THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

Session C6: Open-Source Data and Tools for GNSS Research and Development

Open-Source Multipath Mitigation Technology-integrated GNSS Direct Position Estimation

Sergio Vicenzo, Bing Xu

Email: seergio.vicenzo@connect.polyu.hk

ResearchGate: https://www.researchgate.net/profile/Sergio_Vicenzo

ORCID: https://orcid.org/0000-0003-2974-7899

LinkedIn: http://www.linkedin.com/in/sergio-vicenzo/

Opening Minds • Shaping the Future
啟迪思維 • 成就未來

Background – Current problem

Problem with GNSS in urban environments?

- Multipath (MP) and non-line-of-sight (NLOS) receptions, attenuation, and diffraction from urban structures.

Common Mitigation methods?

- Integration with other systems, including but not limited to 3D mapping, and inertial navigation systems (INS).
- But such methods relied on other positioning systems and does not improve the performance of sole GNSS receivers

Zhang, G., & Hsu, L. T. (2021). Performance assessment of GNSS diffraction models in urban areas. NAVIGATION, 68(2), 369-389. https://doi.org/10.1002/navi.417

Wen, W., Bai, X., Kan, Y. C., & Hsu, L.-T. (2019). Tightly Coupled GNSS/INS Integration via Factor Graph and Aided by Fish-Eye Camera. IEEE Transactions on Vehicular Technology, 68(11), 10651-10662. https://doi.org/10.1109/tvt.2019.2944680

Email: seergio.vicenzo@connect.polyu.hk

Background – Two-step positioning (2SP) receivers

Common Two-Step Positioning (2SP) architectures

Scalar Tracking Loop (STL) – Non-linear Least Square (**LS**) Estimator

- Independent tracking loops estimate pseudorange and pseudorange rates of individual satellites
- No information sharing between tracking loops
- No information fed back from navigation processor (LS)

Vector Tracking Loop (VTL) – Extended Kalman Filter (**EKF**)

- Code discriminator and carrier loop filter outputs are fed towards the EKF
- PVT are fed back into tracking loops to aid in estimating the code phase and carrier frequency => mutual aiding

Lashley, M., Bevly, D. M., & Hung, J. Y. (2010). A valid comparison of vector and scalar tracking loops. IEEE/ION Position, Location and Navigation Symposium, Indian Wells, CA, USA.

Xu, B., Jia, Q., & Hsu, L.-T. (2020). Vector Tracking Loop-Based GNSS NLOS Detection and Correction: Algorithm Design and Performance Analysis. IEEE Transactions on Instrumentation and Measurement, 69(7), 4604-4619. https://doi.org/10.1109/tim.2019.2950578

Direct Position Estimation (DPE)

- A **single-step** positioning algorithm
- <u>**Directly**</u> calculates the positioning estimates in the navigation domain

Main idea:

Satellite signals code delay and Doppler frequencies are functions of the receiver's position, velocity, and time (PVT), $\boldsymbol{\gamma} = [\mathbf{p}^{\mathrm{T}}, \mathbf{v}^{\mathrm{T}}, \delta t, \dot{\delta t}]^{\mathrm{T}}$

Code Delay / Receiver Position

$$\tau \triangleq \mathbf{p}$$

$$f_d \triangleq \mathbf{v}$$

Doppler freq. / Receiver Velocity

$$\hat{\boldsymbol{\gamma}} = \arg\min_{\boldsymbol{\gamma},\mathbf{a}} \sum_{i=1}^{M} \left\| \mathbf{x} - \mathbf{c}_i(\boldsymbol{\gamma}) a^i \right\|^2$$

or equivalently

$$\hat{\boldsymbol{\gamma}} = \arg\max_{\boldsymbol{\gamma}} \sum_{i=1}^{M} \| \mathbf{x}^{\mathrm{H}} \boxed{\mathbf{c}_i(\boldsymbol{\gamma})} \|^2$$

Signal replica for *i-th satellite*, generated as a function of the PVT

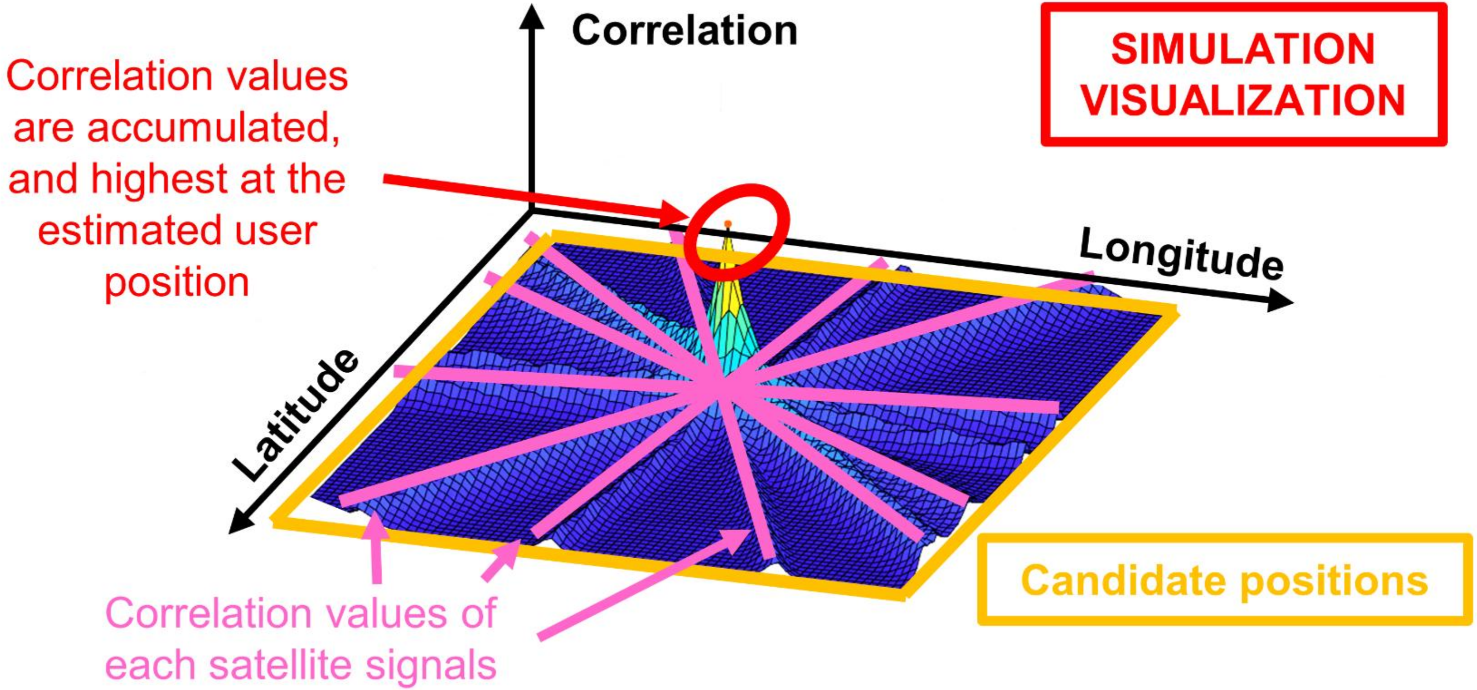
Received baseband signal vector

Closas, P., Fernandez-Prades, C., & Fernandez-Rubio, J. A. (2007). Maximum Likelihood Estimation of Position in GNSS. IEEE Signal Processing Letters, 14(5), 359-362. https://doi.org/10.1109/lsp.2006.888360

Email: seergio.vicenzo@connect.polyu.hk

Background – Direct Position Estimation

Grid-based method - Local signal replicas are for each candidate PVT, and correlated with the incoming signal



Vicenzo S, Xu B, Xu H, Hsu L-T (2024) GNSS direct position estimation-inspired positioning with pseudorange correlogram for urban navigation. GPS Solutions 28(2). https://doi.org/10.1007/s10291-024-01627-5

Email: seergio.vicenzo@connect.polyu.hk

Background – Direct Position Estimation

DPE proven to offer **superior performance against 2SP**

in terms of…

- Weak signal reception
- MP reception

Proven through…

- Theoretical bounds i.e., Cramér-Rao lower bound (CRLB) and Ziv-Zakai bound (ZZB)
- Statistical multipath channel models (DLR)

Closas, P., Fernández-Prades, C., & Fernández-Rubio, J. A. (2009a). Cramér–Rao Bound Analysis of Positioning Approaches in GNSS Receivers. IEEE Transactions on Signal Processing, 57(10), 3775-3786. https://doi.org/10.1109/TSP.2009.2025083
Closas, P., Fernández-Prades, C., Fernández, A., Wis, M., Vecchione, G., Zanier, F., Garcia-Molina, J., & Crisci, M. (2015). Evaluation of GNSS direct position estimation in realistic multipath channels. 28th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2015)
Gusi-Amigó, A., Closas, P., Mallat, A., & Vandendorpe, L. (2018). Ziv-Zakai Bound for Direct Position Estimation. NAVIGATION, 65(3), 463-475. https://doi.org/10.1002/navi.259

Problem Statement

1. GNSS positioning with DPE is mostly **left uninvestigated** and **unapplied commercially**.

2. Though robust, GNSS **DPE positioning still degrades due to Multipath**

- **DPE performance depreciates due to MP and NLOS in urban environments** (Gao et al., 2025; Huang et al., 2025; Jia & Guo, 2025; Vicenzo et al., 2023).
- Xie and Petovello (2015) had previously discovered that the ACF peak does not necessarily correspond to the LOS peak in real urban environments i.e., **a stronger reflected signal than the LOS**.

Gao, W., Yang, R., Huang, J., & Zhan, X. (2025). Quasi-Deep Integration for DPE/INS in GNSS Navigation Domain: Framework Design and Optimization. IEEE Transactions on Aerospace and Electronic Systems, 61(3), 6774-6793. https://doi.org/10.1109/taes.2025.3532891

Vicenzo, S., Xu, B., Dey, A., & Hsu, L.-T. (2023). Experimental Investigation of GNSS Direct Position Estimation in Densely Urban Area. 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023), Denver, Colorado.

Huang, J., Yang, R., & Zhan, X. (2025). GNSS Multi-Frequency Direct Position Estimation Geometric Theory for Multipath Analysis. Proceedings of 2025 IEEE/ION Position, Location and Navigation Symposium (PLANS), Salt Lake City, Utah.

Jia, Q., & Guo, Q. (2025). Design of Dual-Mode DPE Receiver Based on GPS L1/BDS B1C Signals. Proceedings of the 2025 International Technical Meeting of The Institute of Navigation, Long Beach, California.

Xie, P., & Petovello, M. G. (2015). Measuring GNSS Multipath Distributions in Urban Canyon Environments. IEEE Transactions on Instrumentation and Measurement, 64(2), 366-377. https://doi.org/10.1109/tim.2014.2342452

Email: seergio.vicenzo@connect.polyu.hk

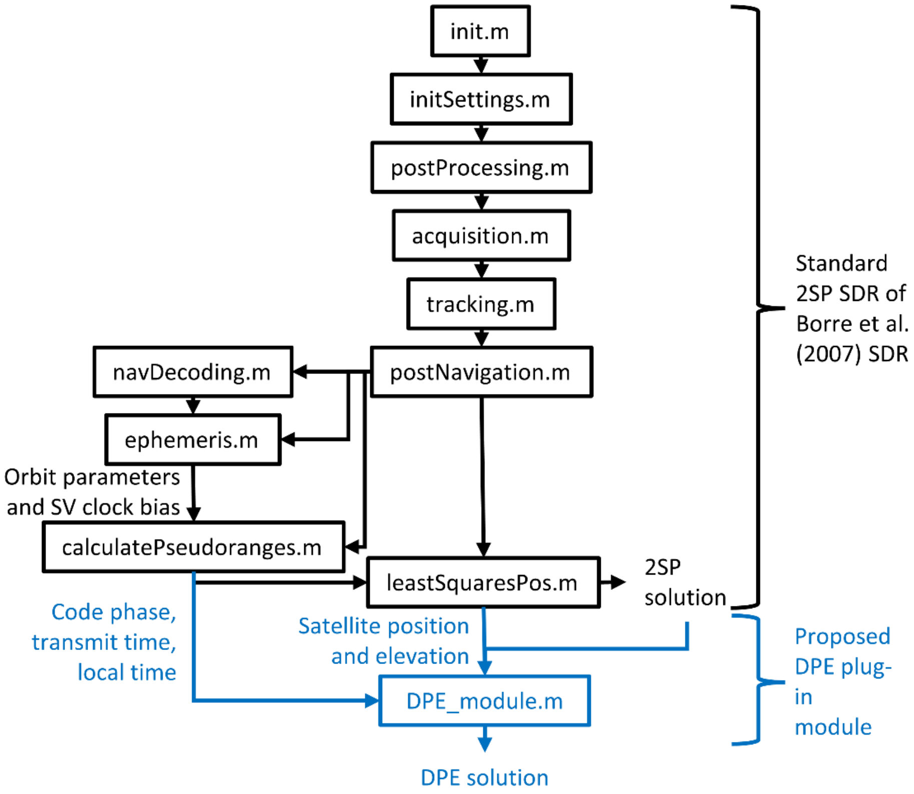
Objective:

1. **Spur research into DPE and its popularization** as a more robust positioning technique compared to 2SP → *New DPE architecture with pseudoranges*

2. **Improving DPE performance against MP** by introducing Multipath Mitigation Technology (MMT)-integrated DPE (MMT-DPE)

Email: seergio.vicenzo@connect.polyu.hk

Methodology – Our Proposed DPE implementation



- Integrated with the GPS L1 C/A **SoftGNSS v3.0** MATLAB 2SP Scalar Tracking Loop (STL) SDR by Borre et al. (2007).
- **2SP tracking for channel propagation**, similar with (Closas et al., 2015) and (Liu et al., 2013).
- **Drawback?** Impaired performance against weak signals. But should still obtain longer position fix in case of DLL/PLL loss of lock (Closas et al., 2015).
- When DLL/PLL maintains lock on signals, DPE MP mitigation capability is maintained.

Borre, K., Akos, D. M., Bertelsen, N., Rinder, P., & Jensen, S. H. (2007). A software-defined GPS and galileo receiver: A single-frequency approach.

Peretic, M. (2019). Development and analysis of a parallelized direct position estimation-based GPS receiver implementation (Doctoral dissertation, University of Illinois at Urbana-Champaign).

Closas, P., Fernández-Prades, C., Fernández, A., Wis, M., Vecchione, G., Zanier, F., & Crisci, M. (2015, September). Evaluation of GNSS direct position estimation in realistic multipath channels. In Proceedings of the 28th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2015) (pp. 3693-3701).

Liu, J., Cui, X., Lu, M., & Feng, Z. (2013). Direct position tracking loop based on linearised signal model for global navigation satellite system receivers. IET Radar, Sonar & Navigation, 7(7), 789-799. https://doi.org/https://doi.org/10.1049/iet-rsn.2012.0307

Email: seergio.vicenzo@connect.polyu.hk

- **Pre-calculate the correlations** per every code phase (Axelrad et al., 2011)
- Pre-calculated correlations would be **referenced to the ranges estimated from 2SP**
- The correlations of the grid of candidate PVT would later be given based on its ranges.

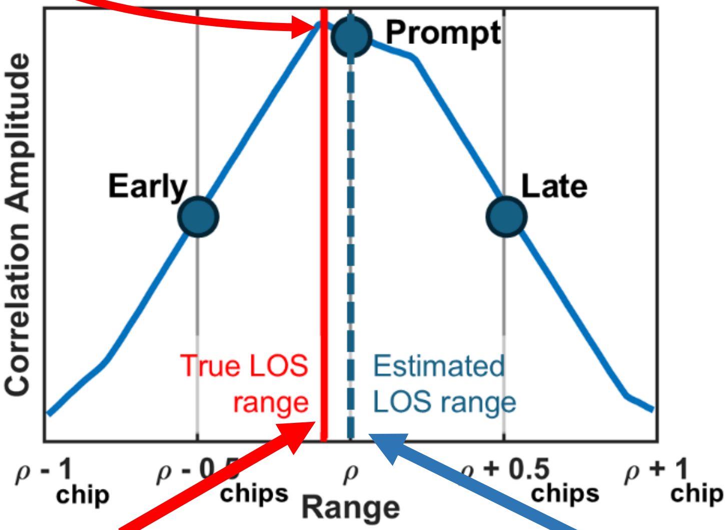
Pseudoranges for DPE? Isn't that **counterintuitive**?

Pseudoranges merely used for indexing the ACF

If the highest correlation value corresponds to the true range, DPE would remain ***robust*** to multipath, but not 100% immune

But highest correlation still falls to the true LOS range

Constructive MP ACF



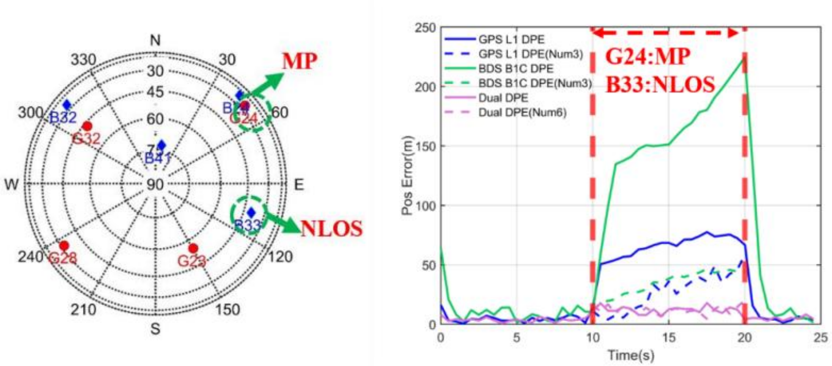
Because of MP, tracking fails to obtain accurate LOS code phase

Prompt correlator referenced to the pseudorange

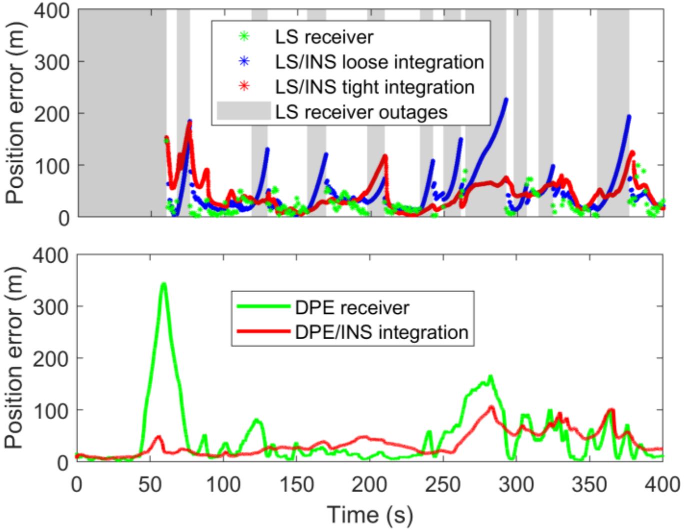
Axelrad, P., Bradley, B. K., Donna, J., Mitchell, M., & Mohiuddin, S. (2011). Collective Detection and Direct Positioning Using Multiple GNSS Satellites. NAVIGATION, 58(4), 305-321. https://doi.org/10.1002/j.2161-4296.2011.tb02588.x

Email: seergio.vicenzo@connect.polyu.hk

Though more robust than 2SP, <u>DPE error still climbs up to tens even hundreds of meters</u> under MP and NLOS



Jia, Q., & Guo, Q. (2025). Design of Dual-Mode DPE Receiver Based on GPS L1/BDS B1C Signals. Proceedings of the 2025 International Technical Meeting of The Institute of Navigation, Long Beach, California.



Gao, W., Yang, R., Huang, J., & Zhan, X. (2025). Quasi-Deep Integration for DPE/INS in GNSS Navigation Domain: Framework Design and Optimization. IEEE Transactions on Aerospace and Electronic Systems, 61(3), 6774-6793. https://doi.org/10.1109/taes.2025.3532891

Ng and Gao (2016) and Strandjord (2020) have introduced 3DMA-based methods for NLOS mitigation for DPE.

But **MP mitigation for DPE** has never been proposed before!

Ng, Y., & Gao, G. X. (2016). Direct position estimation utilizing non-line-of-sight (NLOS) GPS signals. Proceedings of the 29th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2016), Portland, Oregon.
Strandjord, K., Axelrad, P., Akos, D. M., & Mohiuddin, S. (2020). Improved urban navigation with direct positioning and specular matching. Proceedings of the 2020 International Technical Meeting of The Institute of Navigation, San Diego, California.

Email: seergio.vicenzo@connect.polyu.hk

Multipath Mitigation Technology (MMT)*

Additional reflected path signal replica

$$\underbrace{\hat{\boldsymbol{\tau}}^{\mathrm{LOS}},\hat{\boldsymbol{\tau}}^{\mathrm{NLOS}}}_{\text{Code Delays}},\underbrace{\hat{\mathbf{a}}^{\mathrm{LOS}},\hat{\mathbf{a}}^{\mathrm{NLOS}}}_{\text{Complex Amplitudes}} = \arg\min_{\boldsymbol{\tau}^{\mathrm{LOS}},\boldsymbol{\tau}^{\mathrm{NLOS}},\mathbf{a}^{\mathrm{LOS}},\mathbf{a}^{\mathrm{NLOS}}} \sum_{i=1}^{\mathrm{M}} \left\| \mathbf{x} - \mathbf{c}_i\left(\tau_i^{\mathrm{LOS}}\right)\mathrm{a}^{\mathrm{LOS},i} - \boxed{\mathbf{c}_i\left(\tau_i^{\mathrm{NLOS}}\right)\mathrm{a}^{\mathrm{NLOS},i}} \right\|^2$$

Direct Position Estimation (DPE)

$$\hat{\boldsymbol{\gamma}} = \arg\min_{\boldsymbol{\gamma},\mathbf{a}} \sum_{i=1}^{\mathrm{M}} \left\| \mathbf{x} - \mathbf{c}_i(\boldsymbol{\gamma})\mathrm{a}^{i} \right\|^2$$

Integrating MMT cost function into DPE

Since $\boldsymbol{\tau}^{\mathrm{LOS}} \triangleq \mathbf{p}$

MMT-DPE

$$\hat{\boldsymbol{\gamma}} = \arg\min_{\boldsymbol{\gamma},\boldsymbol{\tau}^{\mathrm{NLOS}},\mathbf{a}^{\mathrm{LOS}},\mathbf{a}^{\mathrm{NLOS}}} \sum_{i=1}^{\mathrm{M}} \left\| \mathbf{x} - \mathbf{c}_i(\boldsymbol{\gamma})\mathrm{a}^{\mathrm{LOS},i} - \mathbf{c}_i\left(\tau_i^{\mathrm{NLOS}}\right)\mathrm{a}^{\mathrm{NLOS},i} \right\|^2$$

***Assuming Doppler compensated baseband signals**

Weill, L. R. (2002). Multipath mitigation using modernized GPS signals: how good can it get? Proceedings of the 15th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 2002), Portland, Oregon.

Email: seergio.vicenzo@connect.polyu.hk

Methodology – Proposed MMT-DPE

Multipath Mitigation Technology

$$\hat{\boldsymbol{\tau}}^{\mathrm{LOS}}, \hat{\boldsymbol{\tau}}^{\mathrm{NLOS}}, \hat{\mathbf{a}}^{\mathrm{LOS}}, \hat{\mathbf{a}}^{\mathrm{NLOS}}$$

$$= \arg \min_{\boldsymbol{\tau}^{\mathrm{LOS}}, \boldsymbol{\tau}^{\mathrm{NLOS}}, \mathbf{a}^{\mathrm{LOS}}, \mathbf{a}^{\mathrm{NLOS}}} \sum_{i=1}^{M} \left\| \mathbf{x} - \mathbf{c}_i(\tau_i^{\mathrm{LOS}}) \mathrm{a}^{\mathrm{LOS},i} - \mathbf{c}_i(\tau_i^{\mathrm{NLOS}}) \mathrm{a}^{\mathrm{NLOS},i} \right\|^2$$

1 2

where

3 4

LOS path complex amplitude

$$\mathrm{a}^{\mathrm{LOS},i} = A^{\mathrm{LOS},i}\left(\cos\left(\varphi_{\mathrm{carr}}^{\mathrm{LOS},i}\right) + j\sin\left(\varphi_{\mathrm{carr}}^{\mathrm{LOS},i}\right)\right)$$

Reflected path complex amplitude

$$\mathrm{a}^{\mathrm{NLOS},i} = A^{\mathrm{NLOS},i}\left(\cos\left(\varphi_{\mathrm{carr}}^{\mathrm{NLOS},i}\right) + j\sin\left(\varphi_{\mathrm{carr}}^{\mathrm{NLOS},i}\right)\right)$$

5 6

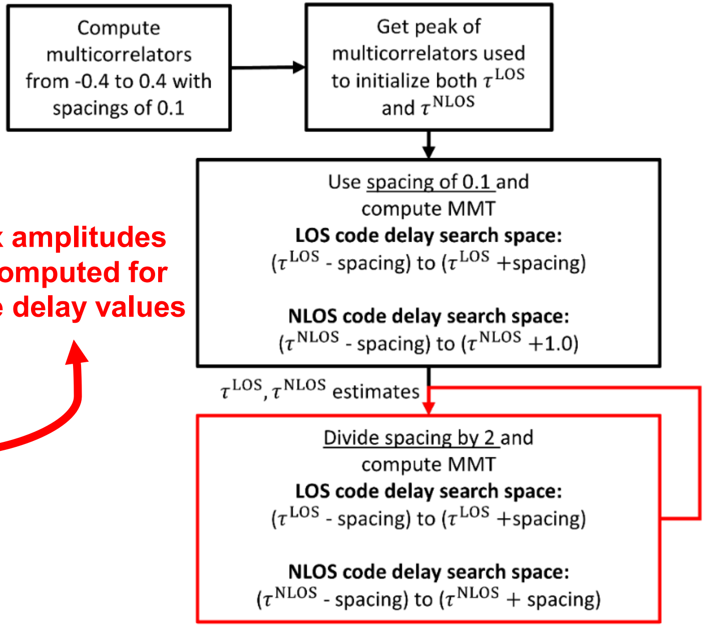
- Six dimensions optimization → Two dimensions only, through *partial derivatives* of complex amplitudes → **Perform MMT w.r.t code delays only**
- A **Lagrange multiplier** is applied, constraining the reflected path's scalar amplitude to be 0.8 of the LOS path

$$\frac{\|\mathrm{a}^{\mathrm{NLOS},i}\|}{\|\mathrm{a}^{\mathrm{LOS,i}}\|} \leq 0.8$$

- **Contracting grid-search algorithm** to reduce computational load (Axelrad et al., 2011)

Complex amplitudes can be computed for each code delay values

Contracting grid-search algorithm for MMT



Weill, L. R. (2002). Multipath mitigation using modernized GPS signals: how good can it get? Proceedings of the 15th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 2002), Portland, Oregon.

Axelrad, P., Bradley, B. K., Donna, J., Mitchell, M., & Mohiuddin, S. (2011). Collective Detection and Direct Positioning Using Multiple GNSS Satellites. NAVIGATION, 58(4), 305-321. https://doi.org/10.1002/j.2161-4296.2011.tb02588.x

Email: seergio.vicenzo@connect.polyu.hk

Methodology – Proposed MMT-DPE

Practical implementation

1. **Run standalone MMT** (tracking_MMT.m) → Obtain $a^{LOS,i}$, $a^{NLOS,i}$, τ_i^{NLOS} and MMT-2SP

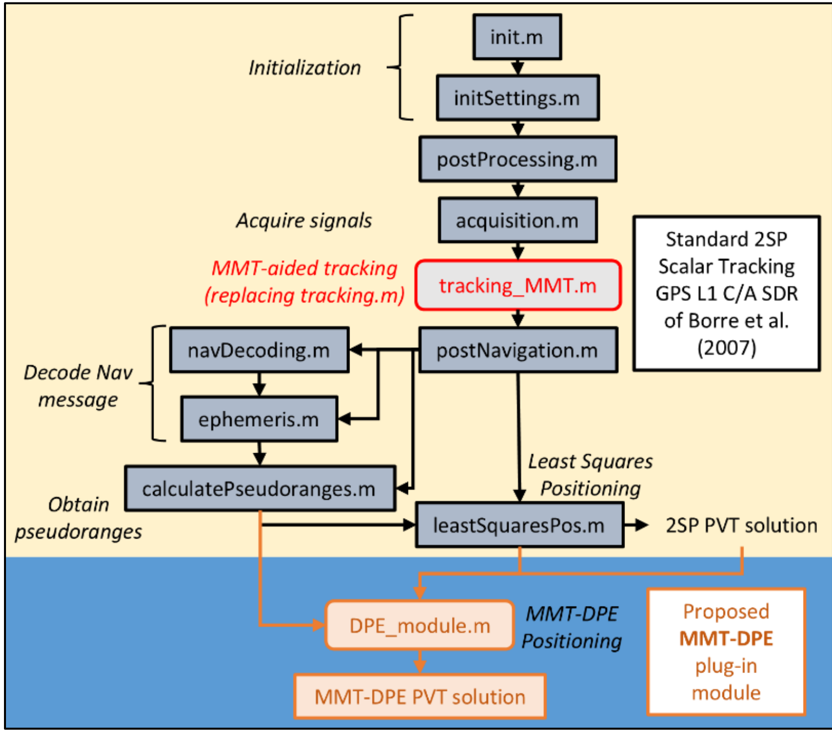
2. **Run MMT-DPE** (DPE_module.m) → Obtain PVT, $\boldsymbol{\gamma}$

$$\hat{\boldsymbol{\gamma}} = \arg\min_{\boldsymbol{\gamma},\boldsymbol{\tau}^{NLOS},\mathbf{a}^{LOS},\mathbf{a}^{NLOS}} \sum_{i=1}^{M} \left\| \mathbf{x} - \mathbf{c}_i(\boldsymbol{\gamma}) a^{LOS,i} - \mathbf{c}_i\left(\tau_i^{NLOS}\right) a^{NLOS,i} \right\|^2$$

which is equivalent to the following <u>maximization</u>

$$\hat{\boldsymbol{\gamma}} = \arg\max_{\boldsymbol{\gamma},\boldsymbol{\tau}^{NLOS},\mathbf{a}^{LOS},\mathbf{a}^{NLOS}} \left\{ \sum_{i=1}^{M} \left(\begin{array}{c} \mathbf{x}^H \mathbf{c}_i(\boldsymbol{\gamma}) a^{LOS,i} + \mathbf{x}^H \mathbf{c}_i\left(\tau_i^{NLOS}\right) a^{NLOS,i} + \\ \left(a^{LOS,i}\right)^H \left(\mathbf{c}_i(\boldsymbol{\gamma})\right)^H \mathbf{x} - \\ \left(a^{LOS,i}\right)^H \left(\mathbf{c}_i(\boldsymbol{\gamma})\right)^H \mathbf{c}_i(\boldsymbol{\gamma}) a^{LOS,i} - \\ \left(a^{LOS,i}\right)^H \left(\mathbf{c}_i(\boldsymbol{\gamma})\right)^H \mathbf{c}_i\left(\tau_i^{NLOS}\right) a^{NLOS,i} + \\ \left(a^{NLOS,i}\right)^H \left(\mathbf{c}_i\left(\tau_i^{NLOS}\right)\right)^H \mathbf{x} - \\ \left(a^{NLOS,i}\right)^H \left(\mathbf{c}_i\left(\tau_i^{NLOS}\right)\right)^H \mathbf{c}_i(\boldsymbol{\gamma}) a^{LOS,i} - \\ \left(a^{NLOS,i}\right)^H \left(\mathbf{c}_i\left(\tau_i^{NLOS}\right)\right)^H \mathbf{c}_i\left(\tau_i^{NLOS}\right) a^{NLOS,i} \end{array} \right) \right\}$$

Flowchart of MMT-DPE



Weill, L., & Fisher, B. (2002). Method for mitigating multipath effects in radio systems. In: Google Patents.

Email: seergio.vicenzo@connect.polyu.hk

Methodology – Data Collection Method

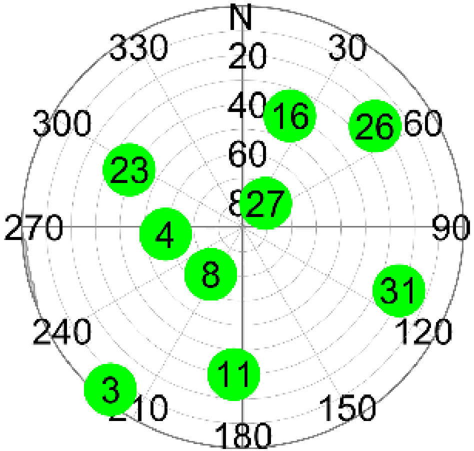
- Evaluation of the proposed DPE module will use real static **GPS L1 C/A** data collected in Hong Kong.
- 2SP scalar tracking uses 0.6 Early-Late correlator spacing
- Coherent integration of **20 ms** used for all algorithms

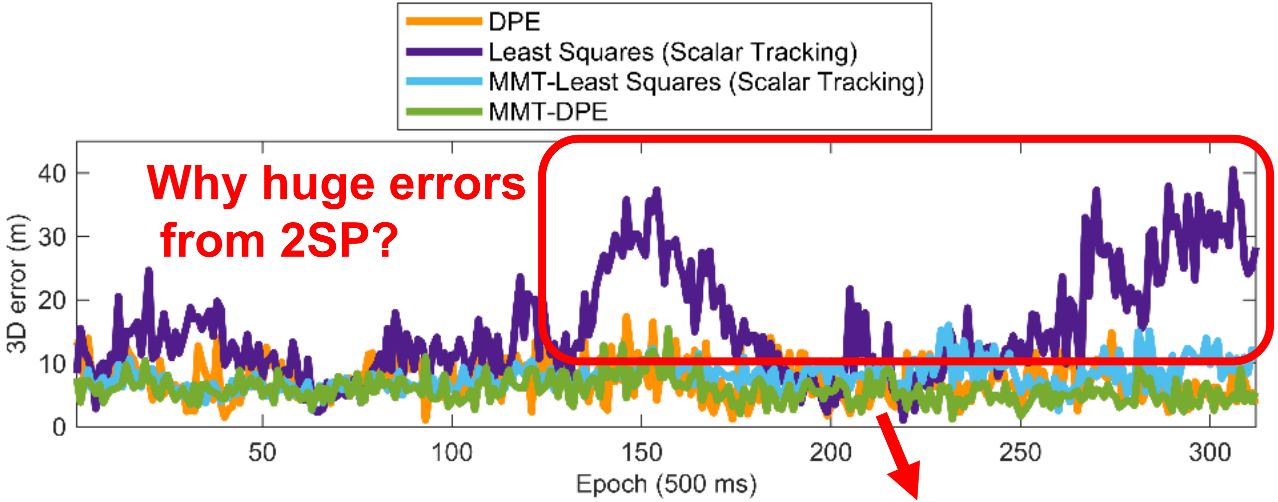
Front-end Configuration

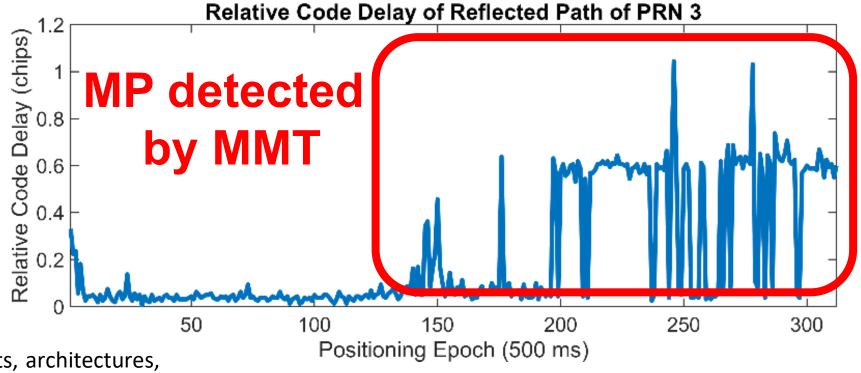
Front-end	**NSL Stereo**
Sampling Frequency	26 MHz
Bandwidth (Double-sided)	8 MHz
Antenna	Allystar AGR6303

Email: seergio.vicenzo@connect.polyu.hk

Results – Open sky scenario



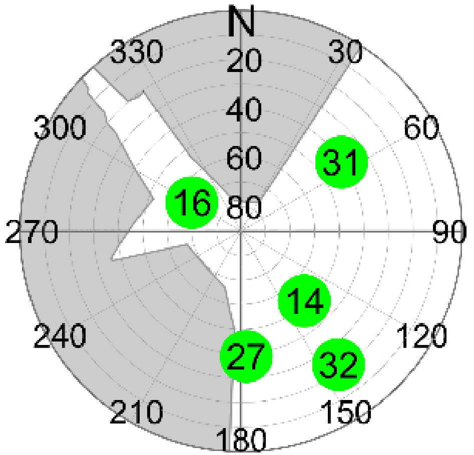


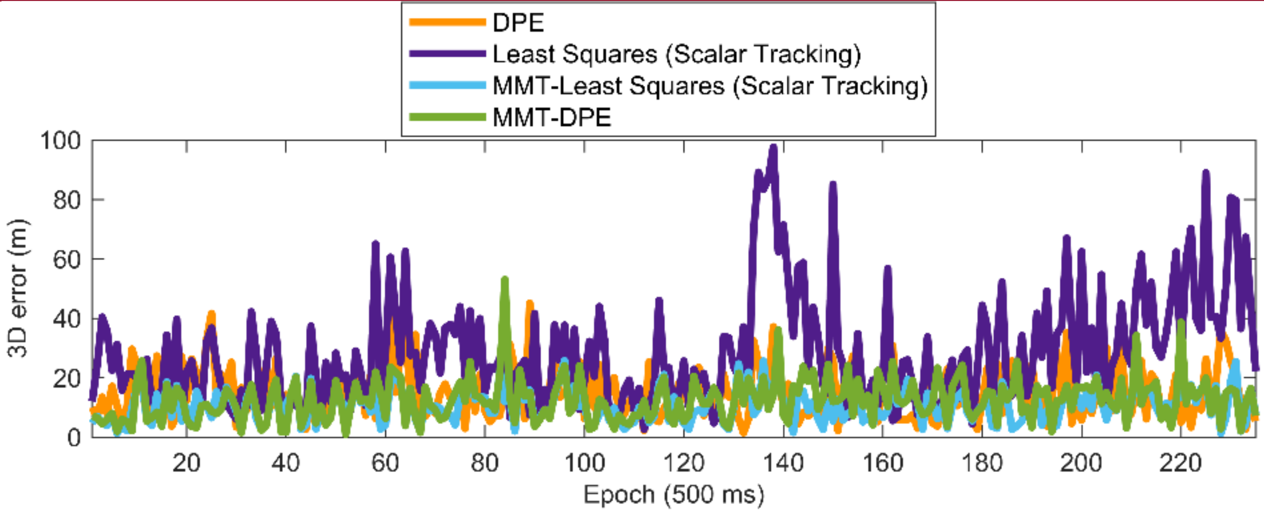


2SP Mean 3D Error (m)	DPE Mean 3D Error (m)	MMT-2SP Mean 3D Error (m)	MMT-DPE Mean 3D Error (m)
15.46	7.28	7.89	5.81

Closas, P., & Gusi-Amigo, A. (2017). Direct position estimation of GNSS receivers: Analyzing main results, architectures, enhancements, and challenges. IEEE Signal Processing Magazine, 34(5), 72-84.

Results – Urban



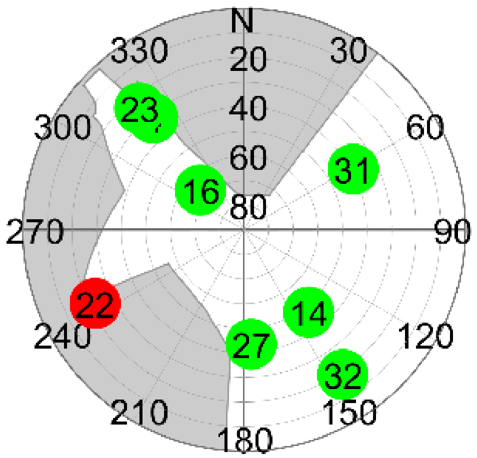


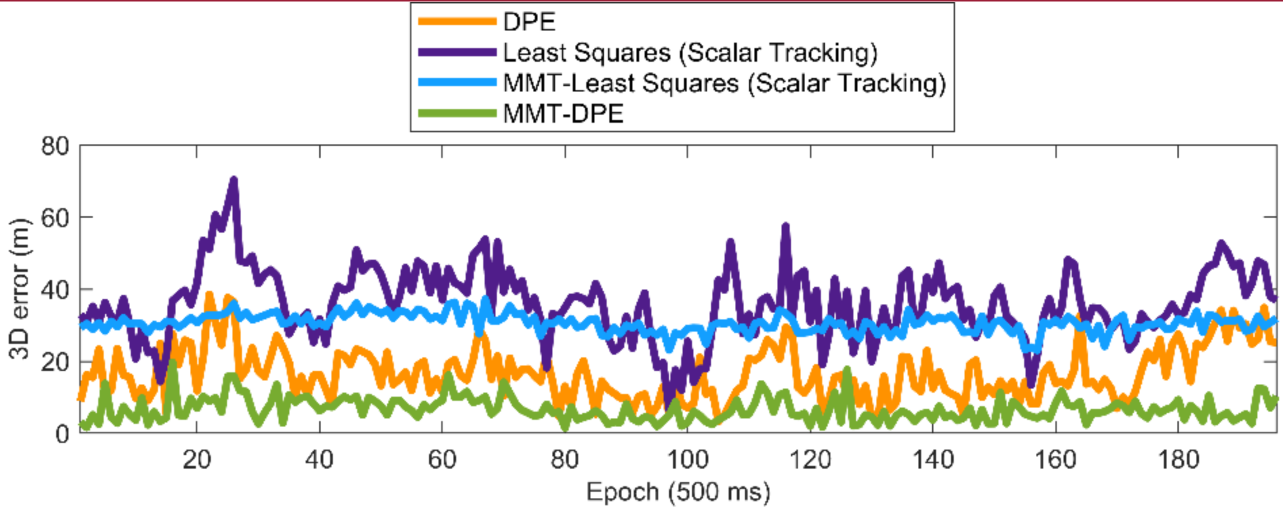
2SP Mean 3D Error (m)	DPE Mean 3D Error (m)	MMT-2SP Mean 3D Error (m)	MMT-DPE Mean 3D Error (m)
28.84	15.10	10.55	12.44

Comparable performance between MMT-2SP and MMT-DPE under MP-only cases

Email: seergio.vicenzo@connect.polyu.hk

Results – Urban with NLOS





2SP Mean 3D Error (m)	DPE Mean 3D Error (m)	MMT-2SP Mean 3D Error (m)	MMT-DPE Mean 3D Error (m)
36.11	16.72	**30.64**	**6.71**

Amar, A., & Weiss, A. J. (2005). Analysis of direct position determination approach in the presence of model errors. IEEE/SP 13th Workshop on Statistical Signal Processing, 2005, Bordeaux, France.

Correlations of an NLOS satellite

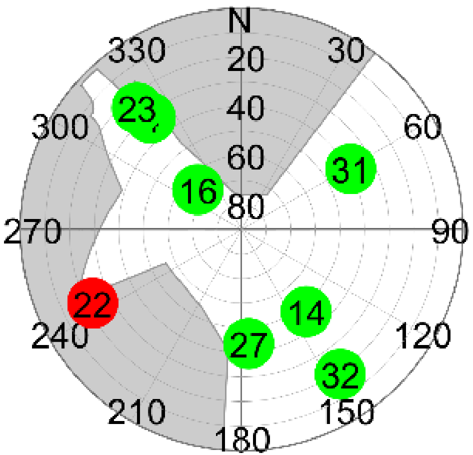
1. **DEVIATE** from the ground truth

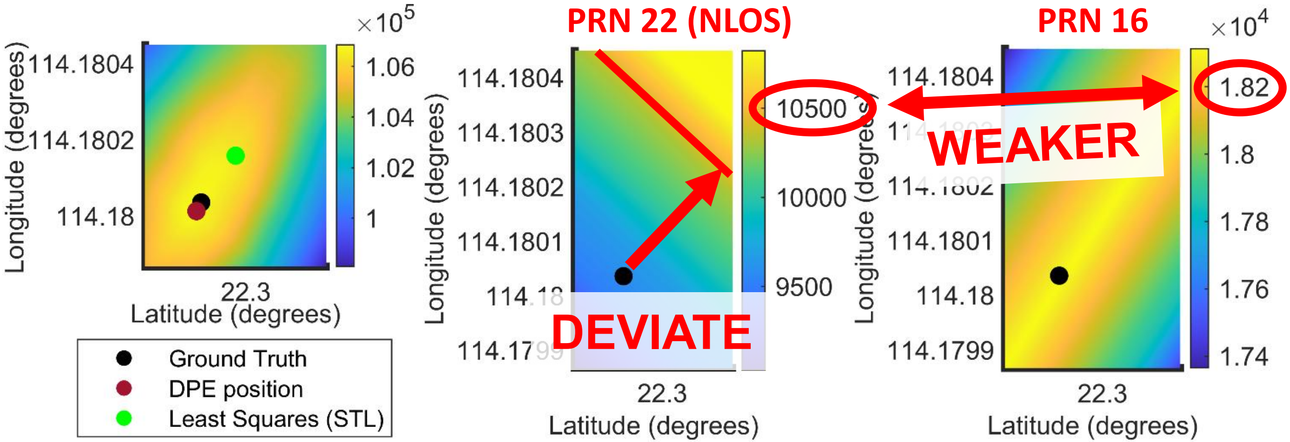
2. Much **WEAKER** compared to that of the highly elevated

Thus, NLOS satellite correlations do not contribute to the global maxima

i.e., **NLOS measurement naturally excluded from PVT estimation**

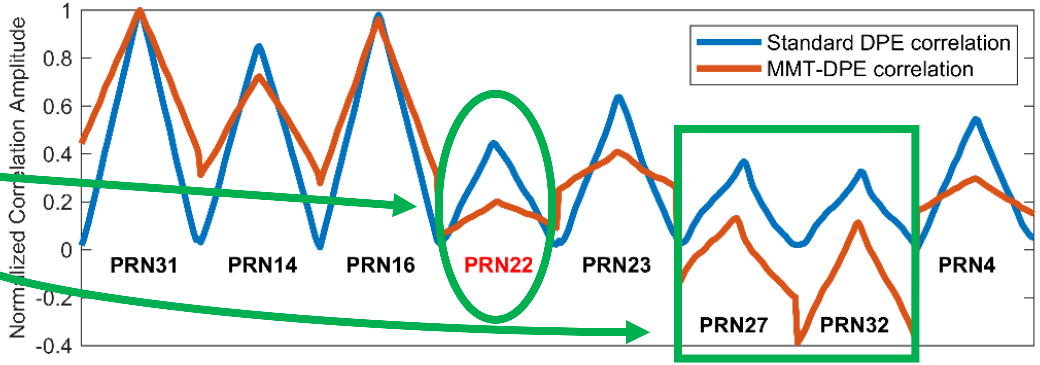
Email: seergio.vicenzo@connect.polyu.hk





Pre-calculated correlations between DPE and MMT-DPE

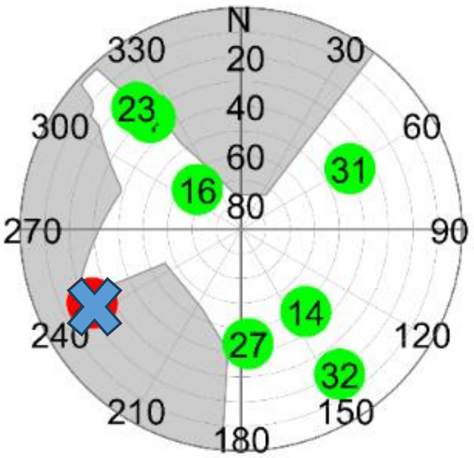
With MMT-DPE, NLOS and low-elevated satellite correlations are further weighed down

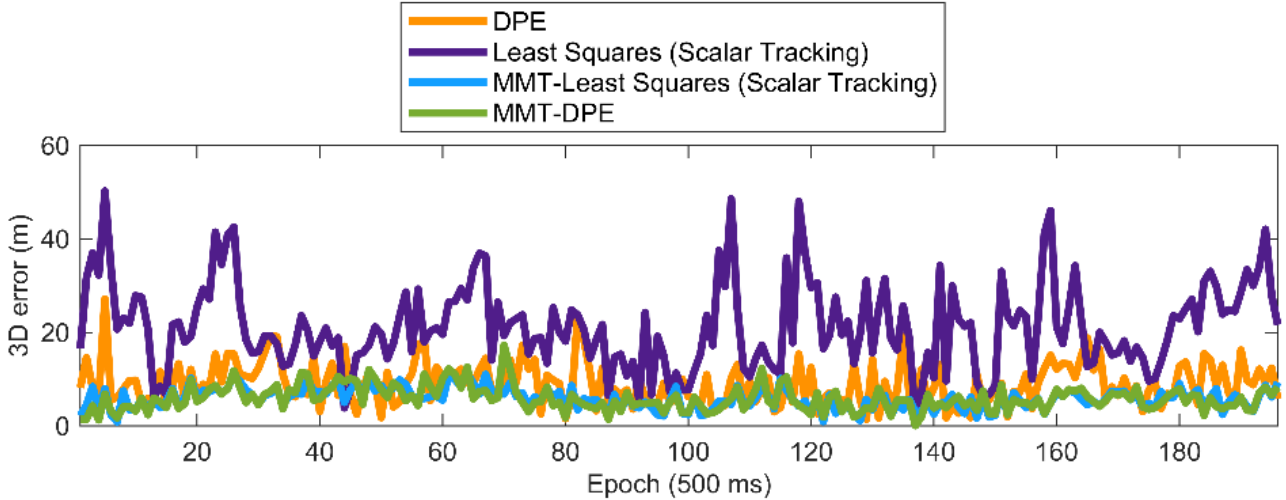


Amar, A., & Weiss, A. J. (2005). Analysis of direct position determination approach in the presence of model errors. IEEE/SP 13th Workshop on Statistical Signal Processing, 2005, Bordeaux, France.

Email: seergio.vicenzo@connect.polyu.hk

Results – Urban





2SP Mean 3D Error (m)	DPE Mean 3D Error (m)	MMT-2SP Mean 3D Error (m)	MMT-DPE Mean 3D Error (m)
21.32	9.03	5.84	5.85

Improvement can be observed for both DPE and 2SP when NLOS is removed

Email: seergio.vicenzo@connect.polyu.hk

Conclusion

Aim

Promotion and **popularization** of DPE research and application

Objectives

1. Proposed our novel implementation of a DPE receiver

- DPE implementation with pseudoranges from 2SP
- Proposed DPE consistently outperforms 2SP in terms of MP mitigation
- **Benefit?** Easier implementation of DPE. The pre-calculated correlations (both for DPE and MMT-DPE) *can* be computed in conjunction with 2SP tracking and later be easily indexed to ranges.

Email: seergio.vicenzo@connect.polyu.hk

Conclusion

2. Improving GNSS DPE performance in urban environments

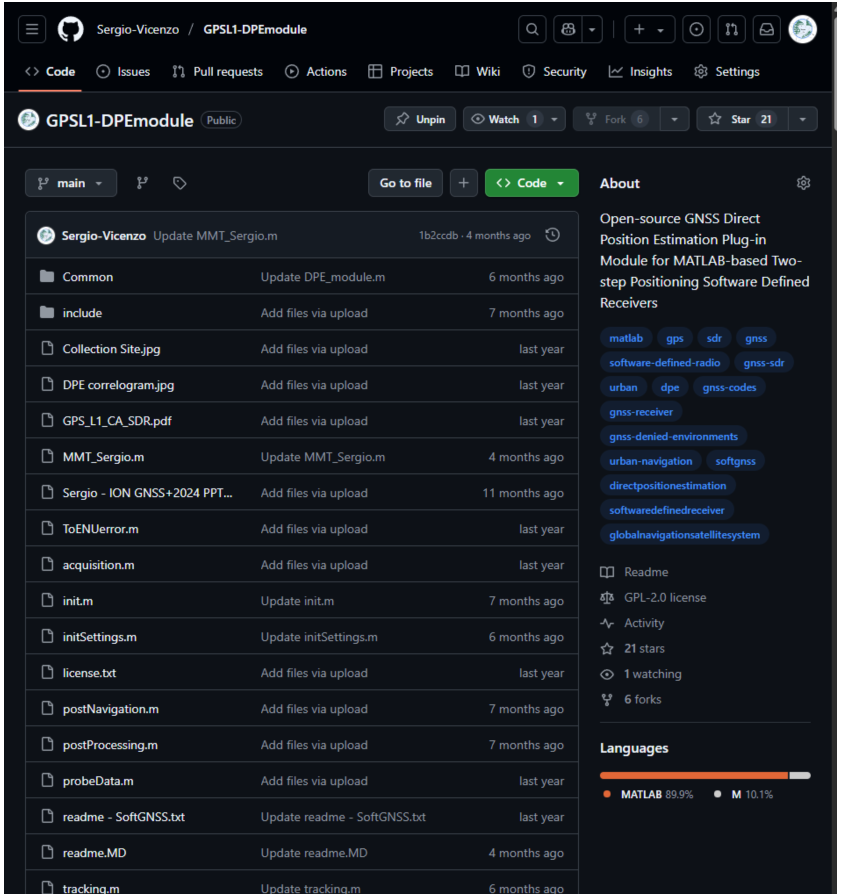
- Proposed incorporating MMT into DPE to compensate for the additional reflected signal from Multipath reception into DPE signal model

- Under MP-only conditions, MMT-2SP and MMT-DPE performs similarly

- But MMT-DPE shows superiority in NLOS conditions

Email: seergio.vicenzo@connect.polyu.hk

Our Proposed MMT-DPE – Available at GitHub



The code is now available at **GitHub**

https://github.com/Sergio-Vicenzo/GPSL1-DPEmodule

Borre, K., Akos, D. M., Bertelsen, N., Rinder, P., & Jensen, S. H. (2007). A software-defined GPS and galileo receiver: A single-frequency approach.

Weill, L. R. (2002). Multipath mitigation using modernized GPS signals: how good can it get? Proceedings of the 15th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 2002), Portland, Oregon.

Email: seergio.vicenzo@connect.polyu.hk

Thank you!

Sergio VICENZO, PhD student, HK PolyU

Email: seergio.vicenzo@connect.polyu.hk (yes 2 Es, but one e is also fine. I own both emails)

Google Scholar: https://scholar.google.com.hk/citations?user=MjNYX3kAAAAJ&hl=en

ResearchGate: https://www.researchgate.net/profile/Sergio_Vicenzo

ORCID: https://orcid.org/0000-0003-2974-7899

LinkedIn: http://www.linkedin.com/in/sergio-vicenzo/

Multipath Mitigation Technology

$$\hat{\tau}_i^{\mathrm{LOS}}, \hat{\tau}_i^{\mathrm{NLOS}}, \hat{\mathrm{a}}^{\mathrm{LOS},i}, \hat{\mathrm{a}}^{\mathrm{NLOS},i} = \arg \min_{\tau_i^{\mathrm{LOS}}, \tau_i^{\mathrm{NLOS}}, \mathrm{a}^{\mathrm{LOS},i}, \mathrm{a}^{\mathrm{NLOS},i}} \underbrace{\left\| \mathbf{x} - \mathbf{c}_i(\tau_i^{\mathrm{LOS}})\mathrm{a}^{\mathrm{LOS},i} - \mathbf{c}_i(\tau_i^{\mathrm{NLOS}})\mathrm{a}^{\mathrm{NLOS},i} \right\|^2}_{\Gamma}$$

minimization of

LOS path signal replica

Reflected path signal replica

$$\Gamma = \int_0^T \left[x(t) - A_1 \cos\phi_1 m(t-\tau_1) - A_2 \cos\phi_2 m(t-\tau_2) \right]^2 dt + \int_0^T \left[y(t) - A_1 \sin\phi_1 m(t-\tau_1) - A_2 \sin\phi_2 m(t-\tau_2) \right]^2 dt \quad (13)$$

Real part

Imaginary part

However, a major breakthrough results by using the invertible transformation

Real part of complex amplitude

$$a = A_1 \cos\phi_1 \qquad b = A_2 \cos\phi_2$$

Imaginary part of complex amplitude

$$c = A_1 \sin\phi_1 \qquad d = A_2 \sin\phi_2 \quad (14)$$

$$R_{xm}(\tau) = \int_0^T x(t) m(t-\tau) dt \qquad R_{ym}(\tau) = \int_0^T y(t) m(t-\tau) dt \qquad R_{mm}(\tau) = \int_0^T m(t) m(t-\tau) dt$$

$$\begin{aligned}\Gamma = &\int_0^T \left[x^2(t) + y^2(t)\right] dt \\ &+ \left(a^2 + b^2 + c^2 + d^2\right) R_{mm}(0) \\ &- 2aR_{xm}(\tau_1) - 2bR_{xm}(\tau_2) + 2abR_{mm}(\tau_1 - \tau_2) \\ &- 2cR_{ym}(\tau_1) - 2dR_{ym}(\tau_2) + 2cdR_{mm}(\tau_1 - \tau_2)\end{aligned} \quad (15)$$

Taking partial derivatives w.r.t real and imaginary parts of the complex amplitudes

$$\begin{aligned}0 &= \frac{\partial \Gamma}{\partial a} = 2aR_{mm}(0) - 2R_{xm}(\tau_1) + 2bR_{mm}(\tau_1 - \tau_2) \\ 0 &= \frac{\partial \Gamma}{\partial b} = 2bR_{mm}(0) - 2R_{xm}(\tau_2) + 2aR_{mm}(\tau_1 - \tau_2) \\ 0 &= \frac{\partial \Gamma}{\partial c} = 2cR_{mm}(0) - 2R_{ym}(\tau_1) + 2dR_{mm}(\tau_1 - \tau_2) \\ 0 &= \frac{\partial \Gamma}{\partial d} = 2dR_{mm}(0) - 2R_{ym}(\tau_2) + 2cR_{mm}(\tau_1 - \tau_2)\end{aligned} \quad (17)$$

The complex amplitudes can be solved for every value of LOS and reflected path code delays

Weill, L. R. (2002). Multipath mitigation using modernized GPS signals: how good can it get? Proceedings of the 15th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 2002), Portland, Oregon.

Email: seergio.vicenzo@connect.polyu.hk