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DEPARTMENT - COMPUTER AND COMMUNICATION ENGINEERING (CCE) A

LAB TITLE AND CODE : SIGNAL PROCESSING LAB 19CCE281
EXPERIMENT NUMBER : 8
DATE : 23/10/2021

FREQUENCY RESPONSE OF LTI SYSTEM

* AIM :

For a given LTI system characterized by an impulse response h[n], determine the frequency response $h(\omega)$ and sketch the same.

* SOFTWARE REQUIRED :

Spyder IDE - Anaconda 3 2021.05 (Python 3.8.8 64-bit)
Publisher - Anaconda, Inc.

* THEORY :

A system is characterized by its impulse h(n) whose DTFT transform is -

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n}$$

And the inverse DTFT is -

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\omega)\, e^{j\omega n}\, d\omega$$

H(w) is called the frequency response or frequency characteristic of the system. It is the frequency characterization of the system whereas the impulse response is the time characterization.

FREQUENCY RESPONSE -

Now we use the time convolution property (or convolution theorem) to map the output y(n) in time domain to its transform Y(w) in the frequency domain -

$$y(n) = x(n) * h(n) \leftrightarrow Y(\omega) = X(\omega) H(\omega)$$

Or
$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

The frequency response $H(\omega)$ is usually a complex quantity, so we write -

$$H(\omega) = H_R(\omega) + jH_I(\omega) = |H(\omega)|\, e^{j\phi(\omega)}$$

where
$$|H(\omega)| = \sqrt{H_R^2(\omega) + H_I^2(\omega)}$$

and
$$\phi(\omega) = \tan^{-1}\frac{H_I(\omega)}{H_R(\omega)}$$

$|H(\omega)|$ is the magnitude response and $\phi(\omega)$ is the phase response.
If the impulse response $h(n)$ is real-valued, then -

$$|H(-\omega)| = |H(\omega)| \quad \text{and} \quad \phi(-\omega) = -\phi(\omega)$$

The frequency response of a system exists if the system is BIBO (Bounded Input Bounded output) stable, i.e.

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

* **GRAPH PLOTTING ALGORITHM :**

The following steps were followed -

① Define the x-axis and corresponding y-axis values as lists.
② Plot them on canvas using .plot() function.
③ Give a name to x-axis and y-axis using .xlabel() and .ylabel() functions.
④ Give a title to your plot using the .title() function.
⑤ Finally, to view your plot, we use the .show() function.

* **PROGRAM WITH COMMENTS :**

```
1  import numpy as np
2  import matplotlib.pyplot as plt
3
4  n = np.arange(0,10) #Get evenly spaced values within the
   interval [0,10)
```

```
5
6  a=[] # u[n] Null List Declaration
7  for sample in range (len(n)):
8      if n[sample] >= 0:
9          temp = 1
10     else:
11         temp = 0
12     a.append (temp)
13 plt.subplot (5,1,1)
14 plt.xlabel ('n')
15 plt.ylabel ('u[n]')
16 plt.title ('Unit step Discrete-Time Function for u[n]')
17 plt.stem (n,a)
18
19 b=[] # u[n-3] Null List Declaration
20 for sample in range (len(n)):
21     if n[sample] >= 3:
22         temp=1
23     else
24         temp=0
25     b.append (temp)
26 plt.subplot (5,1,3)
27 plt.xlabel ('n')
28 plt.ylabel ('u[n-3]')
29 plt.title ('Unit step Discrete-Time Function for u[n-3]')
30 plt.stem (n,b)
31
32 result = [] # u[n-3] - u[n] Null List Declaration
33 for sample in range (len(n))
34     result.append (b[sample] - a[sample])
35 plt.subplot (5,1,5)
36 plt.xlabel ('n')
37 plt.ylabel ('u[n-3] - u[n]')
```

```
38 plt.title('Unit step Discrete-Time Function for u[n-3] - u[n]')
39 plt.stem(n, result)
40 plt.show()
41
42 temp = []  # (u[n-3] - u[n]) * 0.5^n Null List Declaration
43 for sample in range(len(n)):
44     temp.append((b[sample] - a[sample]) * 0.5 ** sample)
45 plt.xlabel('n')
46 plt.ylabel('(u[n-3] - u[n]) * 0.5^n')
47 plt.title('Impulse Response h[n]')
48 plt.stem(n, temp)
49 plt.show()
50
51 # Compute Frequency Response
52 frequency_response = []
53 def summation(f):
54     sum = 0
55     for k in range(len(temp))
56         exponential = np.exp(-2j * np.pi * k * f)
57         sum = sum + float(temp[k]) * exponential
58     return(sum)
59
60 for w in range(len(temp)):
61     frequency_response.append(summation(w))
62
63 # sketch Frequency Response
64 plt.xlabel('w')
65 plt.ylabel('h(w)')
66 plt.title("Frequency Response")
67 plt.plot(np.arange(0, len(frequency_response)),
   np.real(frequency_response))
68 plt.show()
69 print("\n The Frequency Response of h[n] is as follows : {}".
   format(frequency_response))
```

```
70
71  # Compute Magnitude and Phase Spectrum
72  magnitude_spectrum = []
73  phase_spectrum = []
74
75  for sample in range (len (frequency_response)):
76      magnitude_spectrum.append ((frequency_response [sample].
    real **2 + frequency_response [sample]. imag **2) **0.5)
77      phase = np.arctan (frequency_response [sample]. imag /
    frequency_response [sample]. real)
78      phase_spectrum.append (phase)
79
80  # Sketch Magnitude Spectrum
81  plt.xlabel ('w')
82  plt.ylabel ('|H(w)|')
83  plt.title ("Magnitude spectrum : ... {} ... ".format
    (magnitude_spectrum))
84  plt.plot (np.arange (0, len (magnitude_spectrum)),
    magnitude_spectrum)
85  plt.show ()
86  print ("\n The Magnitude Spectrum of h[n] is as follows : {}".
    format (magnitude_spectrum))
87
88  # Sketch Phase Spectrum
89  plt.xlabel ('w')
90  plt.ylabel ('Angle (in radians)')
91  plt.title ("Phase Spectrum $\phi(w)$")
92  plt.plot (np.arange (0, len (phase_spectrum)), phase_spectrum)
93  plt.show ()
94  print ("\n The Phase Spectrum of h[n] is as follows : {}".
    format (phase_spectrum))
95
```

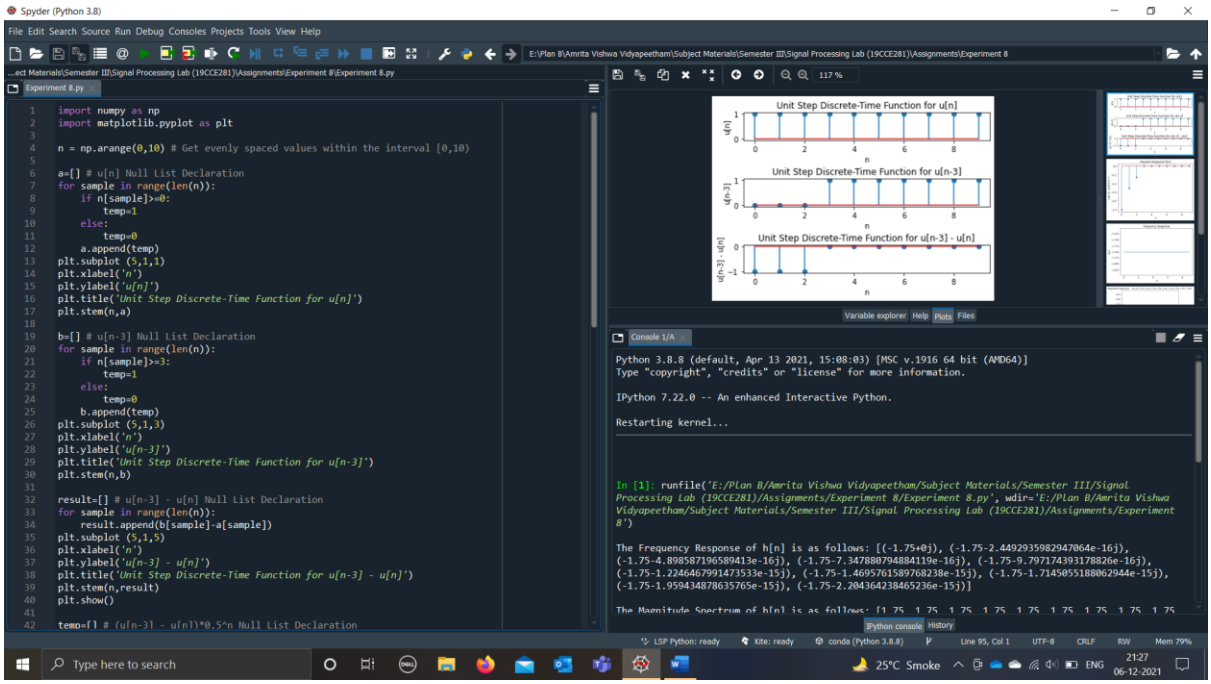
* **INFERENCE:**

For the given LTI system characterized by the impulse response h[n], compute frequency response h(w) and plot the same along with magnitude and phase spectrums.

RESULTS

VERIFIED.

Unit Step Discrete-Time Functions



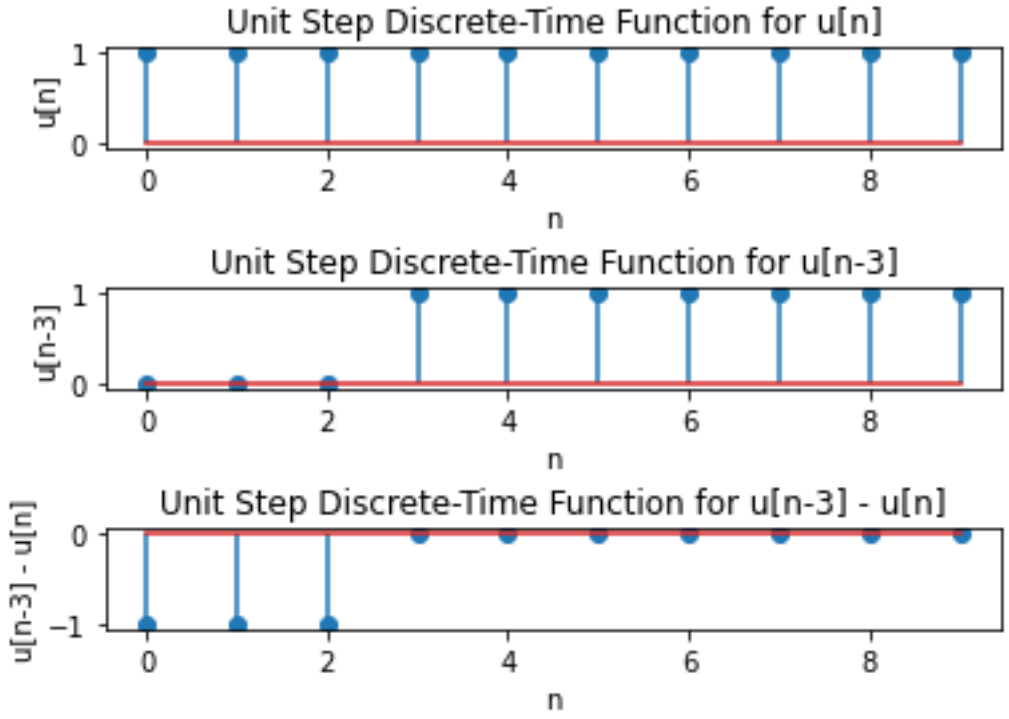

```
import numpy as np
import matplotlib.pyplot as plt

n = np.arange(0,10) # Get evenly spaced values within the interval [0,10)

a=[] # u[n] Null List Declaration
for sample in range(len(n)):
    if n[sample]>=0:
        temp=1
    else:
        temp=0
    a.append(temp)
plt.subplot (5,1,1)
plt.xlabel('n')
plt.ylabel('u[n]')
plt.title('Unit Step Discrete-Time Function for u[n]')
plt.stem(n,a)

b=[] # u[n-3] Null List Declaration
for sample in range(len(n)):
    if n[sample]>=3:
        temp=1
    else:
        temp=0
    b.append(temp)
plt.subplot (5,1,3)
plt.xlabel('n')
plt.ylabel('u[n-3]')
plt.title('Unit Step Discrete-Time Function for u[n-3]')
plt.stem(n,b)

result=[] # u[n-3] - u[n] Null List Declaration
for sample in range(len(n)):
    result.append(b[sample]-a[sample])
plt.subplot (5,1,5)
plt.xlabel('n')
plt.ylabel('u[n-3] - u[n]')
plt.title('Unit Step Discrete-Time Function for u[n-3] - u[n]')
plt.stem(n,result)
plt.show()
```



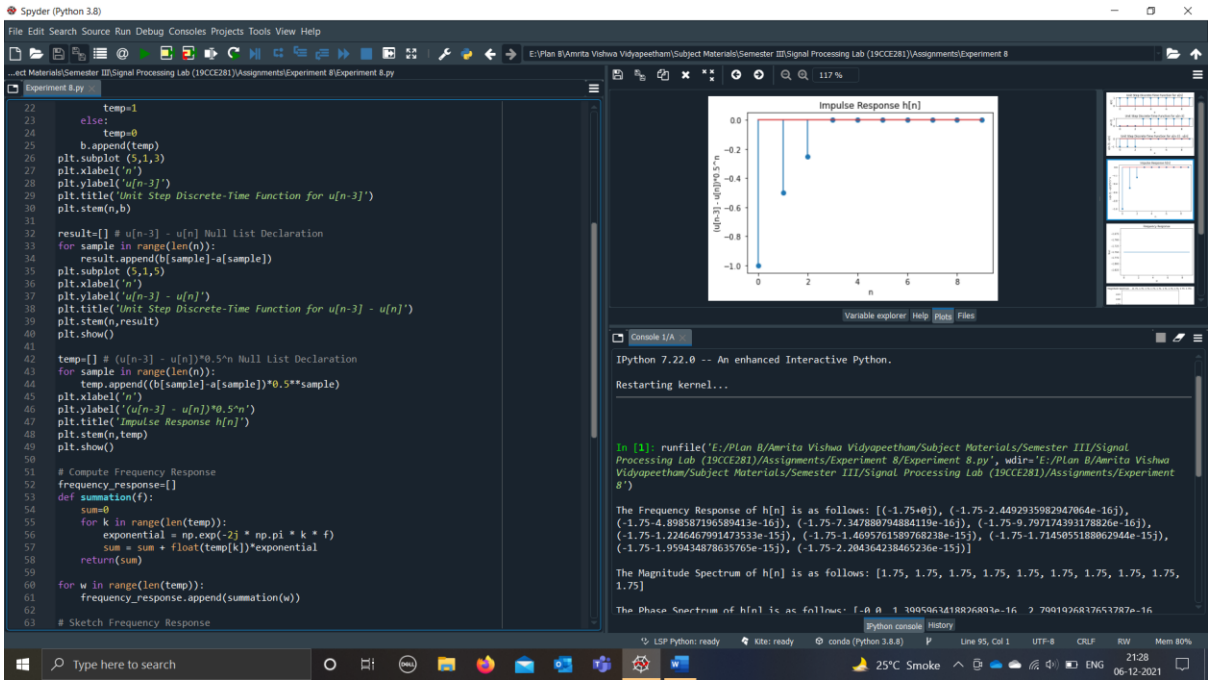
Step 1: Import library source files, numpy and matplotlib.pyplot.

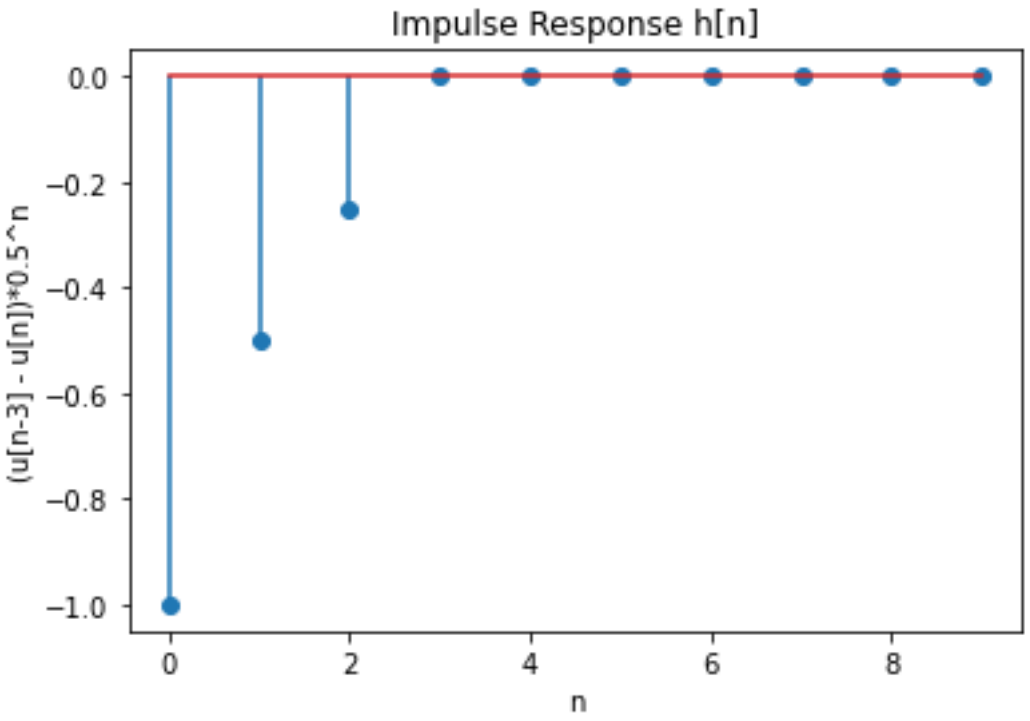
Step 2: Get evenly spaced values within the interval [0,10).

Step 3: Compute u[n] and u[n-3] followed by (u[n-3] - u[n]) and finally, (u[n-3] - u[n])*0.5^n

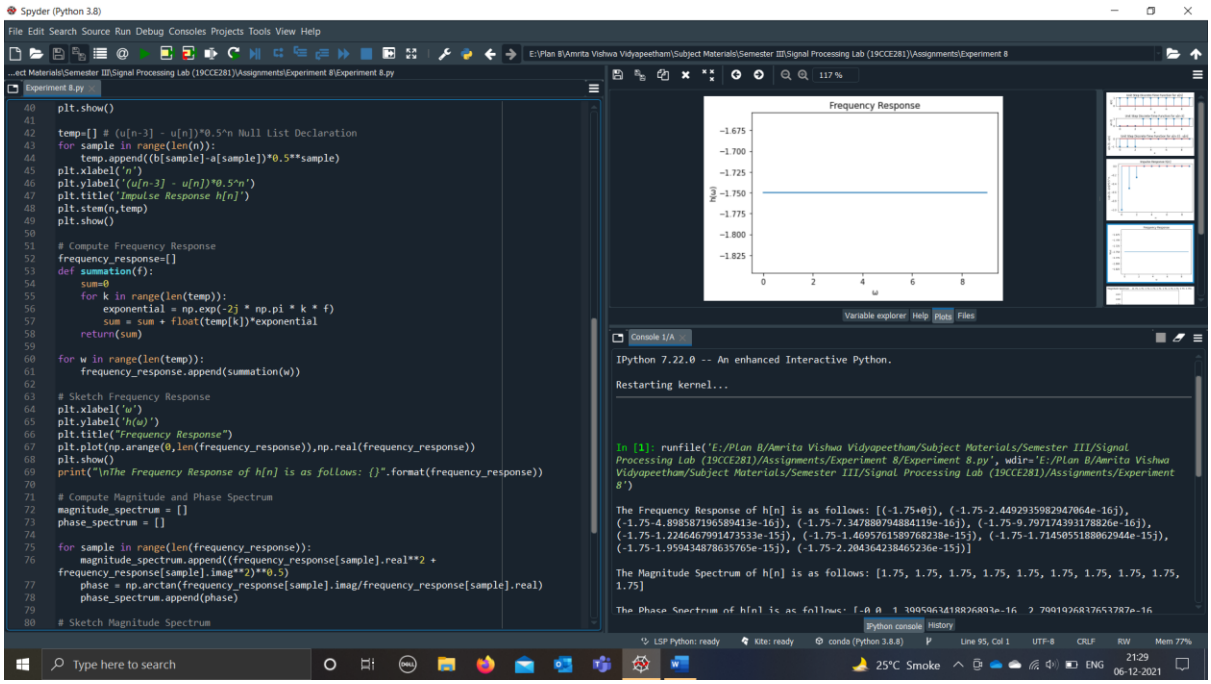
Step 4: Sketch the plot obtained for each function in Step 3.

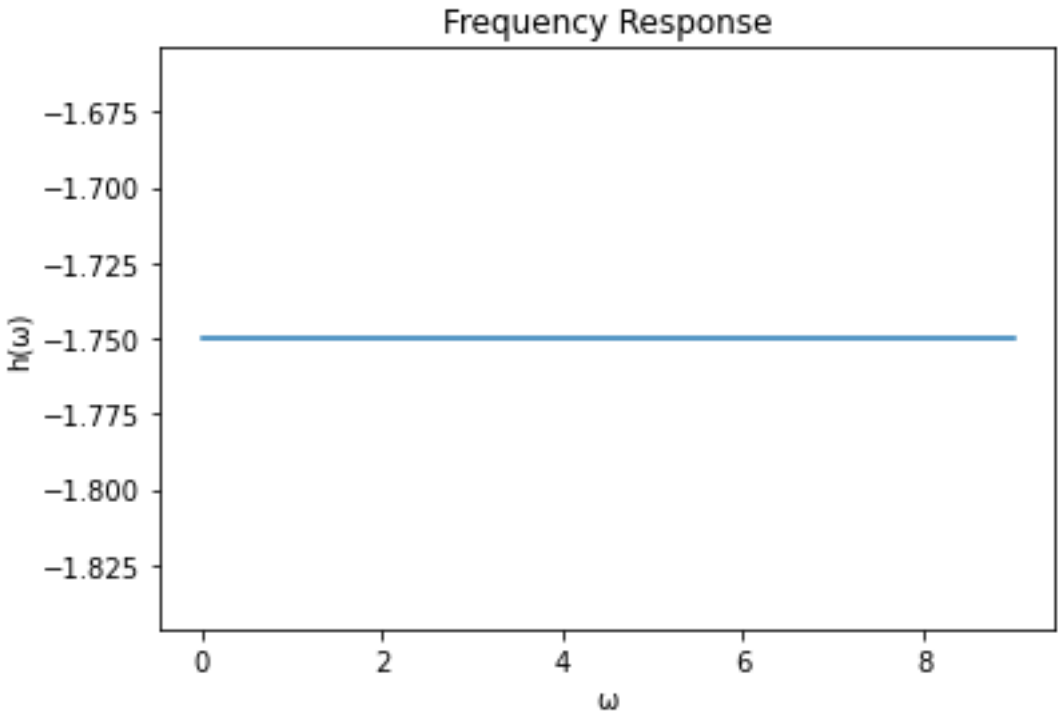
Given Impulse Response h[n]





Frequency Response – h(ω)





Step 1: Compute frequency response for h[n].

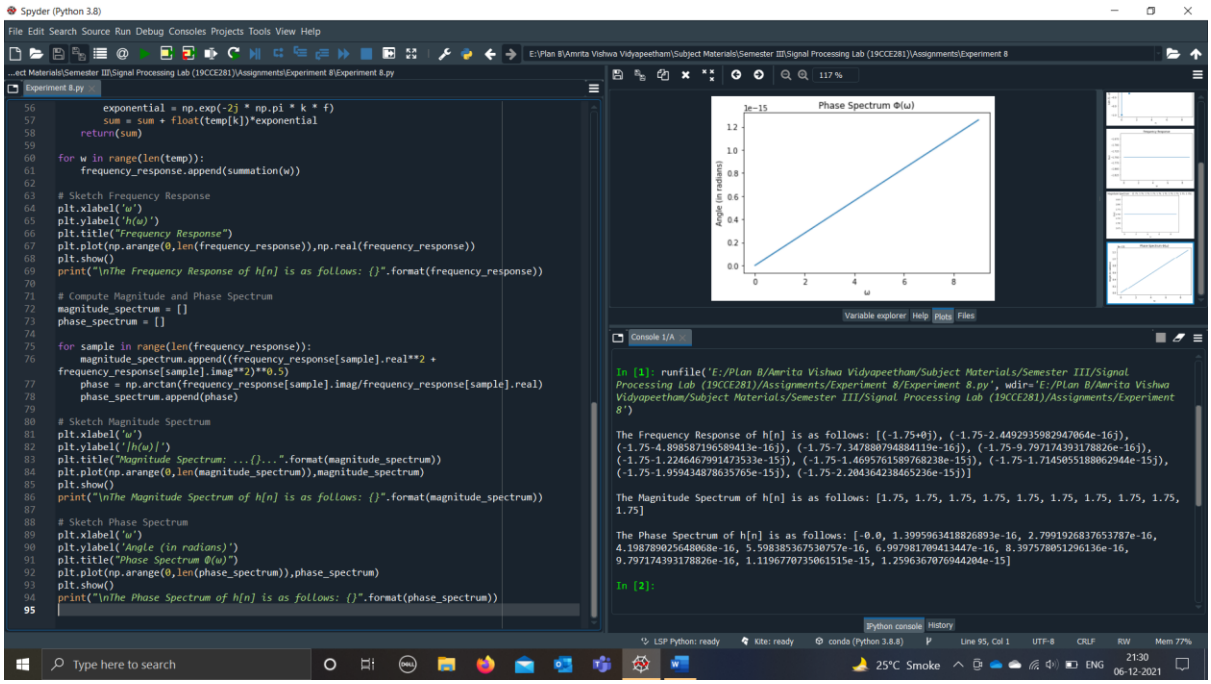
Step 2: Print and sketch the frequency response, h(ω).

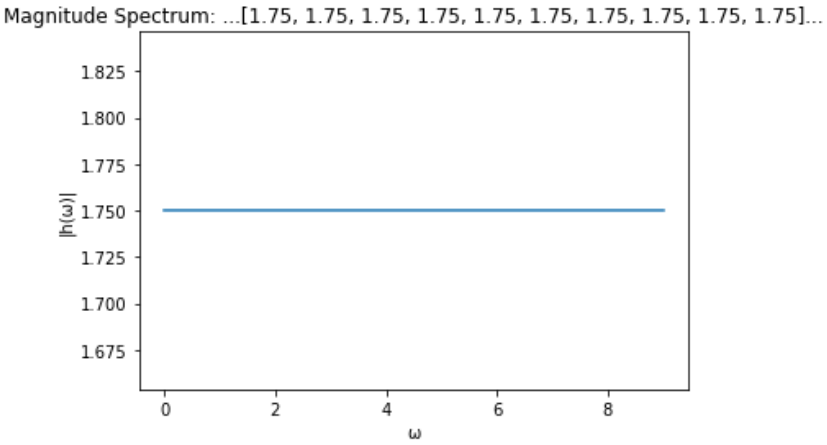
Step 3: Compute magnitude and phase spectrum for h[n].

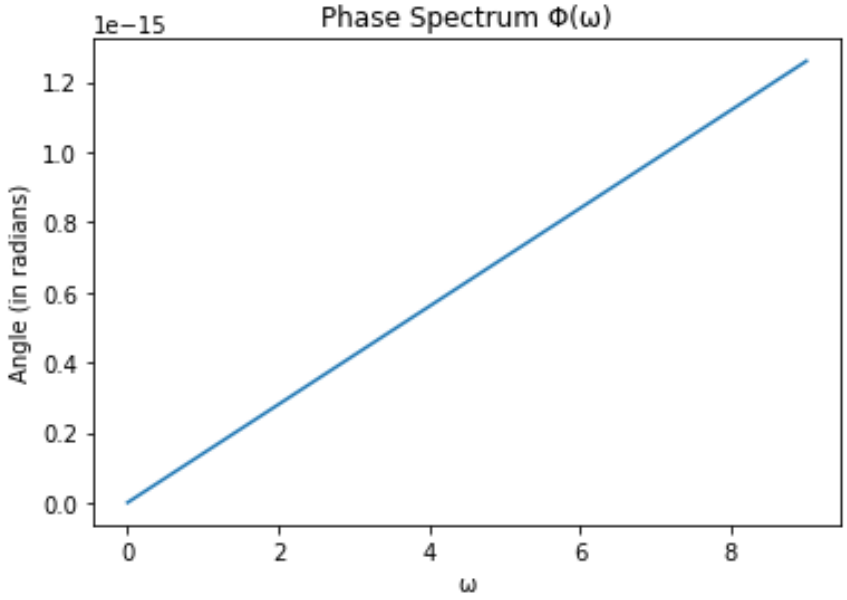
Step 4: Print and sketch magnitude spectrum, |h(ω)|.

Step 5: Print and sketch phase spectrum, Φ(ω).

Magnitude Spectrum – |h(ω)| & Phase Spectrum – Φ(ω)







```
Python 3.9.7 (default, Sep 16 2021, 16:59:28) [MSC v.1916 64 bit (AMD64)]
Type "copyright", "credits" or "license" for more information.

IPython 7.29.0 -- An enhanced Interactive Python.

Restarting kernel...
```

```
In [1]:         'E:/Plan B/Amrita Vishwa Vidyapeetham/Subject Materials/Semester III/
Signal Processing Lab (19CCE281)/Assignments/Experiment 8/Experiment 8.py'      ='E:/Plan
B/Amrita Vishwa Vidyapeetham/Subject Materials/Semester III/Signal Processing Lab
(19CCE281)/Assignments/Experiment 8'

The Frequency Response of h[n] is as follows: [(-1.75+0j),
(-1.75-2.4492935982947064e-16j), (-1.75-4.898587196589413e-16j),
(-1.75-7.347880794884119e-16j), (-1.75-9.797174393178826e-16j),
(-1.75-1.2246467991473533e-15j), (-1.75-1.4695761589768238e-15j),
(-1.75-1.7145055188062944e-15j), (-1.75-1.959434878635765e-15j),
(-1.75-2.204364238465236e-15j)]

The Magnitude Spectrum of h[n] is as follows: [1.75, 1.75, 1.75, 1.75, 1.75, 1.75, 1.75,
1.75, 1.75, 1.75]

The Phase Spectrum of h[n] is as follows: [-0.0, 1.3995963418826893e-16,
2.7991926837653787e-16, 4.198789025648068e-16, 5.598385367530757e-16,
6.997981709413447e-16, 8.397578051296136e-16, 9.797174393178826e-16,
1.1196770735061515e-15, 1.2596367076944204e-15]

In [2]:
```