Fall 2022

Assignment 06

01. (a)

$$I = \int_1^3 e^x - x \quad dx$$

$$= \int_1^3 e^x \, dx - \int_1^3 x \, dx$$

$$= \left[e^x\right]_1^3 - \left[\frac{x^2}{2}\right]_1^3$$

$$= e^3 - e - 4$$

$$= 13.3673$$

(b) $m = 4$, $a = 1$, $b = 3$

$$h = \frac{b-a}{m} = \frac{3-1}{4} = \frac{1}{2}$$

$a = x_0$, $x_1 = a + h$, $x_2 = x_1 + h$, $x_3 = x_2 + h$, $x_4 = b$

$\Rightarrow x_0 = 1$, $x_1 = \frac{3}{2}$, $x_2 = 2$, $x_3 = \frac{5}{2}$. $x_4 = 3$

$$C_{1,4} = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$= \frac{1}{4}\left[1.7183 + 2\times 2.9817 + 2\times 5.3891 + 2\times 9.6825 + 17.0855\right]$$

$$= 13.7276$$

(c)

$$\text{Error} = \left|\frac{I - C_{1,4}}{I}\right| \times 100\%$$

$$= \left|\frac{13.3673 - 13.7276}{13.3673}\right| \times 100\%$$

$$= |-0.02695| \times 100\%$$

$$= 2.695\%$$

we can decrease the error more by increasing the value of m or the number of segments.

(d)

$$I_2(f) = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$= \frac{3-1}{6}\left[1 \cdot 7183 + 4 \times 5 \cdot 3891 + 17 \cdot 0855\right]$$

$$= 13 \cdot 4534$$

02. (a) $a = -2,\ b = 2$

$$I_1(f) = \frac{b-a}{2}\left[f(a) + f(b)\right]$$

$$= \frac{2+2}{2}\left[23 + 7\right]$$

$$= 60$$

(b) $$I = \int_{-2}^{2} 6x^2 - 4x - 9$$

$$= \left[2x^3 - 2x^2 - 9x\right]_{-2}^{2}$$

$$= -10 + 6$$

$$= -4$$

(c) Relative Error $= \left|\frac{I - I_1}{I}\right| \times 100\%$

$$= \left|\frac{-4 - 60}{-4}\right| \times 100\%$$

$$= 16 \times 100\%$$

$$= 1600\%$$