Optimal Marketing Portfolio Strategy

Optimization Methods, 15.093 - Final Paper

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Overview and Purpose

The goal of our project is to design a framework for optimizing a marketing strategy given a fixed budget, multiple marketing options, and varying constraints and optimization goals. We chose to include four marketing options for our project: Targeted Online Advertising, E-mail, Print Media, and Influencer Marketing. However, the frameworks and formulations we provide can be easily extended to fit any combination of marketing options.

Data

The data for this project is synthetically generated. We have four different marketing options for a product: Targeted Online Advertising (max investment value: $990), E-mail (max investment value: $500), Print Media (max investment value: $700), and Influencer Marketing (Social Media) (max investment value: $600). For each of these options, we have the following variables in the data:

- *Investment Amount:* Each row defines an investment amount that generates an expected number of views and purchases made.
- *Number of People Reached:* Number of people who view the marketed advertisement
- *Sales Generated:* Number of people who purchase the product after getting influenced by the marketing through the given platform.

For Data Pre-Processing: We interpolated the data to create a continuous function from our discrete data points, allowing the optimization model to work effectively with any investment amount within the provided ranges.

Methods and Findings

Baseline Model

For our Baseline model, we decided to use an equal allocation of funds for investment among all marketing options for different budget amounts. This model is only constrained by the budget. The results are as follows:

Baseline Model			
Budget	**Investment per Marketing Type**	**Views**	**Buys**
1000	250	25,712	4,750
1500	375	34,665	7,025
2000	500	41,241	9,000

Optimal Views

Here, we aim to maximize the total number of people reached overall, using all marketing options. Our decision variable is the investment amount for each technique. We introduce auxiliary variables to make this a piecewise linear formulation and further consider slopes with breakpoints in the data. We also have a budgetary constraint, so the total investment can not exceed the budget. *The detailed formulation can be found in the appendix[1].*

Optimize Views						
Budget	**Targeted \$***	**Email \$**	**Print \$**	**Influencer \$**	**Views**	**Buys**
\$1000	\$0	\$500 (max)	\$392	\$108	29,509	3,682
\$1500	\$572	\$500 (max)	\$320	\$108	35,509	6,326
\$2000	\$680	\$500 (max)	\$424	\$396	41,509	9,124

* Represents the amount invested in Targeted Online Advertisement

Optimal Buys

Here, we aim to maximize the total number of purchases made overall, using all marketing techniques. The formulation remains similar with two changes. First, the piecewise linear segments are now found on the 'buys' data instead of the 'views' data. Second, the slope will now be calculated for 'buys v/s invested amount' instead of 'views v/s invested amount'.

Optimize Buys						
Budget	**Targeted \$**	**Email \$**	**Print \$**	**Influencer \$**	**Views**	**Buys**
\$1000	\$500	\$0	\$0	\$500	12,224	5,850
\$1500	\$900	\$0	\$0	\$600 (max)	17,524	8,130
\$2000	\$950	\$300	\$150	\$600 (max)	34,301	9,980

MIO Optimal Views and Buys

Now, let's say if we want to invest in any marketing type there is a one-time fixed \$200 onboarding fee we have to pay to an agency to onboard said marketing type. We can model the situation using MIO! First, we would begin by creating binary variables that represent the decision to onboard to an agency. Then we would then need to create BigM constraints that tie our binary variables to our continuous ones. These should state that we can only invest in a marketing type if we have decided to onboard. Finally, we would need to update our budget constraint to include the onboarding costs. The necessary additions to the formulation can be found below:

Define Binary Decision Variables:

$$\begin{aligned} Targeted\_Onboard &\in \{0,1\} \\ Email\_Onboard &\in \{0,1\} \\ Print\_Onboard &\in \{0,1\} \\ Influencer\_Onboard &\in \{0,1\} \end{aligned}$$

Create BigM Linking Constraints:

$$\begin{aligned} Investment\_Targeted &\leq\ Targeted\_Onboard \times \mathbf{M} \\ Investment\_Email &\leq\ Email\_Onboard \times \mathbf{M} \\ Investmen\_Print &\leq\ Print\_Onboard \times \mathbf{M} \\ Investmen\_Influencer &\leq\ Influencer\_Onboard \times \mathbf{M} \end{aligned}$$

Update Budget Constraint:

$$\begin{aligned} Investment\_Targeted +\ Investment\_Email +\ Investment\_Print +\ Investment\_Influencers \\ +200 \times (\ Targeted\_Onboard +\ Email\_Onboard +\ Print\_Onboard +\ Influencers\_Onboard) \leq\ Budget \end{aligned}$$

Below are a selection of outputs from the model with the updates and constraints above:

MIO Model Outputs ($200 Onboard Fee)								
Budget	**Opt. Over**	**Targeted $**	**Email $**	**Print $**	**Influencer $**	**Fees $**	**Views**	**Buys**
$1000	Views	$0	$280	$320	$0	$400	22,678	2,080
$2000	Views	$0	$500 (max)	$495	$405	$600	34,291	5,920
$1000	Buys	$800	$0	$0	$0	$200	9,360	3,880
$2000	Buys	$990 (max)	$0	$0	$600 (max)	$400	18,344	8,450

Additional Constraint and Duals

Many additional constraints could be added to this problem to model more niche and specific scenarios. We have selected a few interesting constraints that could be added to model different requests and have augmented our initial optimal models below to include them as examples.

Fixed Allocation Constraint*:* This constraint models when a fixed portion of the budget must go to a specific marketing type. For example, Let's say your boss is excited about targeted advertising, but still wants to optimize the number of views. Below are the outputs of the original views maximizing model with a constraint that states we must spend $400 on Targeted Advertising. It can be formulated as written below:

$$Investment\_Targeted \geq 400$$

Optimize Views, Given You Must Spend $400 On Target Advertising						
Budget	**Targeted $**	**Email $**	**Print $**	**Influencer $**	**Views**	**Buys**
$1000	$400	$280	$320	$0	27,367	4,080
$1500	$572	$500 (max)	$320	$108	35,509	6,326
$2000	$680	$500 (max)	$424	$396	41,509	9,124

If Then Constraint**:** This constraint models an if-then situation where if one type of marketing is invested in then another type of marketing must be invested in. For example, let's say, if we choose to do influencer marketing then we must also spend at least $100 on print marketing. An executive may want to do this to ensure we maintain some balance in the demographics of customers we market to. We can formulate this by introducing a binary variable and using big M as below:

Define Binary Decision Variables

$$Invest\_In\_Print \in \{0, 1\}$$

Define Big M Constraints

$$Investment\_Influencer \leq \mathbf{M} \times Investment\_In\_Print$$

$$Investment\_Print \geq 100 \times Investment\_In\_Print$$

Optimize Buys, Given If We Do Influencer We Must Do Print						
Budget	**Targeted $**	**Email $**	**Print $**	**Influencer $**	**Views**	**Buys**

$1000	$400	$0	$100	$500	18,324	5,650
$1500	$800	$0	$100	$600 (max)	23,824	8,030
$2000	$950	$300	$150	$600 (max)	34,301	9,980

Balanced Portfolio Constraint: For this set of constraints we want to ensure there is no more than a $200 difference in spend between any two types of marketing. These constraints could be useful if executives are interested in ensuring their portfolio is well-balanced and would like to avoid relying on any specific marketing type. The additional variables and constraints needed to do this are below. Moreover, with some adjustments, a similar variable and constraint structure could be used to ensure the difference in buys or views from two investment types is not too substantial.

Define New Constrained Variables

$$0 \leq Targeted\_Email \leq 200$$
$$0 \leq Targeted\_Print \leq 200$$
$$0 \leq Targeted\_Influencers \leq 200$$
$$0 \leq Email\_Print \leq 200$$
$$0 \leq Email\_Influencers \leq 200$$
$$0 \leq Print\_Influencers \leq 200$$

Define New Constraints for the Difference between all Combinations

$$Targeted\_Email \geq Investment\_Targeted - Investment\_Email$$
$$Targeted\_Email \geq Investment\_Email - Investment\_Targeted$$
$$Targeted\_Print \geq Investment\_Targeted - Investment\_Print$$
$$Targeted\_Print \geq Investment\_Print - Investment\_Targeted$$
$$\vdots$$

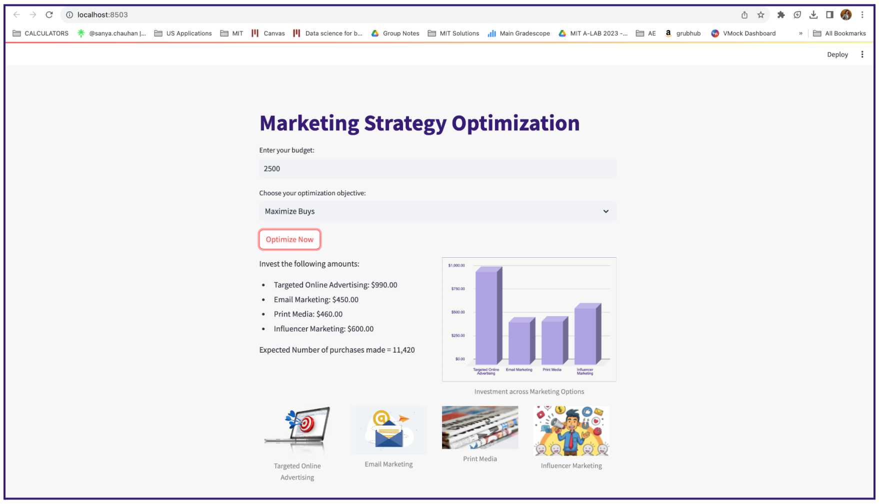
Optimize Views, Given There Must Be No More Than A $200 Difference In Spend Between Any 2 Marketing Types						
Budget	**Targeted $**	**Email $**	**Print $**	**Influencer $**	**Views**	**Buys**
$1000	$151	$300	$351	$198	26,034	4,394
$1500	$399	$300	$495	$306	29,650	5,317
$2000	$500	$300	$495	$500	30,543	8,440

The Dual: The dual interpretation of our project is also a useful framework to consider as we begin adding constraints to our problem. Below we take the fixed allocation constraint example and extract the dual values from our model.

Dual of Optimize Views, Given You Must Spend $400 On Target Advertising			
Budget	**Constraint**	**Dual Value**	**Interpretation**
$1000	Budget Constraint	-19	If we had another dollar to spend on marketing we could get 19 more views
$1000	Minimum Targeted Investment	8	If we lowered our minimum investment amount on targeted advertising by a dollar we could get 8 more views
$1500	Budget Constraint	-12	If we had another dollar to spend on marketing we could get 12 more views
$1500	Minimum Targeted Investment	0	This constraint is not active and does not affect our optimal value
$2000	Budget Constraint	-12	If we had another dollar to spend on marketing we could get 12 more views
$2000	Minimum Targeted Investment	0	This constraint is not active and does not affect our optimal value

Graphic User Interface

To have a user-friendly interface, we created this Streamlit app which would allow the users to input their budget, choose an objective, and get the optimal investment amounts according to the model, as well as expected buys/views (based on the chosen objective). This is a demo for a basic model only with one budgetary constraint. Additional constraints can be added based on the user's needs. [Demo Link]



Extensions

Below we discuss possible future augmentations to this project. If given more data, we could add greater granularity and optimization power to our model.

- ***Time:*** If a data set with time series data was generated we could create a time index that our model loops over to find sums. We could then set constraints that enforce specific requirements to any period or enforce relationships between periods. This model could output time period-specific decisions while optimizing the portfolio as a whole.
- ***Regions:*** A similar principle could be applied to a regional model where an index is generated to represent the different regions and then iterated and summed over to generate region-level decisions that capture region-level trends, while optimizing the portfolio holistically.
- ***Multi-Objective/Weighted Objective Formulation:*** Finally, we can set up a multi-objective function where both the views and the buys are maximized based on a chosen weight. This setup can then be used to create a Pareto frontier to influence and guide decision-makers. *The detailed formulation can be found in the appendix(3).*

Impact and Conclusion

A Deloitte 2023 CMO Survey said that marketing makes up 13.6% of a company's annual expenditure. In the age of data-driven strategy, our modeling framework can help executives and decision-makers efficiently allocate marketing capital given a company's constraints and goals. Our model achieved up to a 14% increase in views for the same budget, and up to a 23% increase in buys, again given the same budget, when compared to the baseline. We also explore how a user could customize these models to best fit their needs using constraints and extensions, such as a time or a region index or a multi/weighted objective function. In conclusion, given the correct data, optimization is a useful framework that can be effectively applied to create business efficiencies in the marketing field.

Appendix

1. Optimization Problem to Maximize Views - Formulation

Decision Variables:

$x_1 =$ investment made in targeted online advertisement,
$x_2 =$ investment made in email marketing,
$x_3 =$ investment made in print media,
$x_4 =$ investment made in influencers marketing.

$$x_1, x_2, x_3, x_4 \geq 0$$

Additional Variables for Piecewise Linear Functions:

$x_{1i} \geq 0$ for $i = 1, 2, 3$ be the segments in Targeted Online Advertising
$x_{2i} \geq 0$ for $i = 1, 2, 3$ be the segments in Email Marketing
$x_{3i} \geq 0$ for $i = 1, 2, 3$ be the segments in Print Media
$x_{4i} \geq 0$ for $i = 1, 2, 3$ be the segments in Influencer Marketing

Objective Function:

$$\text{Maximize} \quad \sum_{i=1}^{3}(s_{1i} \cdot x_{1i}) + \sum_{i=1}^{3}(s_{2i} \cdot x_{2i}) + \sum_{i=1}^{3}(s_{3i} \cdot x_{3i}) + \sum_{i=1}^{3}(s_{4i} \cdot x_{4i})$$

$$Where\ sji\ for\ i = 1,2,3\ and\ j\ =\ 1,2,3,4\ is\ the\ slope\ for\ that\ corresponding\ technique\ for\ view\ v/s\ investment$$

Constraints:

$$x_1 + x_2 + x_3 + x_4 \leq \text{Budget}$$

$$x_1 = \sum_{i=1}^{3} x_{1i}$$

$$\sum_{i=1}^{3} x_{1i} \leq \text{breakpoint}_{\text{targeted, end}}$$

$$x_{1i} \leq \text{breakpoint}_{\text{targeted}, i+1} - \text{breakpoint}_{\text{targeted}, i} \quad \text{for } i = 1, 2, 3$$

$$x_2 = \sum_{i=1}^{3} x_{2i}$$

$$\sum_{i=1}^{3} x_{2i} \leq \text{breakpoint}_{\text{email, end}}$$

$$x_{2i} \leq \text{breakpoint}_{\text{email}, i+1} - \text{breakpoint}_{\text{email}, i} \quad \text{for } i = 1, 2, 3$$

$$x_3 = \sum_{i=1}^{3} x_{3i}$$

$$\sum_{i=1}^{3} x_{3i} \leq \text{breakpoint}_{\text{print, end}}$$

$$x_{3i} \leq \text{breakpoint}_{\text{print}, i+1} - \text{breakpoint}_{\text{print}, i} \quad \text{for } i = 1, 2, 3$$

$$x_4 = \sum_{i=1}^{3} x_{4i}$$

$$\sum_{i=1}^{3} x_{4i} \leq \text{breakpoint}_{\text{influencers, end}}$$

$$x_{4i} \leq \text{breakpoint}_{\text{influencers}, i+1} - \text{breakpoint}_{\text{influencers}, i} \quad \text{for } i = 1, 2, 3$$

2. Multi-Objective Optimization Potential Formulation with weights

$$0.7 \times \left(\sum_{i=1}^{3} \text{slopes\_buys\_targeted}_i \times \text{targeted\_segments}_i + \sum_{i=1}^{3} \text{slopes\_buys\_email}_i \times \text{email\_segments}_i + \sum_{i=1}^{3} \text{slopes\_buys\_print}_i \times \text{print\_segments}_i + \sum_{i=1}^{3} \text{slopes\_buys\_influencers}_i \times \text{influencers\_segments}_i \right) +$$
$$0.3 \times \left(\sum_{i=1}^{3} \text{slopes\_views\_targeted}_i \times \text{targeted\_segments}_i + \sum_{i=1}^{3} \text{slopes\_views\_email}_i \times \text{email\_segments}_i + \sum_{i=1}^{3} \text{slopes\_views\_print}_i \times \text{print\_segments}_i + \sum_{i=1}^{3} \text{slopes\_views\_influencers}_i \times \text{influencers\_segment}_i \right)$$