CHORD: Ptolemy's table of chords

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Overview

CHORD Application[1] for Android (Schrausser, 2023): Famous table of chord lengths according to Ptolemy's *Almagest* (1515, fol. 7r ff.) converted into decimal values and calculated in comparison using the sine function, see Halma (1813, p. 38 ff.), Heiberg (1898, p. 48 ff.) or Toomer (1984, p. 57 ff., 1998, res.).

Chord lengths l_0 are calculated according to *Ptolemy's theorem* (figure 1) within the relation between four sides and two diagonals of a cyclic quadrilateral where

$$AC \cdot BD = AB \cdot CD + BC \cdot AD.$$

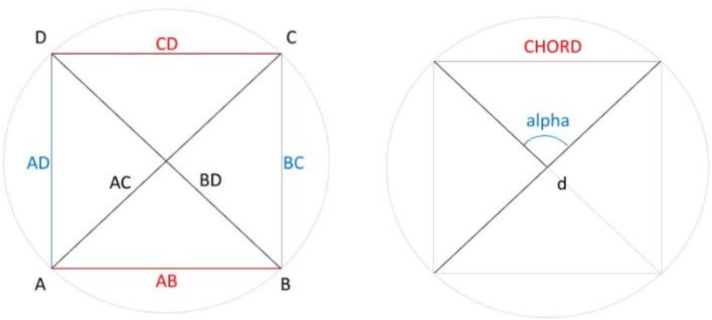

Figure 1: Cyclic quadrilateral with chord length representation.

Chord lengths l_0 (figure 1) are expressed in fractional parts of sexagesimal numerals $x\ y\ z$. Decimal values l_1 are calculated as

$$l_1 = x + {}^{y}/60 + {}^{z}/60^2.$$

Sixtieths is the average interpolation number to be added to length l_0 or l_1 each time angle increases by one minute of arc, that is $n = 30$ times per half angle degree α.

Lengths l_2 to given arcus α and diameter d are calculated using the sine function where

$$l_2 = d \cdot \sin({}^{\alpha \cdot \pi}/_{360}).$$

This is equivalent in terms of content to distance s or radius r determination via angular diameter V with

$$r = s \cdot \tan({}^{V}/_{2}).$$

[1] https://github.com/Schrausser/Ptolemy-s-table-of-chords

In the absence of trigonometric sine functions, however, no *calculation* was made with distance parameters *s*, but tabularized values from previous model calculations with given $d = 120$ by means of the *Pythagorean theorem*

$$a^2 + b^2 = c^2$$

were used and interpolated to the corresponding angle values of expansion:

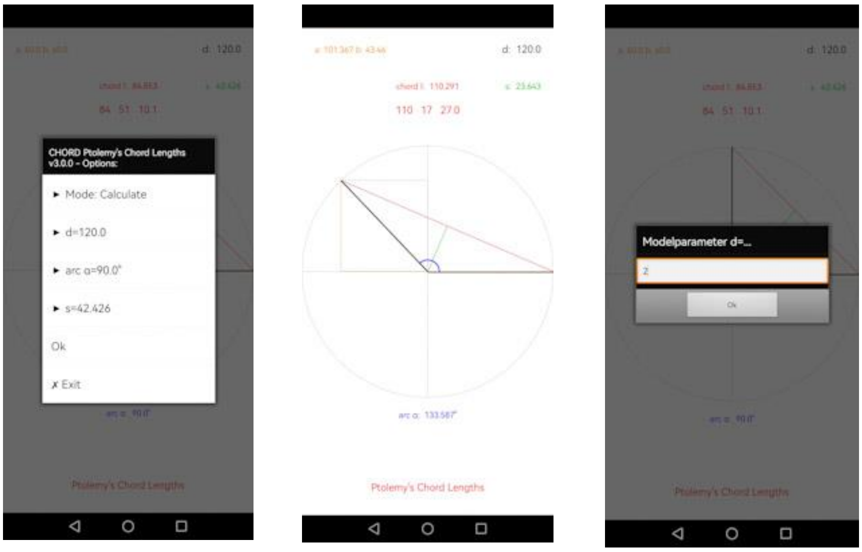

Figure 2: Screenshots from CHORD Application.

Chord parameters $l_{(120)}$ can then be adapted to empirical $l_{(d)}$ proportions by transforming the model parameter with

$$l_{(d)} = l_{(120)} \cdot {}^{d}/_{120}.$$

Chord length values $l_{(e)}$ corresponding to *empirical* distances *s* can be expressed by multiplying $l_{(d)}$ with a ratio factor δ as $l_{(e)} = l_{(d)} \cdot \delta$ to given angle α, where according to *Pythagoras*

$$\delta = s \cdot [({}^{d}/_{2})^2 - ({}^{l}/_{2})^2]^{-1/2}.$$

Differences *diff* show the difference between (1) *sixtieth* and arithmetical interpolation as well as the difference between (2) the calculation types of chord lengths l_1 and l_2, see *chords.md* or *chords.xlsx* tables.

Using this method along with methods for parallax determination, Ptolemy was able to determine e.g. Moon's distance ($d = 59$ Earth radii, *er*) and radius ($r = 0.29$ *er*, where *er* = 6378 *km*) quite accurate, see e.g. Goldstein (1967).

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