Math 410 -Numerical Analysis

Programming Assignment 5

Dec. 10–18, 2013

General Directions

1. Implement a MATLAB function of Runge-Kutta Method for system of Differential equations (algorithm 5.7 page 331)

2. using above program to solve the following second order ODE system with the given initial conditions.

$$\ddot{\mathbf{q}}_i = -\sum_{\substack{1\le j\le 4\\ j\neq i}} \frac{(\mathbf{q}_i - \mathbf{q}_j)}{|\mathbf{q}_i - \mathbf{q}_j|^3}$$

   here each $\mathbf{q}_i = [q_{ix}(t), q_{iy}(t)]$ is 2-D vector function of variable t for $i = 1, \cdots, 4$ and the norm

$$|\mathbf{q}_i - \mathbf{q}_j| = \sqrt{(q_{ix} - q_{jx})^2 + (q_{iy} - q_{jy})^2}.$$

3. For instance,

$$\ddot{\mathbf{q}}_1 = \frac{(q_{1x} - q_{2x}, q_{1y} - q_{2y})}{\sqrt{(q_{1x} - q_{2x})^2 + (q_{1y} - q_{2y})^2}^3} + \frac{(q_{1x} - q_{3x}, q_{1y} - q_{3y})}{\sqrt{(q_{1x} - q_{3x})^2 + (q_{1y} - q_{3y})^2}^3} + \frac{(q_{1x} - q_{4x}, q_{1y} - q_{4y})}{\sqrt{(q_{1x} - q_{4x})^2 + (q_{1y} - q_{4y})^2}^3}$$

4. The time $t \in [0,\ 3]$ and the initial condition

$$\mathbf{q}_1(0) = [1.0597,\ 1.7696], \qquad \dot{\mathbf{q}}_1(0) = [-0.5537,\ -0.3988] \tag{1}$$
$$\mathbf{q}_2(0) = [0,\ \ -0.8094], \qquad \dot{\mathbf{q}}_2(0) = [1.0934,\ \ 0.0000] \tag{2}$$
$$\mathbf{q}_3(0) = [-1.0597,\ 1.7696], \qquad \dot{\mathbf{q}}_3(0) = [-0.5539,\ \ 0.3989] \tag{3}$$
$$\mathbf{q}_4(0) = [0,\ \ -2.7299], \qquad \dot{\mathbf{q}}_4(0) = [0.0142,\ \ 0.000] \tag{4}$$

5. Write an instruction on how to use the program.

6. Plot your solutions in a 2-D graph.

7. This assignment is counted as part of your final exam.