**به نام خدا**

# نظریه زبان‌ها و ماشین‌ها

تمرین اول

## استاد درس: دکتر احمدرضا منتظرالقائم

**دستیاران تدریس:**

فائزه صالحی

نسرین اسحاقیان

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فاطمه سادات شجاعی آرانی

محمدحسین ملکی

-١

L = {ab, aa, baa}

- ab aa baa ab aa ☑
- aa aa baa aa ☑
- baa aa ab aa aa b ☒
- baa aa ab aa ☑

-٢

الف) G = ({S, A}, ∑, S, P)

S → AaA

A → bA | λ

ب) G = ({S, A}, ∑, S, P)

S → AaA

A → aA | bA | λ

ج) G = ({S, A}, ∑, S, P)

S → A | AaA | AaAaA | AaAaAaA

A → bA | λ

د) G = ({S, A}, ∑, S, P)

S → AaAaAaA

A → aA | bA | λ

-٣

$L = \{(ab)^n : n \geq 0\}$

-٤

الف) $L_1 = \{a^n b^m : n \geq 0, m > n\}$

$G = (\{S, A\}, \{a, b\}, S, P_1)$

$S \rightarrow Ab$

$A \rightarrow aAb \mid Ab \mid \lambda$

ب) $L_2 = \{a^n b^{2n} : n \geq 0\}$

$G = (\{S\}, \{a, b\}, S, P_2)$

$S \rightarrow aSbb \mid \lambda$

ج) $L_3 = \{a^{n+2} b^n : n \geq 1\}$

$G = (\{S, A\}, \{a, b\}, S, P_3)$

$S \rightarrow aaA$

$A \rightarrow aAb \mid \lambda$

د) $L_4 = \{a^n b^{n-3} : n \geq 3\}$

$n-3 = m$

$n = m+3$

$n \geq 3$

$m \geq 0$

$L_4 = \{a^{m+3} b^m : m \geq 0\}$

$G = (\{S, A\}, \{a, b\}, S, P_4)$

$S \rightarrow aaaA$

$A \rightarrow aAb \mid \lambda$

ه) $L_1L_2$

$L_5 = \{a^n b^m a^k b^{2k} : (n, k \geq 0) , m > n\}$

$G = (\{S, A, B, C\}, \{a, b\}, S, P_5)$

$S \rightarrow AB$

$A \rightarrow Cb$

$C \rightarrow aCb \mid Cb \mid \lambda$

$B \rightarrow aBbb \mid \lambda$

و) $L_1 \cup L_2$

$L_6 = \{a^n b^m \mid a^k b^{2k} : (n, k \geq 0), m > n\}$

$G = (\{S, A, B\}, \{a, b\}, S, P_6)$

$S \rightarrow Ab \mid B$

$A \rightarrow aAb \mid \lambda$

$B \rightarrow aBbb \mid \lambda$

ز) $L_1^3$

$L_7 = \{a^n b^m a^x b^y a^k b^j : (n, x, k \geq 0), (m, y, j > n, x, k)\}$

$G = (\{S, A\}, \{a, b\}, S, P_7)$

$S \rightarrow AbAbAb$

$A \rightarrow aAb \mid Ab \mid \lambda$

ح) $L_1^*$

$L_8 = \{a^n b^m : n \geq 0, m > n\}^*$

$G = (\{S, A\}, \{a, b\}, S, P_8)$

$S \rightarrow SAb \mid \lambda$

$A \rightarrow aAb \mid Ab \mid \lambda$

ط) $L_1 - \overline{L_4}$

$L_1 - \overline{L_4} = L_1 - (U - L_4) = L_1 + L_4 - U = \emptyset$

۵- $\sum = \{a\}$

الف) $L = \{w : |w| \bmod 3 = 0\}$

$G = (\{S\}, \sum, S, P)$

$S \rightarrow aaaS \mid \lambda$

ب) $L = \{w : |w| \bmod 3 > 0\}$

$G = (\{S, A, B\}, \sum, S, P)$

$S \rightarrow A \mid B$

$A \rightarrow aaaA \mid a$

$B \rightarrow aaaB \mid aa$

۶-

$S \rightarrow aSb \mid ab \mid \lambda$

$L = \{a^n b^n : n \geq 0\}$

$S \rightarrow aAb \mid ab$

$A \rightarrow aAb \mid \lambda$

$L = \{a^n b^n : n > 0\}$

دو گرامر بالا، زبان‌های متفاوت تولید می‌کنند در نتیجه با یک‌دیگر برابر نیستند.

۷- $G = (\{A, B, C, D, E, S\}, \{a\}, S, P)$

$S \rightarrow ABaC$

$Ba \rightarrow aaB$

$BC \rightarrow DC \mid E$

$aD \rightarrow Da$

$AD \rightarrow AB$

$aE \rightarrow Ea$

$AE \rightarrow \lambda$

$S \Rightarrow A\underline{Ba}C \Rightarrow Aaa\underline{BC} \Rightarrow Aa\underline{aE} \Rightarrow A\underline{aE}a \Rightarrow \underline{AE}aa \Rightarrow aa$

$S \Rightarrow A\underline{Ba}C \Rightarrow Aaa\underline{BC} \Rightarrow Aa\underline{aD}C \Rightarrow A\underline{aD}aC \Rightarrow \underline{AD}aaC \Rightarrow A\underline{Ba}aC \Rightarrow Aaa\underline{Ba}C \Rightarrow Aaaaa\underline{BC} \Rightarrow Aaaa\underline{aE} \Rightarrow Aaa\underline{aE}a \Rightarrow Aa\underline{aE}aa \Rightarrow A\underline{aE}aaa \Rightarrow \underline{AE}aaaa \Rightarrow aaaa$

S => A<u>Ba</u>C => Aaa<u>BC</u> => Aa<u>aD</u>C => A<u>aD</u>aC => <u>AD</u>aaC => A<u>Ba</u>aC => Aaa<u>Ba</u>C => Aaaaa<u>BC</u> => Aaaa<u>aD</u>C => Aaa<u>aD</u>aC => Aa<u>aD</u>aaC => A<u>aD</u>aaaC => <u>AD</u>aaaaC => A<u>Ba</u>aaaC => Aaa<u>Ba</u>aaC => Aaaaa<u>Ba</u>aC => Aaaaaaa<u>Ba</u>C => Aaaaaaaaa<u>BC</u> => Aaaaaaaaa<u>E</u> => Aaaaaaaa<u>Ea</u> => Aaaaaaa<u>Eaa</u> => Aaaaaa<u>Eaaa</u> => Aaaaa<u>Eaaaa</u> => Aaaa<u>Eaaaaa</u> => Aaa<u>Eaaaaaa</u> => A<u>aE</u>aaaaaaa => <u>AE</u>aaaaaaaa => aaaaaaaa

L = $\{(aa)^{2\wedge n} : n \geq 0\}$

n = 0 => aa

n = 1 => $(aa)^2$ = aaaa

n = 2 => $(aa)^4$ = aaaaaaaa

n = 3 => $(aa)^8$ = aaaaaaaaaaaaaaaa