We will discuss

- Preprocessing
- Measuring hardness

Preprocessing $\Rightarrow$ cleaning the i/p

$\boxed{I/P} \longrightarrow$ preprocessing $\longrightarrow$ Search tree (hopefully, faster!)

(polynomial time, ofc)

Consider SAT input:

$$(x_1 \vee x_3 \vee x_5) \wedge (\bar{x}_1 \vee x_2 \vee x_4) \wedge (\bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_5) \wedge$$
$$(x_5 \vee x_6) \wedge (x_1) \wedge (x_3 \vee \bar{x}_2 \vee \bar{x}_6) \wedge (\bar{x}_2 \vee \bar{x}_5 \vee x_6) \wedge$$
$$(\bar{x}_5 \vee \bar{x}_6) \wedge (\bar{x}_1 \vee \bar{x}_3)$$

$\Rightarrow$ x_1 must be true (only variable in a clause)

$\Rightarrow$ ~~x_3 must be false (last clause)~~

$\Rightarrow$ x_4 can be true (appears only once)

#2: $$(x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee \bar{x}_3 \vee \bar{x}_4) \wedge (\bar{x}_1 \vee x_3 \vee \bar{x}_5) \wedge$$
$$(\bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_5) \wedge (\bar{x}_1 \vee \bar{x}_4) \wedge (x_1 \vee x_3 \vee x_5)$$

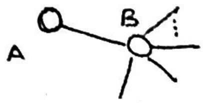
$\Rightarrow$ x_4 ~~must~~ can be false (as it appears as $\bar{x}_4$ always)

<u>Requirements for preprocessing:</u>

- polynomial time
- doesnot affect the soln.

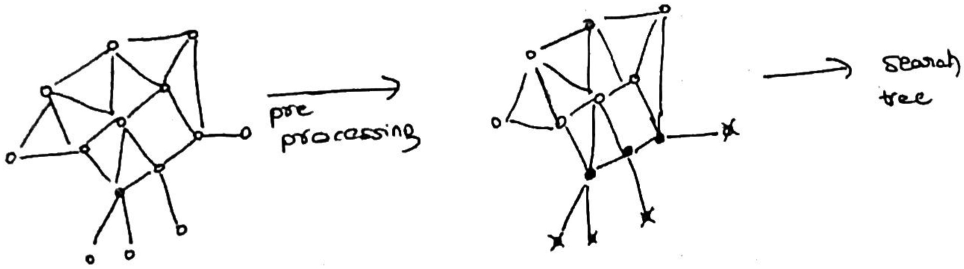
Preprocessing for vertex cover:

Vertex with one neighbor (A)



you can select B

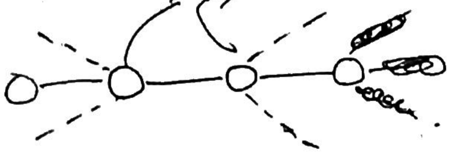
⇒ B covers whatever A does and more.

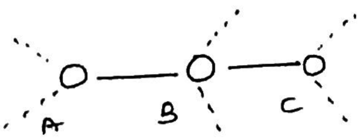


We can improve the search tree

will have more than one neighbor

idea:





2^3 assignments

but only 5 of them make sense

AB & BC have to be covered

~~So either B~~ or

So atleast 2 of 3 should be marked as true

or B should be marked true

branching factor = 5

height ⇒ ?

each level, we ~~add~~ $\cancel{\frac{1}{5}(1+2+2+2+3)}$ mark exactly 3 vertices as true or false

$$\left.\begin{matrix} 0 \\ 3 \\ 6 \\ \vdots \\ n \end{matrix}\right\} n/3$$

Size of the tree $= 5^{n/3} = 1.71^n$ (better than 1.73^n)

Better & complex preprocessing rules can make the tree smaller

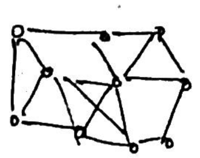
When to apply pre-processing:

- Before starting the search
- Regularly during search (maybe not at each level, too costly)

Measuring hardness

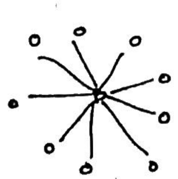
Assumption: Larger instances are harder to solve

V.C.:



n=10

(A)



n=10

(B)

(A) is harder to solve than (B)

↳ preprocessing is enough

It's ~~ex~~ unfair to call them equally hard.

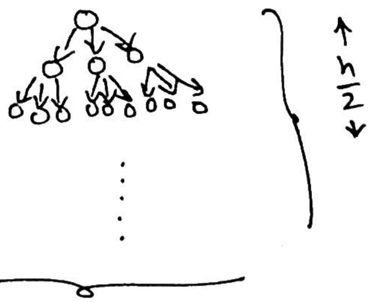
Hardness $O(f(n, k))$

↙ Size of input, n

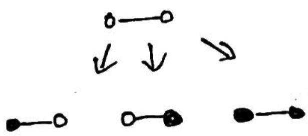
↘ structure of input, k
(↳ can we measure this?

k depends on
→ size of the soln.
→ distance from triviality

<u>Measuring</u> the <u>hardness</u> of <u>v.c.</u>



$3^{n/2} \approx 1.734^n$



on avg. we add ~~1.5~~ $\frac{4}{3}$ vertices at each level

Consider decision variant: does a graph G contain a v.c. of ~~a~~ size at most k?

max. height of search tree $\Rightarrow k$ (add one vertex at each level)

#leaves $= 3^k$

$\frac{4}{3} \cdot h = k$

$h = \frac{3k}{4}$

if done more rigorously,

~~branching factor~~

#leaves $\approx 3^{\frac{3k}{4}} \approx 2.28^k$

D.V. can be solved in $O(2.28^k n^3)$ time

$\Rightarrow$ for any fixed k, vertex cover can be solved in polynomial time

Optimization variant $\Rightarrow$?

$k=1$ $\quad$ $k=2$ $\quad$ $k=3$ $\quad \dots \quad$ k'

$O(2 \cdot 4^1 \cdot n^3)$ $\quad$ $O(2 \cdot 4^2 \cdot n^3$ $\quad\quad$ $O(2 \cdot 4^{k'} \cdot n^3)$

$$\Rightarrow O\left(n^3 \left(2 \cdot 4^1 + 2 \cdot 4^2 + \dots 2 \cdot 4^{k'}\right)\right)$$

$$\Rightarrow O\left(\frac{2 \cdot 4^{k'-1} - 1}{2 \cdot 4 - 1} \cdot n^3\right) \Rightarrow O(2 \cdot 4^{k'} \cdot n^3)$$

$\Downarrow$

Smaller vertex covers are easy to find!!!

This is the idea of fixed parameter tractability.

As long as the parameter that measures the hardness of the problem is fixed, the problem is tractable.

Is every problem F.P.T? No, not every problem.

It depends on how you choose k.

We have discussed size of the soln. What is distance from triviality?

- → lesser choices
- → preprocessing solves many parts of the instance

2 SAT $\in$ P

3 SAT $\in$ NP complete.

i/p1: $(x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_5 \vee x_6) \wedge (\bar{x}_2 \vee x_5 \vee x_7) \wedge (x_3 \vee \bar{x}_6 \vee \bar{x}_7)$

i/p2: $(x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_5 \vee x_6) \wedge (x_2 \vee x_3 \vee _7) \wedge (x_5 \vee \bar{x}_6 \vee \bar{x}_7)$

i/p2
↳ x_1 can be true

i/p1
↳ x_3 can be false
↳ x_1 can be true
↳ x_2 can be false

i/p 1 is much trivial than i/p 2

STATE of THE ART

V.C	I.S. & clique	travelling salesman
≥10,000 vertices	≥1000 vertices	≥ 85000 vertices

Annual competitions are held for SAT.

4 hours & study 7