CHORD: Ptolemy's table of chords calculator

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https://github.com/Schrausser/Ptolemy-s-table-of-chords

Overview

Famous table of chord lengths according to Ptolemy's *Almagest* (e.g. 1515) converted into decimal values and calculated in comparison using the sine function, see e.g. Halma (1813) or Toomer (1984).

Chord lengths l_0 are calculated according to *Ptolemy's theorem* (figure 1) within the relation between four sides and two diagonals of a cyclic quadrilateral where

$$AC \cdot BD = AB \cdot CD + BC \cdot AD.$$

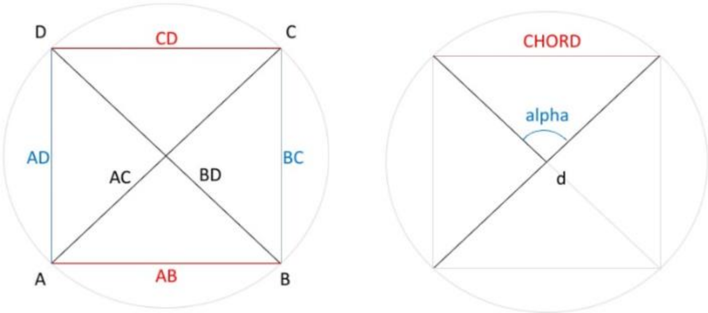

Figure 1: Cyclic quadrilateral with chord length representation.

Chord lengths l_0 (figure 1) are expressed in fractional parts of sexagesimal numerals x y z. Decimal values l_1 are calculated as

$$l_1 = x + y/60 + z/60^2.$$

Sixtieths is the average interpolation number to be added to length l_0 or l_1 each time angle increases by one minute of arc, that is n = 30 times per half angle degree α.

Lengths l_2 to given arcus α and diameter d are calculated using the sine function where

$$l_2 = d \cdot \sin(\alpha \cdot \pi/360).$$

This is equivalent in terms of content to distance s or radius r determination via angular expansion V with

$$r = s \cdot \tan(V/2).$$

In the absence of trigonometric sine functions, however, no *calculation* was made with distance parameters s, but tabularized values from previous model calculations with given $d = 120$ by means of the *Pythagorean theorem*

$$a^2 + b^2 = c^2$$

were used and interpolated to the corresponding angle values of expansion:

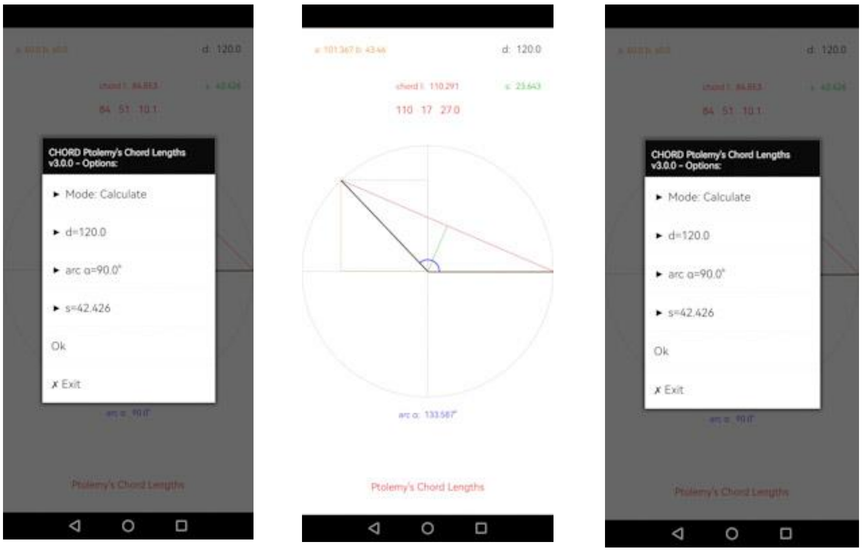

Figure 2: Screenshots from CHORD Application.

Chord parameters $l_{(120)}$ can then be adapted to empirical $l_{(d)}$ proportions by transforming the model parameter with

$$l_{(d)} = l_{(120)} \cdot d/120.$$

Chord length values $l_{(e)}$ corresponding to *empirical* distances s can be expressed by multiplying $l_{(d)}$ with a ratio factor δ as $l_{(e)} = l_{(d)} \cdot \delta$ to given angle α, where according to *Pythagoras*

$$\delta = s \cdot [(^d/_2)^2 - (^l/_2)^2]^{-1/2}.$$

Differences *diff* show the difference between (1) *sixtieth* and arithmetical interpolation as well as the difference between (2) the calculation types of chord lengths l_1 and l_2, see *chords.md* or *chords.xlsx* tables.

Using this method along with methods for parallax determination, Ptolemy was able to determine e.g. Moon's distance ($d = 59$ Earth radii, *er*) and radius ($r = 0.29$ *er*, where *er* = 6378 *km*) quite accurate, see e.g. Goldstein (1967).

References

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