

Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the part of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file.
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

1. Consider A function $f(x) = x^3 - 7x^2 + 4x + 12$.

- (2 marks) Construct two different fixed point functions $g(x)$ such that $f(x) = 0$.
- (3 marks) Compute the convergence rate of each fixed point function $g(x)$ obtained in the previous part, and state which root it is converging to or diverging.
- (3 marks) How many iterations will be required to find the root if the machine epsilon is 1.4×10^{-18} .
- (4 marks) Show 4 iterations using the Bisection Method to find the root of the above function within the interval $[4.25, 8.95]$.
- (4 marks) Starting from $x_0 = 2.26$ find the approximate root of $f(x)$ up to four iterations by applying Aitken acceleration appropriately. Express your result up to five decimal places.

2. A linear system is described by the following equations.

$$\begin{aligned} 2a - 2b + c &= -3 \\ a + 3b - 2c &= 1 \\ 3a - b - c &= 2 \end{aligned}$$

- (2 marks) From the given linear equations, identify the matrices A , x and b such the the linear system can be expressed as a matrix equation.
- (3 marks) Does this system have any unique solution? Explain.
- (3 marks) Evaluate the upper triangular matrix U . Note that you have to show the row multipliers m_{ij} for each step as necessary.
- (4 marks) Using the upper triangular matrix found in the previous question, compute the solution of the given linear system by Gaussian elimination method.

3. A linear system is described by the following equations.

$$\begin{aligned} 3x + 3y + 4z &= 1 \\ -z &= 6 \\ -y - 3z &= 4 \end{aligned}$$

- (2 marks) From the given linear equations, identify the matrices A . Examine if the matrix A has any pivoting problem? Explain why or why not?
- (3 marks) Compute the Frobenius matrices $F^{(1)}$ and $F^{(2)}$ for this system.
- (3 marks) Evaluate the unit lower triangular matrix L , and the upper triangular matrix U .
- (4 marks) Now compute the solution of the given linear system using LU-decomposition method. Use the matrices L and U found in the previous question. Show your works.