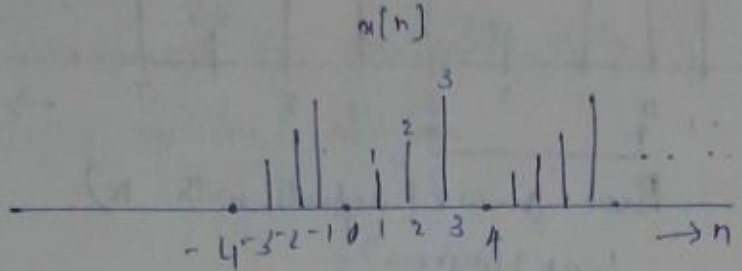
Determine the DTFS coeff for the periodic signal depicted in fig. below & sketch the spectra.



Sol:

$$N = 4; \quad \Omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x(n)\, e^{-jk\Omega_0 n}$$

$$X(k) = \frac{1}{4} \sum_{n=0}^{3} x(n)\, e^{-jk\frac{\pi}{2}n}$$

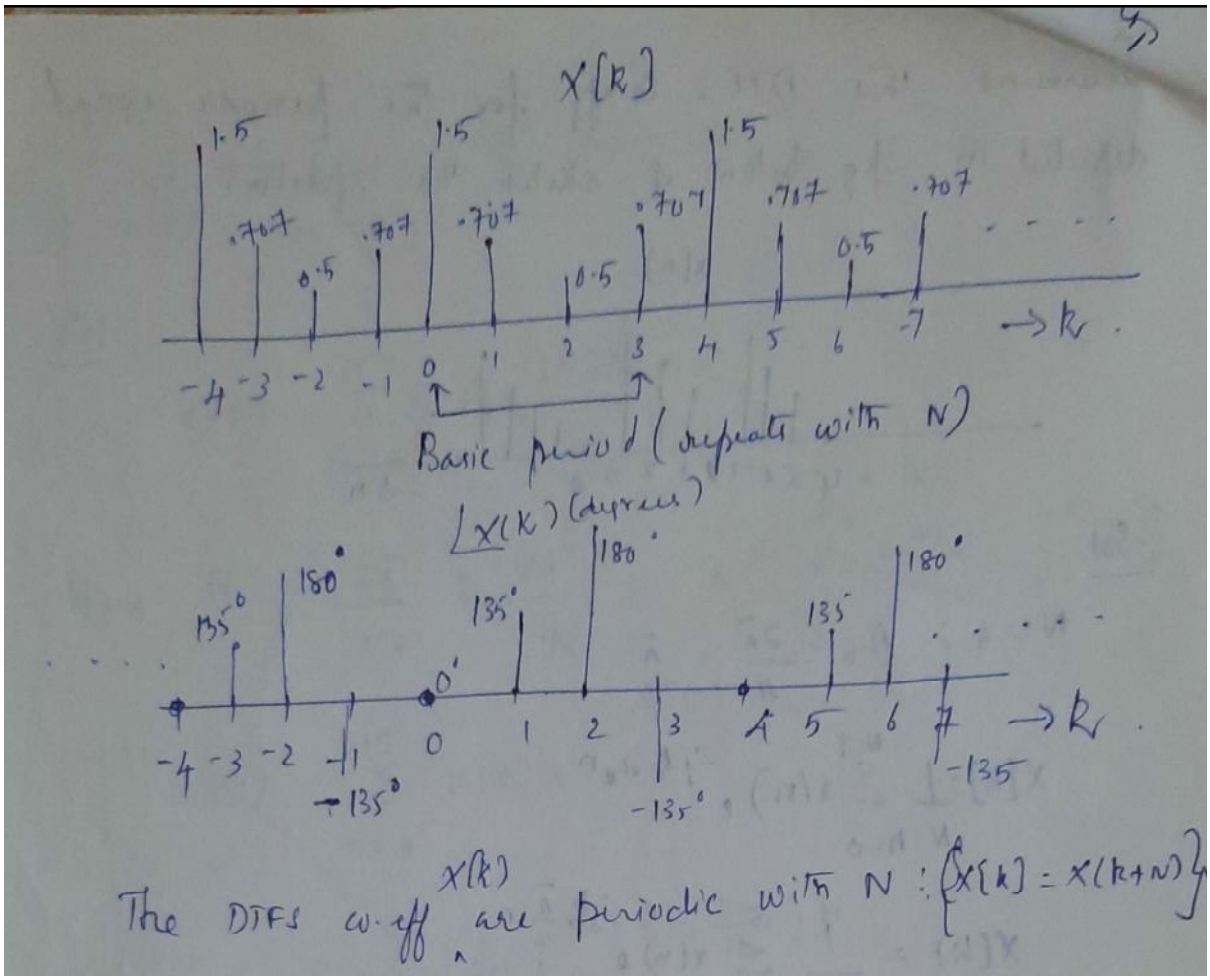
$$= \frac{1}{4}\left[x(0)e^{0} + x(1)e^{-j\frac{\pi}{2}k} + x(2)e^{-jk\frac{\pi}{2}\cdot 2} + x(3)e^{-jk\frac{\pi}{2}\cdot 3} - \right.$$

$$= \frac{1}{4}\left[0 + 1.e^{-jk\frac{\pi}{2}} + 2e^{-jk\pi} + 3e^{-j\frac{3k\pi}{2}}\right] \longleftarrow (1)$$

In this problem, k = 0, 1, 2, 3 (as N = 4).

Hence evaluate x[0], x[1], x[2], x[3] by putting k value in eqn (1). At the same time evaluate |x(k)| & ∠x(k).

k	$X(k)$	$\lvert X(k)\rvert$	$\angle X(k)$
0	1.5	1.5	0
1	$-0.5 + j0.5$	0.707	135° (2.357 radians)
2	-0.5	0.5	180° (π rad)
3	$-0.5 - j0.5$	0.707	$-135°$ (-2.357 radians)



The DTFS coeff $X(k)$ are periodic with N : $\{X[k] = X(k+N)\}$