CS6013: Advanced Data Structures and Algorithms

Programming Assignment III (out of 10 marks)
(Start Date: 11 November 2021)
(Submission Deadline: 11:59 pm, Sunday, 21 November 2021)

0.1 Miller-Rabin_Test(n)

Input: An odd integer $n \geq 3$.
Output: If n is prime, the algo always returns “prime”. If n is composite, the algo with probability at least 1/2 returns “composite”.

Algo:
STEP 0: Check if $n = a^b$ for integers $a, b \geq 2$. If so, return “composite”.
STEP 1: Select $a \in \{1, 2, \ldots, n-1\}$ uniformly at random. Compute $a^{n-1} \bmod n$. If this is not 1, then return “composite”.
STEP 2: Let $n - 1 = 2^k t$, where t is odd. Compute $a^t \bmod n, a^{2t} \bmod n, a^{4t} \bmod n, a^{8t} \bmod n, \ldots,$ until a 1 is seen. If the number before 1 is not -1, then return “composite”; else return “prime”.

Implement the function Miller-Rabin_test(n) described above. Define a function higher_power(a, b) that computes and returns a^b in *polylog*(n) time, where $0 \leq a, b \leq n$ [Do not use any built-in function to compute a^b or $a^b \bmod n$]. Invoke this function *polylog*(n) times to check whether $n = a^b$ in STEP 0. You may use a standard library function to find a random number from the set $\{1, 2, \ldots, n\}$ in STEP 1. Define another function modular_higher_power(a, b, n) that computes and returns $a^b \bmod n$ in *polylog*(n) time, $0 \leq a, b \leq n$. Use this function to compute $a^{n-1} \bmod n$ in STEP 1. Define a function two_factorize(x) that computes and returns in *polylog*(n) time the non-negative integer y such that $x = 2^y z$, where z is odd and $x \leq n$. Invoke two_factorize($n - 1$) to find k in STEP 2. Compute $a^t \bmod n, a^{2t} \bmod n, a^{4t} \bmod n, \ldots$ by repeatedly invoking the modular_higher_power() function with appropriate parameters.

In the main function, read positive integers n and r. Invoke the function Miller-Rabin_Test(n) r times in a loop that runs from 1 to r. If all the r invocations of Miller-Rabin_Test(n) return “prime”, then print “n is a prime number”. Else, print “n is a composite number”. We know that if n is actually prime, this algorithm prints “n is a prime number” with probability 1; if n is actually composite, this algorithm wrongly prints “n is a prime number” with probability at most $\frac{1}{2^r}$.

Sample Output:

```
n = 12000
r = 25
12000 is a composite number.
```

0.2 Program Related Instructions

1. You can write your program in one of C, C++, Java, or Python.

0.3 Submission Guidelines

1. Your submission will be one zip file named <roll-number>.zip , where you replace roll-number by your roll number (e.g. cs20mtech11003.zip), all in small letters. The compressed file should contain the below mentioned files:

(a) Programming files (please do not submit python notebooks or IDE files). **The entire source code has to be in one file named main_prog.c (or main_prog.cpp, or ...).**

(b) **No need to submit a report**. However, if you wish you may submit a text/doc file giving a detailed description of your program. No marks for this.

(c) Upload your zip file in Google Classroom at Classwork→Week 13→Assignment 3. No delays permitted.

2. Failure to comply with instructions (file-naming, upload, input/output specifications) will result in your submission not being evaluated (and you being awarded 0 for the assignment).

3. **Plagiarism policy**: If we find a case of plagiarism in your assignment (i.e. copying of code, either from the internet, or from each other, in part or whole), you will be awarded a zero and will lead to a FR grade for the course in line with the department Plagiarism Policy (https://cse.iith.ac.in/academics/plagiarism-policy.html). Note that we will not distinguish between a person who has copied, or has allowed his/her code to be copied; both will be equally awarded a zero for the submission.

0.4 Evaluation Scheme

Your assignment will be awarded marks based on the following aspects:

- Code clarity (includes comments, indentation, naming of variables and functions, etc.): 1 mark
- Perfect output: 2 mark
- Logic in the code of functions higher_power(), modular_higher_power(), two_factorize(): 1 + 1 + 1 = 3 marks.
- Logic in the code of the function Miller-Rabin_Test(): 4 marks.