Computational Intelligence

Assignment 1

February 2, 2022

1 Task 1

In this task we try to perform various manipulations with planes.

1.1 Task 1.1

Write a procedure to check whether or not two planes represented as $\mathbf{r}_1 = \mathbf{p}_1 + t_1\mathbf{v}_1 + u_1\mathbf{w}_1$ and $\mathbf{r}_2 = \mathbf{p}_2 + t_2\mathbf{v}_2 + u_2\mathbf{w}_2$ intersect each other; $\mathbf{r}_i, \mathbf{p}_i, \mathbf{v}_i, \mathbf{w}_i \in \mathbb{R}^3$, $t_i, u_i \in \mathbb{R}$.

Illustrate the correctness of your procedure by showing examples with graphical output.

1.2 Task 1.2

Given a plane represented as $\mathbf{r} = \mathbf{p} + t\mathbf{v} + u\mathbf{w}$, where $\mathbf{r}, \mathbf{p}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$, find its representation in a form $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$, $\mathbf{n}, \mathbf{r}_0 \in \mathbb{R}^n$. Do it for the cases where:

1. $\mathbf{v} = \begin{bmatrix} 1 \\ 7 \\ 4 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 2 \\ 8 \\ -5 \end{bmatrix}$ and $\mathbf{p} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$;
2. $\mathbf{v} = \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 5 \\ 5 \\ -5 \end{bmatrix}$ and $\mathbf{p} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$;

Plot planes represented in both ways. Fig. 1 illustrates a plane representation in a form $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$.

1.3 Task 1.3

Given a plane s defined by equation $\mathbf{r} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + t \begin{bmatrix} -2 \\ 3 \\ 3 \end{bmatrix} + u \begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix}$, find equation of a line l, perpendicular to s and passing though the origin. Find a projection of the point $\mathbf{g} = \begin{bmatrix} -10 \\ -3 \\ 5 \end{bmatrix}$ on l.

1.4 Task 1.4

Given a plane s defined by equation $\mathbf{r} = \begin{bmatrix} -5 \\ 11 \\ 0.5 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$, and a point $\mathbf{g} = \begin{bmatrix} -10 \\ -3 \\ 5 \end{bmatrix}$, find a point $\mathbf{g}^*$ symmetrical to $\mathbf{g}$ relative to the plane s.

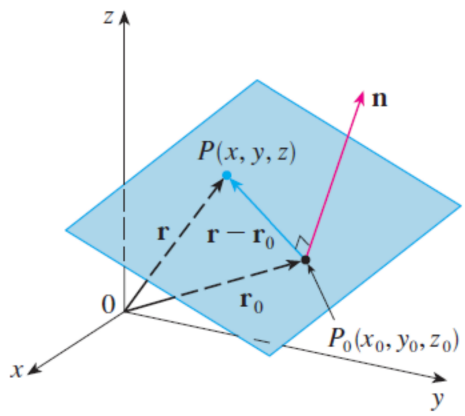

Figure 1: Illustration of a plane

2 Task 2

Given a system of equations

$$\begin{cases} 3x + y + z = 0 \\ 6x + 2y + 2z = 0 \\ -9x - 3y - 3z = 0 \end{cases} \tag{1}$$

we define its space of solutions as V.

2.1 Task 2.1

Find a basis in V. Visualize V as a plane.

2.2 Task 2.2

Given an arbitrary vector $\mathbf{g}$, write a procedure of how to find its orthogonal projection onto V, and onto the orthogonal compliment of V. Prove that the procedure you propose is correct. Show the projection results for $\mathbf{g} = \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$. Visualize the projection.

2.3 Task 2.3

Let $\mathbf{g}^{||}$ be the orthogonal projection of the vector $\mathbf{g}$ onto V, and $\mathbf{g}^{\perp}$ be the orthogonal projection of the vector $\mathbf{g}$ onto the orthogonal compliment of V. With that information, how can we recover $\mathbf{g}$? Prove that your procedure is correct.

3 Task 3

You are given the following optimization problem:

$$
\begin{aligned}
\min_{x_1,x_2} \quad & \frac{1}{2}x_1^2 + 4x_2^2 - 32x_2 + 60 \\
\text{s.t.} \quad & x_1 + x_2 \leq 6 \\
& x_1 + 2x_2 \leq 8 \\
& x_1 \geq 0, x_2 \geq 0, x_2 \leq 9,
\end{aligned} \tag{2}
$$

3.1 Task 3.1

Rearrange the problem (2) in the following form

$$
\begin{aligned}
\min_{\mathbf{x}} \quad & f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T H\mathbf{x} + c\mathbf{x} + c_0 \\
\text{s.t.} \quad & A\mathbf{x} \leq b
\end{aligned} \tag{3}
$$

3.2 Task 3.2

Use CVXPY to solve both (2) and (3).

3.3 Task 3.3

Visualize the domain of the function, its cost function and its solution.

4 Submition

Please upload the single zip file which includes your source code and the report.

5 Deadline

The deadline: April 18, 23:59:59 GMT+3.