

# CHORD: Ptolemy's table of chords calculator

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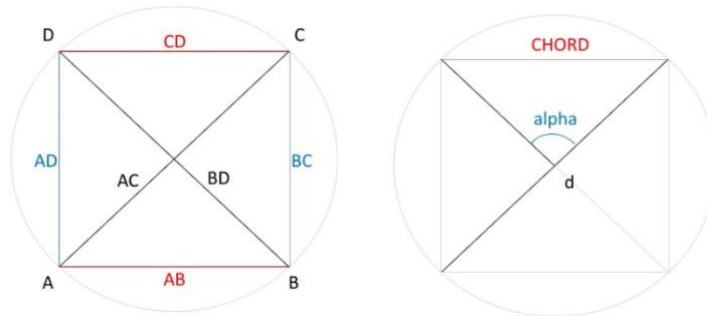
<https://github.com/Schrausser/Ptolemy-s-table-of-chords>

## Overview

CHORD Application for Android (Schrausser, 2023): Famous table of chord lengths according to Ptolemy's *Almagest* (e.g. 1515) converted into decimal values and calculated in comparison using the sine function, see e.g. Halma (1813) or Toomer (1984).

Chord lengths  $l_0$  are calculated according to *Ptolemy's theorem* (figure 1) within the relation between four sides and two diagonals of a cyclic quadrilateral where

$$AC \cdot BD = AB \cdot CD + BC \cdot AD.$$



**Figure 1:** Cyclic quadrilateral with chord length representation.

Chord lengths  $l_0$  (figure 1) are expressed in fractional parts of sexagesimal numerals  $x \ y \ z$ . Decimal values  $l_1$  are calculated as

$$l_1 = x + y/60 + z/60^2.$$

*Sixtieths* is the average interpolation number to be added to length  $l_0$  or  $l_1$  each time angle increases by one minute of arc, that is  $n = 30$  times per half angle degree  $\alpha$ .

Lengths  $l_2$  to given arcus  $\alpha$  and diameter  $d$  are calculated using the sine function where

$$l_2 = d \cdot \sin(\alpha \cdot \pi / 360).$$

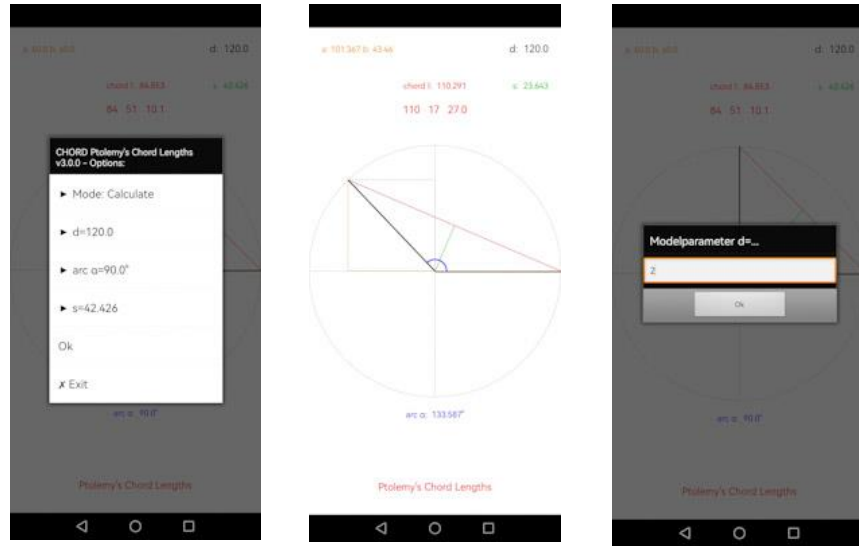
This is equivalent in terms of content to distance  $s$  or radius  $r$  determination via angular expansion  $V$  with

$$r = s \cdot \tan(V/2).$$

In the absence of trigonometric sine functions, however, no *calculation* was made with distance parameters  $s$ , but tabularized values from previous model calculations with given  $d = 120$  by means of the *Pythagorean theorem*

$$a^2 + b^2 = c^2$$

were used and interpolated to the corresponding angle values of expansion:



**Figure 2:** Screenshots from CHORD Application.

Chord parameters  $l_{(120)}$  can then be adapted to empirical  $l_{(d)}$  proportions by transforming the model parameter with

$$l_{(d)} = l_{(120)} \cdot d/120.$$

Chord length values  $l_{(e)}$  corresponding to *empirical* distances  $s$  can be expressed by multiplying  $l_{(d)}$  with a ratio factor  $\delta$  as  $l_{(e)} = l_{(d)} \cdot \delta$  to given angle  $\alpha$ , where according to *Pythagoras*

$$\delta = s \cdot [(d/2)^2 - (l/2)^2]^{-1/2}.$$

Differences *diff* show the difference between (1) *sixtieth* and arithmetical interpolation as well as the difference between (2) the calculation types of chord lengths  $l_1$  and  $l_2$ , see *chords.md* or *chords.xlsx* tables.

Using this method along with methods for parallax determination, Ptolemy was able to determine e.g. Moon's distance ( $d = 59$  Earth radii,  $er$ ) and radius ( $r = 0.29$   $er$ , where  $er = 6378$  km) quite accurate, see e.g. Goldstein (1967).

## References

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