We will discuss

- P vs NP

We have seen 3 problems so far. They are all optimization problems (maximize/minimize a value)

We will now discuss decision problems. The answer to these problems is either Yes/No.

Why switch to decision variants? Easy to work with. Decision and Optimization variants of a problem are equivalent ~~in one way~~.

if optimization problem version is tractable/intractable then decision version is tractable/intractable

AND VICE VERSA.

$$\text{isTractable}(\text{O.V.}(P)) \iff \text{isTractable}(\text{D.V.}(P))$$

eg: Independent S. for $G(E,V)$

O.V. $\Rightarrow$ Find S such that k is maximum where $k = |S|$

D.V. $\Rightarrow$ Is there an S, where $|S| \geq k'$?

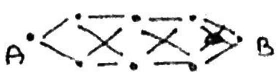
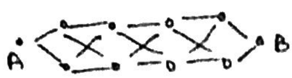
How to show Intractability??!

1. GATHER EVIDENCE? $\Rightarrow$ lots of people worked on a problem and couldn't find a tractable algorithm

2. MATHEMATICAL PROOF? $\Rightarrow$ Intricate, rigorous process showing any algorithm for this problem would need exponential time at least

1. is easier.
2. is convincing.

Many solutions, the cause of intractability? NO!

	# ways from A to B (W)	# vertices (n)
	2	4
	4	6
	8	8
	16	10

$W = 2^{\left(\frac{n-2}{2}\right)}$

But we already know algorithms that are tractab that can find shortest paths.

exponentially many solns. $\not\Rightarrow$ intractability

Since, mathematical proofs are hard, Lets gather some evidence first.

We have already seen that V.C., I.S. and clique are reducible from one to other.

idea: a model of computer, that supports a new instruction called <u>if-better</u>.

- works like if else
- runs in polynomial time
- figueres if its "better" to execute the first block or the second.

```
if better (     ) → what's better for my soln.
{

}
else
{

}
```

Vertex cover (V.C.) with if better

i.p: G with n vertices, k

o.p: Yes if G has v.c of size atmost k

No otherswise.

```
for each vertex in G:
    if-better:
        assign 1 to v
    else:
        assing 0 to v

if assignment is valid:
    if size of assignment ≤ k:
        return YES

return NO
```

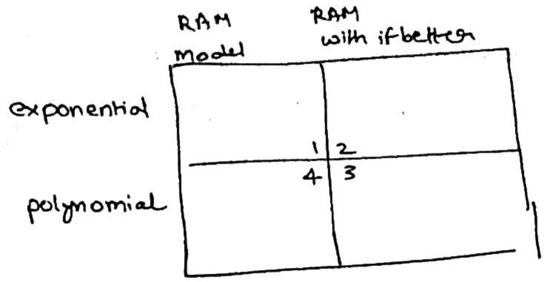
---

runs in polynomial time!!! (not in RAM model though) :(

will work for clique & independent set.

<u>if better</u> can determine what's better correctly always.

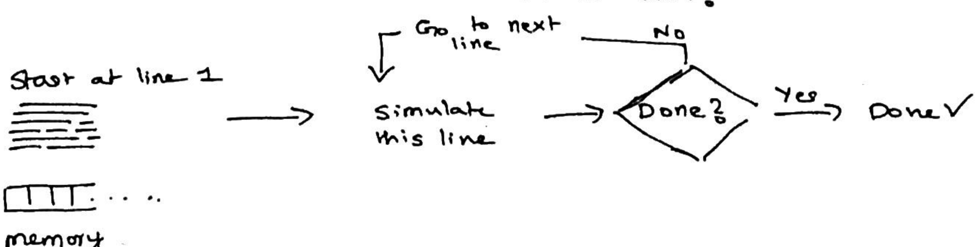
Complexity classes (time)



the 3 problems can be placed in 1,2,3. We don't know if we can put them in 4 yet.

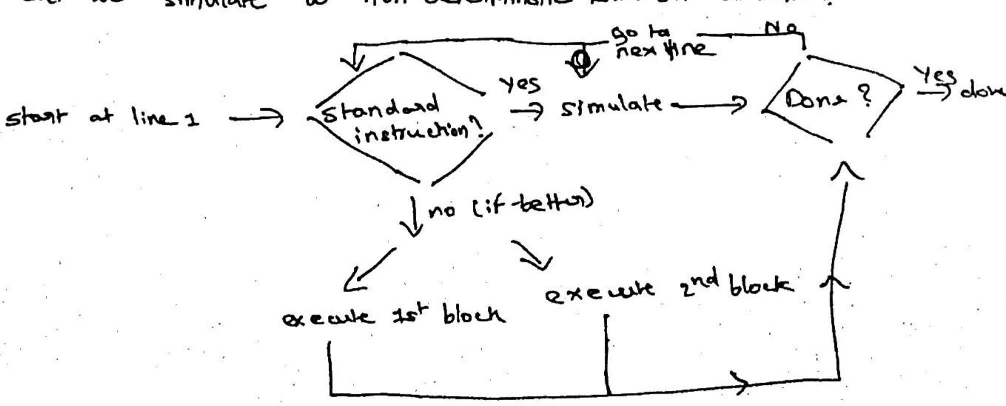
RAM with if-better ⇒
- ↳ extremely powerful
- ↳ mysterious
- ↳ non deterministic (cannot determine the flow of the program)
- ↳ consistent o/p for all inputs

Q1. Can we simulate a RAM on another RAM?



- → takes longer (polynomially)
- → can't give ~~wor~~ wrong results (assume correctness of simulation)

Q2. Can we simulate a non deterministic RAM on a RAM?



if-better is used n times $\Rightarrow 2^n$ simulations

$\Rightarrow$ RAM and non deterministic RAM are equally powerful in that they can compute same set of functions.

But RAM ~~can~~ takes ~~much~~ at least as much time as non deterministic RAM

	RAM	ND-RAM
polynomial time	P	NP
exponential time		

P is a subset of NP

note:

$\rightarrow$ V.c. clique & i.s are in NP

$\rightarrow$ $P \subset NP$

$\rightarrow$ $P = NP$? (who knows)

Let's say we have a problem x in NP.

2 possibilities

$\rightarrow x \in P$

$\rightarrow x \notin P$

if vertex cover is in P, then

i.s. & clique are also in P

(polynomial time reducible)

AND VICE VERSA