Aiolos – A multi-purpose RHD code

M. Schulik & R. Booth

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Why another code?

Why hydrodynamic: Complex, multi-species physics with radiation might give static methods challenges to find solutions

Why finite volumes: Supersonic phenomena can lead to trouble with finite differences

Why well-balancing: Deep gravity wells need to be stabilized, intermediate-mass planets (MNeptunes / SEarths) have their atmospheres accelerated deep in their potentials, need to get this region right

M. Schulik & R. Booth

Physics included

Equations to be solved:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho u) = S_{\text{chem}}$$

$$\frac{\partial \rho u}{\partial t} + \nabla(\rho u^2 + p) = -\rho \frac{\partial \Phi}{\partial x} - \rho \sum_{\text{species i,j}} \alpha_{ij}(v_i - v_j)$$

$$\frac{\partial E}{\partial t} + \nabla(u(E+p)) = -\rho u \frac{\partial \Phi}{\partial x} + \rho \sum_{\text{species i,j}} \alpha'_{ij}(T_i - T_j) - 4\pi \sum_{bands} \rho \kappa_b (J_b - f_b \times \frac{\sigma}{\pi} T^4) + S_b - \Lambda_b$$

$$\frac{1}{c}\frac{\partial J_b}{\partial t} + \frac{1}{4\pi} \nabla F_b = \sum_{species} \rho \kappa_b (J_b - f_b \times \frac{\sigma}{\pi} T^4)$$

i.e. Standard Euler equations, augmented with
(some) photochemistry, gravity, friction and radiation transport.

Physics included

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$$S_b(r) = -\frac{S_{0,b}}{\rho \tilde{e}_v} \times (1 - A) \times \exp(-\tau(r))(\exp(-\Delta\tau(r)) - 1)$$

and closure relations

$$E = \frac{1}{2}\rho u^2 + \rho e$$

$$e = p/\rho(\gamma - 1)$$

$$p = \rho \tilde{c}_v T$$

$$\Phi(r) = GM(r)/r$$

$$F_b = -\frac{\lambda c}{\rho \kappa_{R,b}} \nabla J_b$$

Standard relations for closure:

Physics included

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Potential overlap in cooling functions between Λ and *J-S* term?

Physics included

- Multi-species hydrodynamics: A separate set of Euler equations per species, solved with FV method, HLLC solver for gas, DUST for dust
- Gravity well-balanced with FV pressure gradients
- Species coupled via friction
- Multi-band radiation transport
- Photochemistry
- Dust growth

Finite Volumes

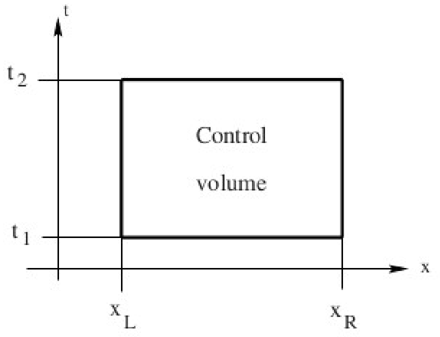


Fig. 2.10. A control volume $V = [x_{\rm L}, x_{\rm R}] \times [t_1, t_2]$ on x–t plane

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = \mathbf{0}$$

$$\mathbf{U} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \equiv \begin{bmatrix} \rho \\ \rho u \\ E \end{bmatrix} \;, \quad \mathbf{F} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \equiv \begin{bmatrix} \rho u \\ \rho u^2 + p \\ u(E+p) \end{bmatrix} \;.$$

Operator-split for advection vs. sources.

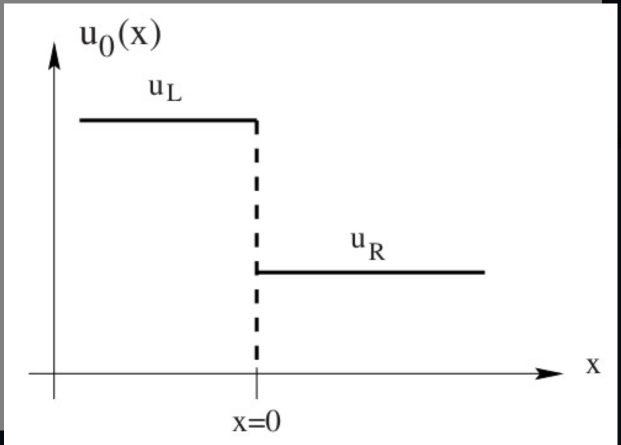
Conservation equations can be solved in a Finite Volume framework, giving <u>exact conservation</u> to machine precision and vastly improved <u>stability</u>@shocks (still has to obey CFL though)

(Many equations and images from Toro, Riemann solvers and numerical methods for Fluid Dynamics)

Finite Volumes

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{x_\mathrm{L}}^{x_\mathrm{R}} \mathbf{U}(x,t)\,\mathrm{d}x = \mathbf{F}(\mathbf{U}(x_\mathrm{L},t)) - \mathbf{F}(\mathbf{U}(x_\mathrm{R},t)) \;, \tag{2.66}$$

$$\left.\begin{aligned}\int_{x_\mathrm{L}}^{x_\mathrm{R}} \mathbf{U}(x,t_2)\,\mathrm{d}x = \int_{x_\mathrm{L}}^{x_\mathrm{R}} \mathbf{U}(x,t_1)\,\mathrm{d}x + \int_{t_1}^{t_2} \mathbf{F}(\mathbf{U}(x_\mathrm{L},t))\,\mathrm{d}t \\ - \int_{t_1}^{t_2} \mathbf{F}(\mathbf{U}(x_\mathrm{R},t))\,\mathrm{d}t \;,\end{aligned}\right\}$$



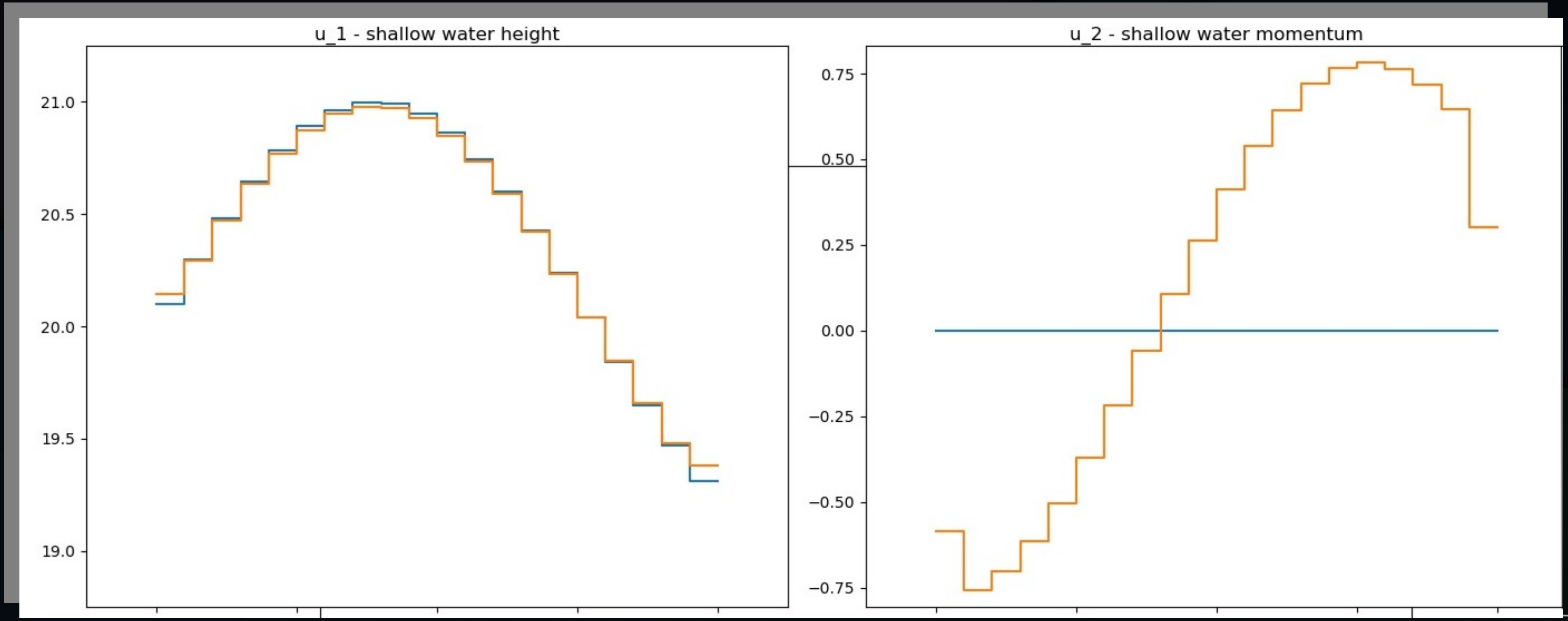
Steps to construct a FV scheme, a.k.a ‘Godunov’s method’:

1. Define dx-integral as new FV variable
2. Find **intermediate state** between u_L and u_R, usually done in Eigenspace
3. \int dt F = $\Delta t\ F^N(u_L,u_R,t_0)$
4. Find ‘good’ approximation $F^N(u_L,u_R,t)$
5. Godunov says: $F^N(u_L,u_R,t) = F^{analytic}(u^*,t)$

Hydrodynamics – Finite Volumes

- Finite Volumes:

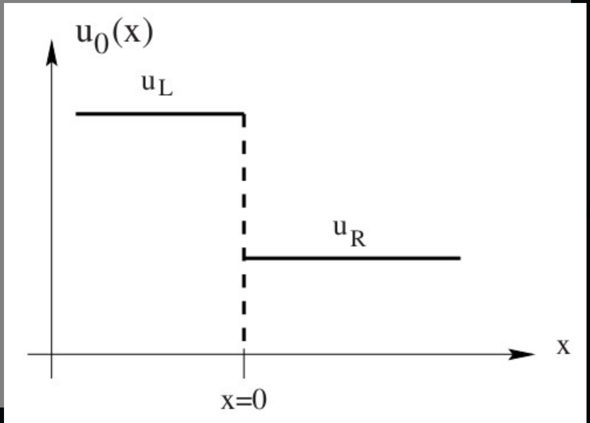
  Instead of fighting with discontinuities – make everything a discontinuity!



Finite Volumes – Characteristics

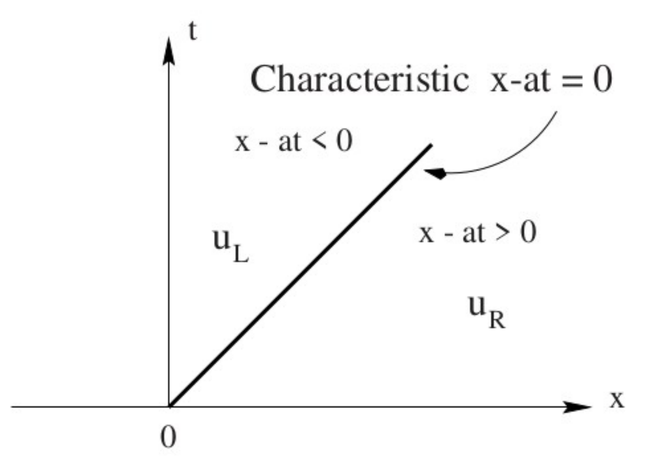
The simple system

$$\left.\begin{array}{ll} \text{PDE:} & u_t + a u_x = 0\ . \\ \text{IC:} & u(x,0) = u_0(x) = \begin{cases} u_{\mathrm{L}} & \text{if } x < 0\ , \\ u_{\mathrm{R}} & \text{if } x > 0\ , \end{cases} \end{array}\right\}$$



Has an analytic solution

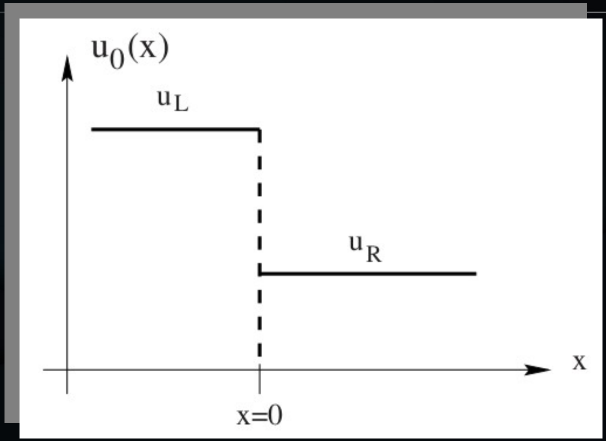
$$u(x,t) = u_0(x - at) = \begin{cases} u_{\mathrm{L}} & \text{if } x - at < 0\ , \\ u_{\mathrm{R}} & \text{if } x - at > 0\ . \end{cases}$$

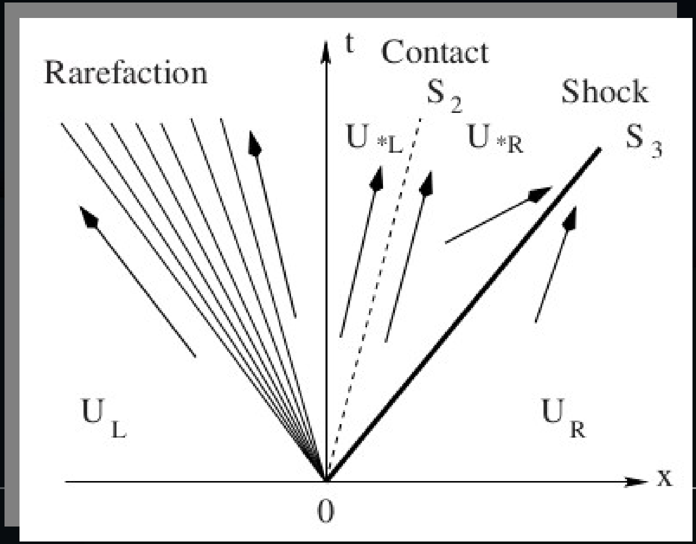


No intermediate state as only one characteristic exists

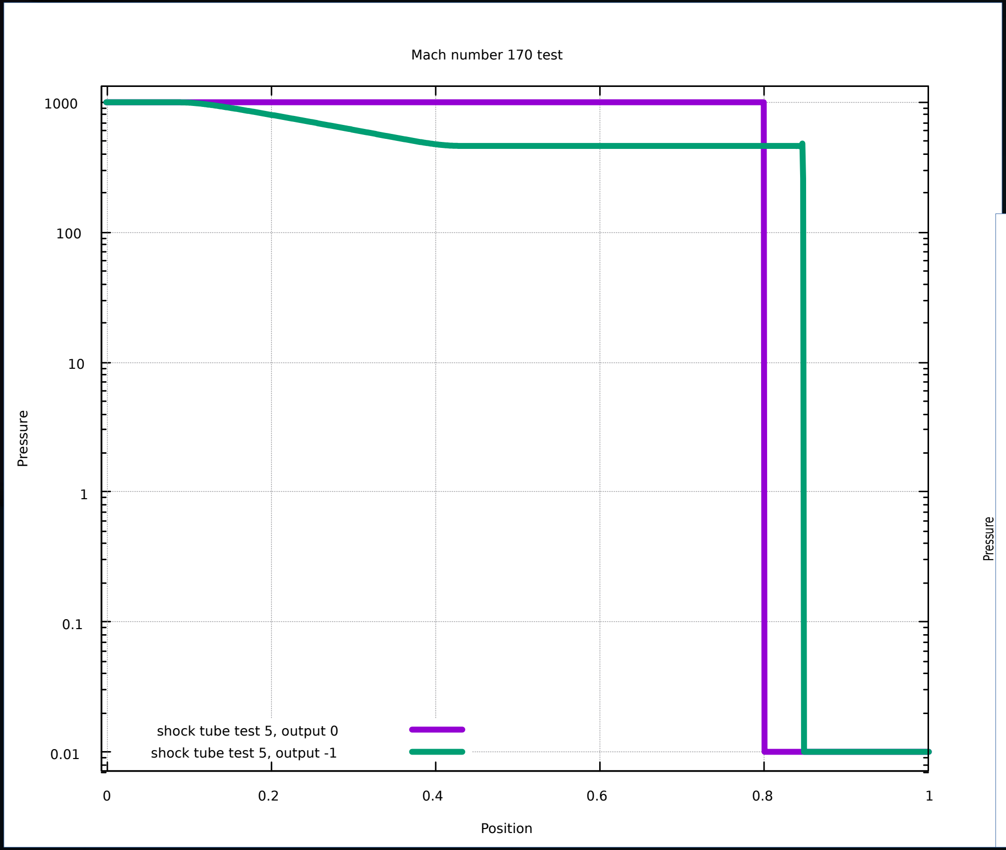
Finite Volumes – Charachteristics

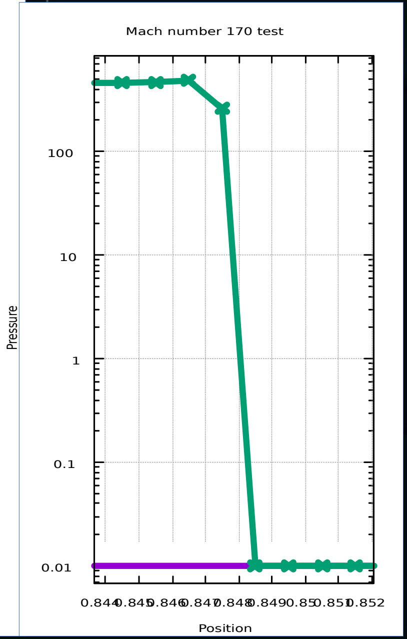
With more complex systems, such as the Euler equations...





Hydrodynamics – HLLC Shock test



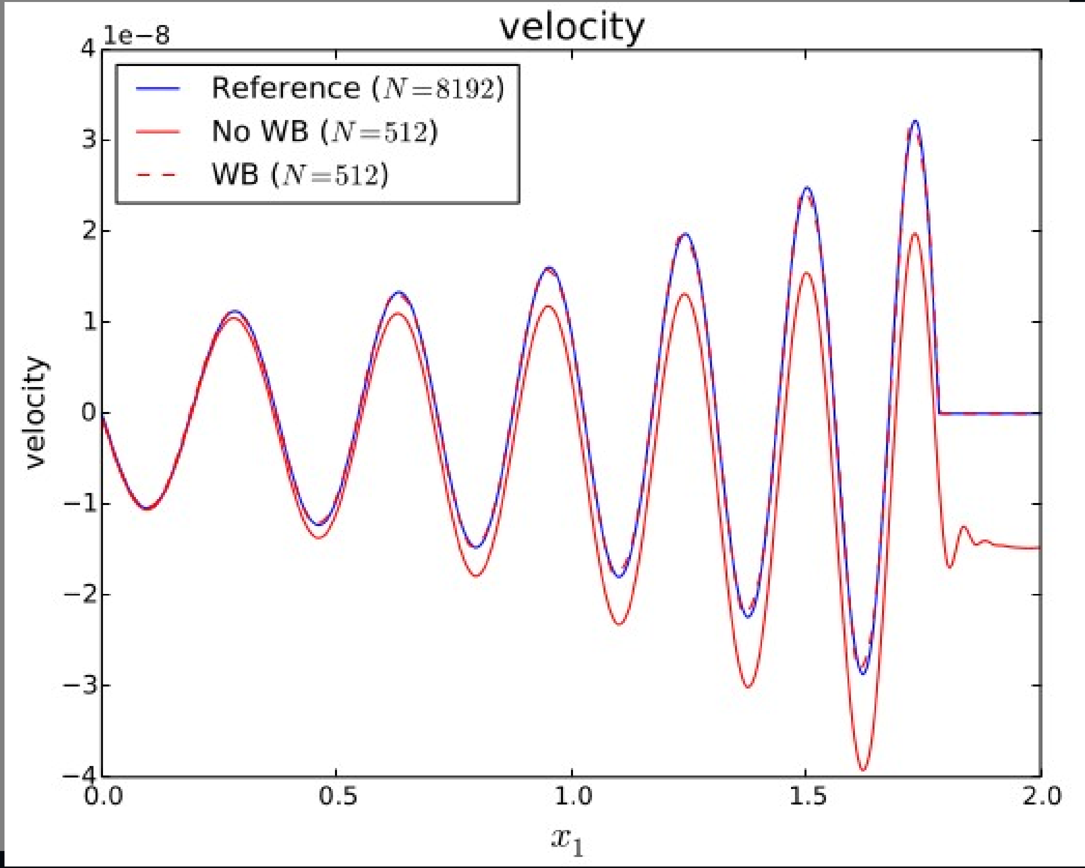


Strong shock ‘surface’ preserved at high Mach number in our shock tube

Well-balancing: Motivation

* Generally, letting a hydrodynamic simulation return to a previous hydrostatic state is nontrivial in a gravity field
* Particularly operator-splits are vulnerable to this

Well-balancing: Motivation

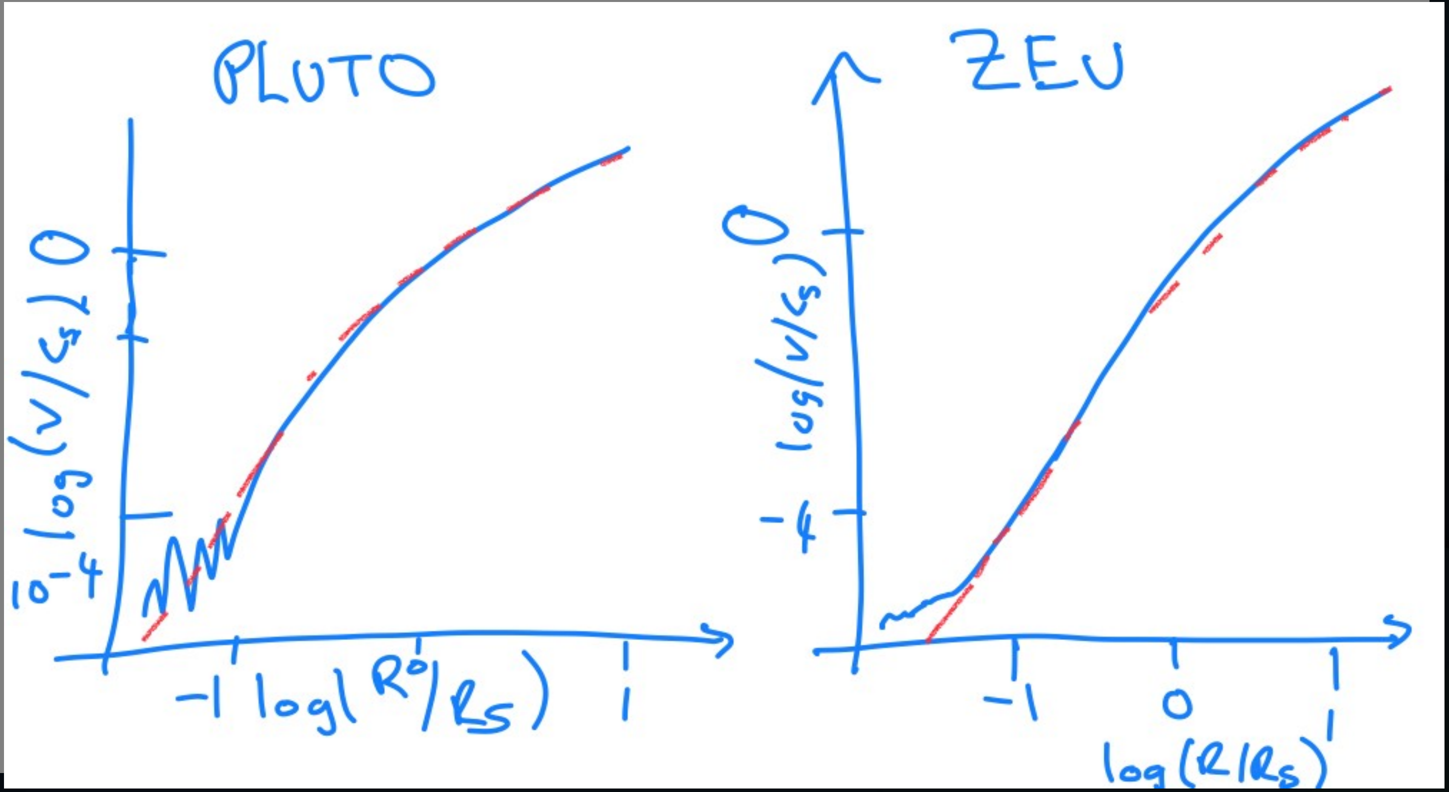


* Generally, letting a hydrodynamic simulation return to a previous hydrostatic state is nontrivial
* Particularly operator-splits are vulnerable to this

From Käppeli&Mishra(2016):

Without well-balancing a passing wave in a G-field leaves a non-zero background state

Well-balancing: Motivation



(c) J. Owen

Well-balancing

The scheme by Käppeli&Mishra(2016):

Leaves the Riemann-solver untouched, only passes a different pressure to it, so that gravity in the same timestep is exactly balanced.

Well-balancing gravity

Step 1: Assure numerical flux function can correctly solve stationary contact waves, i.e.

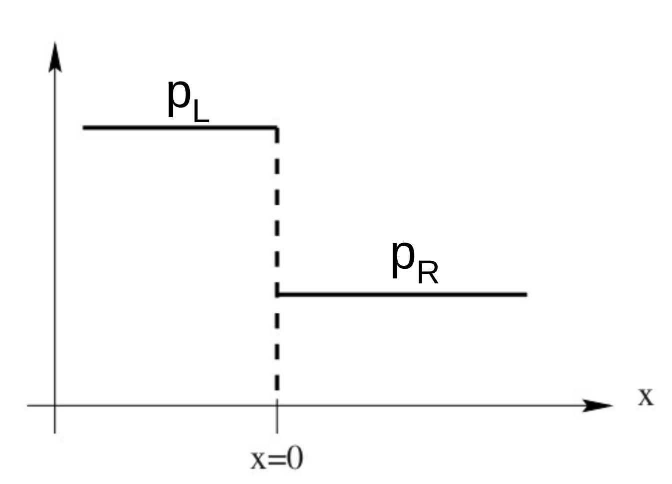
$$\frac{d\boldsymbol{u}_i}{dt} = -\frac{1}{\Delta x}\left(\boldsymbol{F}_{i+1/2} - \boldsymbol{F}_{i-1/2}\right) + \boldsymbol{S}_i,$$

FV update with source

$$\boldsymbol{F}_{i+1/2} = \mathcal{F}(\boldsymbol{w}_{i+1/2-}, \boldsymbol{w}_{i+1/2+}),$$

Numerical flux function

$$\mathcal{F}\left([\rho_L, 0, p]^T, [\rho_R, 0, p]^T\right) = [0, p, 0]^T,$$

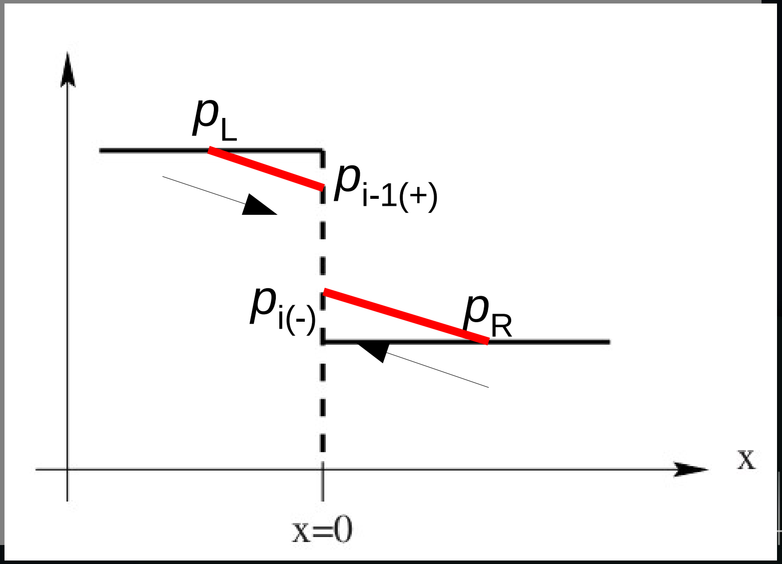


Well-balancing gravity

Step 2: Choose an adequate extrapolation for an interface-pressure
And hand it to the Riemann solver

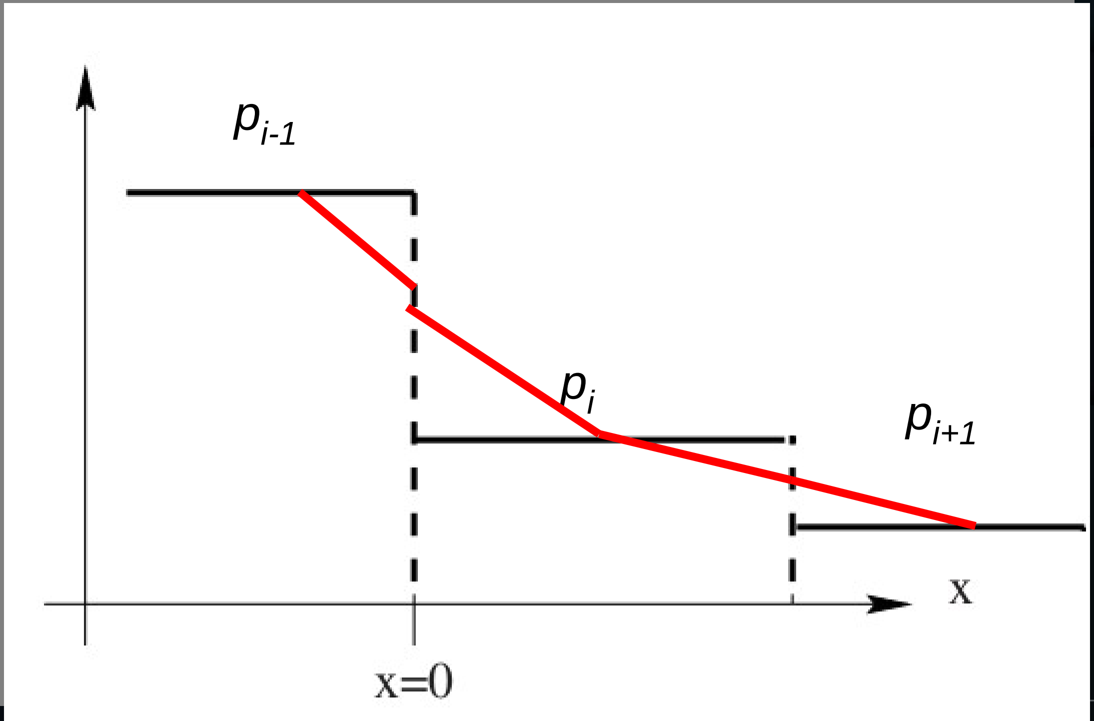
$$p_{0,i}(x_{i-1/2}) = p_i + \rho_i \frac{\phi_i \quad - \phi_{i-1}}{\Delta x} \frac{\Delta x}{2}$$
$$p_{0,i}(x_{i+1/2}) = p_i - \rho_i \frac{\phi_{i+1} - \phi_i}{\Delta x} \frac{\Delta x}{2}.$$

And a standard 2nd order gravitational source function



Well-balancing gravity

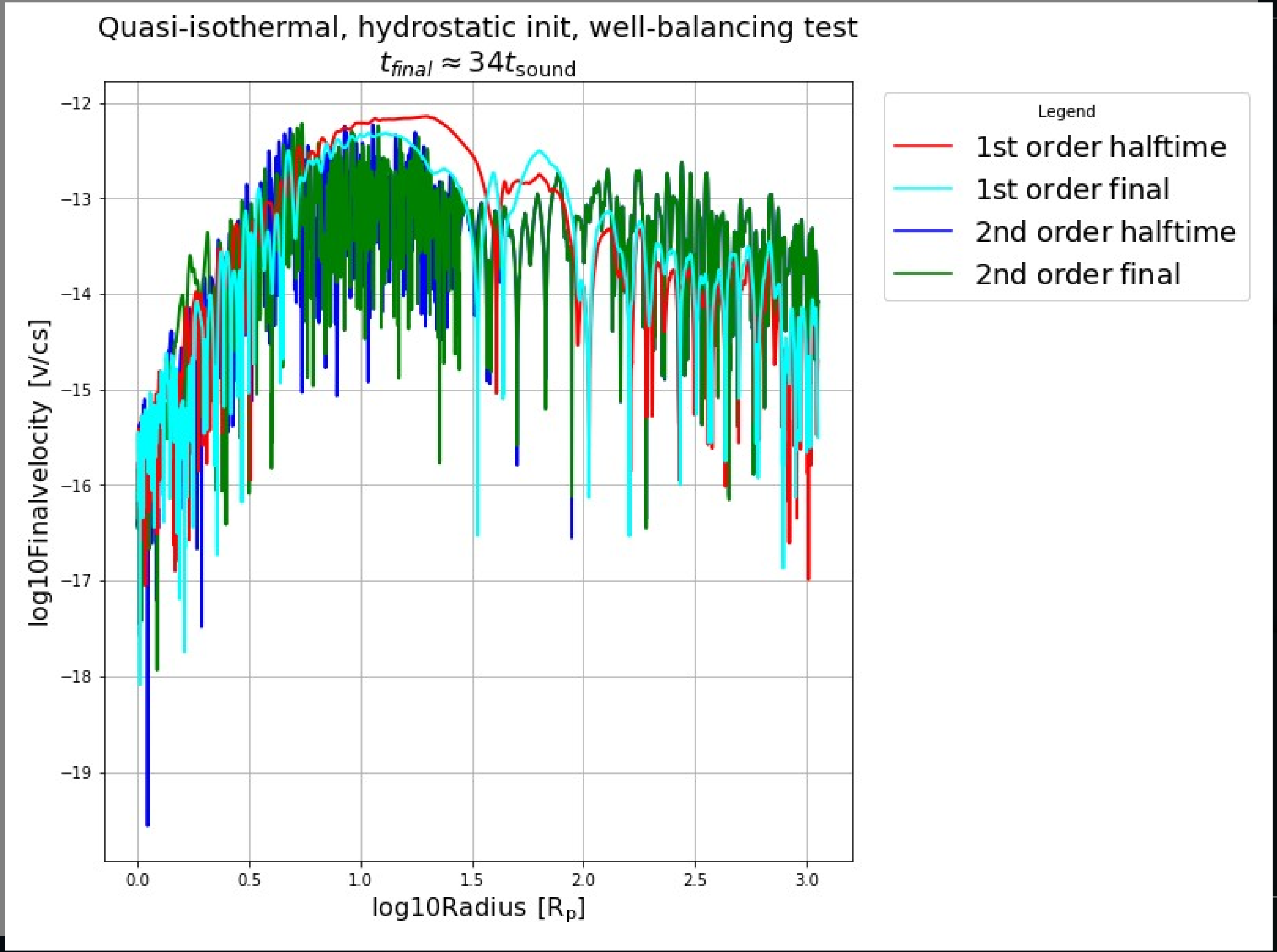
Step 3: For p_i that fulfill the hydrostatic condition, now dF+S=0



$$\frac{\mathrm{d}\boldsymbol{u}_i}{\mathrm{d}t} = -\frac{1}{\Delta x}\left(\boldsymbol{F}_{i+1/2} - \boldsymbol{F}_{i-1/2}\right) + \boldsymbol{S}_i,$$

Doing this via the contact wave is smart, p_i are not changed, extrapolation merely ‘informs’ solver when it’s supposed to be hydrostatic.

Remaining hydrostatic



* Initializing a hydrostatic density profile for T=const=10.000 K, γ_{ad}=1.01 and v=0 everywhere

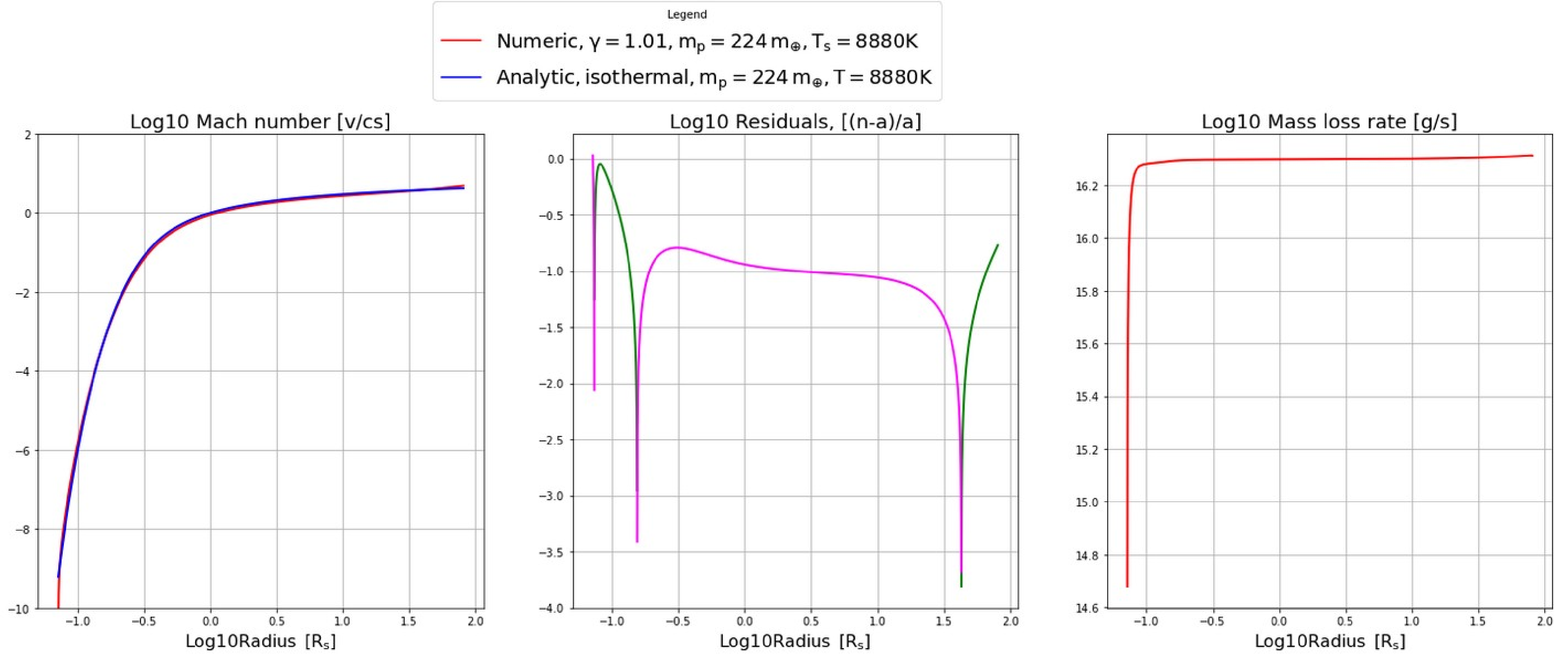
* Check if well-balancing keeps velocity small

Well-balanced winds

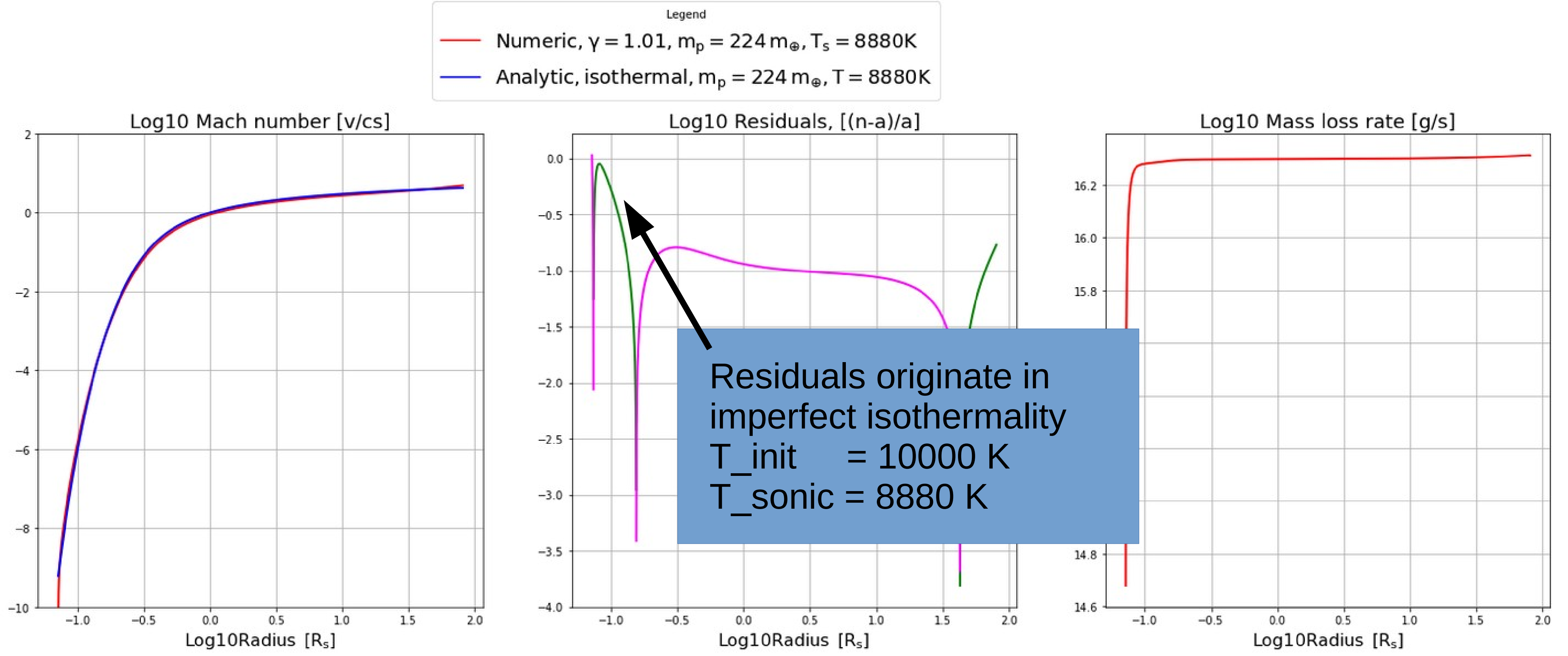
* Now, after checking the v=0 preservation, we can try to launch winds via several methods:

    1.) A step function in density (clearly not hydrostatic)
    2.) A tidal potential
    3.) Irradiation

Winds – Launch via step function

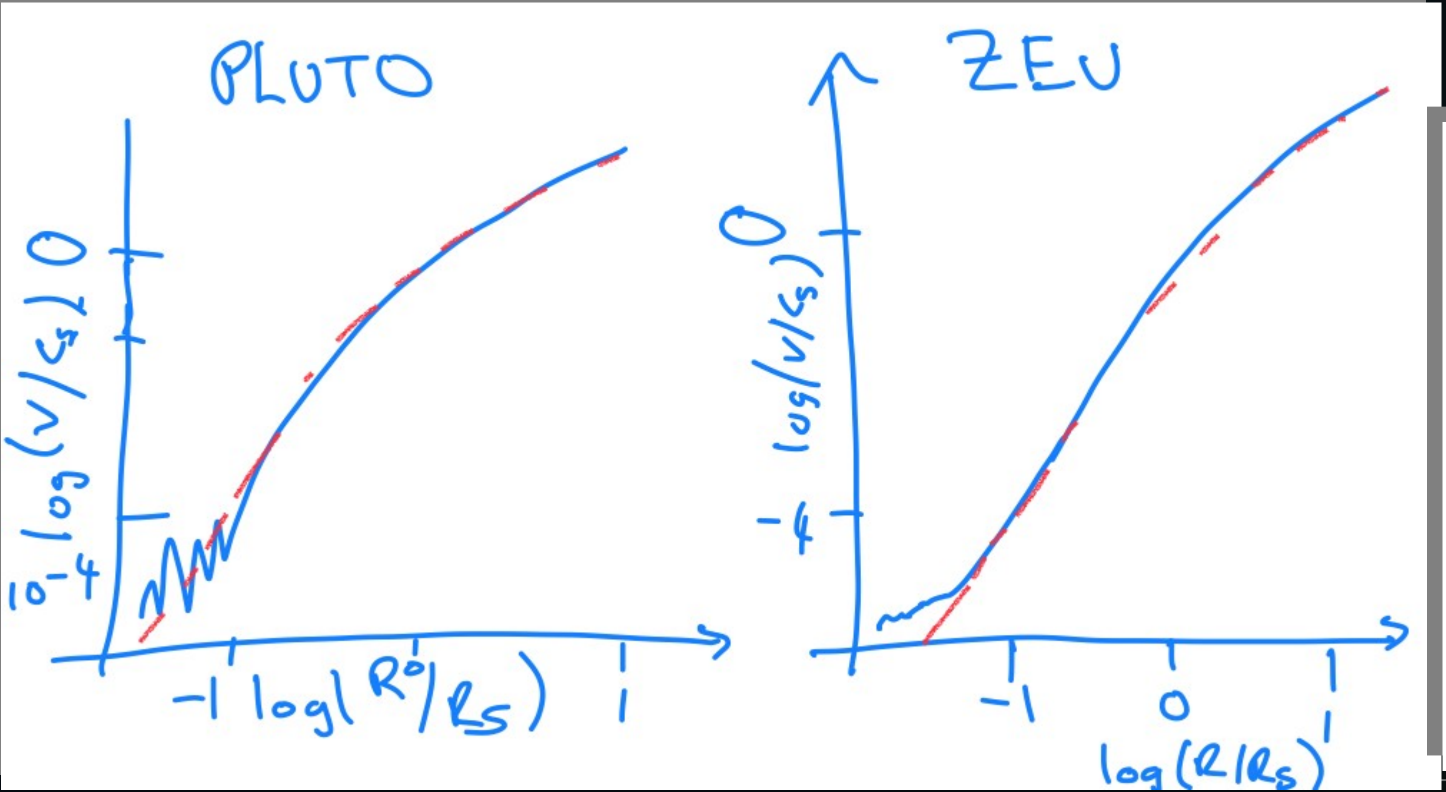


Winds – Launch via step function



Residuals originate in imperfect isothermality
T_init = 10000 K
T_sonic = 8880 K

Well-balancing gravity



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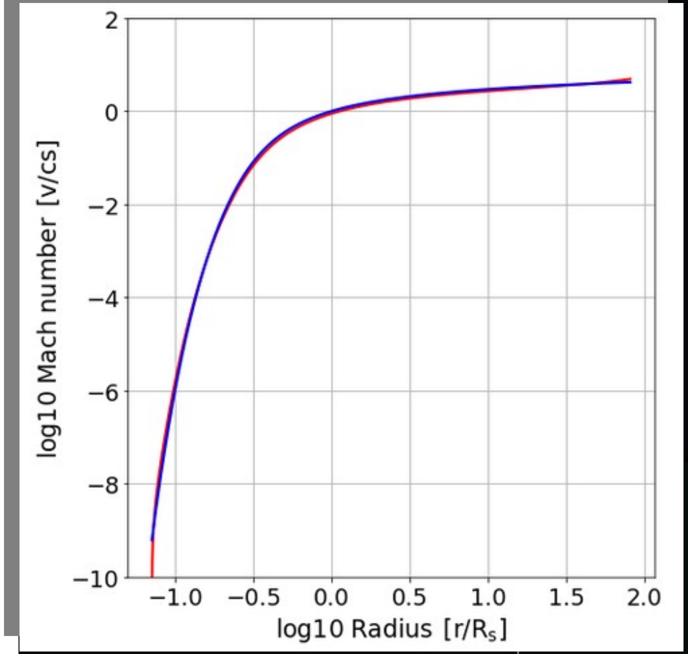


Beyond prescribed isothermality

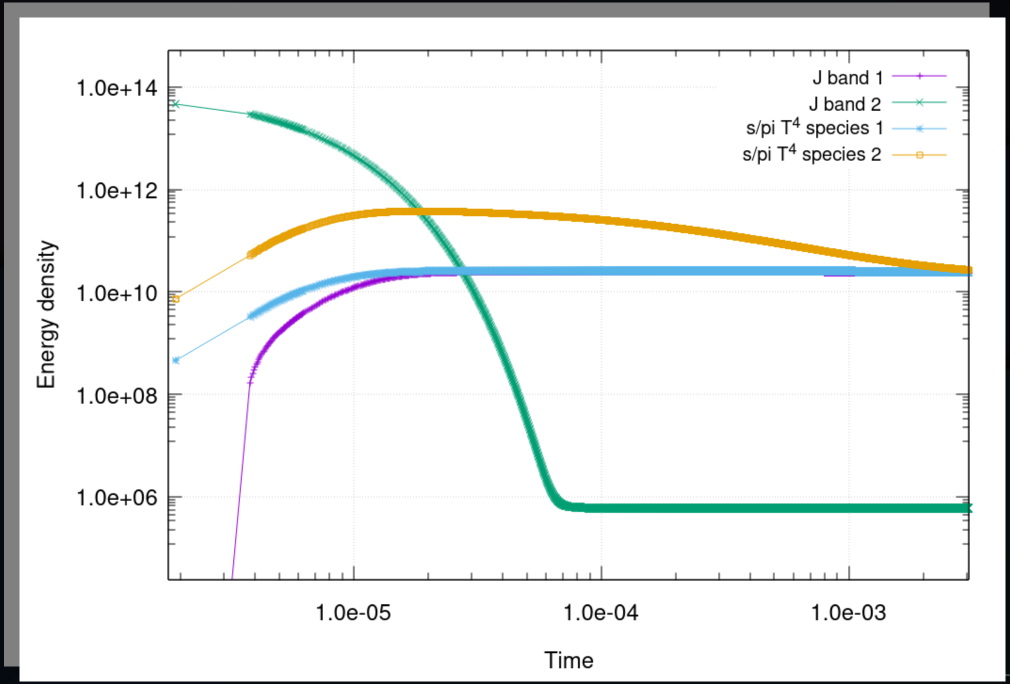
* Using γ_{ad}=1.01 mimicks an isothermal gas
* Real gases have γ_{ad}= 1.1 – 1.67
* Need solar thermal/XUV heating + radiation transport to compensate adiabatic cooling

Two example scenarios of physical interest for γ_{ad}>1 :

* Compact, evolved atmospheres, deep in their potentials → can launch wind
* Protoplanet, shrouded with gas, difficult to jumpstart wind

Radiation transport

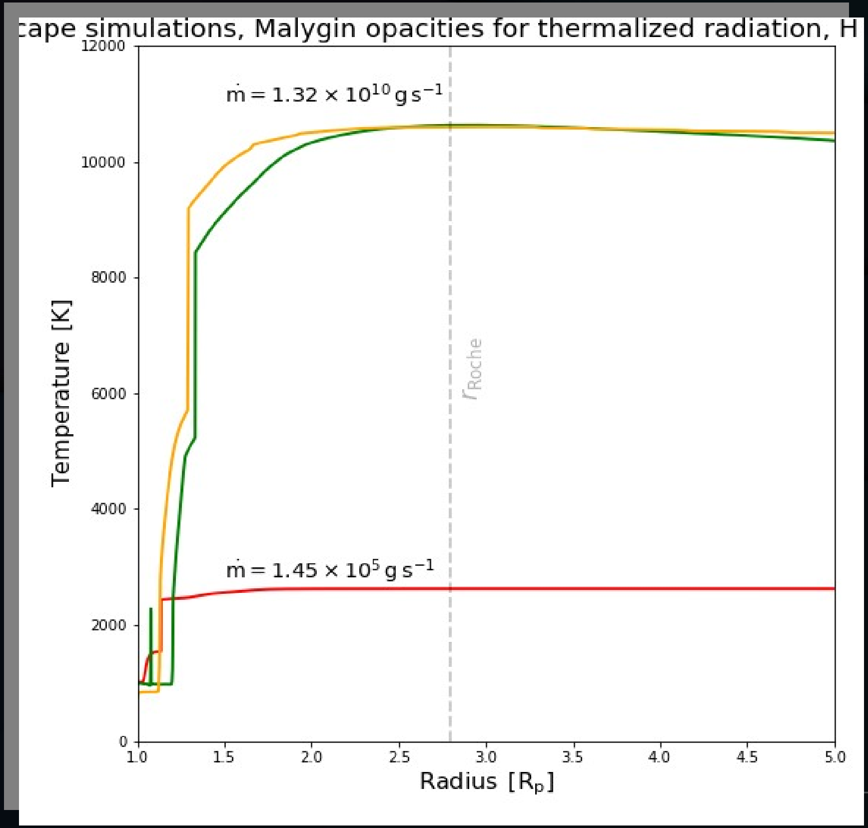
Implicit schemes by Commercon+(2011) and Bitsch+(2013)
with some inspiration by Vaytet+(2011) used to solve for multi-Temperatures and multi-bands.



Example 2-species, 2-band hydrostatic atmosphere attaining thermodynamic equilibrium

(Nearly) Full winds

Steady-state solutions with 3 bands, but **no** photochemistry or high-energy cooling
$F_{UV} = 10^4$ erg/cm²/s @ y=10^5, $F_X = 10^4$ erg/cm²/s @ y=10^7

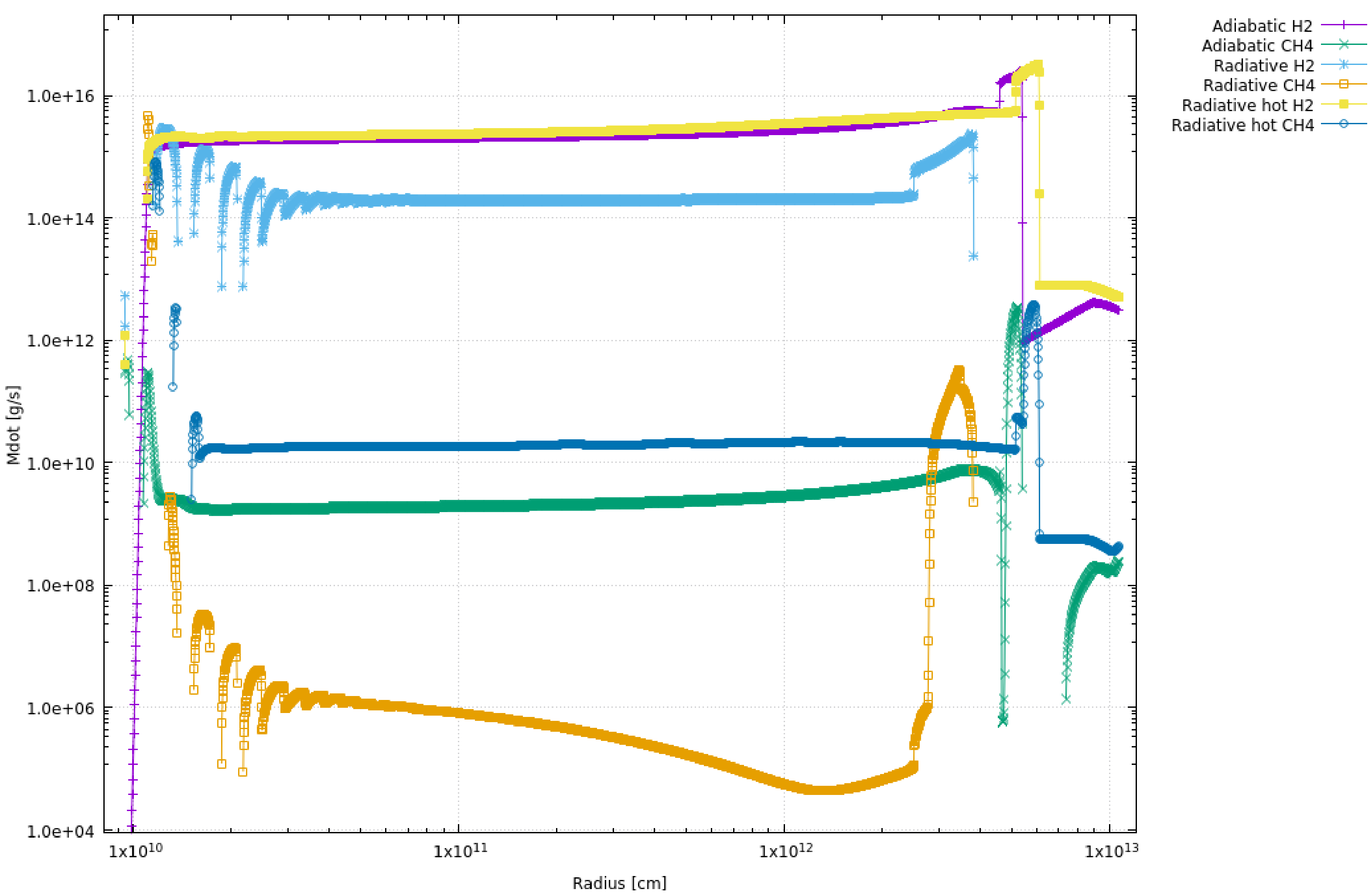


Comparable result to Murray-Clay+(2009) although missing physics is no suprise, as there, pdV cooling dominates, as it does here

Multi-species: Friction coupling

Using Benitez-Llambay(2019) scheme:
 Implicit solution of coupled friction, with total momentum conservation (default) and total energy conservation (Work by from Richard)

2-species winds



To be continued...