# Assignment 02

1 (a) $f(x) = \sin x$

$x_0 = 0 \quad f[x_0] = 0$

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$$f[x_0, x_1] = \frac{2}{\pi}$$

$x_1 = \pi/2 \quad f[x_1] = 1$

$$f[x_0, x_1, x_2] = -\frac{4}{\pi^2}$$

$$f[x_1, x_2] = -2/\pi$$

$x_2 = \pi \quad f[x_2] = 0$

(b)

$$P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

$$= f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

$$= 0 + \frac{2}{\pi}(x - 0) + \left(-4/\pi^2\right)(x - 0)(x - \pi/2)$$

$$= \frac{2}{\pi}x - \frac{4}{\pi^2}x(x - \pi/2)$$

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$$P_2(x = 3\pi/2) = \frac{2}{\pi}\left(\frac{3\pi}{2}\right) - \frac{4}{\pi^2}\left(\frac{3\pi}{2}\right)\left(\frac{3\pi}{2} - \frac{\pi}{2}\right)$$

$$= -3$$

(c) Let. $x_3 = 2\pi$

$x_0 = 0 \quad f[x_0] = 0$

$f[x_0, x_1] = 2/\pi$

$x_1 = \pi/2 \quad f[x_1] = 1$

$f[x_0, x_1, x_2] = -4/\pi^2$

$f[x_1, x_2] = -2/\pi$

$x_2 = \pi \quad f[x_2] = 0$

$f[x_0, x_1, x_2, x_3] = \frac{8}{3\pi^3}$

$f[x_1, x_2, x_3] = 4/3\pi^2$

$f[x_2, x_3] = 0$

$x_3 = 2\pi \quad f[x_3] = 0$

$$P_3(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$

$$= 0 + 2/\pi\, x + (-4/\pi^2)\, x(x - \pi/2) + \frac{8}{3\pi^3} x(x - \pi/2)(x - \pi)$$

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$$= \frac{2}{\pi} x - \frac{4}{\pi^2} x(x - \pi/2) + \frac{8}{3\pi^3} x(x - \pi/2)(x - \pi)$$

(d) $$|f(x) - P_3(x)| = \frac{f^{3+1}(\xi)}{(3+1)!}(x-0)(x-\pi/2)(x-\pi)(x-2\pi)$$

$$= \frac{f^{4}(\xi)}{4!}\, x(x-\pi/2)(x-\pi)(x-2\pi)$$

Here, $$w(x) = x(x-\pi/2)(x-\pi)(x-2\pi)$$

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$$= \left(x^2 - x\pi/2\right)(x-\pi)(x-2\pi)$$

$$= \left(x^3 - \pi x^2 - \frac{x^2\pi}{2} + \frac{3\pi^2 x}{2}\right)(x-2\pi)$$

$$= x^4 - 2\pi x^3 - \pi x^3 + 2\pi^2 x^2 - \frac{x^3\pi}{2} + x^2\pi^2$$
$$+ \frac{\pi^2 x^2}{2} - \pi^3 x$$

$$= x^4 - \frac{7\pi x^3}{2} + \frac{7\pi^2 x^2}{2} - \pi^3 x$$

(e) $|f(x) - P_3(x)| = \frac{f^4(\xi)}{4!} (x-0)(x-\pi/2)(x-\pi)(x-2\pi)$

$f^4(x) = \sin x$

$\max \frac{f^4(\xi)}{4!} = \frac{1}{4!}$

$w(x) = (x-0)(x-\pi/2)(x-\pi)(x-2\pi)$

$= x^4 - \frac{7\pi x^3}{2} + \frac{7\pi^2 x^2}{2} - \pi^3 x$

$w'(x) = 4x^3 - \frac{21\pi x^2}{2} + 7\pi^2 x - \pi^3 = 0$

$\Rightarrow x = 5.225, \ 2.405, \ 0.617$

$\therefore |f(x) - P_3(x)| =$

| $x$ | $w(x)$ |
|---|---|
| 5.225 | -42.093 |
| 2.405 | 5.731 ✓ |
| 0.617 | -8.418 |

$\frac{1}{4!} \times 5.731$

$= 0.2388$

2. (a)

$$l_0(x) = \frac{(x-x_1)}{(x_0-x_1)} = \frac{x-1}{-1-1} \cdot \frac{x-1}{-2}$$

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$$l_1(x) = \frac{(x-x_0)}{(x_1-x_0)} = \frac{x-(-1)}{1-(-1)} = \frac{x+1}{2}$$

(b) $h_0(x) = l_0^2(x)\left(1-2(x-x_0)\, l_0'(x_0)\right)$

$$= \frac{(x-1)^2}{4}\left(1-2(x+1)\left(-\frac{1}{2}\right)\right)$$

$$= \frac{(x-1)^2}{4}(x+2)$$

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$\hat{h}_0(x) = l_0^2(x)(x-x_0)$

$$= \frac{(x-1)^2}{4}(x+1)$$

$h_1(x) = l_1^2(x)\left(1-2(x-x_1)\, l_1^2(x_1)\right)$

$$= \frac{(x+1)^2}{4}\left(1-2(x-1)\left(\frac{1}{2}\right)\right)$$

$$= \frac{(x+1)^2}{4}(-x+2)$$

$$\hat{h}_1(x) = l_1^2(x)(x - x_1)$$

$$= \frac{(x+1)^2}{4}(x-1)$$

(c) $n=1$ , $2n+1 = 3$

$$P_3(x) = f(x_0)h_0(x) + f(x_1)h_1(x) + f'(x_0)\hat{h}_0(x) +$$

$$f'(x_1)\hat{h}_1(x)$$

$$= 0 * h_0(x) + 1 * h_1(x) + 1 * \hat{h}_0(x) + 0 * \hat{h}_1(x)$$

$$= h_1(x) + \hat{h}_0(x)$$

$$= \frac{(x+1)^2}{4}(-x+2) + \frac{(x-1)^2}{4}(x+1)$$

$$= -\frac{(x+1)(x-3)}{4}$$

$$= \frac{3}{4} + \frac{x}{2} - \frac{x^2}{4}$$