BRAC University (Department of Computer Science and Engineering)

CSE 330 (Numerical Methods) for Summer 2022 Semester

Quiz 3 (B)

Student ID: **Full Marks: 15**

Name: **Duration: 25 minutes**

[No extra sheet will be provided. Write your answer to the questions in this answer script.]

1. [CO2] Consider the following linear system:
   $x_2+5x_3 = 5$
   $2x_1+4x_3 = 9$
   $2x_3 = 4$
   (a) ***(2 marks)*** Explain why the Gaussian elimination method fails to solve the system.
   (b) ***(3 marks)*** State how we can remove the problem and solve the system by Gaussian elimination method.
2. [CO2] Consider the following linear system:
   $2x_1+3x_2=5$
   $x_1+4x_2=11$
   (a) ***(3 marks)*** Construct the Frobenius matrix $F^{(1)}$ from this system.
   (b) ***(2 marks)*** Compute the unit lower triangular matrix L.
   (c) ***(5 marks)*** Now find the solution of the linear system using LU decomposition method. Use the unit lower triangular matrix found in the previous question.

1 (a) $\left[\begin{array}{ccc|c} 0 & 1 & 5 & 5 \\ 2 & 0 & 4 & 9 \\ 0 & 0 & 2 & 4 \end{array}\right]$; $m_{21} = \frac{a_{21}}{a_{11}} = \frac{2}{0} \Rightarrow$ undefined & $m_{32} = \frac{0}{0}$

therefore, it fails.

(b) by pivoting.

$\left[\begin{array}{ccc|c} 2 & 0 & 4 & 9 \\ 0 & 1 & 5 & 5 \\ 0 & 0 & 2 & 4 \end{array}\right] \Rightarrow 2x_3 = 4 \Rightarrow x_3 = \frac{4}{2} = 2$ | $x_2 + 5x_3 = 5 \Rightarrow x_2 + (5\times 2) = 5 \Rightarrow x_2 = -5$ | $2x_1 + 4x_3 = 9 \Rightarrow 2x_1 + 8 = 9 \Rightarrow 2x_1 = 1 \Rightarrow x_1 = \frac{1}{2}$

2 (a) $\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 1 & 4 & 11 \end{array}\right]$

$\underbrace{\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}}_{A^{(1)}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 5 \\ 11 \end{pmatrix}}_{b}$

$F^{(1)} = \begin{pmatrix} 1 & 0 \\ -m_{21} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \end{pmatrix}$ $m_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{2}$

$= -\frac{3}{2} + 4$

$= \frac{-3+8}{2} = \frac{11}{2}$

(b) $A^{(2)} = F^{(1)} \times A^{(1)}$

$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 2.5 \end{bmatrix} = U$

$L = \begin{pmatrix} 1 & 0 \\ m_{21} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix}$

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$A = LU$

$Ly = b$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$

$\Rightarrow y_1 = 5$; $\frac{1}{2} y_1 + y_2 = 11 \Rightarrow \frac{5}{2} + y_2 = 11 \Rightarrow y_2 = 11 - \frac{5}{2} = \frac{22-5}{2}$

$= \frac{17}{2}$

$\therefore Ux = y \Rightarrow \begin{bmatrix} 2 & 3 \\ 0 & 2.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 17/2 \end{bmatrix}$

$\Rightarrow x_2 = \frac{17/2}{2.5} = 3.4$

$\therefore 2x_1 + 3x_2 = 5$.

$\Rightarrow 2x_1 = 5 - 10.2 = -5.2$

$\therefore x_1 = -2.6$

BRAC University (Department of Computer Science and Engineering)

CSE 330 (Numerical Methods) for Summer 2022 Semester

Quiz 3 (A)

Student ID: Full Marks: 15

Name: Duration: 25 minutes

[No extra sheet will be provided. Write your answer to the questions in this answer script.]

1. [CO2] Consider the following linear system:
   $2x_2+x_3 = 2$
   $3x_1+4x_3 = 8$
   $3x_3 = 3$
   (a) ***(2 marks)*** Explain why the Gaussian elimination method fails to solve the system.
   (b) ***(3 marks)*** State how we can remove the problem and solve the system by Gaussian elimination method.
2. [CO2] Consider the following linear system:
   $x_1+2x_2=7$
   $2x_1+9x_2=13$
   (a) ***(3 marks)*** Construct the Frobenius matrix $F^{(1)}$ from this system.
   (b) ***(2 marks)*** Compute the unit lower triangular matrix L.
   (c) ***(5 marks)*** Now find the solution of the linear system using LU decomposition method. Use the unit lower triangular matrix found in the previous question.

1 a $\left[\begin{array}{ccc|c} 0 & 2 & 1 & 2 \\ 3 & 0 & 4 & 8 \\ 0 & 0 & 3 & 3 \end{array}\right]$ $m_{21} = \frac{a_{21}}{a_{11}} = \frac{3}{0} \Rightarrow$ undefined & $m_{32} = \frac{0}{0} =$

therefore, it fails.

b by pivoting.

$\left[\begin{array}{ccc|c} 3 & 0 & 4 & 8 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 3 & 3 \end{array}\right] \Rightarrow$ $3x_3 = 3$, $\therefore x_3 = 1$ | $2x_2 + x_3 = 2 \Rightarrow 2x_2 = 2-1 \Rightarrow x_2 = \frac{1}{2}$ | $3x_1 + 4x_3 = 8 \Rightarrow 3x_1 = 8-4 \Rightarrow x_1 = \frac{4}{3}$

2 a $\begin{bmatrix} 1 & 2 \\ 2 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$

$A^{(1)}$ x b

$F^{(1)} = \begin{pmatrix} 1 & 0 \\ -m_{21} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$ $m_{21} = \frac{a_{21}}{a_{11}} = \frac{2}{1} = 2$

b $A^{(2)} = F^{(1)} \times A^{(1)}$

$= \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ 2 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 5 \end{pmatrix}$

$= U$

$L = \begin{pmatrix} 1 & 0 \\ m_{21} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

$\therefore A = LU$

$Ax = b \Rightarrow LUx = b$

$\boxed{Ux = y}$ $\quad \therefore Ly = b$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$$

$\Rightarrow y_1 = 7 \; ; \; 2y_1 + y_2 = 13$

$\Rightarrow \cancel{14-} \; y_2 = 13 - 14 = -1$

$$\therefore \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$$\therefore Ux = y \Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix} \Rightarrow 5x_2 = -1$$

$\therefore x_2 = -\frac{1}{5}$

$x_1 + 2x_2 = 7$

$\Rightarrow x_1 + 2\left(-\frac{1}{5}\right) = 7$

$\Rightarrow x_1 = 7 + \frac{2}{5}$

$= \frac{35+2}{5} = \frac{37}{5}$

$\therefore x_1 = \frac{37}{5}$

$x_2 = -\frac{1}{5}$