# CHORD: Ptolemy's table of chords calculator

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https://github.com/Schrausser/Ptolemy-s-table-of-chords

## Overview

CHORD Application for Android (Schrausser, 2023): Famous table of chord lengths according to Ptolemy's *Almagest* (e.g. 1515) converted into decimal values and calculated in comparison using the sine function, see e.g. Halma (1813) or Toomer (1984).

Chord lengths $l_0$ are calculated according to *Ptolemy's theorem* (figure 1) within the relation between four sides and two diagonals of a cyclic quadrilateral where

$$AC \cdot BD = AB \cdot CD + BC \cdot AD.$$

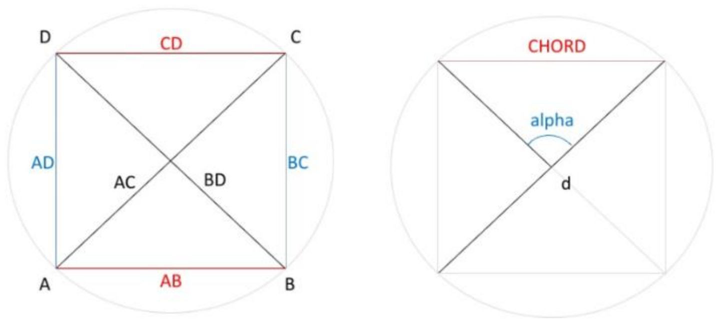


**Figure 1:** Cyclic quadrilateral with chord length representation.

Chord lengths $l_0$ (figure 1) are expressed in fractional parts of sexagesimal numerals $x\ y\ z$. Decimal values $l_1$ are calculated as

$$l_1 = x + y/60 + z/60^2.$$

*Sixtieths* is the average interpolation number to be added to length $l_0$ or $l_1$ each time angle increases by one minute of arc, that is $n$ = 30 times per half angle degree $\alpha$.

Lengths $l_2$ to given arcus $\alpha$ and diameter $d$ are calculated using the sine function where

$$l_2 = d \cdot \sin(\alpha \cdot \pi/360).$$

This is equivalent in terms of content to distance $s$ or radius $r$ determination via angular diameter $V$ with

$$r = s \cdot \tan(V/2).$$

In the absence of trigonometric sine functions, however, no *calculation* was made with distance parameters $s$, but tabularized values from previous model calculations with given $d = 120$ by means of the *Pythagorean theorem*

$$a^2 + b^2 = c^2$$

were used and interpolated to the corresponding angle values of expansion:

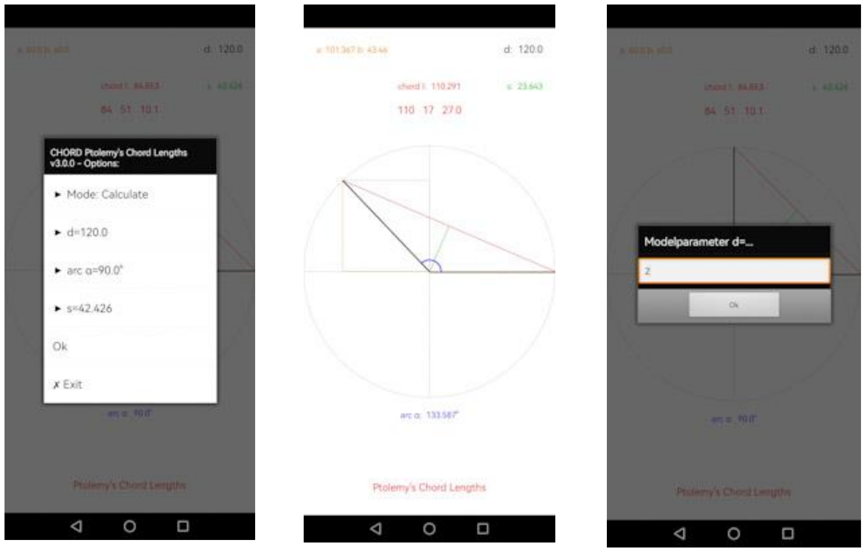


**Figure 2:** Screenshots from CHORD Application.

Chord parameters $l_{(120)}$ can then be adapted to empirical $l_{(d)}$ proportions by transforming the model parameter with

$$l_{(d)} = l_{(120)} \cdot d/120.$$

Chord length values $l_{(e)}$ corresponding to *empirical* distances $s$ can be expressed by multiplying $l_{(d)}$ with a ratio factor $\delta$ as $l_{(e)} = l_{(d)} \cdot \delta$ to given angle $\alpha$, where according to *Pythagoras*

$$\delta = s \cdot [(d/2)^2 - (l/2)^2]^{-1/2}.$$

Differences *diff* show the difference between (1) *sixtieth* and arithmetical interpolation as well as the difference between (2) the calculation types of chord lengths $l_1$ and $l_2$, see *chords.md* or *chords.xlsx* tables.

Using this method along with methods for parallax determination, Ptolemy was able to determine e.g. Moon's distance ($d = 59$ Earth radii, $er$) and radius ($r = 0.29$ $er$, where $er = 6378$ $km$) quite accurate, see e.g. Goldstein (1967).

## References

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