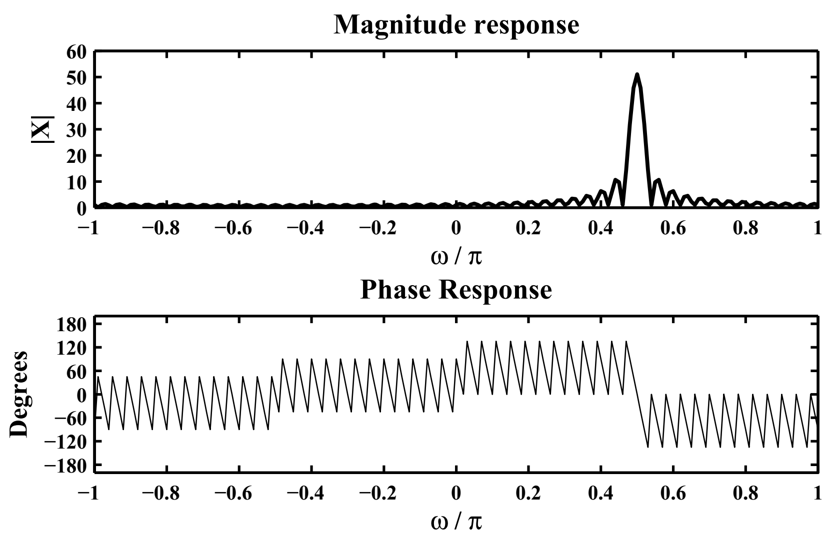

Figure 3.3: Problem P3.1.3 DTFT plots

4. $x(n) = \{\underset{\uparrow}{4}, 3, 2, 1, 1, 2, 3, 4\}$.

```
% P0301d: % DTFT of x4(n) = [4 3 2 1 1 2 3 4] ; n = 0:7;
clc; close all;
%
x4 = [4 3 2 1 1 2 3 4]; n4 = [0:7];
w4 = linspace(-pi,pi,201); X4 = dtft(x4,n4,w4);
magX4 = abs(X4); phaX4 = angle(X4);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0301d');
subplot(2,1,1); plot(w4/pi,magX4,'LineWidth',1.5);
axis([-1,1,0,25]); wtick = [-1:0.2:1]; magtick = [0:5:25];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick);
subplot(2,1,2); plot(w4/pi,phaX4*180/pi,'LineWidth',1.5);
axis([-1,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0301d;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.4.

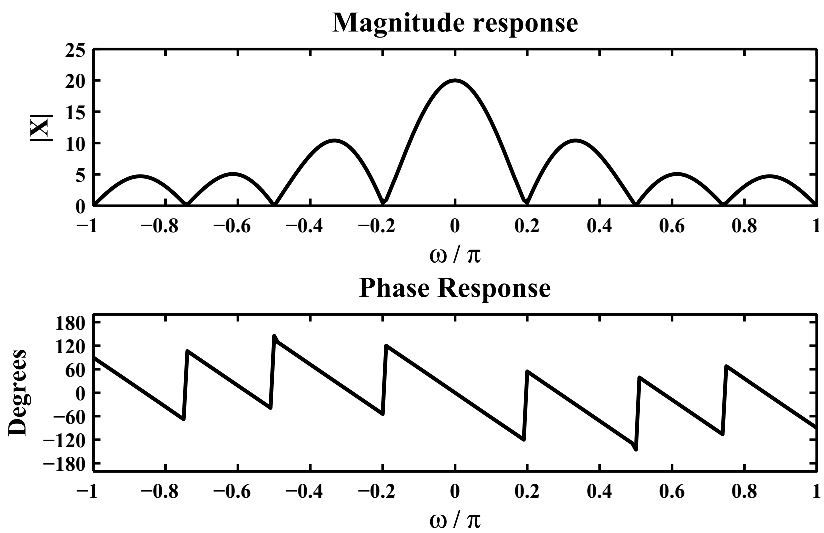

Figure 3.4: Problem P3.1.4 DTFT plots

5. $x(n) = \{\underset{\uparrow}{4}, 3, 2, 1, -1, -2, -3, -4\}$.

```
% P0301e: % DTFT of x5(n) = [4 3 2 1 -1 -2 -3 -4] ; n = 0:7;
clc; close all;
%
x5 = [4 3 2 1 -1 -2 -3 -4]; n5 = [0:7];
w5 = linspace(-pi,pi,201); X5 = dtft(x5,n5,w5);
magX5 = abs(X5); phaX5 = angle(X5);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0301e');
subplot(2,1,1); plot(w5/pi,magX5,'LineWidth',1.5);
wtick = [-1:0.2:1]; magtick = [0:5:20]; axis([-1 1 0 20]);
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick);
subplot(2,1,2); plot(w5/pi,phaX5*180/pi,'LineWidth',1.5);
axis([-1,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0301e;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.5.

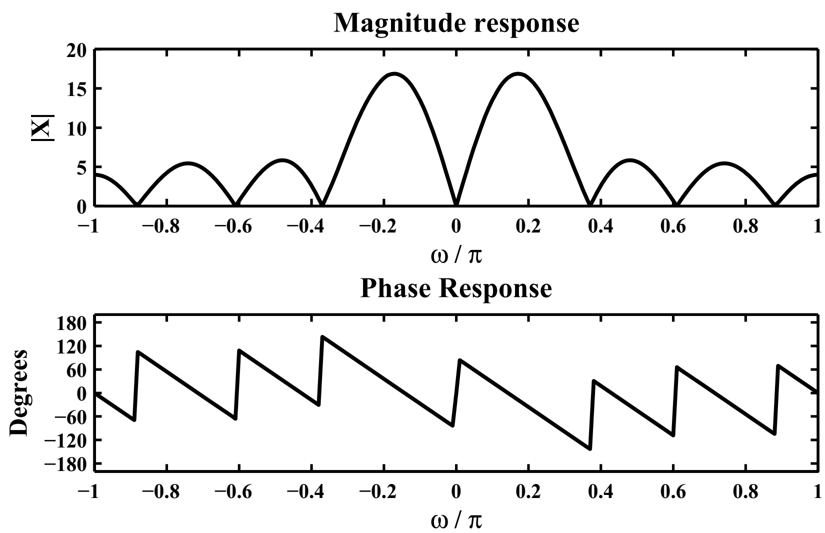

Figure 3.5: Problem P3.1.5 DTFT plots

P3.2 Let $x_1(n) = \{\underset{\uparrow}{1}, 2, 2, 1\}$. A new sequence $x_2(n)$ is formed using

$$x_2(n) = \begin{cases} x_1(n), & 0 \le n \le 3; \\ x_1(n-4), & 4 \le n \le 7; \\ 0, & \text{Otherwise.} \end{cases} \tag{3.1}$$

1. Clearly, $x_2(n) = x_1(n) + x_1(n-4)$. Hence

$$X_2(e^{j\omega}) = X_1(e^{j\omega}) + X_1(e^{j\omega})e^{-j4\omega} = 2e^{-j2\omega}\cos(2\omega)X_1(e^{j\omega})$$

Thus the magnitude $|X_1(e^{j\omega})|$ is scaled by 2 and changed by $|\cos(2\omega)|$ while the phase of $|X_1(e^{j\omega})|$ is changed by 2ω.

2. MATLAB Verification:

```
% P0302b: x1(n) = [1 2 2 1],n = [0:3];
%         x2(n) = x1(n)    ,n = [0:3];
%               = x1(n-4)  ,n = [4:7];
clc; close all;

n1 = [0:3]; x1 = [1 2 2 1]; n2 = [0:7]; x2 = [x1 x1];
w2 = linspace(-pi,pi,201); X1 = dtft(x1,n1,w2); X2 = dtft(x2,n2,w2);
magX1 = abs(X1); phaX1 = angle(X1); magX2 = abs(X2); phaX2 = angle(X2);
wtick = [-1:0.5:1]; phatick = [-180:60 :180];

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0302b');
subplot(2,2,1); plot(w2/pi,magX1,'LineWidth',1.5);
axis([-1 1 0 8]); magtick1 = [0:2:8];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X_1|',FNTSZ,LFS);
title(['Magnitude response' char(10) 'signal x_1'],FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick1);
subplot(2,2,3); plot(w2/pi,phaX1*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title(['Phase response' char(10) 'signal x_1'],FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
subplot(2,2,2); plot(w2/pi,magX2,'LineWidth',1.5);
axis([-1 1 0 16]); magtick2 = [0:4:16];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X_2|',FNTSZ,LFS);
title(['Magnitude response' char(10) 'signal x_2'],FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick2);
subplot(2,2,4); plot(w2/pi,phaX2*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title(['Phase response' char(10) 'signal x_2'],FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0302b;
```

The magnitude and phase plots of $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$ are shown in Figure 3.6 which confirms the observation in part 1. above.

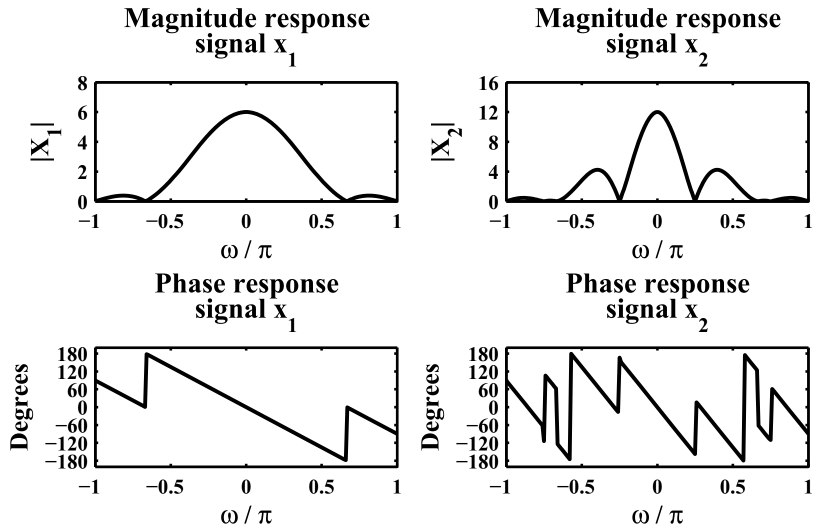

Figure 3.6: Problem P3.2.2 DTFT plots

P3.3 Analytical computation of the DTFTs and plotting of their magnitudes and angles.

1. $x(n) = 2\,(0.5)^n\,u(n+2)$.

$$X\big(e^{j\omega}\big) = 2\sum_{-\infty}^{\infty} 0.5^n u(n+2)e^{-jn\omega} = 2\sum_{-2}^{\infty} 0.5^n e^{-jn\omega} = 2(0.5)^{-2}e^{j2\omega}\sum_{0}^{\infty} 0.5^n e^{-jn\omega} = 8\frac{e^{j2\omega}}{1-0.5e^{-j\omega}}$$

MATLAB Verification:

```
% P0303a: DTFT of x1(n) = 2*((0.5)^n)*u(n+2) = 8*exp(j*2*w)/(1-0.5*exp(-j*w))
clc; close all;

w1 = linspace(0,pi,501); X1 = 8*exp(j*2*w1)./(1-0.5*exp(-j*w1));
magX1 = abs(X1); phaX1 = angle(X1);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0303a');
subplot(2,1,1); plot(w1/pi,magX1,'LineWidth',1.5);
wtick = [0:0.2:1]; magtick = [0:4:20]; axis([0,1,0,20]);
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w1/pi,phaX1*180/pi,'LineWidth',1.5);
axis([0,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick); set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0303a;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.7.

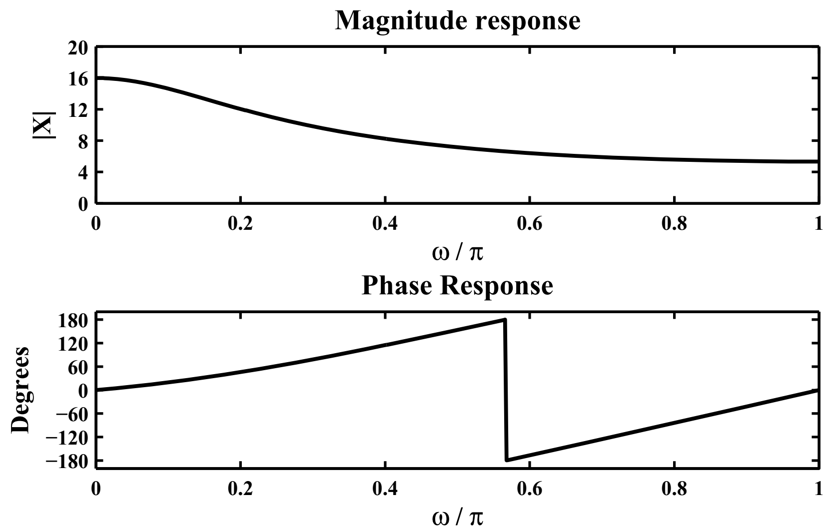

Figure 3.7: Problem P3.3.1 DTFT plots

2. $x(n) = (0.6)^{|n|}\,[u(n+10) - u(n-11)]$.

$$X\left(e^{j\omega}\right) = \sum_{-\infty}^{\infty} (0.6)^{|n|}\,[u(n+10) - u(n-11)]\,e^{-jn\omega} = \sum_{-10}^{10} 0.6^{|n|} e^{-jn\omega}$$

$$= \sum_{-10}^{0} 0.6^{-n} e^{-jn\omega} + \sum_{0}^{10} 0.6^{n} e^{-jn\omega} - 1 = \frac{0.64 - 2(0.6)^{11}\cos(11\omega) + 2(0.6)^{12}\cos(10\omega)}{1.36 - 1.2\cos(\omega)}$$

MATLAB Verification:

```
% P0303b: DTFT of x2(n) = (0.6) ^ |n|*[u(n+10)-u(n-11)]
clc; close all;
w2 = linspace(0,pi,501);
X2 = (0.64-2*(0.6)^11*cos(11*w2)+2*(0.6)^12*cos(10*w2))./(1.36-1.2*cos(w2));
magX2 = abs(X2); phaX2 = angle(X2);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0303b');
subplot(2,1,1); plot(w2/pi,magX2,'LineWidth',1.5);
axis([0,1,0,5]); wtick = [0:0.2:1]; magtick = [0:1:5];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w2/pi,phaX2*180/pi,'LineWidth',1.5);
axis([0,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase Response','FontSize',TFS); set(gca,'XTick',wtick);
set(gca,'YTick',phatick); print -deps2 ../EPSFILES/P0303b;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.8.

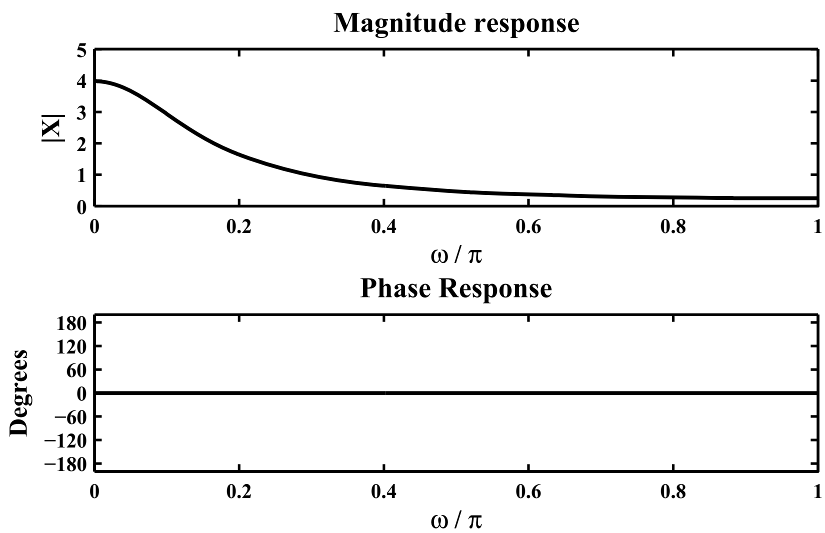

Figure 3.8: Problem P3.3.2 DTFT plots

3. $x(n) = n\,(0.9)^n\,u(n+3)$.

$$X\big(e^{j\omega}\big) = \sum_{-\infty}^{\infty} n\,(0.9)^n\,u(n+3)e^{-jn\omega} = \sum_{-3}^{\infty} n\,(0.9)^n\,e^{-jn\omega}$$
$$= -3(0.9)^{-3}e^{j3\omega} - 2(0.9)^{-2}e^{j2\omega} - (0.9)^{-1}e^{j1\omega} + \sum_{0}^{\infty} n\,(0.9)^n\,e^{-jn\omega}$$
$$= -4.1152e^{j3\omega} - 2.4691e^{j2\omega} - 1.1111e^{j\omega} + \frac{0.9e^{-j\omega}}{(1-0.9e^{-j\omega})^2} = \frac{-4.1151e^{j3\omega} + 4.9383e^{j2\omega}}{1 - 1.8e^{-j\omega} + 0.81e^{-j2\omega}}$$

MATLAB Verification:

```
% P0303c: DTFT of x3(n) = n*((0.9) ^ n)*u(n+3);
clc; close all;
w3 = linspace(0,pi,501); X3_num = (-4.1151*exp(j*3*w3)+4.9383*exp(j*2*w3));
X3_den = 1-1.8*exp(-j*w3)+0.81*exp(-j*2*w3); X3 = X3_num./X3_den;
magX3 = abs(X3); phaX3 = angle(X3);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0303c');
subplot(2,1,1); plot(w3/pi,magX3,'LineWidth',1.5);
axis([0,1,0,100]); wtick = [0:0.2:1]; magtick = [0:20:100];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w3/pi,phaX3*180/pi,'LineWidth',1.5);
axis([0,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase Response','FontSize',TFS); set(gca,'XTick',wtick);
set(gca,'YTick',phatick); print -deps2 ../EPSFILES/P0303c;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.9.

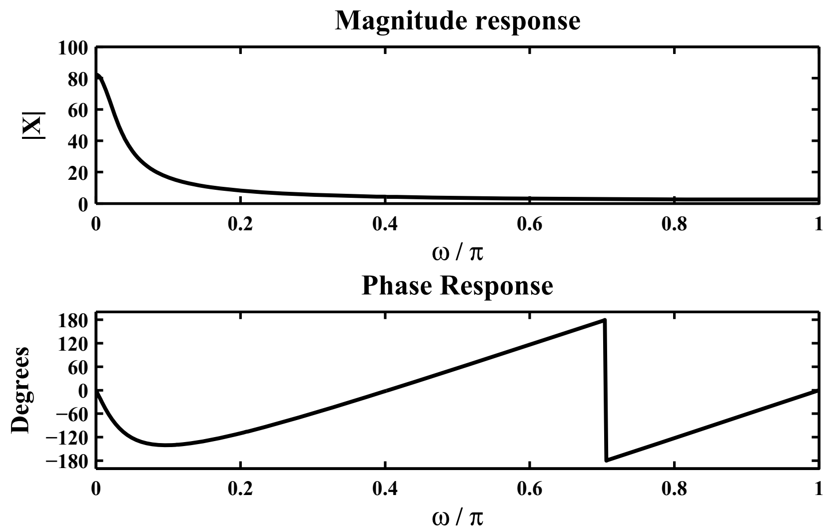

Figure 3.9: Problem P3.3.3 DTFT plots

4. $x(n) = \sum_{-\infty}^{\infty} (n+3)\,(0.8)^{n-1}\,u(n-2)$.

$$X\big(e^{j\omega}\big) = \sum_{-\infty}^{\infty} (n+3)\,(0.8)^{n-1}\,u(n-2)e^{-jn\omega} = \sum_{-\infty}^{\infty} (n+5)\,(0.8)^{n+1}\,u(n)e^{-j(n+2)\omega}$$

$$= (0.8)e^{-j2\omega}\sum_{0}^{\infty} n(0.8)^n e^{-jn\omega} + 4e^{-j2\omega}\sum_{0}^{\infty}(0.8)^n e^{-jn\omega}$$

$$= \frac{0.64e^{-j3\omega}}{\left(1-0.8e^{-j\omega}\right)^2} + \frac{4e^{-j2\omega}}{1-0.8e^{-j\omega}} = \frac{4e^{-j2\omega} - 2.56e^{-j3\omega}}{1-1.6e^{-j\omega}+0.64e^{-j2\omega}}$$

MATLAB Verification:

```
% P0303d: DTFT of x4(n) = (n+3)*((0.8) ^ (n-1))*u(n-2);
clc; close all;
w4 = linspace(0,pi,501); X4_num = 4*exp(-2*j*w4)-2.56*exp(-3*j*w4);
X4_den = 1-1.6*exp(-1*j*w4)+0.64*exp(-2*j*w4); X4 = X4_num./X4_den;
magX4 = abs(X4); phaX4 = angle(X4);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0303d');
subplot(2,1,1); plot(w4/pi,magX4,'LineWidth',1.5);
axis([0 1 0 40]); wtick = [0:0.2:1]; magtick = [0:5:40];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w4/pi,phaX4*180/pi,'LineWidth',1.5);
axis([0,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase Response','FontSize',TFS); set(gca,'XTick',wtick);
set(gca,'YTick',phatick); print -deps2 ../EPSFILES/P0303d;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.10.

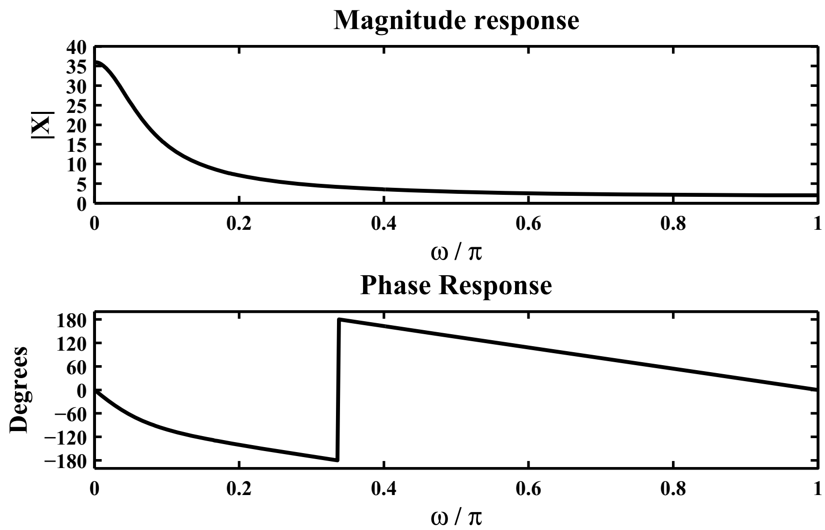

Figure 3.10: Problem P3.3.4 DTFT plots

5. $x(n) = 4\,(-0.7)^n \cos(0.25\pi n)u(n)$.

$$X\big(e^{j\omega}\big) = \sum_{-\infty}^{\infty} 4\,(-0.7)^n \cos(0.25\pi n)u(n)e^{-jn\omega} = 4\sum_{0}^{\infty} (-0.7)^n \cos(0.25\pi n)e^{-jn\omega}$$
$$= 4\frac{1-(-0.7)\cos(0.25\pi)e^{-j\omega}}{1-2(-0.7)e^{-j\omega}+(-0.7)^2e^{-j2\omega}} = 4\frac{1+0.495e^{-j\omega}}{1+1.4e^{-j\omega}+0.49e^{-j2\omega}}$$

MATLAB Verification:

```
% P0303e: DTFT of x5(n) = 4*((-0.7) ^ n)*cos(0.25*pi*n)*u(n)
clc; close all;
w5 = [0:500]*pi/500; X51 = 4*(ones(size(w5))+0.7*cos(0.25*pi)*exp(-j*w5));
X52 = ones(size(w5))+1.4*cos(0.25*pi)*exp(-j*w5)+0.49*exp(-j*2*w5);
X5 = X51./X52; magX5 = abs(X5); phaX5 = angle(X5);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0303e');
subplot(2,1,1); plot(w5/pi,magX5,'LineWidth',1.5);
axis([0 1 0 10]); wtick = [0:0.2:1]; magtick = [0:2:10];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w5/pi,phaX5*180/pi,'LineWidth',1.5);
axis([0,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase Response','FontSize',TFS); set(gca,'XTick',wtick);
set(gca,'YTick',phatick); print -deps2 ../EPSFILES/P0303e;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.11.

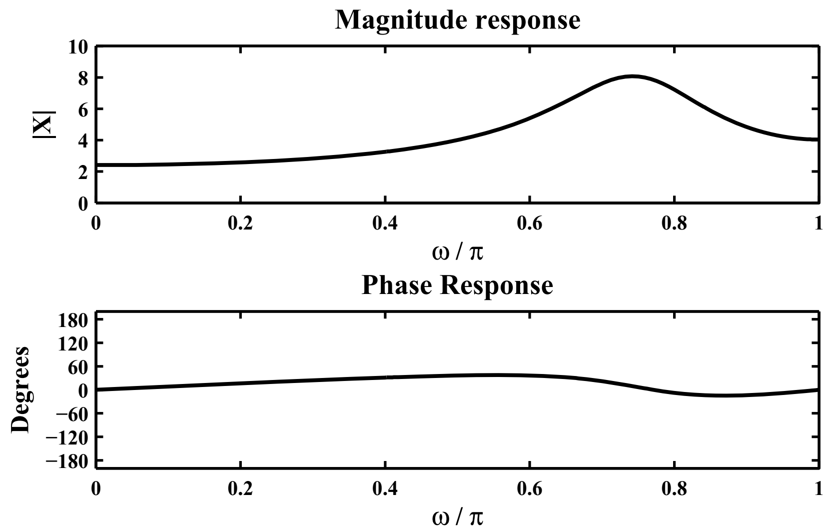

Figure 3.11: Problem P3.3.5 DTFT plots

P3.4 Window function DTFTs:

Rectangular Window: $\mathcal{R}_M(n) = u(n) - u(n - M)$

MATLAB script:

```
% P0304a: DTFT of a Rectangular Window, M = 10,25,50,101
clc; close all;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0304a');
w = linspace(-pi,pi,501); wtick = [-1:0.5:1]; magtick = [0:0.5:1.1];
% M = 10
M = 10; n = 0:M; x = ones(1,length(n));
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
ylabel('|X|','FontSize',LFS); title(['M = 10'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick);
% M = 25
M = 25; n = 0:M; x = ones(1,length(n));
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,2); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
title(['M = 25'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 50
M = 50; n = 0:M; x = ones(1,length(n));
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,3); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['M = 50'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 101
M = 101; n = 0:M; x = ones(1,length(n));
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,4); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); title(['M = 101'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0304a;
```

The magnitude plots of the DTFTs are shown in Figure 3.12.

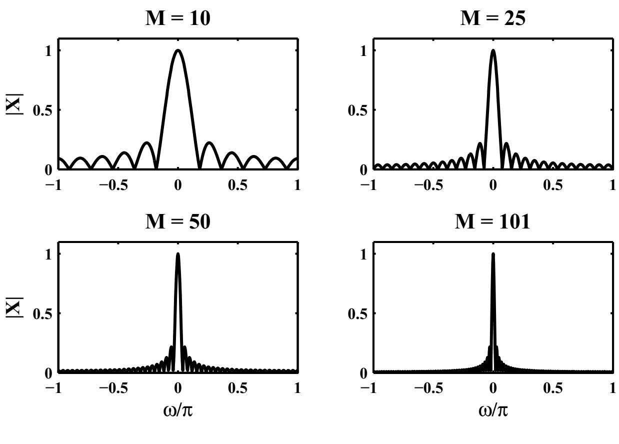

Figure 3.12: Problem P3.4 Rectangular window DTFT plots

Triangular Window: $\mathcal{T}_M(n) = \left[1 - \dfrac{|M-1-2n|}{M-1}\right]\mathcal{R}_M(n)$

MATLAB script:

```
% P0304b: DTFT of a Triangular Window,M = 10,25,50,101
clc; close all;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0304b');
w = linspace(-pi,pi,501); wtick = [-1:0.5:1]; magtick = [0:0.5:1.1];
% M = 10
M = 10; n = 0:M; x = (1-(abs( M-1-(2*n) )/(M+1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
ylabel('|X|','FontSize',LFS); title(['M = 10'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick);
% M = 25
M = 25; n = 0:M; x = (1-(abs( M-1-(2*n) )/(M+1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,2); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
title(['M = 25'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 50
M = 50; n = 0:M; x = (1-(abs( M-1-(2*n) )/(M+1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,3); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['M = 50'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 101
M = 101; n = 0:M; x = (1-(abs( M-1-(2*n) )/(M+1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,4); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); title(['M = 101'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0304b;
```

The magnitude plots of the DTFTs are shown in Figure 3.13.

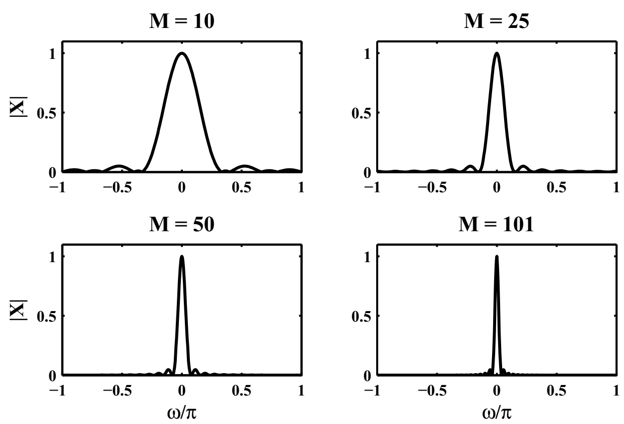

Figure 3.13: Problem P3.4 Triangular window DTFT plots

Hann Window: $\mathcal{C}_M(n) = 0.5\left[1 - \cos\dfrac{2\pi n}{M-1}\right]\mathcal{R}_M(n)$

MATLAB script:

```
% P0304c: DTFT of a Hann Window,M = 10,25,50,101
clc; close all;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0304c');
w = linspace(-pi,pi,501); wtick = [-1:0.5:1]; magtick = [0:0.5:1.1];
% M = 10
M = 10; n = 0:M; x = 0.5*(1-cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
ylabel('|X|','FontSize',LFS); title(['M = 10'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick);
% M = 25
M = 25; n = 0:M; x = 0.5*(1-cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,2); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
title(['M = 25'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 50
M = 50; n = 0:M; x = 0.5*(1-cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,3); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['M = 50'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 101
M = 101; n = 0:M; x = 0.5*(1-cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,4); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); title(['M = 101'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0304c;
```

The magnitude plots of the DTFTs are shown in Figure 3.14.

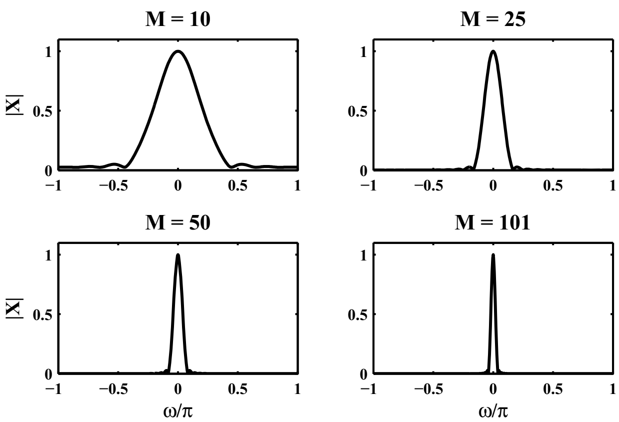

Figure 3.14: Problem P3.4 Hann window DTFT plots

Hamming Window: $\mathcal{H}_M(n) = \left[0.54 - 0.46\cos\dfrac{2\pi n}{M-1}\right]\mathcal{R}_M(n)$

MATLAB script:

```
% P0304d: DTFT of a Hamming Window,M = 10,25,50,101
clc; close all;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0304d');
w = linspace(-pi,pi,501); wtick = [-1:0.5:1]; magtick = [0:0.5:1.1];
% M = 10
M = 10; n = 0:M; x = (0.54-0.46*cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
ylabel('|X|','FontSize',LFS); title(['M = 10'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick);
% M = 25
M = 25; n = 0:M; x = (0.54-0.46*cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,2); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
title(['M = 25'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 50
M = 50; n = 0:M; x = (0.54-0.46*cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,3); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['M = 50'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 101
M = 101; n = 0:M; x = (0.54-0.46*cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,4); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); title(['M = 101'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0304d;
```

The magnitude plots of the DTFTs are shown in Figure 3.15.

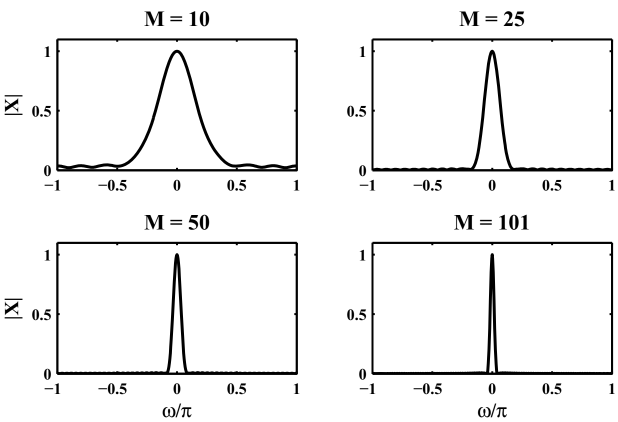

Figure 3.15: Problem P3.4 Hamming window DTFT plots

P3.5 Inverse DTFTs using the definition of the DTFT:

1. $X\left(e^{j\omega}\right) = 3 + 2\cos(\omega) + 4\cos(2\omega)$: Using the Euler identity

$$X\left(e^{j\omega}\right) = 3 + 2\frac{e^{j\omega} + e^{-j\omega}}{2} + 4\frac{e^{j2\omega} + e^{-j2\omega}}{2} = 2e^{j2\omega} + e^{j\omega} + 3 + e^{-j\omega} + 2e^{-j2\omega}$$

Hence $x(n) = \{2, 1, \underset{\uparrow}{3}, 1, 2\}$.

2. $X\left(e^{j\omega}\right) = [1 - 6\cos(3\omega) + 8\cos(5\omega)]\, e^{-j3\omega}$: Using the Euler identity

$$\begin{aligned} X\left(e^{j\omega}\right) &= \left[1 - 6\frac{e^{j3\omega} + e^{-j3\omega}}{2} + 8\frac{e^{j5\omega} + e^{-j5\omega}}{2}\right] e^{-j3\omega} \\ &= 4e^{j2\omega} - 3 + e^{-j3\omega} - 3e^{-j6\omega} + 4e^{-j8\omega} \end{aligned}$$

Hence $x(n) = \{4, 0, \underset{\uparrow}{-3}, 0, 0, 1, 0, 0, -3, 0, 4\}$.

3. $X\left(e^{j\omega}\right) = 2 + j4\sin(2\omega) - 5\cos(4\omega)$: Using the Euler identity

$$X\left(e^{j\omega}\right) = 2 + j4\frac{e^{j2\omega} - e^{-j2\omega}}{2j} - 5\frac{e^{j4\omega} + e^{-j4\omega}}{2} = -\tfrac{5}{2}e^{j4\omega} + 2e^{j2\omega} + 2 - 2e^{-j2\omega} - \tfrac{5}{2}e^{-j4\omega}$$

Hence $x(n) = \{-\tfrac{5}{2}, 0, 2, 0, \underset{\uparrow}{2}, 0, -2, 0, -\tfrac{5}{2}\}$.

4. $X\left(e^{j\omega}\right) = [1 + 2\cos(\omega) + 3\cos(2\omega)]\cos(\omega/2)e^{-j5\omega/2}$: Using the Euler identity

$$\begin{aligned} X\left(e^{j\omega}\right) &= \left[1 + 2\frac{e^{j\omega} + e^{-j\omega}}{2} + 3\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right] \frac{e^{j\frac{1}{2}\omega} + e^{-j\frac{1}{2}\omega}}{2} e^{-j5\omega/2} \\ &= \left[\tfrac{3}{2}e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + \tfrac{3}{2}e^{-j2\omega}\right] \frac{e^{-j2\omega} + e^{-j3\omega}}{2} \\ &= \tfrac{3}{4} + \tfrac{5}{4}e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + \tfrac{5}{4}e^{-j4\omega} + \tfrac{3}{4}e^{-j5\omega} \end{aligned}$$

Hence $x(n) = \{\underset{\uparrow}{\tfrac{3}{4}}, \tfrac{5}{4}, 1, 1, \tfrac{5}{4}, \tfrac{3}{4}\}$.

5. $X\left(e^{j\omega}\right) = j\,[3 + 2\cos(\omega) + 4\cos(2\omega)]\sin(\omega)e^{-j3\omega}$: Using the Euler identity

$$\begin{aligned} X\left(e^{j\omega}\right) &= j\left[3 + 2\frac{e^{j\omega} + e^{-j\omega}}{2} + 4\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right] \frac{e^{j\omega} - e^{-j\omega}}{2j} e^{-j3\omega} \\ &= 2 + e^{-j\omega} + e^{-j2\omega} - e^{-j4\omega} - e^{-j5\omega} - 2e^{-j6\omega} \end{aligned}$$

Hence $x(n) = \{\underset{\uparrow}{2}, 1, 1, 0, -1, -1, -2\}$.

P3.6 Inverse DTFTs using the definition of the IDTFT::

1. $X\left(e^{j\omega}\right) = \begin{cases} 1, & 0 \le |\omega| \le \pi/3; \\ 0, & \pi/3 < |\omega| \le \pi. \end{cases}$

   Solution: Consider

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) e^{jn\omega}\, d\omega = \frac{1}{2\pi}\int_{-\pi/3}^{\pi/3} e^{jn\omega}\, d\omega = \left.\frac{e^{jn\omega}}{j2\pi n}\right|_{-\pi/3}^{\pi/3} = \frac{\sin\left(\frac{\pi n}{3}\right)}{\pi n} = \frac{1}{3}\text{sinc}\left(\frac{n}{3}\right)$$

2. $X\left(e^{j\omega}\right) = \begin{cases} 0, & 0 \le |\omega| \le 3\pi/4; \\ 1, & 3\pi/4 < |\omega| \le \pi. \end{cases}$

   Solution: Consider

$$\begin{aligned} x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) e^{jn\omega}\, d\omega = \frac{1}{2\pi}\int_{-\pi}^{-3\pi/4} e^{jn\omega}\, d\omega + \frac{1}{2\pi}\int_{3\pi/4}^{\pi} e^{jn\omega}\, d\omega \\ &= \frac{2}{2\pi}\int_{3\pi/4}^{\pi} \cos(n\omega)\, d\omega = \frac{1}{\pi}\left.\frac{\sin(n\omega)}{n}\right|_{3\pi/4}^{\pi} = \delta(n) - \frac{3}{4}\text{sinc}\left(\frac{3n}{4}\right) \end{aligned}$$

3. $X\left(e^{j\omega}\right) = \begin{cases} 2, & 0 \le |\omega| \le \pi/8; \\ 1, & \pi/8 < |\omega| \le 3\pi/4. \\ 0, & 3\pi/4 < |\omega| \le \pi. \end{cases}$

   Solution: Consider

$$\begin{aligned} x(n) &= \frac{2}{2\pi}\left[\int_{0}^{\pi/8} 2\cos(n\omega)\, d\omega + \int_{\pi/8}^{3\pi/4} \cos(n\omega)\, d\omega\right] = \frac{1}{\pi}\left[2\left.\frac{\sin(n\omega)}{n}\right|_{0}^{\pi/8} + \left.\frac{\sin(n\omega)}{n}\right|_{\pi/8}^{3\pi/4}\right] \\ &= \frac{1}{n\pi}\left[2\sin\left(\frac{n\pi}{8}\right) + \sin\left(\frac{3n\pi}{4}\right) - \sin\left(\frac{n\pi}{8}\right)\right] = \frac{1}{n\pi}\left[\sin\left(\frac{n\pi}{8}\right) + \sin\left(\frac{3n\pi}{4}\right)\right] \\ &= \frac{1}{8}\text{sinc}\left(\frac{n}{8}\right) + \frac{3}{4}\text{sinc}\left(\frac{3n}{4}\right) \end{aligned}$$

4. $X\left(e^{j\omega}\right) = \begin{cases} 0, & -\pi \le \omega < \pi/4; \\ 1, & \pi/4 \le |\omega| \le 3\pi/4. \\ 0, & 3\pi/4 < |\omega| \le \pi. \end{cases}$

   Solution: Consider

$$x(n) = \frac{2}{2\pi}\int_{\pi/4}^{3\pi/4} \cos(n\omega)\, d\omega = \left.\frac{\sin(n\omega)}{n\pi}\right|_{\pi/4}^{3\pi/4} = \frac{\sin\left(\frac{3n\pi}{4}\right) - \sin\left(\frac{n\pi}{4}\right)}{n\pi} = \frac{3}{4}\text{sinc}\left(\frac{3n}{4}\right) - \frac{1}{4}\text{sinc}\left(\frac{n}{4}\right)$$

5. $X\left(e^{j\omega}\right) = \omega\, e^{j(\pi/2 - 10\omega)}$.

   Solution: Consider

$$\begin{aligned} x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \omega\, e^{j(\pi/2-10\omega)} e^{jn\omega} d\omega = \frac{j}{2\pi}\int_{-\pi}^{\pi} \omega\, e^{j(n-10)\omega} d\omega \\ &= \frac{j}{2\pi}\left[\left.\frac{\omega e^{j(n-10)\omega}}{j(n-10)}\right|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{e^{j(n-10)\omega}}{j(n-10)}\right] = \cos[(n-10)\pi] - \frac{\sin[(n-10)\pi]}{\pi(n-10)^2} \end{aligned}$$

P3.7 A complex-valued sequence $x(n)$ can be decomposed into a conjugate symmetric part $x_e(n)$ and an conjugate anti-symmetric part $x_o(n)$ as

$$x_e(n) = \frac{1}{2}\left[x(n) + x^*(-n)\right]; \qquad x_o(n) = \frac{1}{2}\left[x(n) - x^*(-n)\right]$$

Consider

$$\begin{aligned}\mathcal{F}\left[x_e(n)\right] &= \sum_{-\infty}^{\infty} x_e(n)e^{-jn\omega} = \sum_{-\infty}^{\infty} \frac{1}{2}\left[x(n) + x^*(-n)\right]e^{-jn\omega} = \frac{1}{2}\left[\sum_{-\infty}^{\infty} x(n)e^{-jn\omega} + \sum_{-\infty}^{\infty} x^*(-n)e^{-jn\omega}\right] \\ &= \frac{1}{2}\left[X\left(e^{j\omega}\right) + X^*\left(e^{j\omega}\right)\right] = X_R\left(e^{j\omega}\right)\end{aligned}$$

Similarly,

$$\begin{aligned}\mathcal{F}\left[x_o(n)\right] &= \sum_{-\infty}^{\infty} x_o(n)e^{-jn\omega} = \sum_{-\infty}^{\infty} \frac{1}{2}\left[x(n) - x^*(-n)\right]e^{-jn\omega} = \frac{1}{2}\left[\sum_{-\infty}^{\infty} x(n)e^{-jn\omega} - \sum_{-\infty}^{\infty} x^*(-n)e^{-jn\omega}\right] \\ &= \frac{1}{2}\left[X\left(e^{j\omega}\right) - X^*\left(e^{j\omega}\right)\right] = jX_I\left(e^{j\omega}\right)\end{aligned}$$

MATLAB Verification using $x(n) = 2(0.9)^{-n}\left[\cos(0.1\pi n) + j\sin(0.9\pi n)\right]\left[u(n) - u(n-10)\right]$:

```
% P0307: DTFT after even and odd part decomposition of x(n)
% x(n) = 2*(0.9)^(-n)*(cos(0.1*pi*n)+j*sin(0.9*pi*n))(u(n)-u(n-10))
clc; close all;
%
[x1,n1] = stepseq(0,0,10); [x2,n2] = stepseq(10,0,10);
[x3,n3] = sigadd(x1,n1,-x2,n2);
n = n3; x = 2*(0.9 .^ (-n)).*(cos(0.1*pi*n)+j*sin(0.9*pi *n)).*x3;
[xe,xo,m] = evenodd(x,n);
w = [-500:500]*pi/500; X = dtft(x,n,w); realX = real(X); imagX = imag(X);
Xe = dtft(xe,m,w); Xo = dtft(xo,m,w);
diff_e = max(abs(realX-Xe)); diff_o = max(abs(j*imagX-Xo));
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0307');
subplot(2,2,1); plot(w/pi,real(Xe),'LineWidth',1.5);
axis([-1 1 -30 20]); wtick = sort([-1:0.4:1 0]); magtick = [-30:10:20];
xlabel('\omega/\pi','FontSize',LFS); ylabel('X_e','FontSize',LFS);
title('DTFT of even part of x(n)','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,realX,'LineWidth',1.5); axis([-1 1 -30 20]);
wtick = sort([-1:0.4:1 0] ); magtick = [-30:10:20];
xlabel('\omega/\pi','FontSize',LFS); ylabel('X_R','FontSize',LFS);
title('Real part:DTFT of x(n)','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,imag(Xo),'LineWidth',1.5); axis([-1 1 -30 20]);
wtick = sort([-1:0.4:1 0]); magtick = [-30:10:20];
xlabel('\omega/\pi','FontSize',LFS); ylabel('X_o','FontSize',LFS);
title('DTFT of odd part of x(n)','FontSize',TFS);
```

```
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,imagX,'LineWidth',1.5); axis([-1 1 -30 20]);
wtick = sort([-1:0.4:1 0]); magtick = [-30:10:20];
xlabel('\omega/\pi','FontSize',LFS); ylabel('X_I','FontSize',LFS);
title('Imaginary part:DTFT of x(n)','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0307;
```

The magnitude plots of the DTFTs are shown in Figure 3.16.

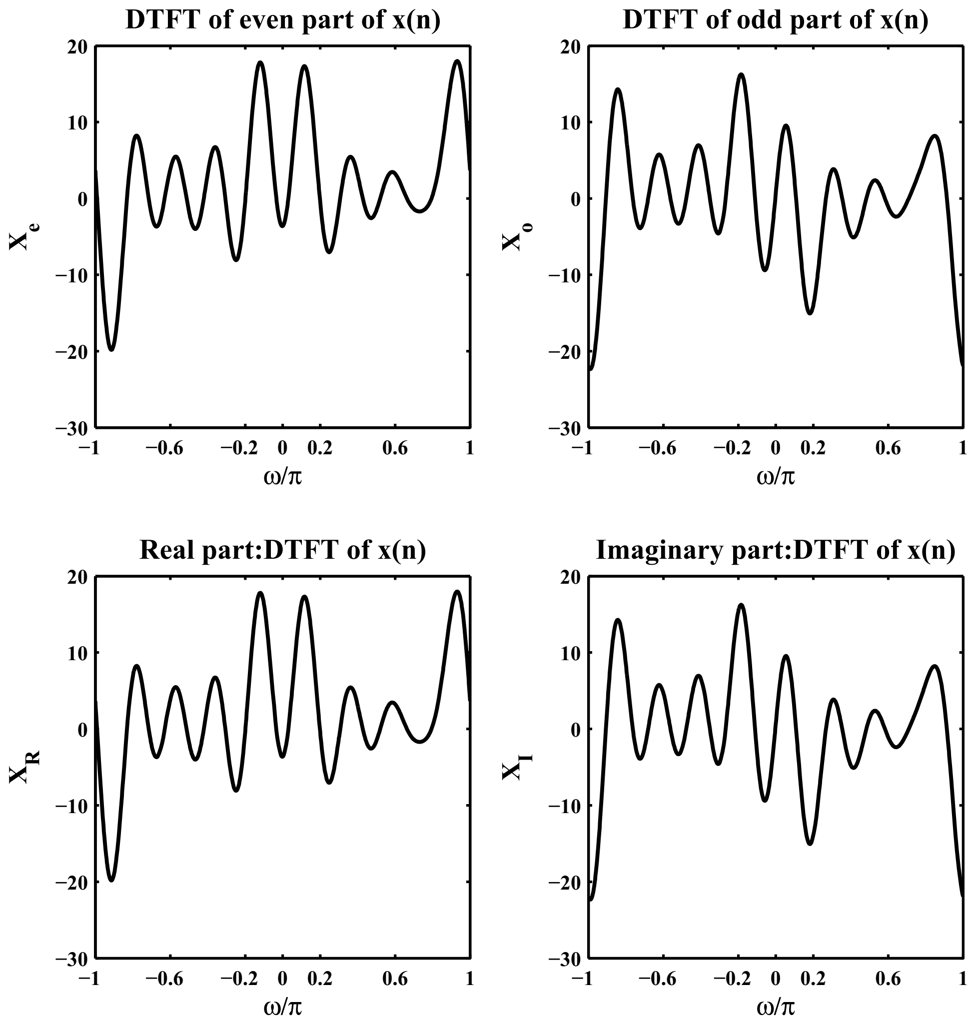

Figure 3.16: Problem P3.7 DTFT plots

P3.8 A complex-valued DTFT $X\left(e^{j\omega}\right)$ can be decomposed into its conjugate symmetric part $X_e\left(e^{j\omega}\right)$ and conjugate anti-symmetric part $X_o\left(e^{j\omega}\right)$, as

$$X\left(e^{j\omega}\right) = X_e\left(e^{j\omega}\right)+X_o\left(e^{j\omega}\right);\ X_e\left(e^{j\omega}\right) = \tfrac{1}{2}\left[X\left(e^{j\omega}\right) + X^*\left(e^{-j\omega}\right)\right],\ X_0\left(e^{j\omega}\right) = \tfrac{1}{2}\left[X\left(e^{j\omega}\right) - X^*\left(e^{-j\omega}\right)\right]$$

Consider

$$\begin{aligned}\mathcal{F}^{-1}\left[X_e\left(e^{j\omega}\right)\right] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X_e\left(e^{j\omega}\right)e^{jn\omega}\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}\tfrac{1}{2}\left[X\left(e^{j\omega}\right) + X^*\left(e^{-j\omega}\right)\right]e^{jn\omega}\,d\omega \\ &= \tfrac{1}{2}[x(n) + x^*(-n)] = x_R(n)\end{aligned}$$

Similarly,

$$\begin{aligned}\mathcal{F}^{-1}\left[X_o\left(e^{j\omega}\right)\right] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X_o\left(e^{j\omega}\right)e^{jn\omega}\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}\tfrac{1}{2}\left[X\left(e^{j\omega}\right) - X^*\left(e^{-j\omega}\right)\right]e^{jn\omega}\,d\omega \\ &= \tfrac{1}{2}[x(n) - x^*(-n)] = jx_I(n)\end{aligned}$$

MATLAB Verification using $x(n) = e^{j0.1\pi n}\left[u(n) - u(n-20)\right]$:

```
% P0308: x(n) = exp(0.1*j*pi*n)*(u(n)-u(n-20));
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,6]);
%
[x1,n1] = stepseq(0,0,20); [x2,n2] = stepseq(20,0,20);
[x3,n3] = sigadd(x1,n1,-x2,n2); n = n3; x = exp(0.1*j*pi*n).*x3;
w1 = [-500:500]*pi/500; X = dtft(x,n,w1); [Xe,Xo,w2] = evenodd(X,[-500:500]);
w2 = w2*pi/500; xr = real(x); xi = imag(x); Xr = dtft(xr,n,w1);
Xi = dtft(j*xi,n,w1); diff_r = max(abs(Xr-Xe)); diff_i = max(abs(Xi-Xo));
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0308');
subplot(4,2,1); plot(w1/pi,abs(Xr),'LineWidth',1.5);
ylabel('|X_r|','FontSize',LFS);
title('Magnitude response of x_R','FontSize',TFS);
subplot(4,2,2); plot(w1/pi,angle(Xr)*180/pi,'LineWidth',1.5);
axis([-1 1 -200 200]); magtick = [-180:90:180];
ylabel('Degrees','FontSize',LFS);
title('Phase response of x_R','FontSize',TFS); set(gca,'YTick',magtick);
subplot(4,2,3); plot(w1/pi,abs(Xe),'LineWidth',1.5); axis([-1 1 0 15]);
ytick = [0:5:15]; ylabel('|X_e|','FontSize',LFS);
title([ 'Magnitude part of X_e' ],'FontSize',TFS); set(gca,'YTick',ytick);
subplot(4,2,4); plot(w1/pi,angle(Xe)*180/pi,'LineWidth',1.5);
axis([-1 1 -200 200]); magtick = [-180:90:180]; ylabel('Degrees','FontSize',LFS);
title([ 'Phase part of X_e' ],'FontSize',TFS); set(gca,'YTick',magtick);
subplot(4,2,5); plot(w1/pi,abs(Xi),'LineWidth',1.5);
ytick = [0:5:15]; axis([-1 1 0 15]); ylabel('|X_i|','FontSize',LFS);
title([ 'Magnitude response of j*x_I' ],'FontSize',TFS);
set(gca,'YTick',ytick);
subplot(4,2,6); plot(w1/pi,angle(Xi)*180/pi,'LineWidth',1.5);
axis([-1 1 -200 200]); magtick = [-180:90:180];
ylabel('Degrees','FontSize',LFS);
```

```
title([ 'Phase response of j*x_I' ],'FontSize',TFS); set(gca,'YTick',magtick);
subplot(4,2,7); plot(w1/pi,abs(Xo),'LineWidth',1.5);
ytick = [0:5:15]; axis([-1 1 0 15]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X_o|','FontSize',LFS);
title(['Magnitude part of X_o'],'FontSize',TFS); set(gca,'YTick',ytick);
subplot(4,2,8); plot(w1/pi,angle(Xo)*180/pi,'LineWidth',1.5);
axis([-1 1 -200 200]); magtick = [-180:90:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase part of X_o'],'FontSize',TFS); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0308;
```

The magnitude plots of the DTFTs are shown in Figure 3.17.

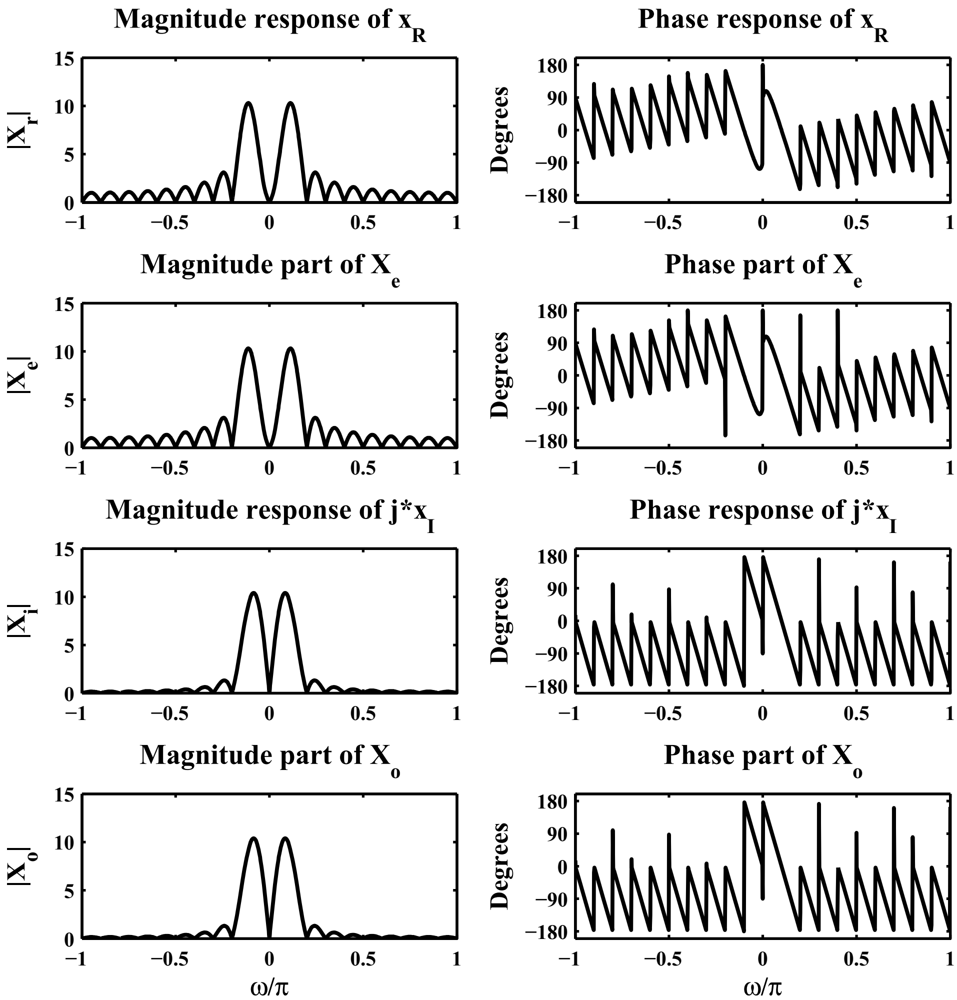

Figure 3.17: Problem P3.8 DTFT plots

P3.9 The real-part of the DTFT of a sinusoidal pulse $x(n) = (\cos\omega_0 n)\,\mathcal{R}_M(n)$:

First note that if the sequence $x(n)$ is a real-valued sequence, then the real part of its DTFT $X\left(e^{j\omega}\right)$ is given by

$$X_{\mathrm{R}}\left(e^{j\omega}\right) = \sum_{-\infty}^{\infty} x(n)\cos(n\omega)$$

Hence for the given sinusoidal pulse, we have

$$X_{\mathrm{R}}\left(e^{j\omega}\right) = \sum_{0}^{M-1}\cos(\omega_0 n)\cos(n\omega) = \frac{1}{2}\sum_{0}^{M-1}\cos[(\omega-\omega_0)n] + \frac{1}{2}\sum_{0}^{M-1}\cos[(\omega+\omega_0)n] \tag{3.2}$$

Consider the first sum in (3.2),

$$\begin{aligned}
\sum_{0}^{M-1}\cos[(\omega-\omega_0)n] &= \frac{1}{2}\sum_{0}^{M-1}\left\{e^{j(\omega-\omega_0)n} + e^{-j(\omega-\omega_0)n}\right\} = \frac{1}{2}\left\{\frac{1-e^{j(\omega-\omega_0)M}}{1-e^{j(\omega-\omega_0)}} + \frac{1-e^{-j(\omega-\omega_0)M}}{1-e^{-j(\omega-\omega_0)}}\right\}\\
&= \frac{1}{2}\left(\frac{1-\cos(\omega-\omega_0)-\cos[(\omega-\omega_0)M]+\cos[(\omega-\omega_0)(M-1)]}{1-\cos(\omega-\omega_0)}\right)\\
&= \frac{1}{2}\left(\frac{2\sin^2[(\omega-\omega_0)/2]+2\sin[(\omega-\omega_0)/2]\sin[(\omega-\omega_0)(M-\frac{1}{2})]}{2\sin^2[(\omega-\omega_0)/2]}\right)\\
&= \frac{1}{2}\left(\frac{\sin[(\omega-\omega_0)/2]+\sin[(\omega-\omega_0)(M-\frac{1}{2})]}{\sin[(\omega-\omega_0)/2]}\right)\\
&= \frac{\cos[(\omega-\omega_0)(M-1)]\sin[(\omega-\omega_0)M/2]}{\sin[(\omega-\omega_0)/2]}
\end{aligned} \tag{3.3}$$

Similarly,

$$\begin{aligned}
\sum_{0}^{M-1}\cos[(\omega+\omega_0)n] &= \sum_{0}^{M-1}\cos[\{\omega-(2\pi-\omega_0)\}n]\\
&= \frac{\cos[\{\omega-(2\pi-\omega_0)\}(M-1)]\sin[\{\omega-(2\pi-\omega_0)\}M/2]}{\sin[\{\omega-(2\pi-\omega_0)\}/2]}
\end{aligned} \tag{3.4}$$

Substituting (3.3) and (3.4) in (3.2), we obtain the desired result.

MATLAB Computation and plot of $X_{\mathrm{R}}\left(e^{j\omega}\right)$ for $\omega_0 = \pi/2$ and $M = 5,\ 15,\ 25,\ 100$:

```
% P0309: DTFT of sinusoidal pulse for different values of M
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,9]);
%% M = 5
M = 5; w0 = pi/2; w = [-500:500]*pi/500;
X = 0.5*(exp(-j*(w-w0)*((M-1)/2)).*sin((w-w0)*M/2)./sin((w-w0+eps)/2)) + ...
    0.5* (exp(-j*(w+w0)*((M-1)/2)).*sin((w+w0)*M/2)./sin((w+w0+eps)/2));
magX = abs(X); phaX = angle(X);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0309');
subplot(4,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 4]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Magnitude Response M = 5'],'FontSize',TFS);
```

```
subplot(4,2,2); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase Response M = 5'],'FontSize',TFS);
ytick = [-180 0 180]; set(gca,'YTickmode','manual','YTick',ytick);
%% M = 15
M = 15; w0 = pi/2; w = [-500:500]*pi/500;
X = 0.5*(exp(-j*(w-w0)*((M-1)/2)).*sin((w-w0)*M/2)./sin((w-w0+eps)/2)) + ...
    0.5* (exp(-j*(w+w0)*((M-1)/2)).*sin((w+w0)*M/2)./sin((w+w0+eps)/2));
magX = abs(X); phaX = angle(X);
%
subplot(4,2,3); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 10]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title([char(10) 'Magnitude Response M = 15' char(10)],'FontSize',TFS);
subplot(4,2,4); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title([char(10) 'Phase Response M = 15' char(10)],'FontSize',TFS);
ytick = [-180 0 180]; set(gca,'YTickmode','manual','YTick',ytick);
%% M = 25
M = 25; w0 = pi/2; w = [-500:500]*pi/500;
X = 0.5*(exp(-j*(w-w0)*((M-1)/2)).*sin((w-w0)*M/2)./sin((w-w0+eps)/2)) + ...
    0.5* (exp(-j*(w+w0)*((M-1)/2)).*sin((w+w0)*M/2)./sin((w+w0+eps)/2));
magX = abs(X); phaX = angle(X);
%
subplot(4,2,5); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 15]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title([char(10) 'Magnitude Response M = 25' char(10)],'FontSize',TFS);
subplot(4,2,6); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title([char(10) 'Phase Response M = 25' char(10)],'FontSize',TFS);
ytick = [-180 0 180];set(gca,'YTickmode','manual','YTick',ytick);
%%M = 101
M = 101; w0 = pi/2; w = [-500:500]*pi/500;
X = 0.5*(exp(-j*(w-w0)*((M-1)/2)).*sin((w-w0)*M/2)./sin((w-w0+eps)/2)) + ...
    0.5* (exp(-j*(w+w0)*((M-1)/2)).*sin((w+w0)*M/2)./sin((w+w0+eps)/2));
magX = abs(X); phaX = angle(X);

subplot(4,2,7); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 75]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title([char(10) 'Magnitude Response M = 101' char(10)],'FontSize',TFS);
ytick = [0 50 75];set(gca,'YTickmode','manual','YTick',ytick);
subplot(4,2,8); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title([char(10) 'Phase Response M = 101' char(10)],'FontSize',TFS);
ytick = [-180 0 180];set(gca,'YTickmode','manual','YTick',ytick);
print -deps2 ../EPSFILES/P0309;
```

The plots of $X_{\mathrm{R}}\left(e^{j\omega}\right)$ are shown in Figure 3.18.

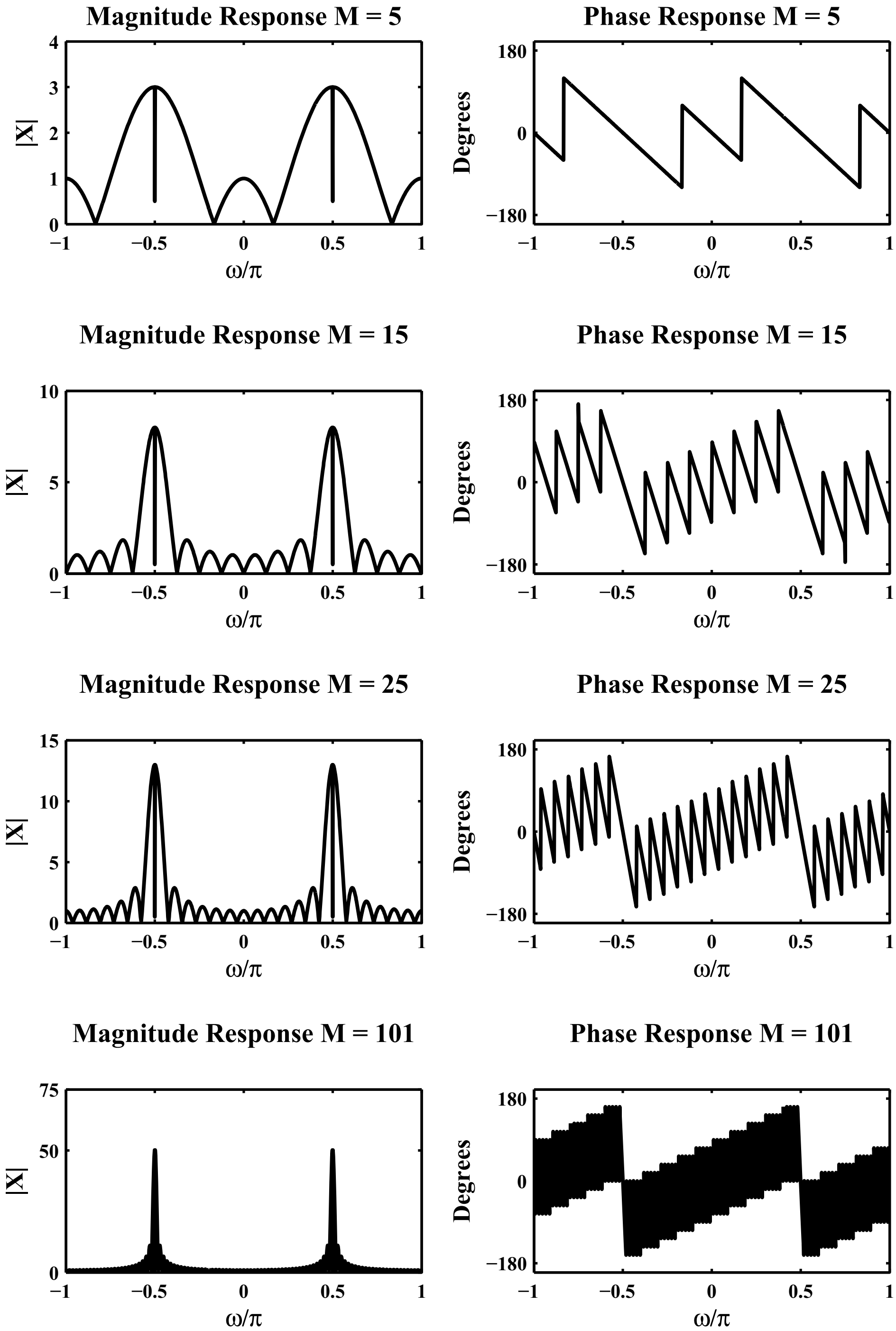

Figure 3.18: Problem P3.9 plots

P3.10 $x(n) = \mathcal{T}_{10}(n)$ is a triangular pulse given in Problem P4.3. DTFT calculations and plots using properties of the DTFT:

1. $x(n) = \mathcal{T}_{10}(-n)$:

$$X\left(e^{j\omega}\right) = \mathcal{F}[x(n)] = \mathcal{F}[\mathcal{T}_{10}(-n)] = \mathcal{F}[\mathcal{T}_{10}(n)]|_{\omega\to-\omega}$$
$$= \mathcal{F}[\mathcal{T}_{10}(n)]^* \ (\because \text{ real signal}$$

MATLAB script:

```
% P0310a: DTFT of x(n) = T_10(-n)
clc; close all; set(0,'defaultfigurepaperposition',[0,0,7,4]);
%
% Triangular Window T_10(n) & its DTFT
M = 10; n = 0:M; Tn = (1-(abs(M-1-2*n)/(M+1)));
w = linspace(-pi,pi,501); Tw = dtft(Tn,n,w); magTw = abs(Tw);
magTw = magTw/max(magTw); phaTw = angle(Tw)*180/pi;
% x(n) & its DTFT
[x,n] = sigfold(Tn,n); X = dtft(x,n,w); magX = abs(X);
magX = magX/max(magX); phaX = angle(X)*180/pi;
% DTFT of x(n) from the Property
Y = fliplr(Tw); magY = abs(Y)/max(abs(Y)); phaY = angle(Y)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0310a');
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,phaX,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.5:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magY,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,phaY,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.4:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0310a;
```

The property verification using plots of $X\left(e^{j\omega}\right)$ is shown in Figure 3.19.

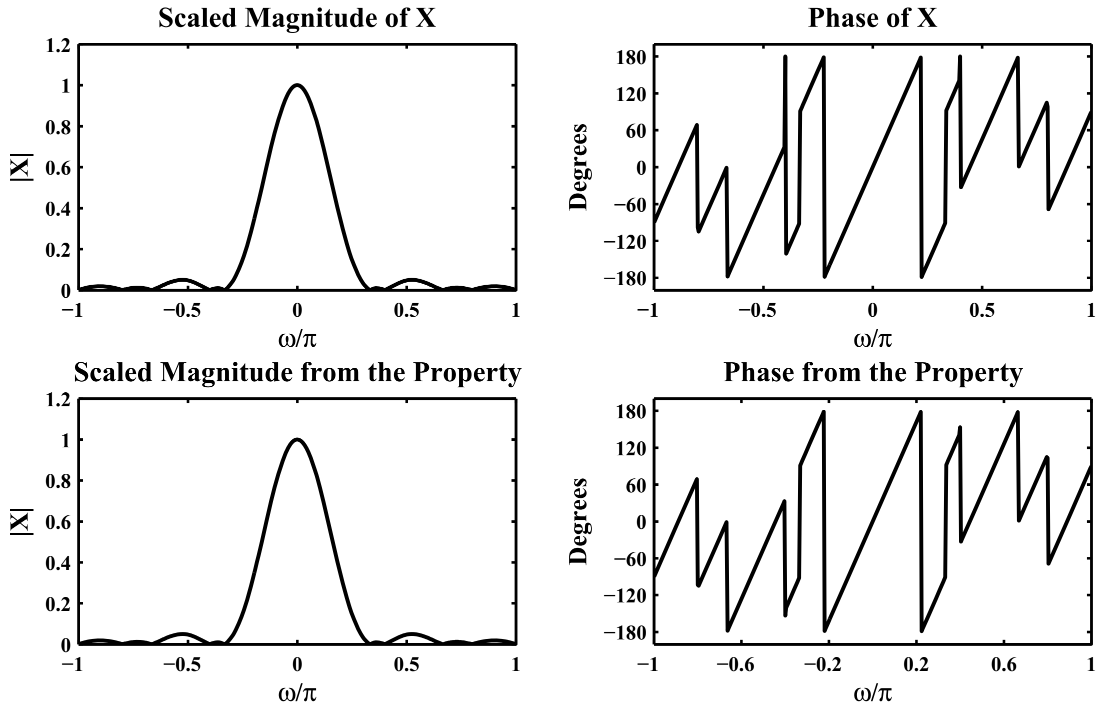

Figure 3.19: Problem P3.10a plots

2. $x(n) = \mathcal{T}_{10}(n) - \mathcal{T}_{10}(n-10)$:

$$\begin{aligned} X\left(e^{j\omega}\right) &= \mathcal{F}[x(n)] = \mathcal{F}[\mathcal{T}_{10}(n) - \mathcal{T}_{10}(n-10)] = \mathcal{F}[\mathcal{T}_{10}(n)] - \mathcal{F}[\mathcal{T}_{10}(n)]e^{-j10\omega} \\ &= \left(1 - e^{-j10\omega}\right)\mathcal{F}[\mathcal{T}_{10}(n)] \end{aligned}$$

MATLAB script:

```
% P0310b: DTFT of T_10(n)-T_10(n-10);
%         T_10(n) = [1-(abs(M-1-2*n)/(M+1)) ]*R_M(n), M = 10
clc; close all; set(0,'defaultfigurepaperposition',[0,0,7,4]);
%
% Triangular Window T_10(n) & its DTFT
M = 10; n = 0:M; Tn = (1-(abs(M-1-2*n)/(M+1)));
w = linspace(-pi,pi,501); Tw = dtft(Tn,n,w); magTw = abs(Tw);
magTw = magTw/max(magTw); phaTw = angle(Tw)*180/pi;
% x(n) & its DTFT
[x1,n1] = sigshift(Tn,n,10); [x,n] = sigadd(Tn,n,-x1,n1);
X = dtft(x,n,w); magX = abs(X);
magX = magX/max(magX); phaX = angle(X)*180/pi;
% DTFT of x(n) from the Property
Y = Tw-exp(-j*w*10).*Tw; magY = abs(Y)/max(abs(Y)); phaY = angle(Y)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0310b');
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,phaX,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.5:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magY,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,phaY,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.4:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0310b;
```

The property verification using plots of $X\left(e^{j\omega}\right)$ is shown in Figure 3.20.

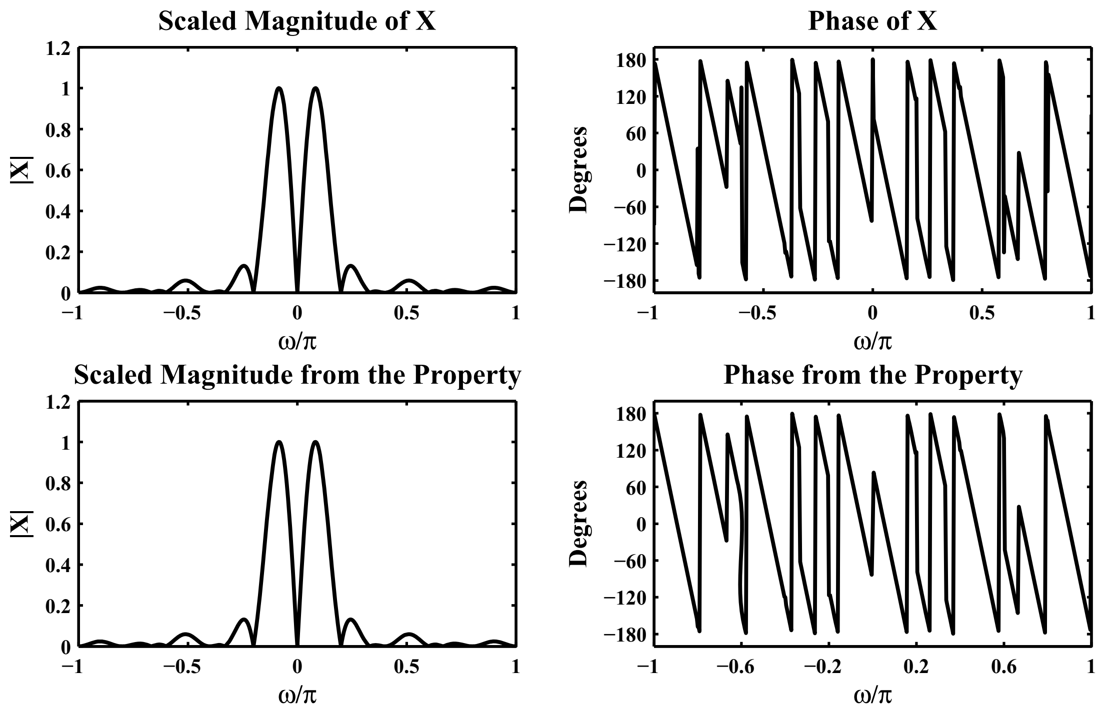

Figure 3.20: Problem P3.10b plots

3. $x(n) = \mathcal{T}_{10}(n) * \mathcal{T}_{10}(-n)$:

$$X\left(e^{j\omega}\right) = \mathcal{F}[x(n)] = \mathcal{F}[\mathcal{T}_{10}(n) * \mathcal{T}_{10}(-n)] = \mathcal{F}[\mathcal{T}_{10}(n)] * \mathcal{F}[\mathcal{T}_{10}(n)]^* \\ = |\mathcal{F}[\mathcal{T}_{10}(n)]|^2$$

MATLAB script:

```
% P0310c: DTFT of T_10(n) Conv T_10(-n)
%         T_10(n) = [1-(abs(M-1-2*n)/(M+1)) ]*R_M(n), M = 10
clc; close all; set(0,'defaultfigurepaperposition',[0,0,7,4]);
%
% Triangular Window T_10(n) & its DTFT
M = 10; n = 0:M; Tn = (1-(abs(M-1-2*n)/(M+1)));
w = linspace(-pi,pi,501); Tw = dtft(Tn,n,w); magTw = abs(Tw);
magTw = magTw/max(magTw); phaTw = angle(Tw)*180/pi;
% x(n) & its DTFT
[x1,n1] = sigfold(Tn,n); [x,n] = conv_m(Tn,n,x1,n1);
X = dtft(x,n,w); magX = abs(X);
magX = magX/max(magX); phaX = angle(X)*180/pi;
% DTFT of x(n) from the Property
Y = Tw.*fliplr(Tw); magY = abs(Y)/max(abs(Y)); phaY = angle(Y)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0310c');
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,phaX,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.5:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magY,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,phaY,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.4:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0310c;
```

The property verification using plots of $X\left(e^{j\omega}\right)$ is shown in Figure 3.21.

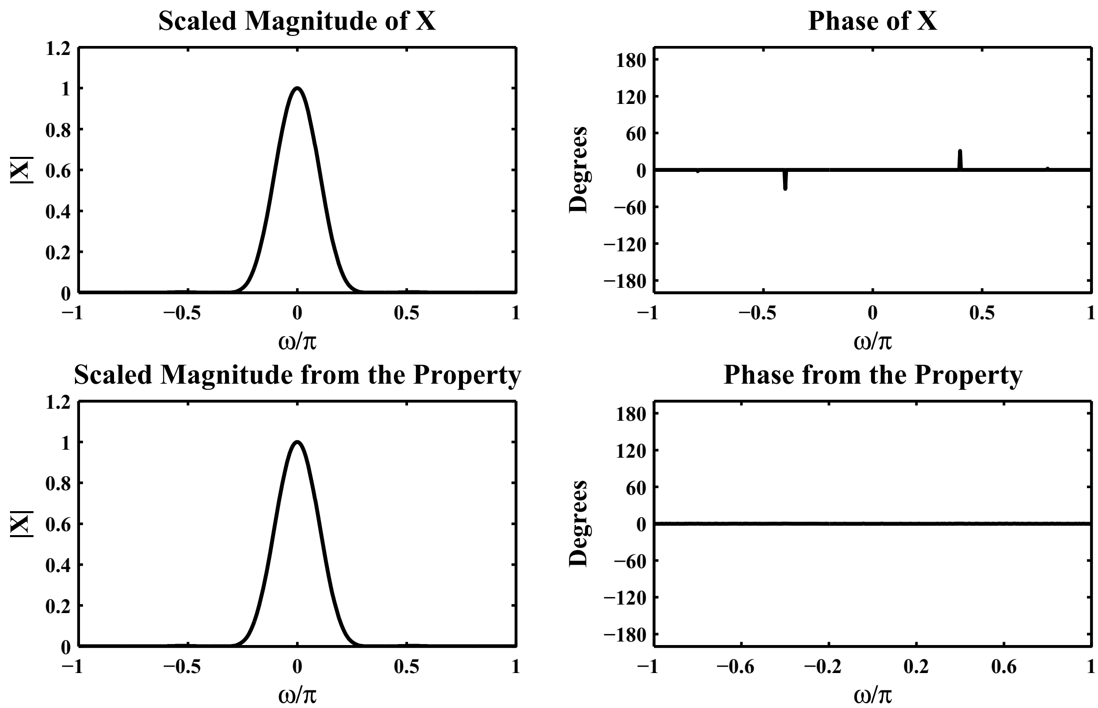

Figure 3.21: Problem P3.10c plots

4. $x(n) = \mathcal{T}_{10}(n)e^{j\pi n}$:

$$X\left(e^{j\omega}\right) = \mathcal{F}[x(n)] = \mathcal{F}[\mathcal{T}_{10}(n)e^{j\pi n}] = \mathcal{F}[\mathcal{T}_{10}(n)]|_{\omega\to(\omega-\pi)}$$

MATLAB script:

```
% P0310d: DTFT of T_10(n)*exp(j*pi*n)
%         T_10(n) = [1-(abs(M-1-2*n)/(M+1)) ]*R_M(n), M = 10
clc; close all; set(0,'defaultfigurepaperposition',[0,0,7,4]);
%
% Triangular Window T_10(n) & its DTFT
M = 10; n = 0:M; Tn = (1-(abs(M-1-2*n)/(M+1)));
w = linspace(-pi,pi,501); Tw = dtft(Tn,n,w); magTw = abs(Tw);
magTw = magTw/max(magTw); phaTw = angle(Tw)*180/pi;
% x(n) & its DTFT
x = Tn.*exp(j*pi*n); X = dtft(x,n,w); magX = abs(X);
magX = magX/max(magX); phaX = angle(X)*180/pi;
% DTFT of x(n) from the Property
Y = [Tw(251:501),Tw(1:250)]; magY = abs(Y)/max(abs(Y)); phaY = angle(Y)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0310d');
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,phaX,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.5:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magY,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,phaY,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.4:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0310d;
```

The property verification using plots of $X\left(e^{j\omega}\right)$ is shown in Figure 3.22.

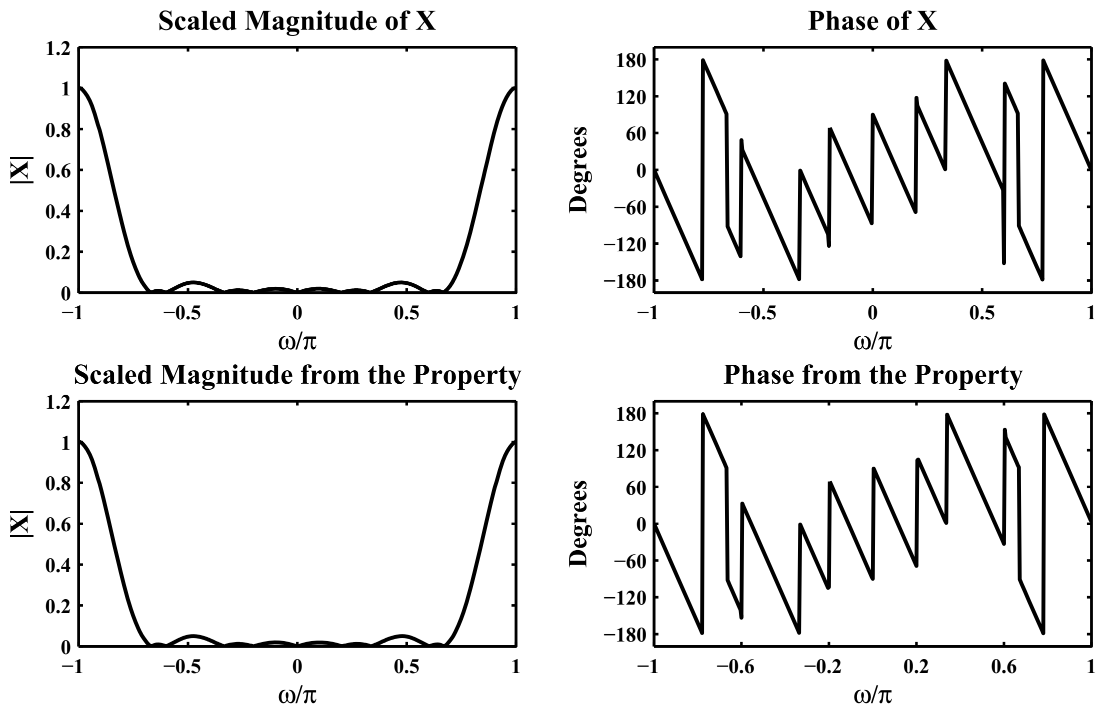

Figure 3.22: Problem P3.10d plots

5. $x(n) = \cos(0.1\pi n)\mathcal{T}_{10}(n)$:

$$X\left(e^{j\omega}\right) = \mathcal{F}[x(n)] = \mathcal{F}[\cos(0.1\pi n)\mathcal{T}_{10}(n)] = \mathcal{F}\left[\frac{1}{2}\left\{e^{j0.1\pi n} + e^{-j0.1\pi n}\right\}\mathcal{T}_{10}(n)\right]$$
$$= \frac{1}{2}\,\mathcal{F}[\mathcal{T}_{10}(n)]|_{\omega\to(\omega-0.1\pi)} + \frac{1}{2}\,\mathcal{F}[\mathcal{T}_{10}(n)]|_{\omega\to(\omega+0.1\pi)}$$

MATLAB script:

```
% P0310e: DTFT of x(n) = T_10(-n)
clc; close all; set(0,'defaultfigurepaperposition',[0,0,7,4]);
%
% Triangular Window T_10(n) & its DTFT
M = 10; n = 0:M; Tn = (1-(abs(M-1-2*n)/(M+1)));
w = linspace(-pi,pi,501); Tw = dtft(Tn,n,w); magTw = abs(Tw);
magTw = magTw/max(magTw); phaTw = angle(Tw)*180/pi;
% x(n) & its DTFT
x = cos(0.1*pi*n).*Tn; X = dtft(x,n,w); magX = abs(X);
magX = magX/max(magX); phaX = angle(X)*180/pi;
% DTFT of x(n) from the Property
Y = 0.5*([Tw(477:501),Tw(1:476)]+[Tw(26:501),Tw(1:25)]);
magY = abs(Y)/max(abs(Y)); phaY = angle(Y)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0310e');
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,phaX,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.5:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magY,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,phaY,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.4:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0310e;
```

The property verification using plots of $X\left(e^{j\omega}\right)$ is shown in Figure 3.23.

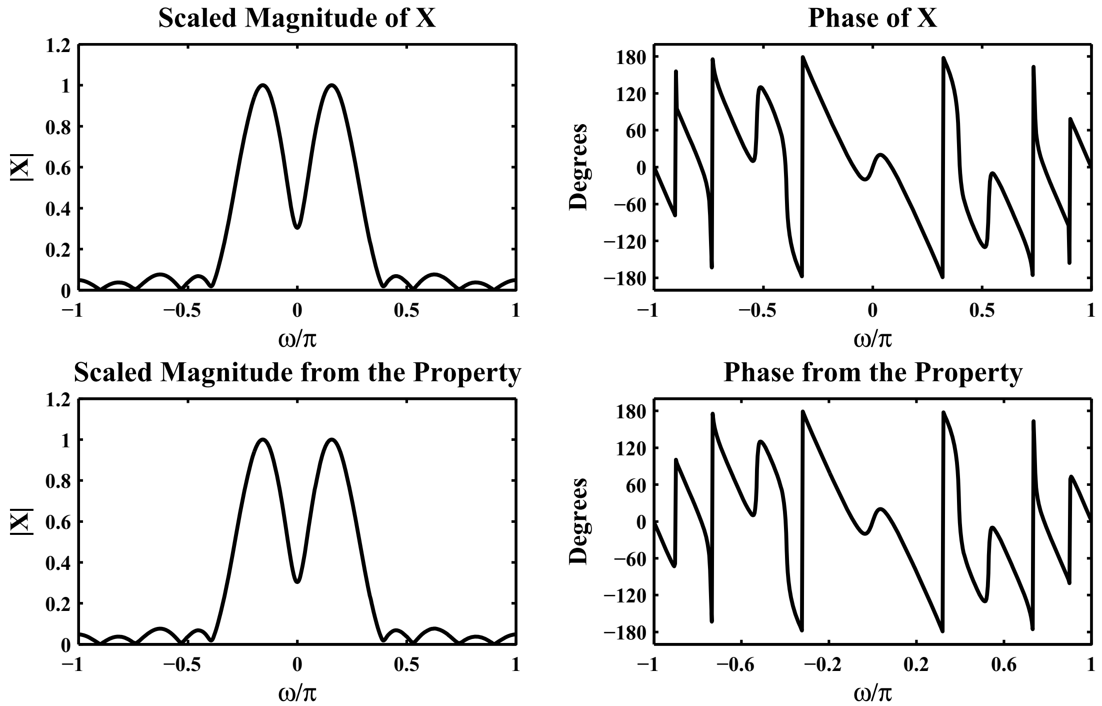

Figure 3.23: Problem P3.10e plots

P3.11 Determination and plots of the frequency response function $H\left(e^{j\omega}\right)$.

1. $h(n) = (0.9)^{|n|}$

$$
\begin{aligned}
H\left(e^{j\omega}\right) &= \sum_{h=-\infty}^{\infty} (n) e^{-jn\omega} = \sum_{0=-\infty}^{-1} .9^{-n} e^{-jn\omega} + \sum_{0=0}^{\infty} .9^{n} e^{-jn\omega} = \sum_{0=0}^{\infty} .9^{n} e^{jn\omega} - 1 + \sum_{0=0}^{\infty} .9^{n} e^{-jn\omega} \\
&= \frac{1}{1-0.9e^{j\omega}} + \frac{1}{1-0.9e^{-j\omega}} - 1 \\
&= \frac{0.19}{1.81 - 1.8\cos(\omega)}
\end{aligned}
$$

MATLAB script:

```
% P0311a: h(n) = (0.9)^|n|; H(w) = 0.19/(1.81-1.8*cos(w));
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
w = [-300:300]*pi/300; H = 0.19*ones(size(w))./(1.81-1.8*cos(w));
magH = abs(H); phaH = angle(H)*180/pi;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0311a');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 20]);
wtick = [-1:0.2:1]; magtick = [0:5:20];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title('Magnitude response of h(n) = (0.9)^{|n|}','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response of h(n) = (0.9)^{|n|}','FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0311a;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.24.

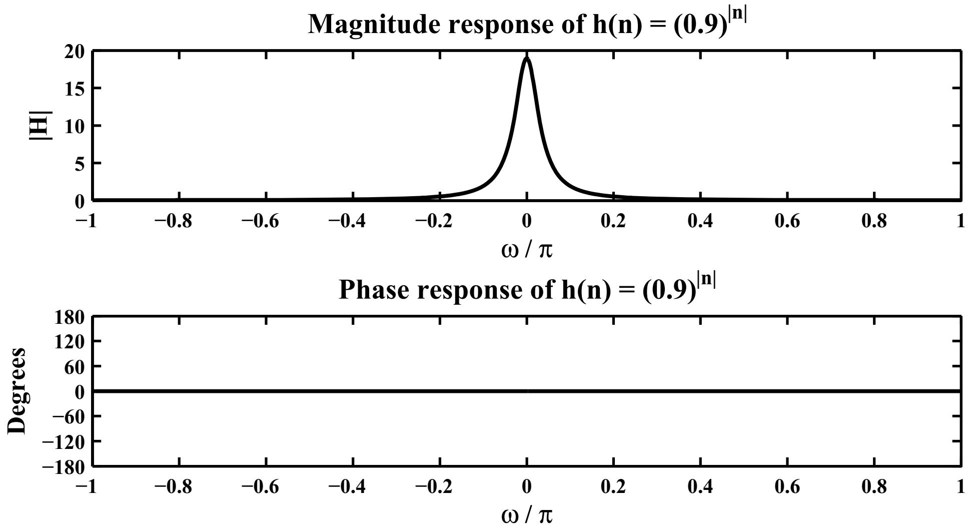

Figure 3.24: Problem P3.11a plots

2. $h(n) = \text{sinc}\,(0.2n)\,[u\,(n+20) - u\,(n-20)]$, where $\text{sinc}\,0 = 1$.
   MATLAB script:

```
% P0311b: h(n) = sinc(0.2*n)*[u(n+20)-u(n-20)]
clc; close all;set(0,'defaultfigurepaperposition',[0,0,6,3]);
[h1,n1] = stepseq(-20,-20,20); [h2,n2] = stepseq(20,-20,20);
[h3,n3] = sigadd(h1,n1,-h2,n2); n = n3; h = sinc(0.2*n).*h3;
w = [-300:300]*pi/300; H = dtft(h,n,w); magH = abs(H); phaH = angle(H)*180/pi;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0311b');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 6]);
wtick = [-1:0.2:1]; magtick = [0:1:6];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response of h(n) = sinc(0.2 \times n)\times' ...
    '[u(n+20)-u(n-20)]'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase response of h(n) = sinc(0.2 n)' ...
    '\times[u(n+20)-u(n-20)]'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0311b;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.25.

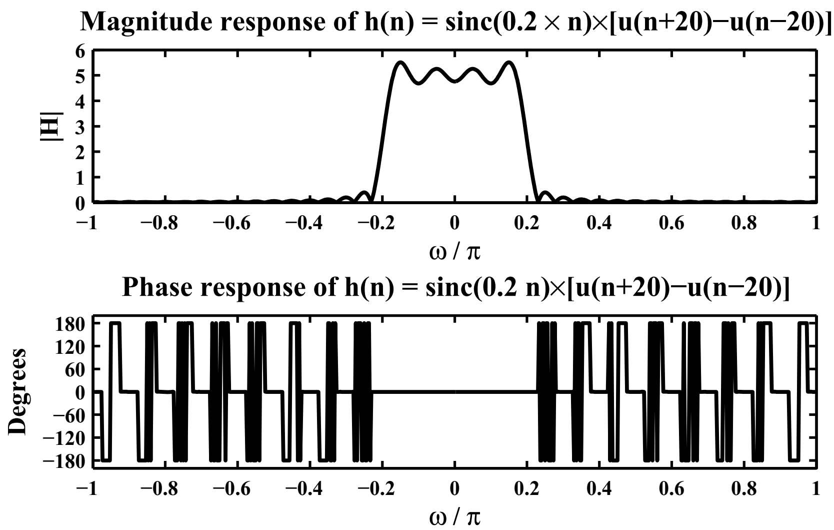

Figure 3.25: Problem P3.11b plots

3. $h(n) = \text{sinc}\,(0.2n)\,[u(n) - u\,(n-40)]$

   MATLAB script:

```
% P0311c: h(n) = sinc(0.2*n)*[u(n)-u(n-40)]
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
[h1,n1] = stepseq(0,0,40); [h2,n2] = stepseq(40,0,40);
[h3,n3] = sigadd(h1,n1,-h2,n2); n = n3; h = sinc(0.2*n).*h3;
w = [-300:300]*pi/300; H = dtft(h,n,w); magH = abs(H); phaH = angle(H)*180/pi;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0311c');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 5]);
wtick = [-1:0.2:1]; magtick = [0:1:5];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response of h(n) = sinc(0.2n)\times' ...
    '[u(n)-u(n-40)]'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase response of h(n) = sinc(0.2)\times' ...
    '[u(n)-u(n-40)]'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0311c;
```

   The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.26.

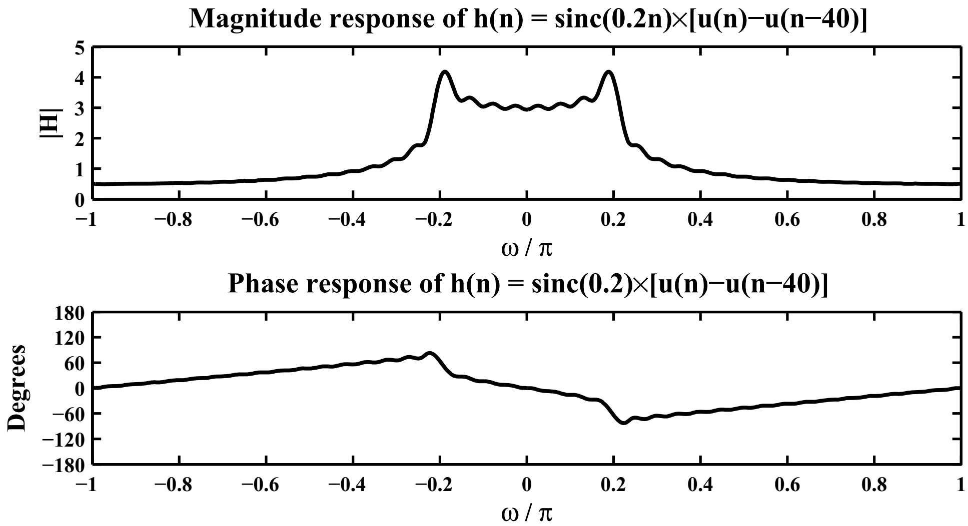

Figure 3.26: Problem P3.11c plots

4. $h(n) = \left[(0.5)^n + (0.4)^n\right] u(n)$

$$H\left(e^{j\omega}\right) = \sum_{h=-\infty}^{\infty} (n) e^{-jn\omega} = \sum_{0=0}^{\infty} .5^n e^{-jn\omega} + \sum_{0=0}^{\infty} .4^n e^{-jn\omega} = \frac{1}{1-0.5e^{-j\omega}} + \frac{1}{1-0.4e^{-j\omega}}$$
$$= \frac{2-0.9e^{-j\omega}}{1-0.9e^{-j\omega}+0.2e^{-j2\omega}}$$

MATLAB script:

```
% P0311d: h(n) = ((0.5)^n+(0.4)^n) u(n);
%         H(w) = (2-0.9*exp(-j*w))./(1-0.9*exp(-j*w)+0.2*exp(-j*2*w))
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
w = [-300:300]*pi/300; H = (2-0.9*exp(-j*w))./(1-0.9*exp(-j*w)+0.2*exp(-j*2*w));
magH = abs(H); phaH = angle(H)*180/pi;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0311d');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 1 4]);
wtick = [-1:0.2:1]; magtick = [0:0.5:4];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title('Magnitude response:h(n) = [(0.5)^n+(0.4)^n] u(n)','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response:h(n) = [(0.5)^n+(0.4)^n] u(n)','FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0311d;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.27.

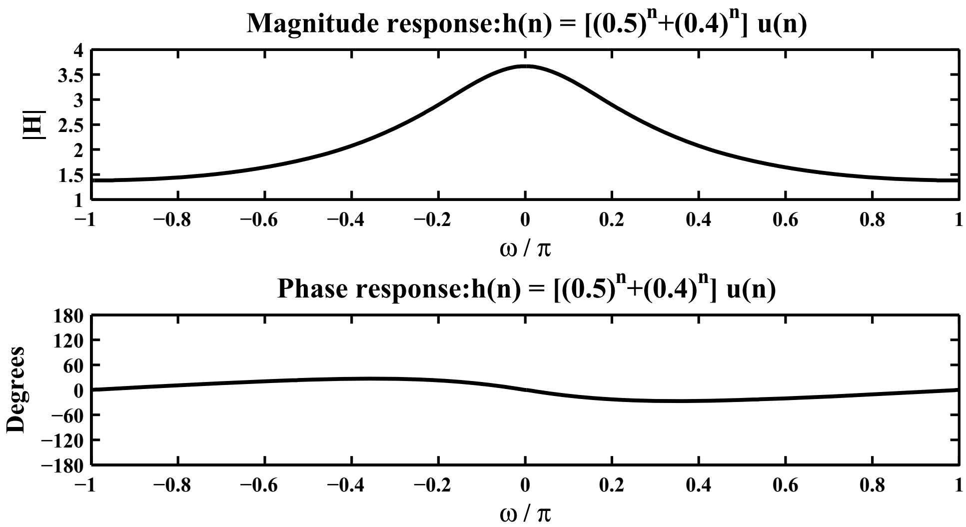

Figure 3.27: Problem P3.11d plots

5. $h(n) = (0.5)^{|n|}\cos(0.1\pi n) = \frac{1}{2}0.5^{|n|}e^{j0.1\pi n} + \frac{1}{2}0.5^{|n|}e^{-j0.1\pi n}$

$$H\left(e^{j\omega}\right) = \sum_{h=-\infty}^{\infty}(n)e^{-jn\omega} = \frac{1}{2}\left[\sum_{0=-\infty}^{\infty}.5^{|n|}e^{-j(\omega-0.1\pi)n} + \sum_{0=-\infty}^{\infty}.5^{|n|}e^{-j(\omega+0.1\pi)n}\right]$$
$$= \frac{0.5\times 0.75}{1.25-\cos(\omega-0.1\pi)} + \frac{0.5\times 0.75}{1.25-\cos(\omega+0.1\pi)}$$

MATLAB script:

```
% P0311e: h(n) = (0.5)^|n|*cos(0.1*pi*n);
%         H(w) = 0.5*0.75*ones(size(w)) ./(1.25-cos(w-(0.1*pi)))+
%                0.5*0.75*ones(size(w)) ./(1.25-cos(w+(0.1*pi)))
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
w = [-300:300]*pi/300; H = 0.5*0.75*ones(size(w))./(1.25-cos(w-(0.1*pi)))+...
      0.5*0.75*ones(size(w))./(1.25-cos(w+(0.1*pi)));
magH = abs(H); phaH = angle(H)*180/pi;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0311e');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 3]);
wtick = [-1:0.2:1]; magtick = [0:0.5:3];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase response'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0311e;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.28.

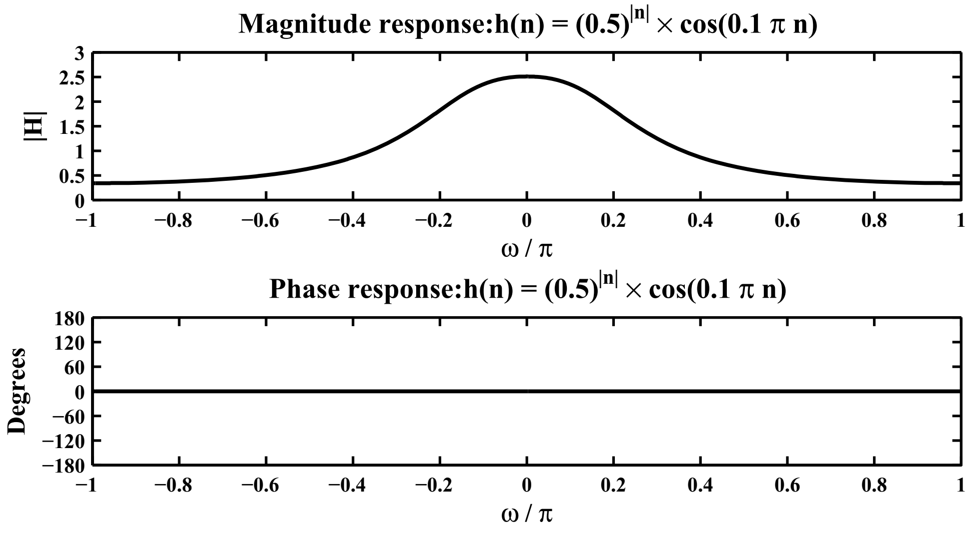

Figure 3.28: Problem P3.11e plots

P3.12 Let $x(n) = A\cos(\omega_0 n + \theta_0)$ be an input sequence to an LTI system described by the impulse response $h(n)$. Then the output sequence $y(n)$ is given by

$$
\begin{aligned}
y(n) = h(n) * x(n) &= A \sum_{h=-\infty}^{\infty} (k) \cos[\omega_0(n-k) + \theta_0] \\
&= \frac{A}{2} \sum_{h=-\infty}^{\infty} (k) \exp\left[j\omega_0(n-k) + \theta_0\right] + \frac{A}{2} \sum_{h=-\infty}^{\infty} (k) \exp\left[-j\omega_0(n-k) - \theta_0\right] \\
&= \frac{A}{2} e^{j\theta_0} \left[\sum_{h=-\infty}^{\infty} (k) e^{-j\omega_0 k} \right] e^{j\omega_0 n} + \frac{A}{2} e^{-j\theta_0} \left[\sum_{h=-\infty}^{\infty} (k) e^{j\omega_0 k} \right] e^{-j\omega_0 n} \\
&= \frac{A}{2} e^{j\theta_0} H\left(e^{j\omega_0}\right) e^{j\omega_0 n} + \frac{A}{2} e^{-j\theta_0} H^*\left(e^{j\omega_0}\right) e^{-j\omega_0 n} \\
&= \frac{A}{2} e^{j\theta_0} |H\left(e^{j\omega_0}\right)| e^{j\angle H\left(e^{j\omega_0}\right)} e^{j\omega_0 n} + \frac{A}{2} e^{-j\theta_0} |H\left(e^{j\omega_0}\right)| e^{-j\angle H\left(e^{j\omega_0}\right)} e^{-j\omega_0 n} \\
&= \frac{A}{2} |H\left(e^{j\omega_0}\right)| \left[\exp\left\{j\left[\omega_0 n + j\theta_0 + j\angle H\left(e^{j\omega_0}\right)\right]\right\} + \exp\left\{-j\left[\omega_0 n + j\theta_0 + j\angle H\left(e^{j\omega_0}\right)\right]\right\}\right] \\
&= A|H\left(e^{j\omega_0}\right)| \cos\left[\omega_0 n + j\theta_0 + j\angle H\left(e^{j\omega_0}\right)\right]
\end{aligned}
$$

P3.13 Sinusoidal steady-state responses

1. The input to the system $h(n) = (0.9)^{|n|}$ is $x(n) = 3\cos(0.5\pi n + 60°) + 2\sin(0.3\pi n)$. The steady-state response $y(n)$ is computed using MATLAB.

```
% P0313a: x(n) = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n); i.e.
%         x(n) = 3*cos(0.5*pi*n+pi/3)+2*cos(0.3*pi*n-pi/2)
%         h(n) = (0.9)^|n|
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
n = [-20:20]; x = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n);
w1 = 0.5*pi; H1 = 0.19*w1/(1.81-1.8*cos(w1));
w2 = 0.3*pi; H2 = 0.19*w2/(1.81-1.8*cos(w2));
magH1 = abs(H1); phaH1 = angle(H1); magH2 = abs(H2); phaH2 = angle(H2);
y = 3*magH1*cos(w1*n+pi/3+phaH1)+...
    2*magH2*cos(w2*n-pi/2+phaH2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0313a');
subplot(2,1,1); Hs = stem(n,x,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS); axis([-22 22 -5 5]);
title(['Input sequence x(n): 3*cos(0.5{\pi}n + \pi/3) + 2sin(0.3{\pi}n)'],...
    'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); axis([-22 22 -5 5]);
title('Output sequence y(n) for h(n) = (0.9)^{|n|}','FontSize',TFS);
print -deps2 ../EPSFILES/P0313a;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.29.

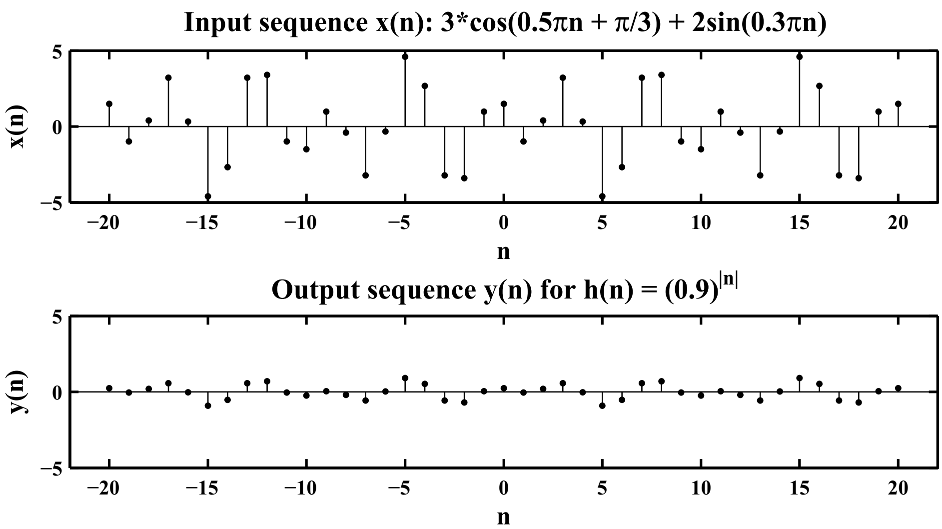

Figure 3.29: Problem P3.13a plots

2. The input to the system $h(n) = \text{sinc}(0.2n)[u(n+20) - u(n-20)]$, where $\text{sinc}\,0 = 1$. The steady-state response $y(n)$ is computed using MATLAB.

```
% P0313b: x(n) = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n); i.e.
%         x(n) = 3*cos(0.5*pi*n+pi/3)+2*cos(0.3*pi*n-pi/2)
%         h(n) = sinc(0.2*n)*[u(n+20)-u(n-20)]
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
n = [-20:20]; x = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n);
[h1,n1] = stepseq(-20,-20,20); [h2,n2] = stepseq(20,-20,20);
[h3,n3] = sigadd(h1,n1,-h2,n2); n = n3; h = sinc(0.2*n).*h3;
w1 = 0.5*pi; H1 = dtft(h,n,w1); w2 = 0.3*pi; H2 = dtft(h,n,w2);
magH1 = abs(H1); phaH1 = angle(H1); magH2 = abs(H2); phaH2 = angle(H2);
y = 3*magH1*cos(w1*n+pi/3+phaH1)+2*magH2*cos(w2*n-pi/2+phaH2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0313b');
subplot(2,1,1); Hs = stem(n,x,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS); axis([-22 22 -5 5]);
title(['Input sequence x(n): 3*cos(0.5{\pi}n + \pi/3) + 2sin(0.3{\pi}n)'],...
    'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); axis([-22 22 -5 5]);
title('Output sequence y(n) for h(n) = sinc(0.2 n)[u(n+20) - u(n-20)]',...
    'FontSize',TFS); print -deps2 ../EPSFILES/P0313b;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.30.

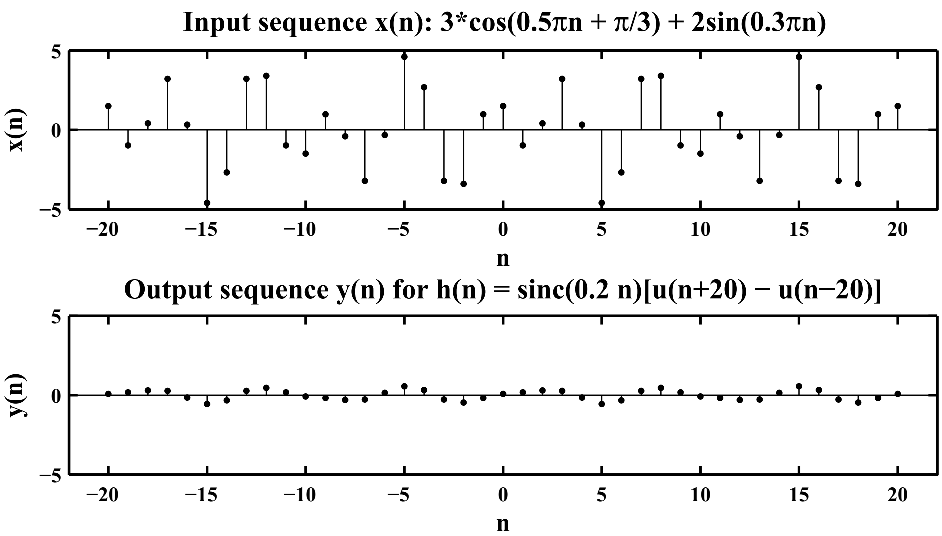

Figure 3.30: Problem P3.13b plots

3. The input to the system $h(n) = \text{sinc}\,(0.2n)\,[u(n) - u\,(n-40)]$. The steady-state response $y(n)$ is computed using MATLAB.

```
% P0313c: x(n) = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n); i.e.
%         x(n) = 3*cos(0.5*pi*n+pi/3)+2*cos(0.3*pi*n-pi/2)
%         h(n) = sinc(0.2*n)*[u(n)-u(n-40)]
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
n = [-20:20]; x = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n);
[h1,n1] = stepseq(0,0,40); [h2,n2] = stepseq(40,0,40);
[h3,n3] = sigadd(h1,n1,-h2,n2); h = sinc(0.2*n3).*h3;
w1 = 0.5*pi; w2 = 0.3*pi; H1 = dtft(h,n3,w1); H2 = dtft(h,n3,w2);
magH1 = abs(H1); phaH1 = angle(H1); magH2 = abs(H2); phaH2 = angle(H2);
y = 3*magH1*cos(w1*n+pi/3+phaH1)+2*magH2*cos(w2*n-pi/2+phaH2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0313c');
subplot(2,1,1); Hs = stem(n,x,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS); axis([-22 22 -5 5]);
title(['Input sequence x(n): 3*cos(0.5{\pi}n + \pi/3) + 2sin(0.3{\pi}n)'],...
    'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); axis([-22 22 -5 5]);
title('Output sequence y(n) for h(n) = sinc(0.2 n)[u(n) - u(n-40)]',...
    'FontSize',TFS); print -deps2 ../EPSFILES/P0313c;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.31.

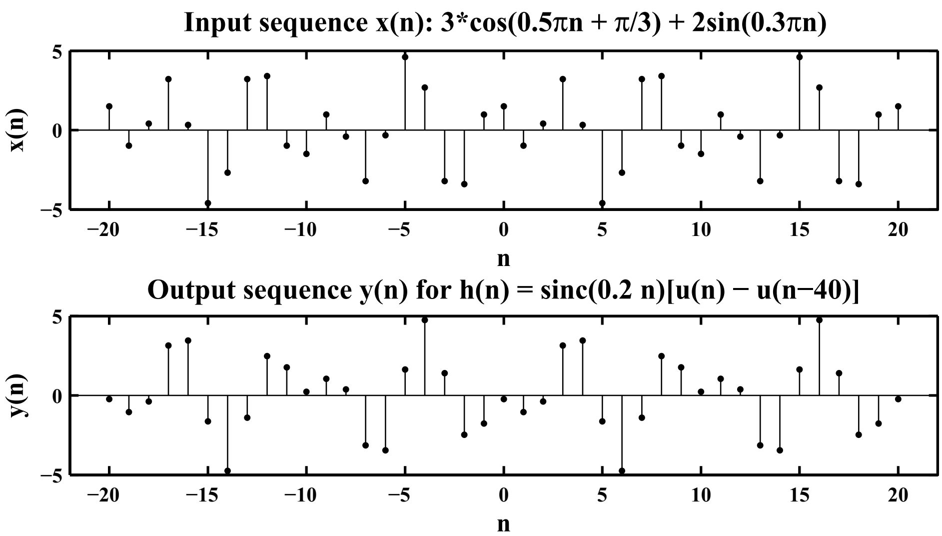

Figure 3.31: Problem P3.13c plots

4. The input to the system $h(n) = \left[(0.5)^n + (0.4)^n\right] u(n)$. The steady-state response $y(n)$ is computed using MATLAB.

```
% P0313d: x(n) = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n); i.e.
%         x(n) = 3*cos(0.5*pi*n+pi/3)+2*cos(0.3*pi*n-pi/2)
%         h(n) = ((0.5)^(n)+(0.4)^(n)).*u(n)
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
n = [-20:20]; x = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n);
w1 = 0.5*pi; H1 = (2-0.9*exp(-j*w1))./(1-0.9*exp(-j*w1)+0.2*exp(-j*2*w1));
w2 = 0.3*pi; H2 = (2-0.9*exp(-j*w2))./(1-0.9*exp(-j*w2)+0.2*exp(-j*2*w2));
magH1 = abs(H1); phaH1 = angle(H1); magH2 = abs(H2); phaH2 = angle(H2);
y = 3*magH1*cos(w1*n+pi/3+phaH1)+2*magH2*cos(w2*n-pi/2+phaH2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0313d');
subplot(2,1,1); Hs = stem(n,x,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS); axis([-22 22 -10 10]);
title(['Input sequence x(n): 3*cos(0.5{\pi}n + \pi/3) + 2sin(0.3{\pi}n)'],...
    'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); axis([-22 22 -10 10]);
title('Output sequence y(n) for h(n) = [(0.5)^n+(0.4)^n] u(n)]',...
    'FontSize',TFS); print -deps2 ../EPSFILES/P0313d;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.32.

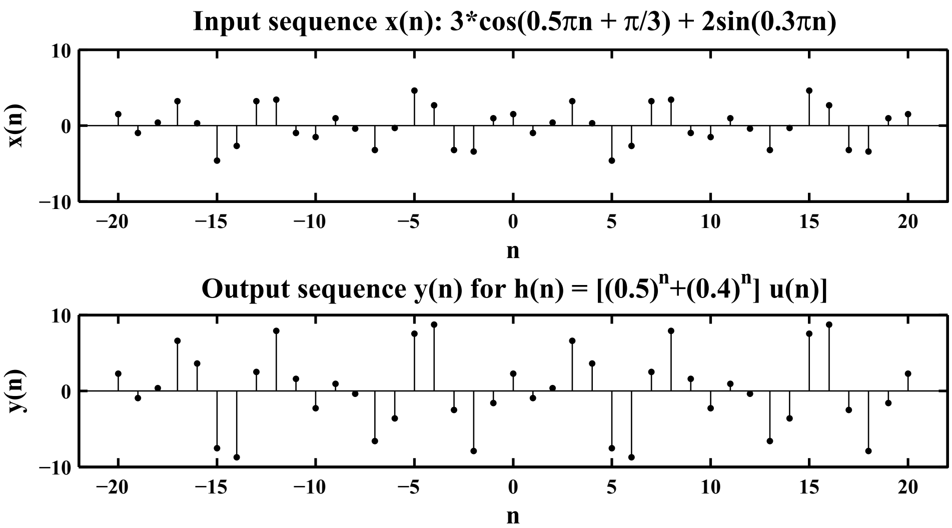

Figure 3.32: Problem P3.13d plots

5. The input to the system $h(n) = (0.5)^{|n|} \cos(0.1\pi n)$. The steady-state response $y(n)$ is computed using MATLAB.

```
% P0313e: x(n) = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n); i.e.
%         x(n) = 3*cos(0.5*pi*n+pi/3)+2*cos(0.3*pi*n-pi/2)
%         h(n) = (0.5)^|n|*cos(0.1*pi*n);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
n = [-20:20]; x = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n);
w1 = 0.5*pi; H1 = 0.5*0.75*w1/(1.25-cos(w1-(0.1*pi)))+...
    0.5*0.75*w1/(1.25-cos(w1+(0.1*pi)));
w2 = 0.3*pi; H2 = 0.5*0.75*w2/(1.25-cos(w2-(0.1*pi)))+...
    0.5*0.75*w2/(1.25-cos(w2+(0.1*pi)));
magH1 = abs(H1); phaH1 = angle(H1); magH2 = abs(H2); phaH2 = angle(H2);
y = 3*magH1*cos(w1*n+pi/3+phaH1)+2*magH2*cos(w2*n-pi/2+phaH2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0313e');
subplot(2,1,1); Hs = stem(n,x,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS); axis([-22 22 -6 6]);
title(['Input sequence x(n): 3*cos(0.5{\pi}n + \pi/3) + 2sin(0.3{\pi}n)'],...
    'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); axis([-22 22 -6 6]);
title('Output sequence y(n) for h(n) = (0.5)^{|n|}cos(0.1{\pi}n)] u(n)]',...
    'FontSize',TFS); print -deps2 ../EPSFILES/P0313e;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.33.

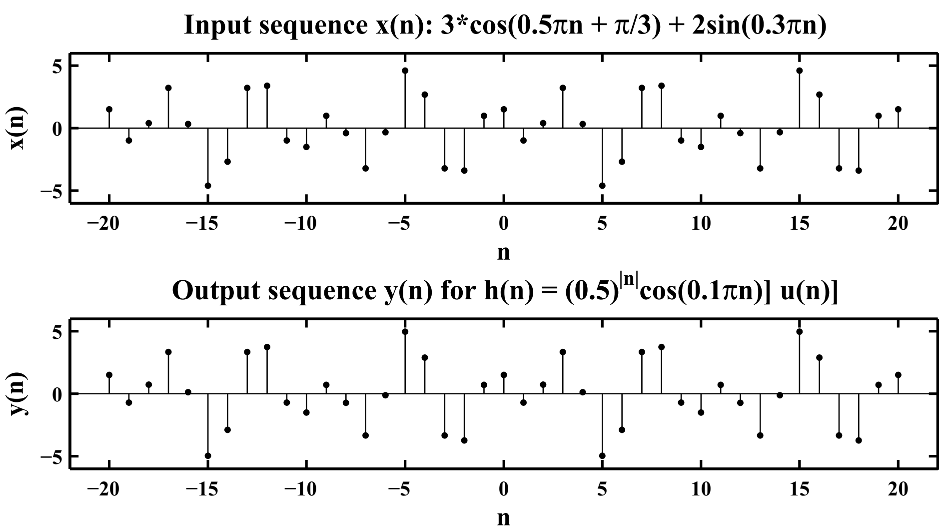

Figure 3.33: Problem P3.13e plots

P3.14 An ideal lowpass filter is described in the frequency-domain by

$$H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\alpha\omega}, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

where ω_c is called the cutoff frequency and α is called the phase delay.

1. The ideal impulse response $h_d(n)$ using the IDTFT relation (3.2):

$$h_d(n) = \mathcal{F}^{-1}\left[H_d(e^{j\omega})\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_d(e^{j\omega})e^{jn\omega}d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\alpha\omega}e^{jn\omega}d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j(n-\alpha)\omega}d\omega$$
$$= \frac{1}{2\pi}\left.\frac{e^{j(n-\alpha)\omega}}{j(n-\alpha)}\right|_{-\omega_c}^{\omega_c} = \frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}$$

2. Plot of the truncated impulse response:

$$h(n) = \begin{cases} h_d(n), & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \dfrac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

for $N = 41$, $\alpha = 20$, and $\omega_c = 0.5\pi$. MATLAB script: MATLAB script:

```
% P0314b: Truncated Ideal Lowpass Filter; h(n) = h_d(n) ,0 <= n <= N-1
%                                              = 0      ,otherwise
clc; close all; set(0,'defaultfigurepaperposition',[0,0,5,2]);
%
n = [0:40]; alpha = 20; wc = 0.5*pi;
fc = wc/(2*pi); h = 2*fc*sinc(2*fc*(n-alpha));
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0314b');
Hs = stem(n,h,'filled'); set(Hs,'markersize',2); axis([-2 42 -0.2 0.6]);
xlabel('n','FontSize',LFS); ylabel('h(n)','FontSize',LFS);
title('Truncated Impulse Response h(n)','FontSize',TFS);
set(gca,'YTick',[-0.2:0.1:0.6]); print -deps2 ../EPSFILES/P0314b;
```

The truncated impulse response plot of $h_d(n)$ is shown in Figure 3.34.

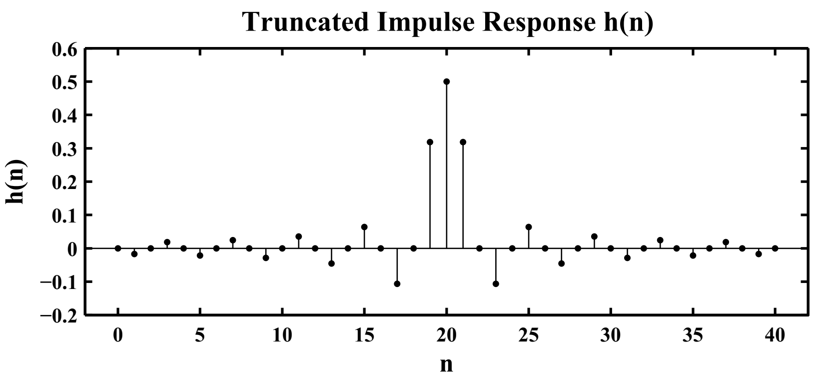

Figure 3.34: Problem P3.14b plot

3. Plot of the frequency response function $H\left(e^{j\omega}\right)$ and comparison with the ideal lowpass filter response $H_d\left(e^{j\omega}\right)$: MATLAB script:

```
% P0314c: Freq Resp of truncated and ideal impulse responses for lowpass filter
clc; close all;  set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
K = 500; w = [-K:K]*pi/K; H = dtft(h,n,w); magH = abs(H); phaH = angle(H);
H_d = zeros(1,length(w)); H_d(K/2+1:3*K/2+1) = exp(-j*alpha*w(K/2+1:3*K/2+1));
magH_d = abs(H_d); phaH_d = angle(H_d); wtick = sort([-1:0.4:1 0]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0314c');
subplot(2,2,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 1.2]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title('Magnitude of H(e^{j\omega})','FontSize',TFS); set(gca,'XTick',wtick);
subplot(2,2,2); plot(w/pi,phaH*180/pi,'LineWidth',1.5);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase of H(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); magtick = [-180:60:180];
set(gca,'YTick',magtick); set(gca,'XTick',wtick);
subplot(2,2,3); plot(w/pi,magH_d,'LineWidth',1.5); axis([-1 1 0 1.2]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H_d|','FontSize',LFS);
title('Magnitude of H_d(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); ytick = [0:0.2:1.2]; set(gca,'YTick',ytick);
subplot(2,2,4); plot(w/pi,phaH_d*180/pi,'LineWidth',1.5);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase of H_d(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); magtick = [-180:60:180]; set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0314c;
```

The frequency responses are shown in Figure 3.35 from which we observe that the truncated response is a smeared or blurred version of the ideal response.

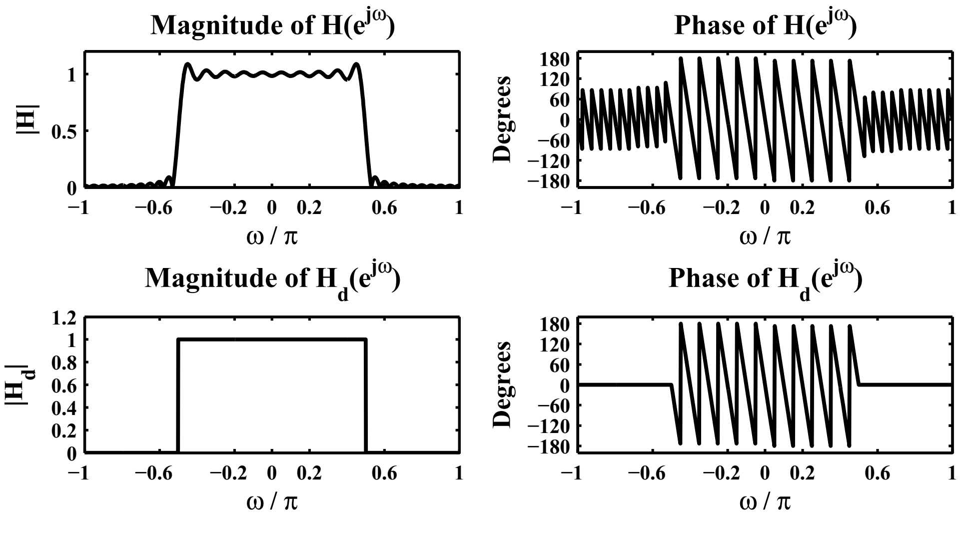

Figure 3.35: Problem P3.14c plots

P3.15 An ideal highpass filter is described in the frequency-domain by

$$H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\alpha\omega}, & \omega_c < |\omega| \le \pi \\ 0, & |\omega| \le \omega_c \end{cases}$$

where ω_c is called the cutoff frequency and α is called the phase delay.

1. The ideal impulse response $h_d(n)$ using the IDTFT relation (3.2):

$$h_d(n) = \mathcal{F}^{-1}\left[H_d(e^{j\omega})\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_d(e^{j\omega})e^{jn\omega}d\omega = \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{-j\alpha\omega}e^{jn\omega}d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{-j\alpha\omega}e^{jn\omega}d\omega$$
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(n-\alpha)\omega}d\omega - \frac{1}{2\pi}\int_{-\omega_c}^{\omega} e^{j(n-\alpha)\omega}d\omega = \frac{\sin[\pi(n-\alpha)]}{\pi(n-\alpha)} - \frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}$$

2. Plot of the truncated impulse response:

$$h(n) = \begin{cases} h_d(n), & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \dfrac{\sin[\pi(n-\alpha)]}{\pi(n-\alpha)} - \dfrac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

for $N = 31$, $\alpha = 15$, and $\omega_c = 0.5\pi$. MATLAB script: MATLAB script:

```
% P0315b: Ideal Highpass Filter; h(n) = h_d(n) ,0 <= n <= N-1
%                                          = 0      ,otherwise
clc; close all; set(0,'defaultfigurepaperposition',[0,0,5,2]);
%
n = [0:40]; alpha = 20; wc = 0.5*pi; fc = wc/(2*pi);
h = sinc(n-alpha)-2*fc*sinc(2*fc*(n-alpha));
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0315b');
Hs = stem(n,h,'filled'); set(Hs,'markersize',2); axis([-2 42 -0.4 0.6]);
xlabel('n','FontSize',LFS); ylabel('h(n)','FontSize',LFS);
title('Truncated Impulse Response h(n)','FontSize',TFS);
set(gca,'YTick',[-0.4:0.1:0.6]); print -deps2 ../EPSFILES/P0315b;
```

The truncated impulse response plot of $h_d(n)$ is shown in Figure 3.36.

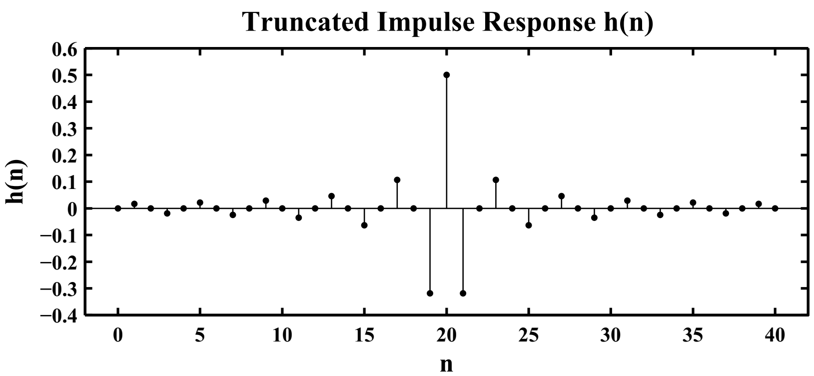

Figure 3.36: Problem P3.15b plot

3. Plot of the frequency response function $H\left(e^{j\omega}\right)$ and comparison with the ideal lowpass filter response $H_d\left(e^{j\omega}\right)$: MATLAB script:

```
% P0315c: Freq Resp of truncated and ideal impulse responses for highpass filter
clc; close all;  set(0,'defaultfigurepaperposition',[0,0,6,3]);
K = 500; w = [-K:K]*pi/K; H = dtft(h,n,w); magH = abs(H); phaH = angle(H);
H_d = zeros(1,length(w)); H_d(1:K/2+1) = exp(-j*alpha*w(1:K/2+1));
H_d(3*K/2+1:end) = exp(-j*alpha*w(3*K/2+1:end));
magH_d = abs(H_d); phaH_d = angle(H_d); wtick = sort([-1:0.4:1 0]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0315c');
subplot(2,2,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 1.2]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title('Magnitude of H(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'XTick',wtick);
subplot(2,2,2); plot(w/pi,phaH*180/pi,'LineWidth',1.5);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase of H(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); magtick = [-180:60:180]; set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magH_d,'LineWidth',1.5); axis([ -1 1 0 1.2]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H_d|','FontSize',LFS);
title('Magnitude of H_d(e^{j\omega})','FontSize',TFS);
set(gca,'YTick',ytick); ytick = [0:0.2:1.2];set(gca,'XTick',wtick);
subplot(2,2,4); plot(w/pi,phaH_d*180/pi,'LineWidth',1.5);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase of H_d(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); magtick = [-180:60:180]; set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0315c;
```

The frequency responses are shown in Figure 3.37 from which we observe that the truncated response is a smeared or blurred version of the ideal response.

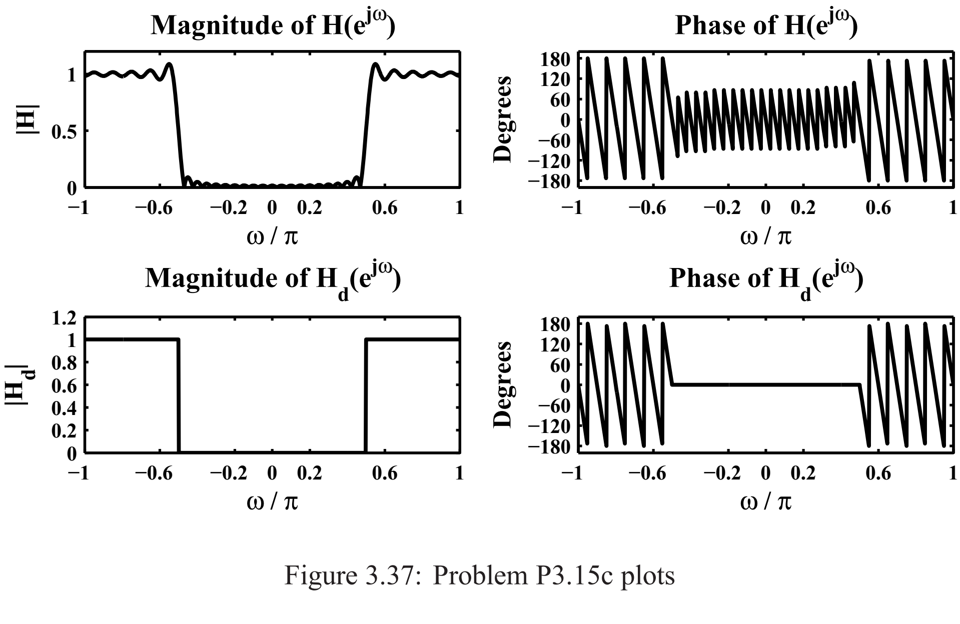

Figure 3.37: Problem P3.15c plots

P3.16 MATLAB function `freqresp`.

```
function [H] = freqresp(b,a,w)
% Frequency response function from difference equation
% [H] = freqresp(b,a,w)
%  H  = frequency response array evaluated at w frequencies
%  b  = numerator coefficient array
%  a  = denominator coeeficient array (a(1) = 1)
%  w  = frequency location array
%
b = reshape(b,1,length(b));
a = reshape(a,1,length(a));
w = reshape(w,1,length(w));
m = 0:length(b)-1; num = b*exp(-j*m'*w);
l = 0:length(a)-1; den = a*exp(-j*l'*w);
H = num./den;
```

P3.17 Computation and plot of the frequency response $H(e^{j\omega})$ using MATLAB for each of the following systems:

1. $y(n) = \frac{1}{5}\sum_{m=0}^{4} x(n-m)$:

   MATLAB script:

```
% P0317a: y(n) = (1/5) sum_{0}^{4} x(n-m)
clc; close all;
%
w = [-300:300]*pi/300; a = [1]; b = [0.2 0.2 0.2 0.2 0.2];
[H] = freqresp(b,a,w); magH = abs(H); phaH = angle(H)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0317a');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = [-1:0.2:1]; magtick = [0:0.2:1.2];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -220 220]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase Response ','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0317a;
```

   The frequency responses are shown in Figure 3.38

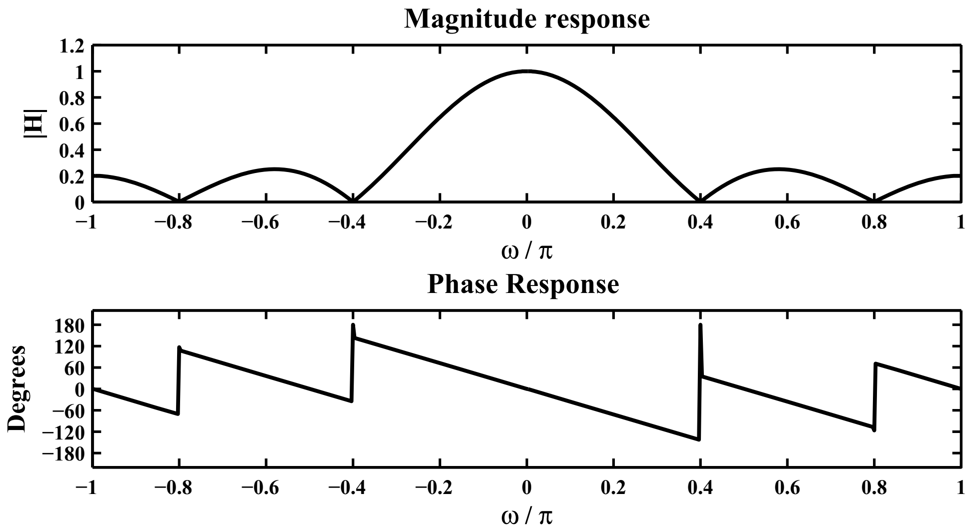

Figure 3.38: Frequency response plots in Problem P3.17a

2. $y(n) = x(n) - x(n-2) + 0.95y(n-1) - 0.9025y(n-2)$

MATLAB script:

```
% P0317b: y(n) = x(n)-x(n-2)+0.95*y(n-1)-0.9025*y(n-2)
clc; close all;
%
w = [-300:300]*pi/300; a = [1 -0.95 0.9025]; b = [1 0 -1];
[H] = freqresp(b,a,w); magH = abs(H); phaH = angle(H)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0317b');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 25]);
wtick = [-1:0.2:1]; magtick = [0:5:25];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0317b;
```

The frequency responses are shown in Figure 3.39

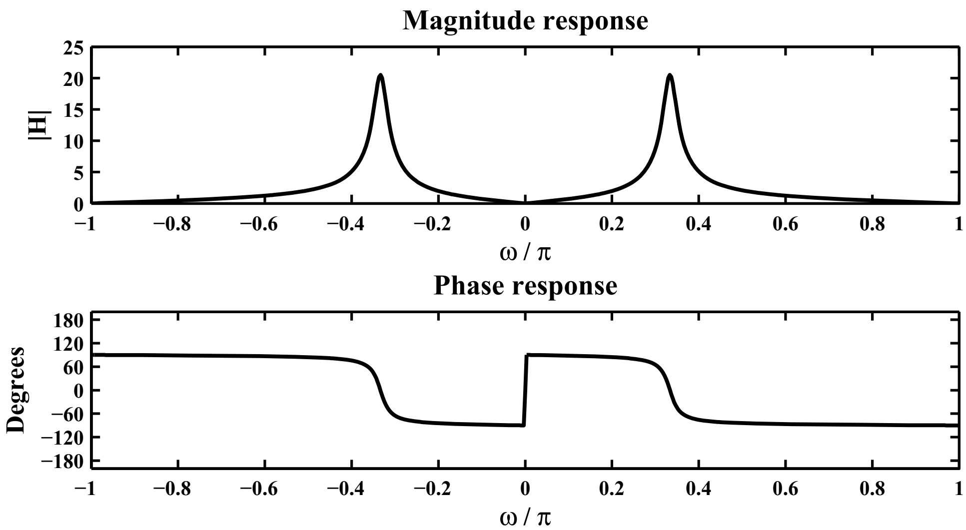

Figure 3.39: Frequency response plots in Problem P3.17b

3. $y(n) = x(n) - x(n-1) + x(n-2) + 0.95y(n-1) - 0.9025y(n-2)$

MATLAB script:

```
% P0317c: y(n) = x(n)-x(n-1)+x(n-2)+0.95*y(n-1)-0.9025*y(n-2)
clc; close all;
%
w = [-300:300]*pi/300; a = [1 -0.95 0.9025]; b = [1 -1 1];
[H] = freqresp(b,a,w); magH = abs(H); phaH = angle(H)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0317c');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 1.4]);
wtick = [-1:0.2:1]; magtick = [0:0.2:1.4];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0317c;
```

The frequency responses are shown in Figure 3.40

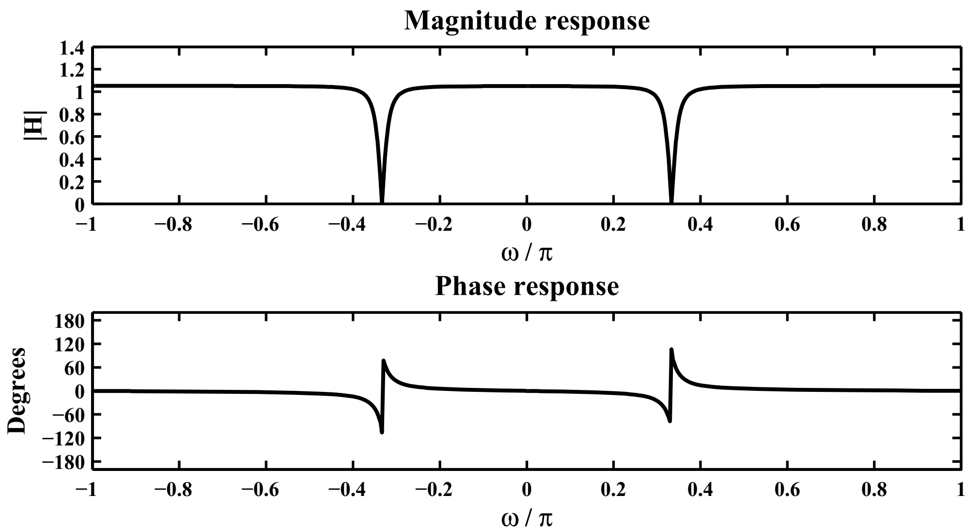

Figure 3.40: Frequency response plots in Problem P3.17c

4. $y(n) = x(n) - 1.7678x(n-1) + 1.5625x(n-2) + 1.1314y(n-1) - 0.64y(n-2)$

   MATLAB script:

```
% P0317d: y(n) = x(n)-1.7678*x(n-1)+1.5625*x(n-2)+1.1314*y(n-1)-  0.64*y(n-2)
clc; close all;
%
w = [-300:300]*pi/300; a = [1 -1.1314 0.64]; b = [1 -1.7678 1.5625];
[H] = freqresp(b,a,w); magH = abs(H); phaH = angle(H)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0317d');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5);
wtick = [-1:0.2:1]; magtick = [1.5:0.02:1.6];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response'],'FontSize',TFS);
set(gca,'XTick',wtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0317d;
```

   The frequency responses are shown in Figure 3.41

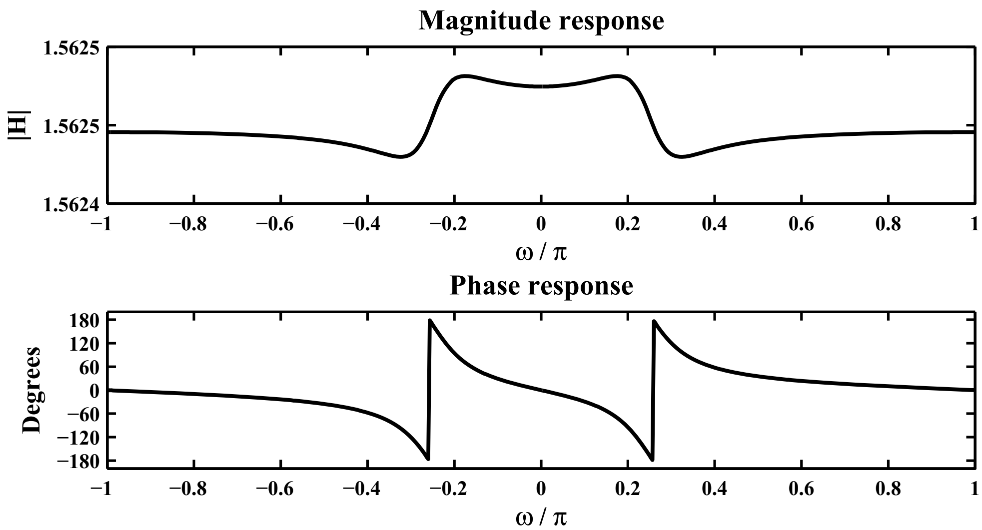

Figure 3.41: Frequency response plots in Problem P3.17d

5. $y(n) = x(n) - \sum_{\ell=1}^{5} (0.5)^{\ell}\, y\,(n-\ell)$

MATLAB script:

```
% P0317e: y(n) = x(n)-sum _ {l = 1} ^ {5} (0.5) ^ l*y(n-l);
clc; close all;
%
w = [-300:300]*pi/300; l = [0:5]; a = 0.5 .^ l; b = [1]; [H] = freqresp(b,a,w);
magH = abs(H); phaH = angle(H)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0317e');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 1.8]);
wtick = [-1:0.2:1]; magtick = [0:0.2:1.8];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude Response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase Response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0317e;
```

The frequency responses are shown in Figure 3.42

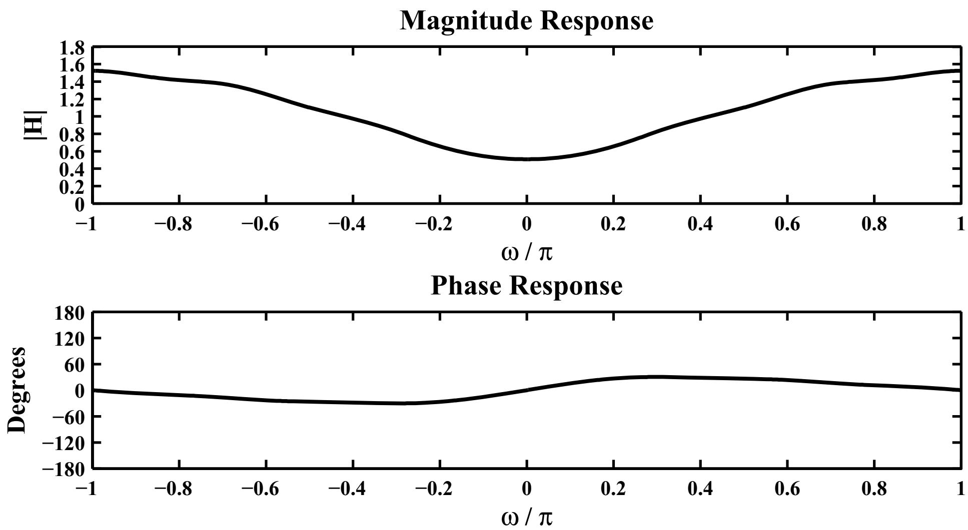

Figure 3.42: Frequency response plots in Problem P3.17e

P3.18 A linear, shift-invariant system is described by the difference equation

$$y(n) = \sum_{m=0}^{3} x(n-2m) - \sum_{\ell=1}^{3} (0.81)^{\ell} y(n-2\ell) \Rightarrow H\left(e^{j\omega}\right) = \frac{\sum_{m=0}^{3} e^{-j2m\omega}}{\sum_{\ell=0}^{3} (0.81)^{2\ell} e^{-j2\ell\omega}}$$

1. $x(n) = 5 + 10(-1)^n = 5 + 10\cos(n\pi)$: We need frequency responses at $\omega = 0$ and $\omega = \pi$.

$$H\left(e^{j0}\right) = \frac{\sum_{m=0}^{3} e^{-j0}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j0}} = 1.6885 \text{ and } H\left(e^{j\pi}\right) = \frac{\sum_{m=0}^{3} e^{-j2\pi\ell}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j2\pi\ell}} = 1.6885$$

Hence the steady-state response is $y(n) = 1.6885x(n) = 8.4424 + 16.8848(-1)^n$. MATLAB script:

```
% P0318a: y(n) = sum_{m=0}^{3} x(n-2m)-sum_{l=1}^{3} (0.81)^l y(n-2l)
%         x(n) = 5+10(-1) ^ n;
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
n = [0:50]; a = [1  0  0.81^2  0  0.81^4  0  0.81^6]; b = [1 0 1 0 1 0 1];
w = [0 pi]; A = [5 10]; theta = [0 0]; [H] = freqresp(b,a,w);
magH = abs(H); phaH = angle(H); mag = A.*magH; pha = phaH+theta;
term1 = w'*n; term2 = pha'*ones(1,length(n)); cos_term = cos(term1+term2);
y1 = mag*cos_term; x = 5+10*(-1) .^ n; y2 = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0318a');
subplot(2,1,1); Hs = stem(n,y1,'filled'); axis([-1 51 -10 30]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title('Steady state response y_{ss}(n) for x(n) = 5+10(-1)^{n}',...
      'FontSize',TFS); ytick = [-10:5:25]; set(gca,'YTick',ytick);
subplot(2,1,2); Hs = stem(n,y2,'filled'); axis([-1 51 -10 30]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title(['Output response y(n) using the filter function for x(n) = ' ...
       '5+10(-1)^{n}'],'FontSize',TFS); set(gca,'YTick',ytick);
print -deps2 ../EPSFILES/P0318a;
```

The steady-state responses are shown in Figure 3.43.

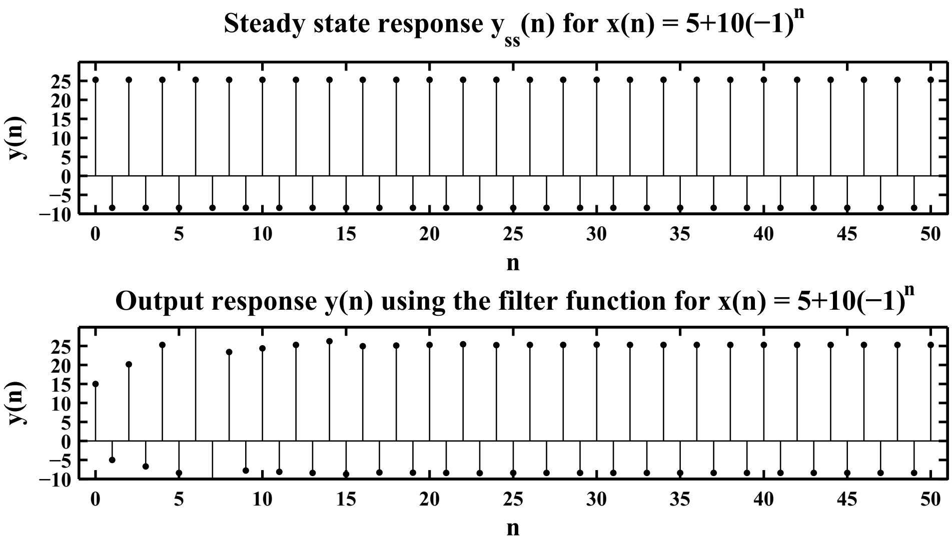

Figure 3.43: Steady-state response plots in Problem P3.18a

2. $x(n) = 1 + \cos(0.5\pi n + \pi/2)$: We need responses at $\omega = 0$ and $\omega = 0.5\pi$.

$$H\left(e^{j0}\right) = \frac{\sum_{m=0}^{3} e^{-j0}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j0}} = 1.6885 \text{ and } H\left(e^{j0.5\pi}\right) = \frac{\sum_{m=0}^{3} e^{-j\pi\ell}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j\pi\ell}} = 0$$

Hence the steady-state response is $y(n) = 1.6885$. MATLAB script:

```
% P0318b: y(n) = sum_{m=0}^{3} x(n-2m)-sum_{l=1}^{3} (0.81)^l y(n-2l)
%         x(n) = 1+cos(0.5*pi*n+pi/2);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
n = [0:50]; a = [1  0  0.81^2  0  0.81^4  0  0.81^6]; b = [1 0 1 0 1 0 1];
w = [0 pi/2]; A = [1 1]; theta = [0 pi/2]; [H] = freqresp(b,a,w);
magH = abs(H); phaH = angle(H); mag = A.*magH; pha = phaH+theta;
term1 = w'*n; term2 = pha'*ones(1,length(n)); cos_term = cos(term1+term2);
y1 = mag*cos_term; x = 1+cos(0.5*pi*n+pi/2); y2 = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0318b');
subplot(2,1,1); Hs = stem(n,y1,'filled'); axis([-1 51 0 2.5]);
ytick = [0:0.5:2.5]; set(Hs,'markersize',2); set(gca,'YTick',ytick);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title(['SS response y_{ss}(n): x(n) = 1+cos(0.5{\pi}n+\pi/2)'],'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y2,'filled'); set(Hs,'markersize',2);
axis([-1 51 0 2.5]); ytick = [0:0.5:2.5]; set(gca,'YTick',ytick);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title(['Output response y(n) using the filter function'],'FontSize',TFS);
print -deps2 ../EPSFILES/P0318b;
```

The steady-state responses are shown in Figure 3.44.

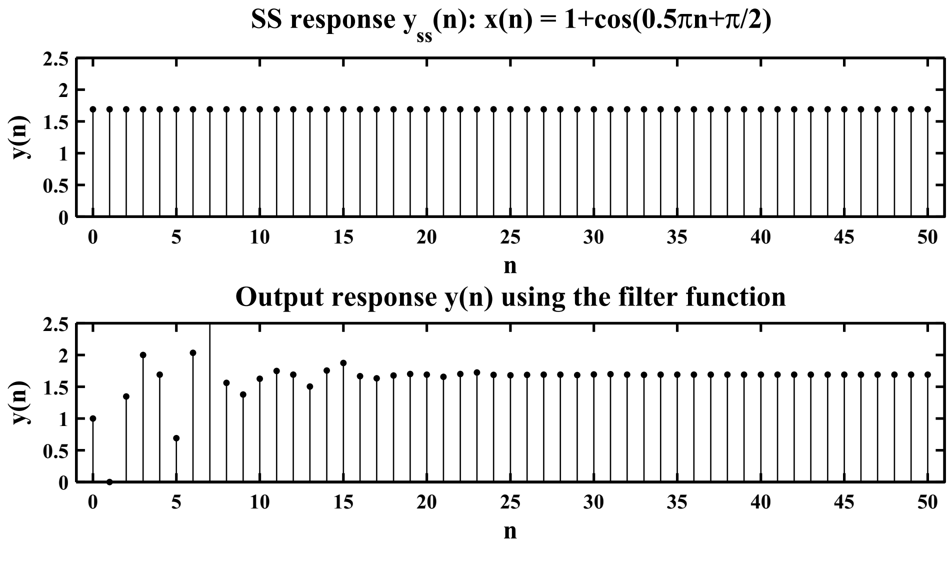

Figure 3.44: Steady-state response plots in Problem P3.18b

3. $x(n) = 2\sin(\pi n/4) + 3\cos(3\pi n/4)$: We need responses at $\omega = \pi/4$ and $\omega = 3\pi/4$.

$$H\left(e^{j0.25\pi}\right) = \frac{\sum_{m=0}^{3} e^{-j0.5\pi m}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j0.5\pi m}} = 0 \text{ and } H\left(e^{j0.75\pi}\right) = \frac{\sum_{m=0}^{3} e^{-j1.5\pi\ell}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j1.5\pi\ell}} = 0$$

Hence the steady-state response is $y(n) = 0$. MATLAB script:

```
% P0318c: y(n) = sum_{m=0}^{3} x(n-2m)-sum_{l=1}^{3} (0.81)^l y(n-2l)
%         x(n) = 2 * sin(\pi n/4)+3 * cos(3 \pi n/4);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
n = [0:50]; a = [1  0  0.81^2  0  0.81^4  0  0.81^6]; b = [1 0 1 0 1 0 1];
w = [pi/4 3*pi/4]; A = [2 3]; theta = [-pi/2 0]; [H] = freqresp(b,a,w);
magH = abs(H); phaH = angle(H); mag = A.*magH; pha = phaH+theta;
term1 = w'*n; term2 = pha'*ones(1,length(n)); cos_term = cos(term1+term2);
y1 = mag*cos_term; x = 2*sin(pi*n/4)+3*cos(3*pi*n/4); y2 = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0318c'); subplot(2,1,1);
Hs = stem(n,y1,'filled'); axis([-1 51 -3 4]); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title(['SS response y_{ss}(n): x(n) = 2sin({\pi}n/4)+3cos(3 \pi n/4)'],...
       'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y2,'filled'); axis([-1 51 -3 4]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS); ylabel('y(n)',...
    'FontSize',LFS);
title(['Output response y(n) using the filter function'],'FontSize',TFS);
print -deps2 ../EPSFILES/P0318c;
```

The steady-state responses are shown in Figure 3.45.

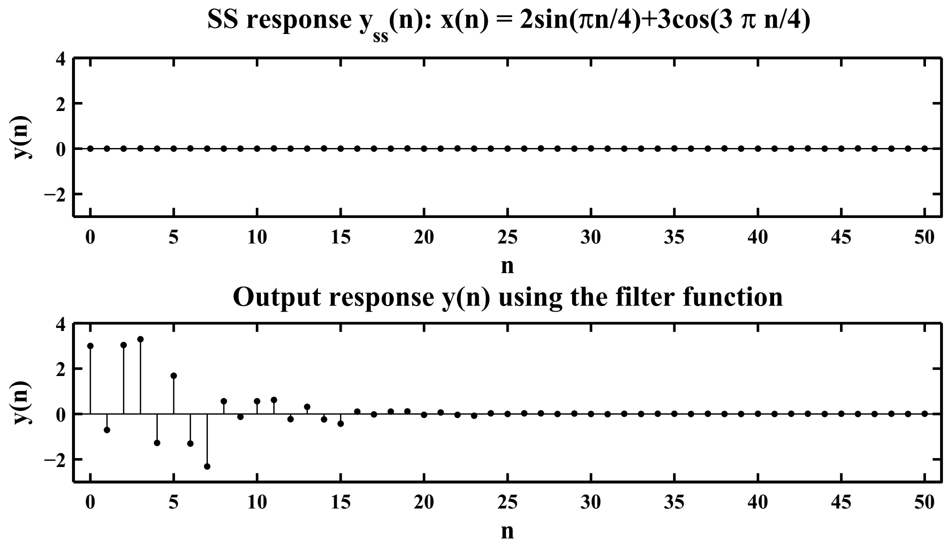

Figure 3.45: Steady-state response plots in Problem P3.18c

4. $x(n) = \sum_{k=0}^{5} (k+1)\cos(\pi kn/4)$: We need responses at $\omega = k\pi/4$, $k = 0, 1, 2, 3, 4, 5$.

$$H\left(e^{j0}\right) = \frac{\sum_{m=0}^{3} e^{-j0}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j0}} = 1.6885 = H\left(e^{j\pi}\right) \text{ and}$$
$$H\left(e^{j0.25\pi}\right) = H\left(e^{j0.5\pi}\right) = H\left(e^{j0.75\pi}\right) = H\left(e^{j1.25\pi}\right) = 0$$

Hence the steady-state response is $y(n) = 1.6885 + 8.4425\cos(n\pi)$. MATLAB script:

```
% P0318d: y(n) = sum_{m=0}^{3} x(n-2m)-sum_{l=1}^{3} (0.81)^l y(n-2l)
%         x(n) = sum_{k = 0}^{5} (k+1) cos(pi*k*n/4);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
n = [0:50]; a = [1 0 0.81^2 0 0.81^4 0 0.81^6]; b = [1 0 1 0 1 0 1];
k = [0:5]; w = pi /4*k; A = (k+1); theta = zeros(1,length(k));
[H] = freqresp(b,a,w); magH = abs(H); phaH = angle(H); mag = A.*magH;
pha = phaH+theta; term1 = w'*n; term2 = pha'*ones(1,length(n)); cos_term = ...
    cos(term1+term2); y1 = mag*cos_term; x = A*cos(term1); y2 = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0318d');
subplot(2,1,1); Hs = stem(n,y1,'filled'); axis([-1 51 -10 15]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['SS response y_{ss}(n): x(n) = sum_{0}^{5} (k+1)cos({\pi}kn/4)'],...
       'FontSize',TFS);
ytick = [-20:5:30]; set(gca,'YTick',ytick); ylabel('y(n)','FontSize',LFS);
subplot(2,1,2); Hs = stem(n,y2,'filled'); axis([-1 51 -10 15]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['Output response y(n) using the filter function'],'FontSize',TFS);
ytick = [-20:5:30]; set(gca,'YTick',ytick); ylabel('y(n)','FontSize',LFS);
print -deps2 ../EPSFILES/P0318d;
```

The steady-state responses are shown in Figure 3.46.

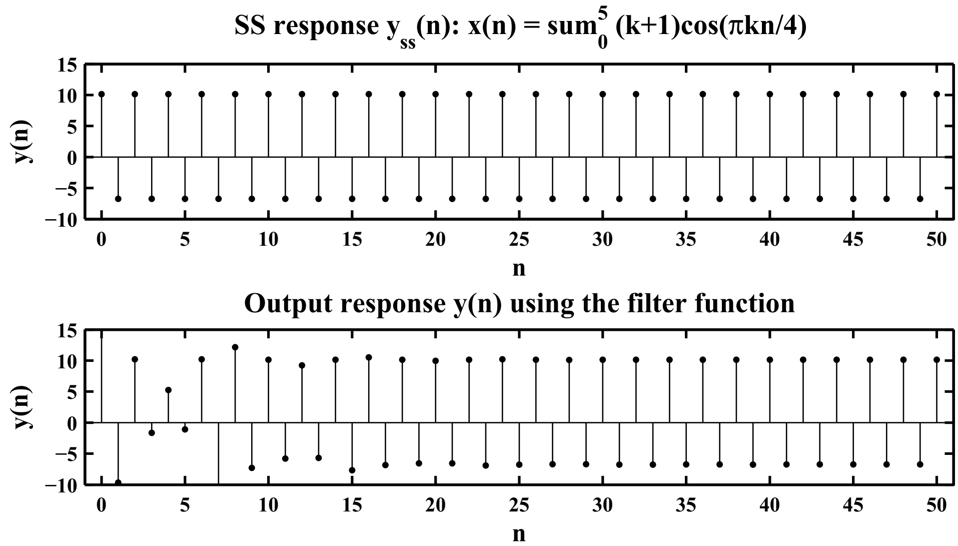

Figure 3.46: Steady-state response plots in Problem P3.18d

5. $x(n) = \cos(\pi n)$: We need response at $\omega = \pi$.

$$H\left(e^{j\pi}\right) = \frac{\sum_{m=0}^{3} e^{-j2\pi m}}{\sum_{\ell=0}^{3} (0.81)^{2\ell} e^{-j2\pi m}} = 1.6885$$

Hence the steady-state response is $y(n) = 1.6885 \cos(\pi n)$. MATLAB script:

```
% P0318e: y(n) = sum_{m=0}^{3} x(n-2m)-sum_{l=1}^{3} (0.81)^l y(n-2l)
% x(n) = cos(pi*n);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
n = [0:50]; a = [1  0  0.81^2  0  0.81^4  0  0.81^6]; b = [1 0 1 0 1 0 1];
w = [pi]; A =  [1]; theta = [0]; [H] = freqresp(b,a,w); magH = abs(H);
phaH = angle(H); mag = A.*magH; pha = phaH+theta; term1 = w'*n;
term2 = pha'*ones(1,length(n)); cos_term = cos(term1+term2); y1 = mag*cos_term;
x = cos(pi*n); y2 = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0318e');
subplot(2,1,1); Hs = stem(n,y1,'filled'); axis([-1 51 -2 2]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['SS response y_{ss}(n) for x(n) = cos(\pi \times n)'],'FontSize',TFS);
ytick = [-2:0.5:2]; set(gca,'YTick',ytick); ylabel('y(n)','FontSize',LFS);
subplot(2,1,2); Hs = stem(n,y2,'filled'); axis([-1 51 -2 2]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['Output response y(n) using the filter function'],'FontSize',TFS);
ytick = [-2:0.5:2]; set(gca,'YTick',ytick); ylabel('y(n)','FontSize',LFS);
print -deps2 ../EPSFILES/P0318e;
```

The steady-state responses are shown in Figure 3.47.

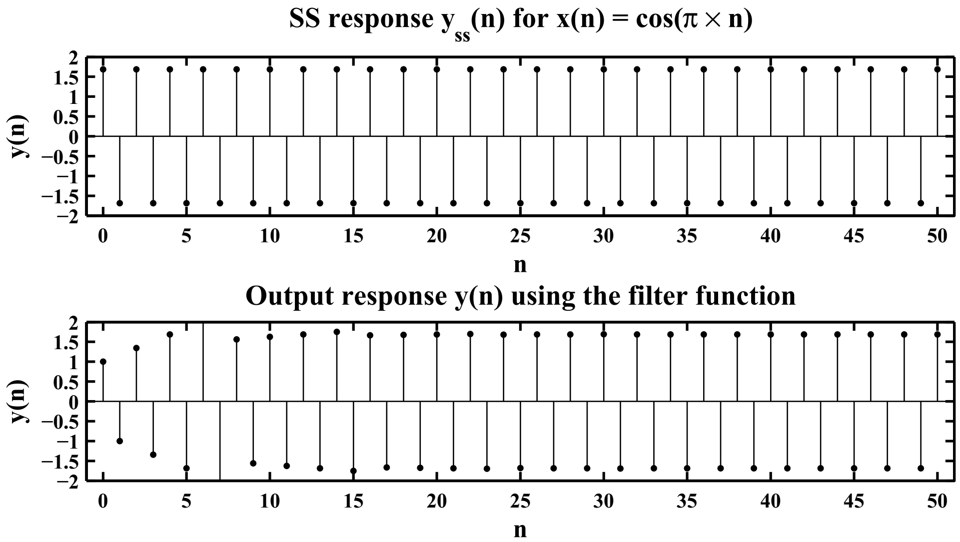

Figure 3.47: Steady-state response plots in Problem P3.18e

P3.19 An analog signal $x_a(t) = \sin(1000\pi t)$ is sampled using the following sampling intervals.

1. $T_s = 0.1$ ms: MATLAB script:

```
% P0319a: x_a(t) = sin(1000*pi*t); T_s = 0.1 ms;
clc; close all;
%
Ts = 0.0001; n = [-250:250]; x = sin(1000*pi*n*Ts); w = linspace(-pi,pi,501);
X = dtft(x,n,w); magX = abs(X); phaX = angle(X);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0319a');
subplot(2,1,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 300]);
wtick = [-1:0.2:1]; magtick = [0:100:300]; set(gca,'XTick',wtick);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response x_1(n) = sin(1000 \pi n T_s), T_s = 0.1 msec'...
     ,'FontSize',TFS); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response x_1(n) = sin(1000 \pi n T_s), T_s = 0.1 msec'...
     ,'FontSize',TFS); set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0319a;
```

The spectra are shown in Figure 3.48.

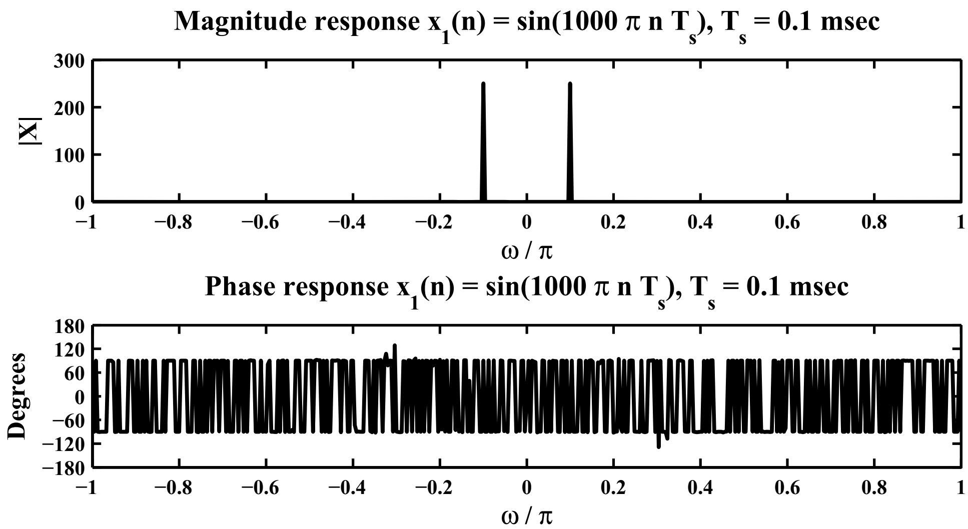

Figure 3.48: Spectrum plots in Problem P3.19a

2. $T_s = 1$ ms: MATLAB script:

```
% P0319b: x_a(t) = sin(1000*pi*t); T_s = 1 ms;
clc; close all;
%
Ts = 0.001; n = [-25:25]; x = sin(1000*pi*n*Ts); w = [-500:500]*pi/500;
X = dtft(x,n,w); magX = abs(X); phaX = angle(X);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0319b');
subplot(2,1,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 -1 1]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response x_1(n) = sin(1000 \pi n T_s), T_s = 1 msec'...
     ,'FontSize',TFS);
wtick = [-1:0.2:1]; set(gca,'XTick',wtick);
subplot(2,1,2); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -180 180]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response x_1(n) = sin(1000 \pi n T_s), T_s = 1 msec'...
     ,'FontSize',TFS); magtick = [-180:60:180];
wtick = [-1:0.2:1]; set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0319b;
```

The spectra are shown in Figure 3.49.

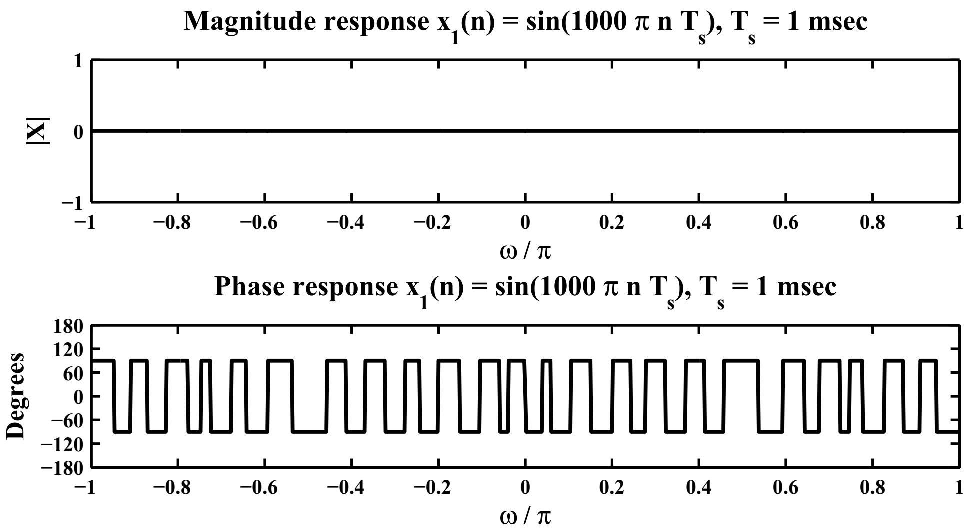

Figure 3.49: Spectrum plots in Problem P3.19b

3. $T_s = 0.01$ sec: MATLAB script:

```
% P0319c: x_a(t) = sin(1000*pi*t); T_s = 0.01 sec;
clc; close all;
%
Ts = 0.01; n = [-25:25]; x = sin(1000*pi*n*Ts); w = [-500:500]*pi/500;
X = dtft(x,n,w); magX = abs(X); phaX = angle(X);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0319c');
subplot(2,1,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 -1 1]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response x_1(n) = sin(1000 \pi n T_s),T_s = 0.01 sec'...
     ,'FontSize',TFS); wtick = [-1:0.2:1]; set(gca,'XTick',wtick);
subplot(2,1,2); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -180 180]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response x_1(n) = sin(1000 \pi n T_s),T_s = 0.01 sec'...
     ,'FontSize',TFS); wtick = [-1:0.2:1]; set(gca,'XTick',wtick);
magtick = [-180:60:180]; set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0319c;
```

The spectra are shown in Figure 3.50.

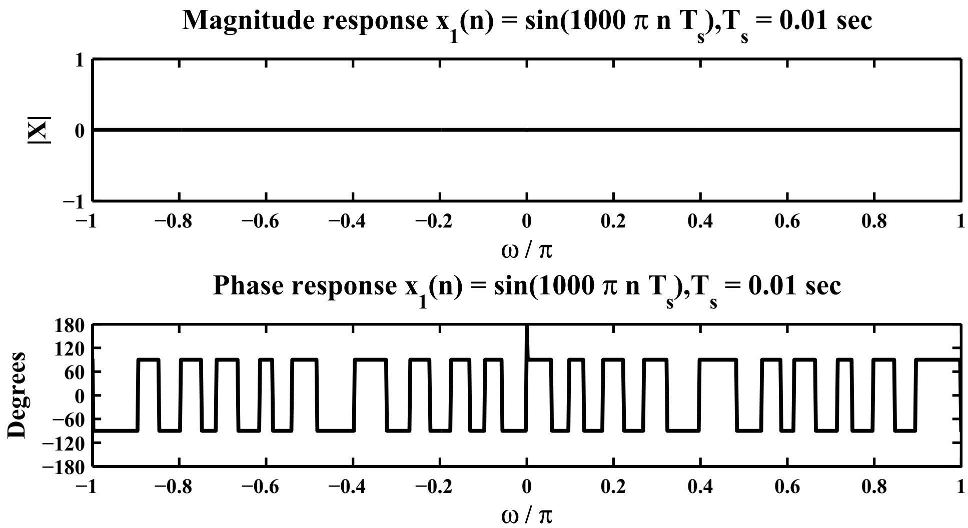

Figure 3.50: Spectrum plots in Problem P3.19c

P3.20 Sampling frequency $F_s = 8000$ sam/sec (or sampling interval $T_s = 0.125$ msec/sam) and impulse response $h(n) = (-0.9)^n u(n)$.

(a) $x_a(t) = 10\cos(10000\pi t)$. Hence $x(n) = x_a(nT_s) = 10\cos(10000\pi n 0.000125) = 10\cos(1.25\pi n)$. Therefore, the digital frequency is $(1.25 - 2)\pi = -0.75\pi$ rad/sam.

(b) The steady-state response when $x(n) = 10\cos(-0.75\pi n) = 10\cos(0.75\pi n$: The frequency response is

$$H\left(e^{j\omega}\right) = \mathcal{F}[h(n)] = \mathcal{F}\left[(-0.9)^n u(n)\right] = \frac{1}{1+0.9e^{j\omega}}.$$

At $\omega = -0.75\pi$, the response is

$$H\left(e^{j0.75\pi}\right) = \frac{1}{1+0.9e^{j0.75\pi}} = 0.7329\left(\angle 1.0517^c\right).$$

Hence

$$y_{ss}(n) = 10\,(0.7329)\cos(0.75\pi n + 1.0517)$$

which after D/A conversion gives $y_{ss}(t)$ as

$$y_{ss,a}(t) = 7.329\cos(6000\pi t 1.0517).$$

(c) The steady-state DC gain is obtained by setting $\omega = 0$ which is equal to $H\left(e^{j0}\right) = 1/(1+0.9) = 0.5263$. Hence $y_{ss}(n) = 10\,(0.5263) = y_{ss,a}(t) = 5.263$.

(d) Aliased frequencies of F_0 for the given sampling rate F_s are $F_0 + kF_s$. Now for $F_0 = 5$ KHz and $F_s = 8$ KHz, the aliased frequencies are $5 + 8k = \{13, 21, \ldots\}$ KHz. Therefore, two other $x_a(t)$'s are

$$10\cos(26000\pi t) \text{ and } 10\cos(42000\pi t).$$

(e) The prefilter should be a lowpass filter with the cutoff frequency of 4 KHz.

P3.21 Consider an analog signal $x_a(t) = \cos(20\pi t)$, $0 \le t \le 1$. It is sampled at $T_s = 0.01$, 0.05, and 0.1 sec intervals to obtain $x(n)$.

1. Plots of $x(n)$ for each T_s. MATLAB script:

```
% P0321a: plot x(n) for T_s = 0.01 sec,0.05 sec,0.1 sec
%         x_a(t) = cos(20*pi*t);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,4]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0321a');
T_s1 = 0.01; n1 = [0:100]; x1 = cos(20*pi*n1*T_s1);
subplot(3,1,1); Hs = stem(n1,x1,'filled'); axis([-5 105 -1.2 1.2]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.01 sec'],'FontSize',TFS);
ylabel('x(n)','FontSize',LFS);
T_s2 = 0.05; n2 = [0:20]; x2 = cos(20*pi*n2*T_s2);
subplot(3,1,2); Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
set(gca,'XTick',[0:20]); axis([-2 22 -1.2 1.2]);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.05 sec'],'FontSize',TFS);
T_s3 = 0.1; n3 = [0:10]; x3 = cos(20*pi*n3*T_s3);
```

```
subplot(3,1,3); Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
set(gca,'XTick',[0:10]); axis([-1 11 -1.2 1.2]);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.1 sec'],'FontSize',TFS);
print -deps2 ../EPSFILES/P0321a;
```

The plots are shown in Figure 3.51.

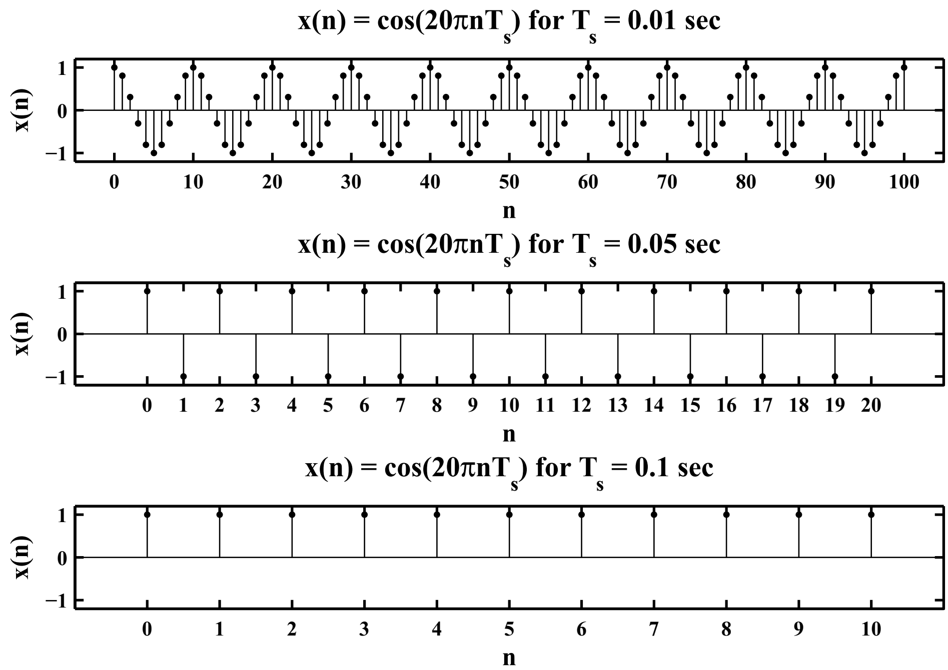

Figure 3.51: Plots of $x(n)$ for various T_s in Problem P3.21a.

2. Reconstruction from $x(n)$ using the sinc interpolation. MATLAB script:

```
% P0321a: plot x(n) for T_s = 0.01 sec,0.05 sec,0.1 sec
%         x_a(t) = cos(20*pi*t);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,4]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0321a');
T_s1 = 0.01; n1 = [0:100]; x1 = cos(20*pi*n1*T_s1);
subplot(3,1,1); Hs = stem(n1,x1,'filled'); axis([-5 105 -1.2 1.2]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.01 sec'],'FontSize',TFS);
ylabel('x(n)','FontSize',LFS);
T_s2 = 0.05; n2 = [0:20]; x2 = cos(20*pi*n2*T_s2);
subplot(3,1,2); Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
set(gca,'XTick',[0:20]); axis([-2 22 -1.2 1.2]);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.05 sec'],'FontSize',TFS);
T_s3 = 0.1; n3 = [0:10]; x3 = cos(20*pi*n3*T_s3);
subplot(3,1,3); Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
```

```
set(gca,'XTick',[0:10]); axis([-1 11 -1.2 1.2]);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.1 sec'],'FontSize',TFS);
print -deps2 ../EPSFILES/P0321a;
```

The reconstruction is shown in Figure 3.52.

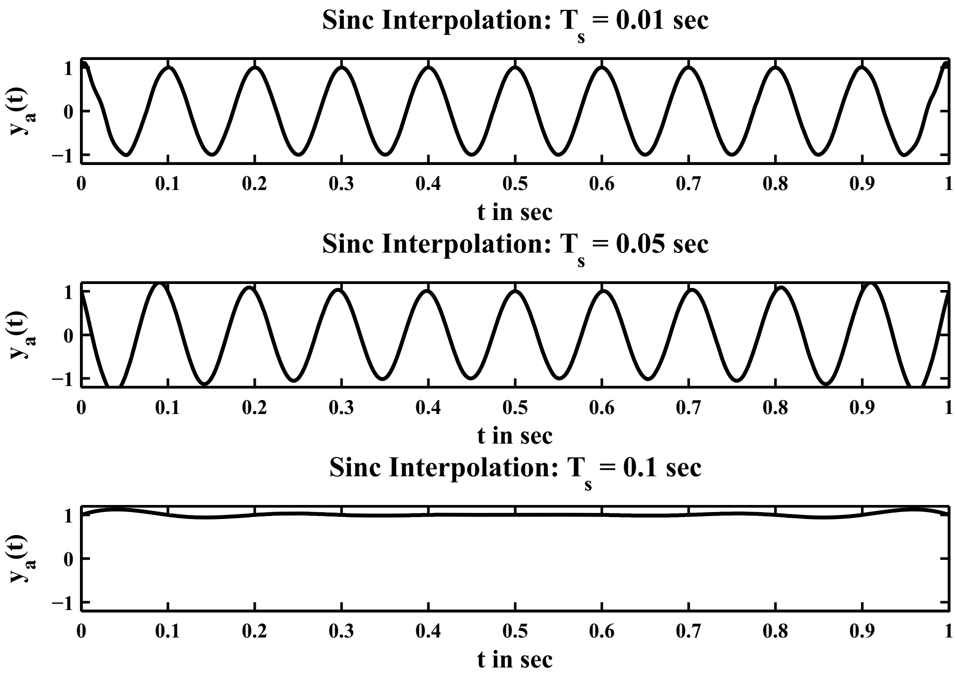

Figure 3.52: The sinc interpolation in Problem P3.21b.

3. Reconstruction from $x(n)$ using the spline interpolation. MATLAB script:

```
% P0321c Spline Interpolation: x_a(t) = cos(20*pi*t); 0 <= t <= 1;
%                                T_s = 0.01 sec,0.05 sec and 0.1 sec;
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,4]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0321c');
%
Ts1 = 0.01; Fs1 = 1/Ts1; n1 = [0:100]; nTs1 = n1*Ts1;
x1 = cos(20*pi*nTs1); Dt = 0.001; t = 0:Dt:1; xa1 = spline(nTs1,x1,t);
subplot(3,1,1); plot(t,xa1,'LineWidth',1.5); axis([0 1 -1.2 1.2]);
xlabel('t in sec','FontSize',LFS); ylabel('y_a(t)','FontSize',LFS);
title(['Spline Interpolation: T_s = 0.01 sec'],'FontSize',TFS);
%
Ts2 = 0.05; Fs2 = 1/Ts2; n2 = [0:20]; nTs2 = n2*Ts2;
x2 = cos(20*pi*nTs2); Dt = 0.001; t = 0:Dt:1; xa2 = spline(nTs2,x2,t);
subplot(3,1,2); plot(t,xa2,'LineWidth',1.5); axis([0 1 -1.2 1.2]);
xlabel('t in sec','FontSize',LFS); ylabel('y_a(t)','FontSize',LFS);
title(['Spline Interpolation: T_s = 0.05 sec'],'FontSize',TFS); grid;
%
```

```
Ts3 = 0.1; Fs3 = 1/Ts3; n3 = [0:10]; nTs3 = n3*Ts3; x3 = cos(20*pi*nTs3);
Dt = 0.001; t = 0:Dt:1; xa3 = spline(nTs3,x3,t);
subplot(3,1,3); plot(t,xa3,'LineWidth',1.5); axis([0 1 -1.2 1.2]);
xlabel('t in sec','FontSize',LFS); ylabel('y_a(t)','FontSize',LFS);
title(['Spline Interpolation: T_s = 0.1 sec'],'FontSize',TFS); grid;
print -deps2 ../EPSFILES/P0321c;
```

The reconstruction is shown in Figure 3.53.

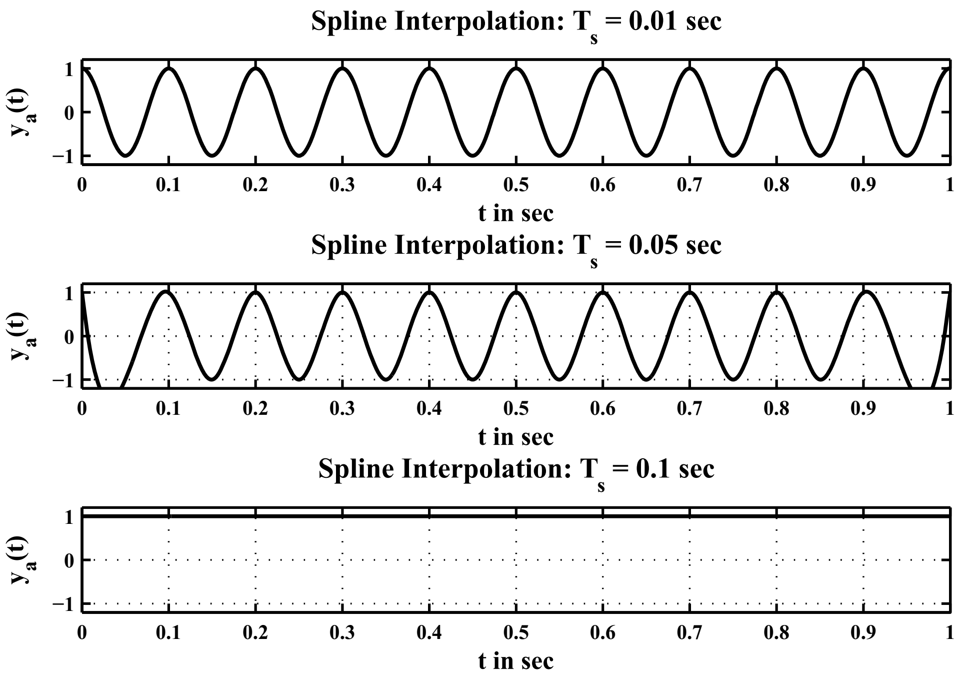

Figure 3.53: The sinc interpolation in Problem P3.21c.

4. Comments: From the plots in Figures it is clear that reconstructions from samples at $T_s = 0.01$ and 0.05 depict the original frequency (excluding end effects) but reconstructions for $T_s = 0.1$ show the original frequency aliased to zero. Furthermore, the cubic spline interpolation is a better reconstruction than the sinc interpolation, that is, the sinc interpolation is more susceptible to boundary effect.

P3.22 Consider the analog signal $x_a(t) = \cos(20\pi t + \theta)$, $0 \le t \le 1$. It is sampled at $T_s = 0.05$ sec intervals to obtain $x(n)$. Let $\theta = 0,\ \pi/6,\ \pi/4,\ \pi/3,\ \pi/2$. For each of these θ values, perform the following.

(a) Plots of $x_a(t)$ and $x(n)$ for $\theta = 0,\ \pi/6,\ \pi/4,\ \pi/3,\ \pi/2$. MATLAB script:

```
% P0322a: x_a(t) = cos(20*pi*t+theta); x(n) for theta = 0,pi/6,pi/4,pi/3, pi/2
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,7]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0322a');
Ts = 0.05; Fs = 1/Ts; Dt = 0.001; t = 0:Dt:1; n = [0:20]; nTs = n*Ts;
theta1 = 0; x_a1 = cos(20*pi*t+theta1); x1 = cos(20*pi*nTs+theta1);
subplot(5,1,1); plot(t,x_a1,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x1,'o'); xlabel('t in sec','FontSize',LFS);
title('x_a(t) and x(n) for \theta = 0','FontSize',TFS);
```

```
ylabel('Amplitude','FontSize',LFS);
theta2 = pi/6; x_a2 = cos(20*pi*t+theta2); x2 = cos(20*pi*nTs+theta2);
subplot(5,1,2); plot(t,x_a2,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x2,'o'); xlabel('t in sec','FontSize',LFS);
title('x_a(t) and x(n) for \theta = \pi/6','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta3 = pi/4; x_a3 = cos(20*pi*t+theta3); x3 = cos(20*pi*nTs+theta3);
subplot(5,1,3); plot(t,x_a3,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x3,'o'); xlabel('t in sec','FontSize',LFS);
title('x_a(t) and x(n) for \theta = \pi/4','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta4 = pi/3; x_a4 = cos(20*pi*t+theta4); x4 = cos(20*pi*nTs+theta4);
subplot(5,1,4); plot(t,x_a4,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x4,'o'); xlabel('t in sec','FontSize',LFS);
title('x_a(t) and x(n) for \theta = \pi/3','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta5 = pi/2; x_a5 = cos(20*pi*t+theta5); x5 = cos(20*pi*nTs+theta5);
subplot(5,1,5); plot(t,x_a5,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x5,'o'); xlabel('t in sec','FontSize',LFS);
title('x_a(t) and x(n) for \theta = \pi/2','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS); print -deps2 ../EPSFILES/P0322a;
```

The reconstruction is shown in Figure 3.54.

(b) Reconstruction of the analog signal $y_a(t)$ from the samples $x(n)$ using the sinc interpolation (for $\theta =$ 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$. MATLAB script:

```
% P0322b: Sinc Interpolation for theta = 0,pi/6,pi/4,pi/3, pi/2
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,7]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0322b');
Ts = 0.05; Fs = 1/Ts; Dt = 0.001; t = 0:Dt:1; n = [0:20]; nTs = n*Ts;
theta1 = 0; x1 = cos(20*pi*nTs+theta1);
y_a1 = x1*sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
subplot(5,1,1); plot(t,y_a1,'LineWidth',1.5); hold on;
plot(nTs,x1,'o'); axis([0 1 -1.2 1.2]); xlabel('t in sec','FontSize',LFS);
title('Sinc Interpolation for \theta = 0','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta2 = pi/6; x2 = cos(20*pi*nTs+theta2);
y_a2 = x2*sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
subplot(5,1,2); plot(t,y_a2,'LineWidth',1.5); hold on; axis([0 1 -1.2 1.2])
plot(nTs,x2,'o'); xlabel('t in sec','FontSize',LFS);
title('Sinc Interpolation for \theta = \pi/6','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta3 = pi/4; x3 = cos(20*pi*nTs+theta3);
y_a3 = x3*sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
subplot(5,1,3); plot(t,y_a3,'LineWidth',1.5); hold on; axis([0 1 -1.2 1.2])
plot(nTs,x3,'o'); xlabel('t in sec','FontSize',LFS);
title('Sinc Interpolation for \theta = \pi/4','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta4 = pi/3; x4 = cos(20*pi*nTs+theta4);
```

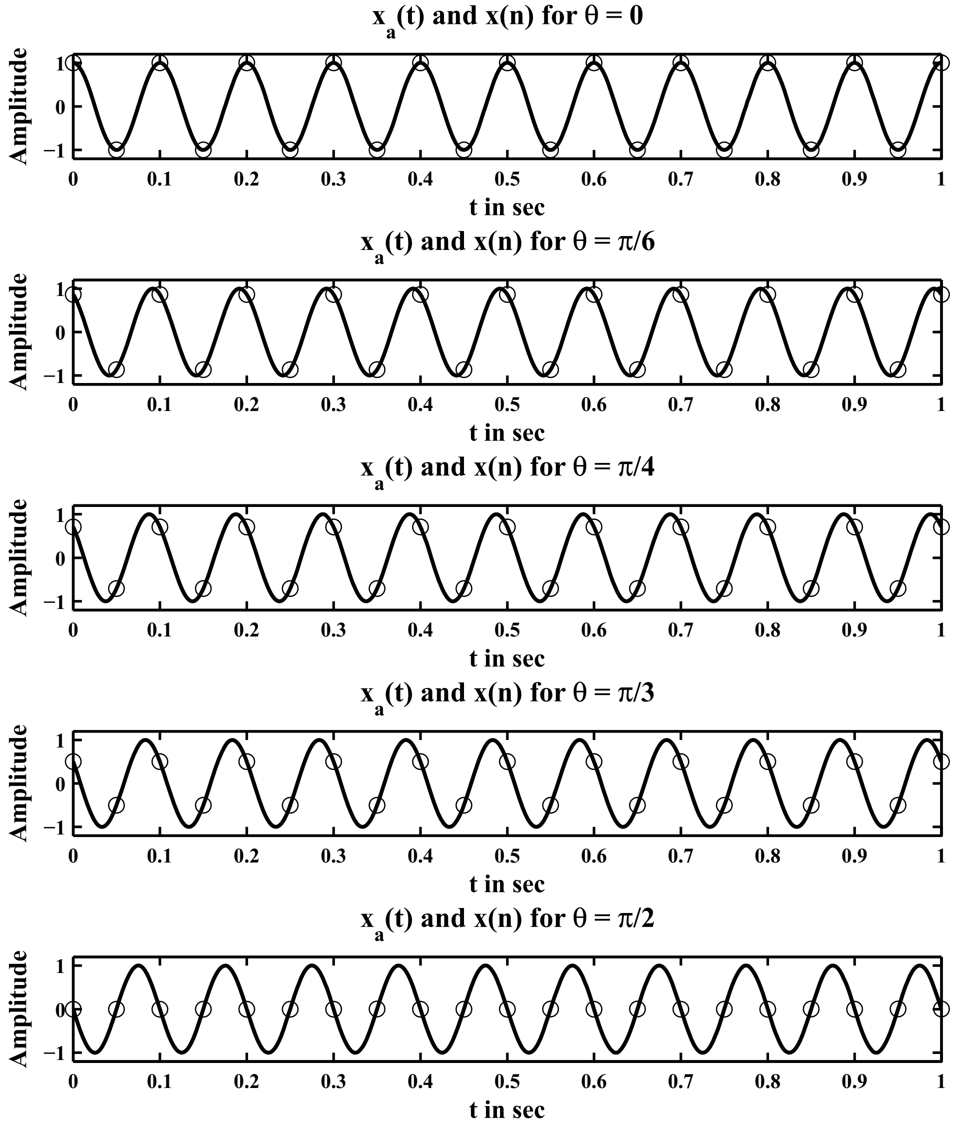

Figure 3.54: The sinc interpolation in Problem P3.22a.

```
y_a4 = x4*sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
subplot(5,1,4); plot(t,y_a4,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x4,'o'); xlabel('t in sec','FontSize',LFS);
title('Sinc Interpolation for \theta = \pi/3','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta5 = pi/2; x5 = cos(20*pi*nTs+theta5);
y_a5 = x5*sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
subplot(5,1,5); plot(t,y_a5,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x5,'o'); xlabel('t in sec','FontSize',LFS);
title('Sinc Interpolation for \theta = \pi/3','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS); print -deps2 ../EPSFILES/P0322b;
```

The reconstruction is shown in Figure 3.55.

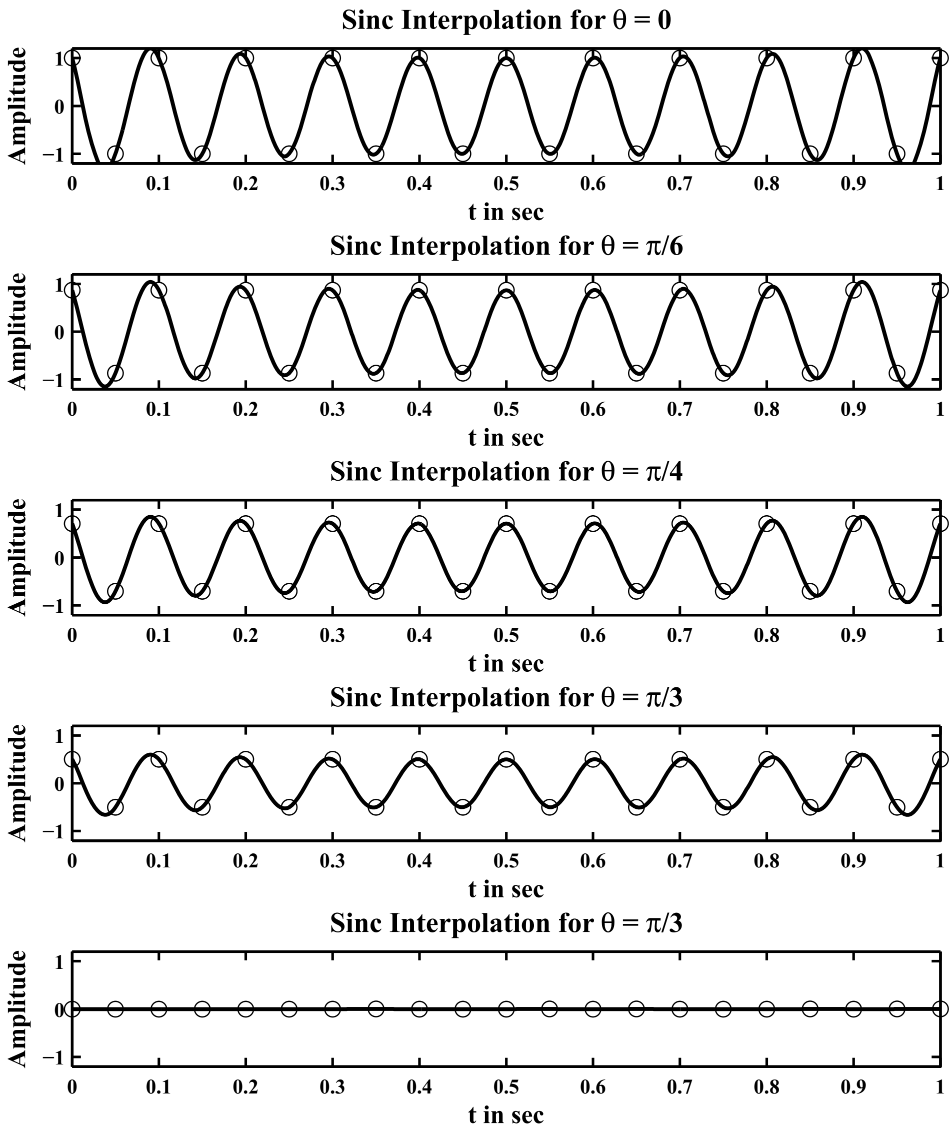

Figure 3.55: The sinc interpolation in Problem P3.22b.

(c) Reconstruction of the analog signal $y_a(t)$ from the samples $x(n)$ using the spline interpolation (for $\theta = 0,\ \pi/6,\ \pi/4,\ \pi/3,\ \pi/2$. MATLAB script:

```
% P0322c: Spline Interpolation for theta = 0,pi/6,pi/4,pi/3, pi/2
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,7]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0322c');
Ts = 0.05; Fs = 1/Ts; Dt = 0.001; t = 0:Dt:1; n = [0:20]; nTs = n*Ts;
theta1 = 0; x1 = cos(20*pi*nTs+theta1); y_a1 = spline(nTs,x1,t);
subplot(5,1,1); plot(t,y_a1,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x1,'o'); xlabel('t in sec','FontSize',LFS);
title('Spline Interpolation for theta = 0','FontSize',TFS);
```

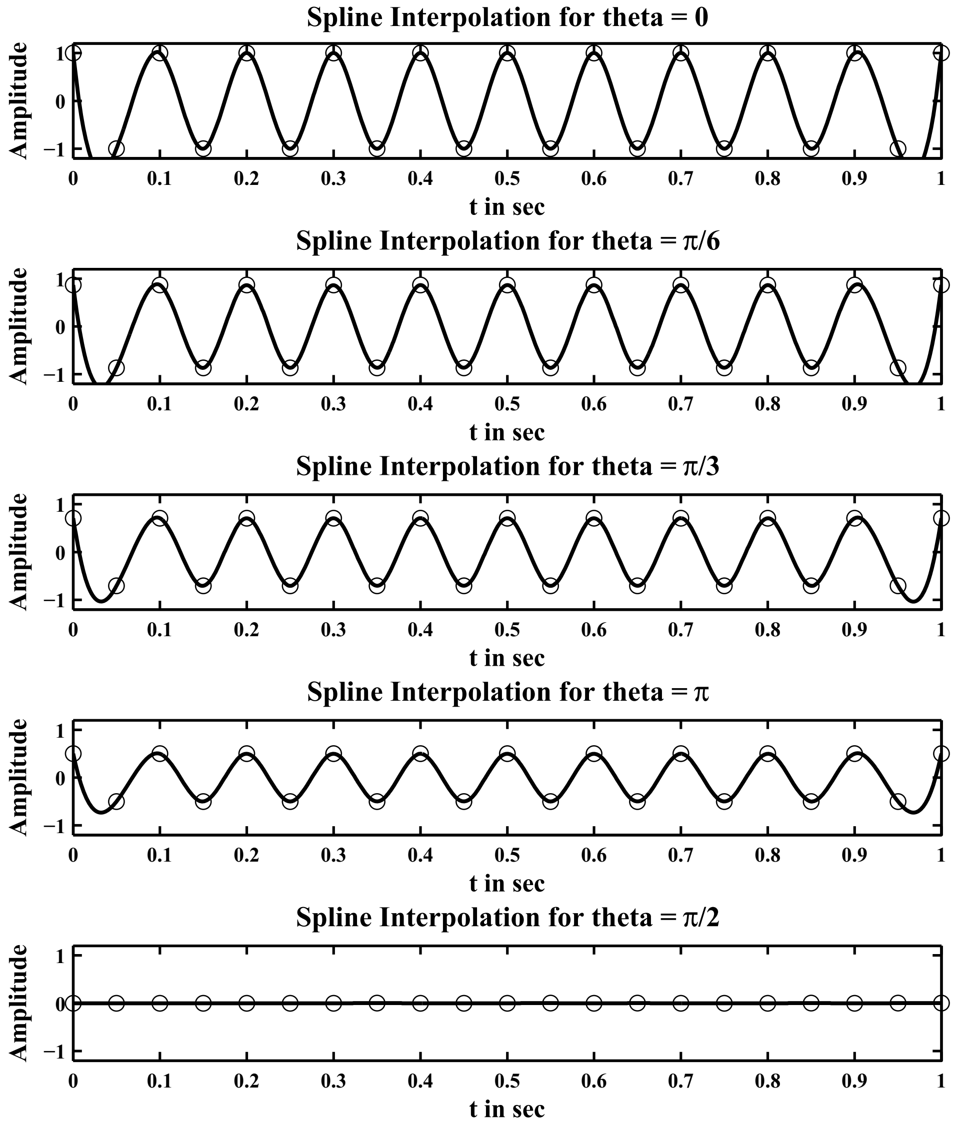

Figure 3.56: The sinc interpolation in Problem P3.22c.

```
ylabel('Amplitude','FontSize',LFS);
theta2 = pi/6; x2 = cos(20*pi*nTs+theta2); y_a2 = spline(nTs,x2,t);
subplot(5,1,2); plot(t,y_a2,'LineWidth',1.5); hold on; axis([0 1 -1.2 1.2]);
plot(nTs,x2,'o'); xlabel('t in sec','FontSize',LFS);
title('Spline Interpolation for theta = \pi/6','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta3 = pi/4; x3 = cos(20*pi*nTs+theta3); y_a3 = spline(nTs,x3,t);
subplot(5,1,3); plot(t,y_a3,'LineWidth',1.5); hold on; axis([0 1 -1.2 1.2]);
plot(nTs,x3,'o'); xlabel('t in sec','FontSize',LFS);
title('Spline Interpolation for theta = \pi/3','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta4 = pi/3; x4 = cos(20*pi*nTs+theta4); y_a4 = spline(nTs,x4,t);
```

```
subplot(5,1,4); plot(t,y_a4,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x4,'o'); ylabel('Amplitude','FontSize',LFS);
title('Spline Interpolation for theta = \pi','FontSize',TFS);
xlabel('t in sec','FontSize',LFS);
theta5 = pi/2; x5 = cos(20*pi*nTs+theta5); y_a5 = spline(nTs,x5,t);
subplot(5,1,5); plot(t,y_a5,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x5,'o'); ylabel('Amplitude','FontSize',LFS);
title('Spline Interpolation for theta = \pi/2','FontSize',TFS);
xlabel('t in sec','FontSize',LFS); print -deps2 ../EPSFILES/P0322c;
```

The reconstruction is shown in Figure 3.56.

(d) When a sinusoidal signal is sampled at $f = 2$ samples per cycle as is the case in this problem, then the resulting samples $x(n)$ has the amplitude that depends on the phase of the signal. In particular note that this amplitude is given by $\cos(\theta)$. Thus the amplitude of the reconstructed signal $y(t)$ is also equal to $\cos(\theta)$.