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LAB TITLE AND CODE : SIGNAL PROCESSING LAB 19CCE281

EXPERIMENT NUMBER : 6

DATE : 26/10/2021

FOURIER SERIES REPRESENTATION OF PERIODIC SIGNALS

* **AIM :**

Obtain the frequency domain representation of the given discrete-time periodic sequence and sketch the input sequence and Fourier coefficients magnitude and phase spectrum

* **SOFTWARE REQUIRED :**

Spyder IDE (Anaconda3) - Python 3.9.7 (64-bit)

* **THEORY :**

① **Discrete-time signals -**

A discrete-time signal of fundamental period N can consist of frequency components $f = \frac{1}{N}, \frac{2}{N}, \ldots, \frac{(N-1)}{N}$ besides $f=0$, the DC component. Therefore, the Fourier series representation of the discrete-time periodic signal contains only N complex exponential basis functions.

② **Fourier series for discrete-time periodic signals -**

Given a periodic sequence $x[k]$ with period N, the Fourier series representation for $x[k]$ uses N harmonically related exponential functions -

$$e^{j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

The Fourier series is expressed as -

$$\boxed{x[k] = \sum_{n=0}^{N-1} c_n e^{j2\pi kn/N}}$$

③ Fourier coefficients-

The Fourier coefficients $\{c_n\}$ are given by -

$$\boxed{c_n = \frac{1}{N}\sum_{k=0}^{N-1} x[k]\, e^{-j2\pi kn/N}}$$

* GRAPH PLOTTING ALGORITHM :

The following steps are followed -

① Define the x-axis and corresponding y-axis values as lists.

② Plot them on canvas using .plot() function.

③ Give a name to x-axis and y-axis using .xlabel() and .ylabel() functions.

④ Give a title to your plot using the .title() function.

⑤ Finally, to view your plot, we use the .show() function.

* THEORETICAL CALCULATION :

Given periodic sequence,

$x[n] = [\ldots.\ 5\ 10\ 15\ 20\ 25\ \mathbf{5}\ 10\ 15\ 20\ 25\ 5\ 10\ 15\ 20\ 25 \ldots.]$

(Note: Boldface indicates n=0 index).

By keen observation, we find that fundamental period (N)=5

Angular frequency $(\Omega_0) = \frac{2\pi}{N} = \frac{2\pi}{5}$

$$x[k] = \frac{1}{N}\sum_{n=0}^{4} x[n]\, e^{-jk\frac{2\pi}{5}n} = \frac{1}{5}\sum_{n=0}^{4} x[n]\, e^{-jk\frac{2\pi}{5}n}$$

$$= \frac{1}{5}\Big[x[0]e^{0} + x[1]e^{-jk\frac{2\pi}{5}} + x[2]e^{-jk\frac{4\pi}{5}} + x[3]e^{-jk\frac{6\pi}{5}} + x[4]e^{-jk\frac{8\pi}{5}}\Big]$$

$$= \frac{1}{5}\Big[5 + 10e^{-jk\frac{2\pi}{5}} + 15e^{-jk\frac{4\pi}{5}} + 20e^{-jk\frac{6\pi}{5}} + 25e^{-jk\frac{8\pi}{5}}\Big]$$

Taking 5 common, we get -

$$x[k] = 1 + 2e^{-jk\frac{2\pi}{5}} + 3e^{-jk\frac{4\pi}{5}} + 4e^{-jk\frac{6\pi}{5}} + 5e^{-jk\frac{8\pi}{5}}$$

Substituting the values of k in $x[k]$, we find that -

$x[0] = 15 + 0j$, for $k = 0$

$x[1] = -2.5 + 3.44j$, for $k = 1$

$x[2] = -2.5 + 0.81j$, for $k = 2$

$x[3] = -2.5 - 0.81j$, for $k = 3$

$x[4] = -2.5 - 3.44j$, for $k = 4$

Upon computing the magnitude spectrum, we find that -

$|x[0]| = \sqrt{15^2 + 0^2} = 15$

$|x[1]| = \sqrt{(-2.5)^2 + (3.44)^2} = 4.25$

$|x[2]| = \sqrt{(-2.5)^2 + (0.81)^2} = 2.63$

$|x[3]| = \sqrt{(-2.5)^2 + (-0.81)^2} = 2.63$

$|x[4]| = \sqrt{(-2.5)^2 + (-3.44)^2} = 4.25$

Upon computing the phase spectrum, we find that -

$(x[0]) = \tan^{-1}\left(\frac{0}{15}\right) = 0^\circ = 0 \times \frac{\pi}{180}$ radians $= 0$ radians

$(x[1]) = \tan^{-1}\left(\frac{3.44}{-2.5}\right) = -54^\circ = -\cancel{54}^{3} \times \frac{\pi}{\cancel{180}_{10}}$ radians $= -0.94$ radians

$(x[2]) = \tan^{-1}\left(\frac{0.81}{-2.5}\right) = -18^\circ = -\cancel{18}^{1} \times \frac{\pi}{\cancel{180}_{10}}$ radians $= -0.31$ radians

$(x[3]) = \tan^{-1}\left(\frac{+0.81}{+2.5}\right) = 54^\circ = \cancel{54}^{3} \times \frac{\pi}{\cancel{180}_{10}}$ radians $= 0.94$ radians

$(x[4]) = \tan^{-1}\left(\frac{+3.44}{+2.5}\right) = 18^\circ = \cancel{18}^{1} \times \frac{\pi}{\cancel{180}_{10}}$ radians $= 0.31$ radians

* **PROGRAM WITH COMMENTS:**

```
# import library source files
import numpy as np
import matplotlib.pyplot as plt

# Given Periodic Sequence
```

PTO

```
size_input = int (input ("Enter the size of input x[n] : "))
user_defined_input = [0] * (size_input)
print ("Enter the elements of the input x[n] one-by-one as follows:
for sample in range (0, size_input, 1):
    user_defined_input [sample] = input ("element " + str (
user_defined_input) + "\n")
print ("\n The entered input x[n] is : - " + str (user_defined_input)
+ "\n")

# Null Array Declaration
x = []
fourier_series = []

# Compute Fundamental Time Period
for sample in range (1, size_input - 1):
    x. append (user_defined_input [sample-1])
    if user_defined_input [0] == user_defined_input [sample]:
        breakpoint = sample -2 # How many times does the
sequence repeat?
        break

# Obtain Frequency Domain Representation
boldface = int (input (" Enter the element position that
indicates n=0 index: "))
index = int ((boldface -1) / len (x))

# Sketch Input Sequence
plt. xlabel ('n')
plt. ylabel ('x[n]')
plt. title (" Input sequence : ... {} ... ". format (x))
plt. stem (np. arange (- (index * len (x)), len (x) * (breakpoint -
index)), x * breakpoint)
```

```
plt.show()

# Compute Fourier series
def summation():
    sum = 0
    for n in range(len(x)):
        exponential = np.exp(-2j * np.pi * k * n/len(x))
        sum = sum + float(x[n]) * exponential
    return (sum)

# Compute Fourier Series Coefficients
for k in range(len(x)):
    fourier_series.append(summation(k)/len(x))
print("\nThe Fourier Series Coefficients are as follows: {}".
 format(fourier_series))

# Compute Magnitude and Phase spectrum
magnitude_spectrum = []
phase_spectrum = []
for sample in range(len(fourier_series)):
    magnitude_spectrum.append((fourier_series[sample].
real ** 2 + fourier_series[sample].imag ** 2) ** 0.5)
    phase = np.arctan(fourier_series[sample].imag /
fourier_series[sample].real)
    phase_spectrum.append(phase)

# sketch magnitude Spectrum
plt.xlabel('k')
plt.ylabel('|x[k]|')
plt.title("magnitude spectrum: ...{}...".format
  (magnitude_spectrum))
plt.stem(np.arange(0, len(magnitude_spectrum)),
  magnitude_spectrum)
```

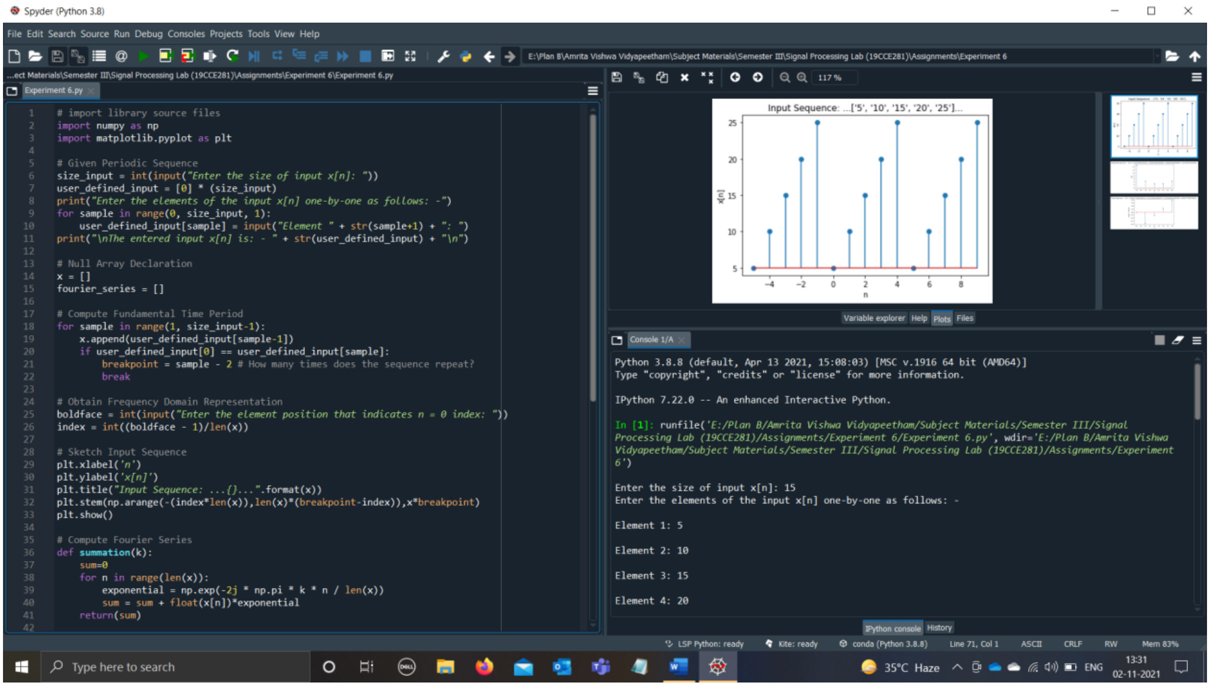
```
plt.show()
print("\n The Magnitude Spectrum is as follows : {}".
   format(magnitude_spectrum))

# sketch Phase Spectrum
plt.xlabel('k')
plt.ylabel('Angle (in radians)')
plt.title('Phase spectrum : ...{}...'.format(phase_spectrum))
plt.stem(np.arange(0, len(phase_spectrum)), phase_spectrum)
plt.show()
print("\n The Phase Spectrum is as follows : {}".format
   (phase_spectrum))
```

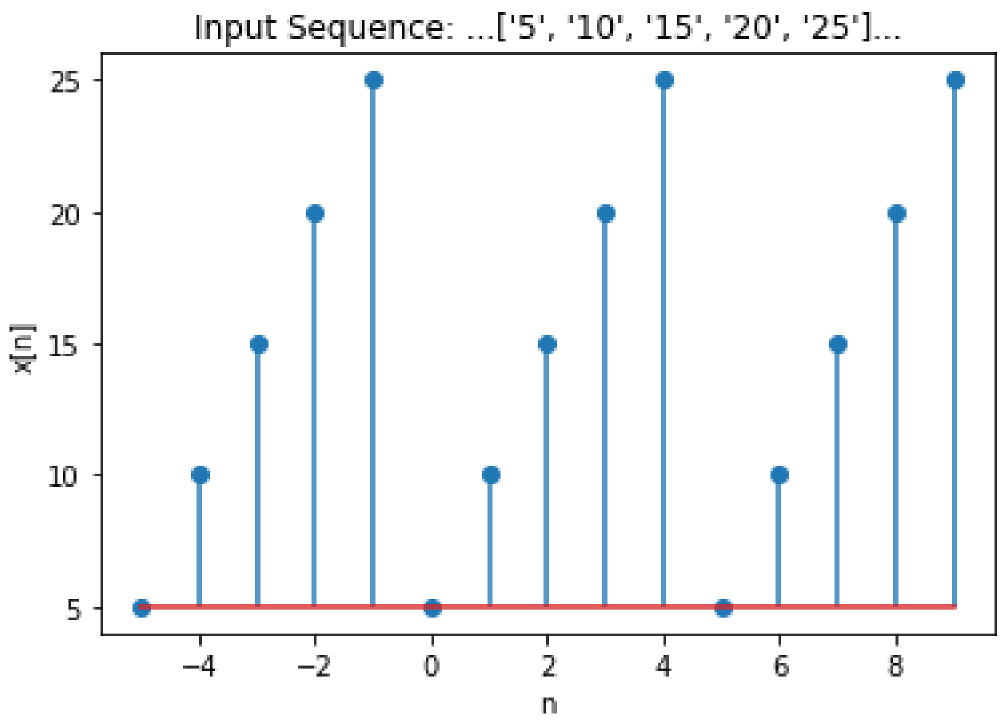
* **INFERENCE :**

For the given input periodic sequence x[n], obtain frequency domain representation along with fundamental time-period and compute Fourier series, magnitude and phase spectrum.

RESULTS
VERIFIED.

Given Discrete Periodic Sequence





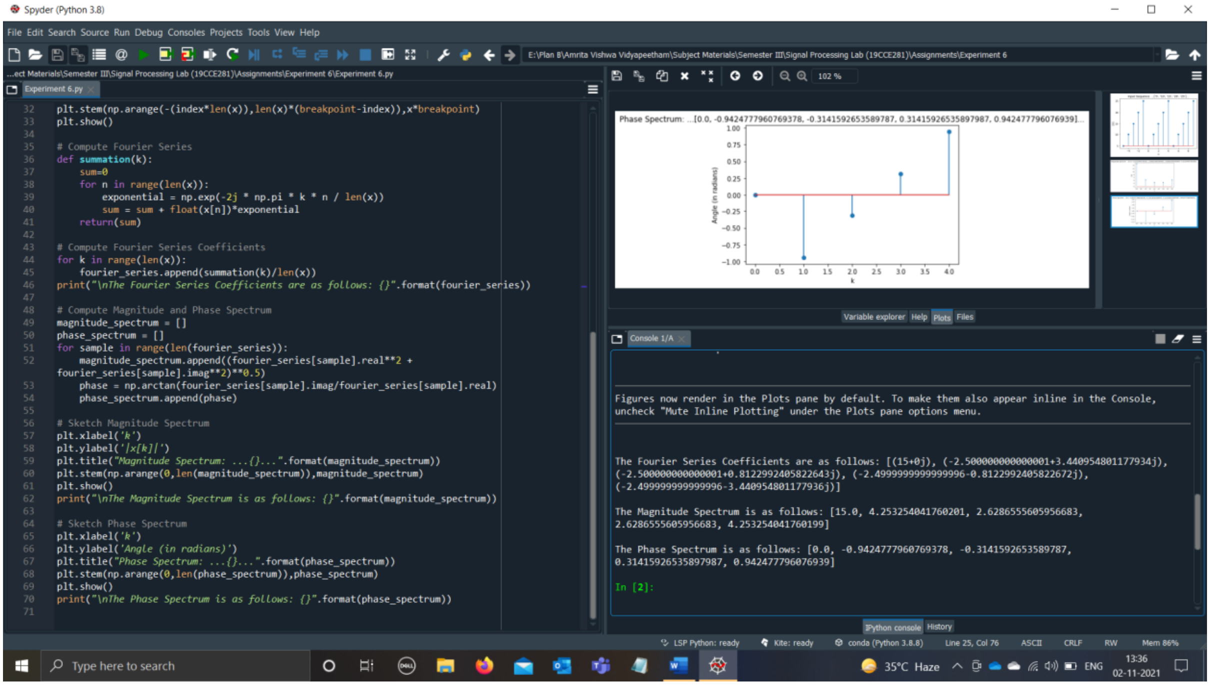
Step 1: Enter the size of input x[n] and declare an array with the number of elements equal to the value of size.

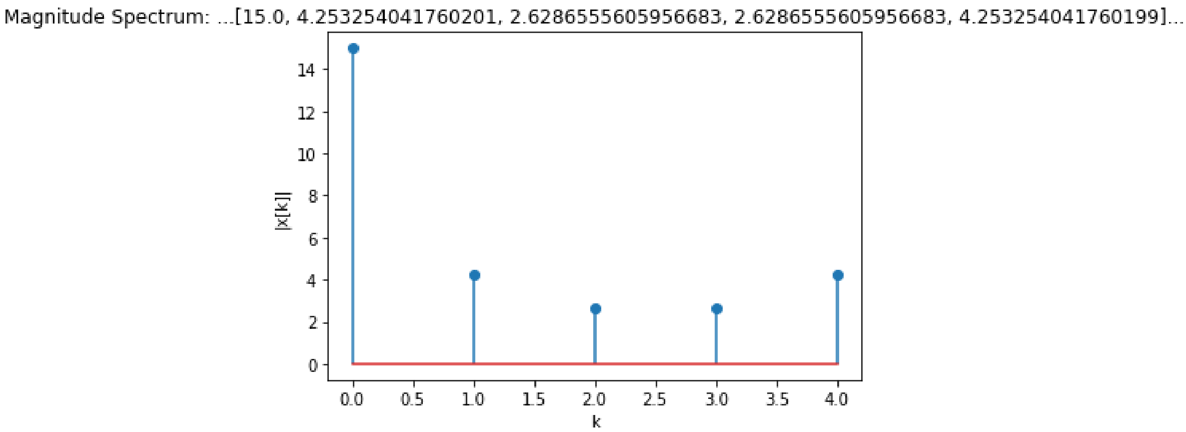
Step 2: Enter the elements of the input x[n] along with the element position that indicates n = 0 index.

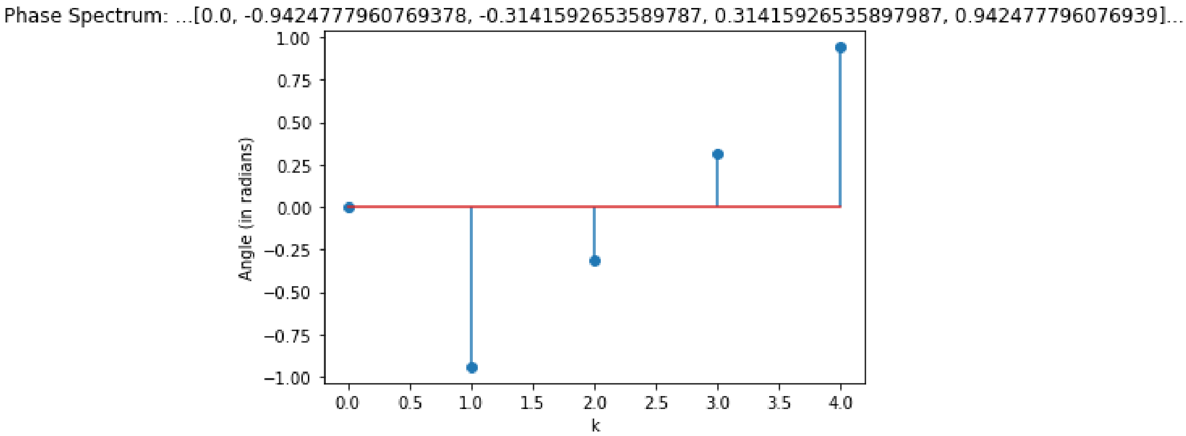
Step 3: Compute fundamental time period and obtain frequency domain representation.

Step 4: Print the input sequence and show its labelled plot x[n].

Plot Magnitude and Phase Spectrum







Step 1: Compute Fourier series and their coefficients.

Step 2: Compute magnitude and phase spectrum.

Step 3: Sketch the two spectrums and print them.

```
Python 3.9.7 (default, Sep 16 2021, 16:59:28) [MSC v.1916 64 bit (AMD64)]
Type "copyright", "credits" or "license" for more information.

IPython 7.29.0 -- An enhanced Interactive Python.

Restarting kernel...
```

```
In [1]:         'E:/Plan B/Amrita Vishwa Vidyapeetham/Subject Materials/Semester III/Signal Processing Lab (19CCE281)/Assignments/Experiment 6/Experiment 6.py'      ='E:/Plan B/Amrita Vishwa Vidyapeetham/Subject Materials/Semester III/Signal Processing Lab (19CCE281)/Assignments/Experiment 6'

Enter the size of input x[n]: 15
Enter the elements of the input x[n] one-by-one as follows: -

Element 1: 5

Element 2: 10

Element 3: 15

Element 4: 20

Element 5: 25

Element 6: 5

Element 7: 10

Element 8: 15

Element 9: 20

Element 10: 25

Element 11: 5

Element 12: 10

Element 13: 15

Element 14: 20

Element 15: 25

The entered input x[n] is: - ['5', '10', '15', '20', '25', '5', '10', '15', '20', '25',
'5', '10', '15', '20', '25']


Enter the element position that indicates n = 0 index: 6

The Fourier Series Coefficients are as follows: [(15+0j),
(-2.500000000000001+3.440954801177934j), (-2.500000000000001+0.8122992405822643j),
```

```
(-2.4999999999999996-0.8122992405822672j), (-2.499999999999996-3.440954801177936j)]

The Magnitude Spectrum is as follows: [15.0, 4.253254041760201, 2.6286555605956683,
2.6286555605956683, 4.253254041760199]

The Phase Spectrum is as follows: [0.0, -0.9424777960769378, -0.3141592653589787,
0.31415926535897987, 0.942477796076939]
```

In [2]: