Solution 1: $\beta = 2$, $m = 4$, $e_{min} = -3$; $e_{max} = 6$

1. convention -1 : $(0.1111)_2 \times 2^6$

3   " -2 : $(1.1111)_2 \times 2^6$

" -3 : $(0.11111)_2 \times 2^6$

2. " -1 : $(0.1000)_2 \times 2^{-3}$

3   " -2 : $(1.0000)_2 \times 2^{-3}$

" -3 : $(0.10000)_2 \times 2^{-3}$

(6) for $e = -1$

$-1\ -2\ -1\ -4$

$(0.1000) \times 2^{-1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$(0.1001) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^4}\right) \times \frac{1}{2} = \frac{9}{32}$

$(0.1010) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^3}\right) \times \frac{1}{2} = \frac{5}{16}$

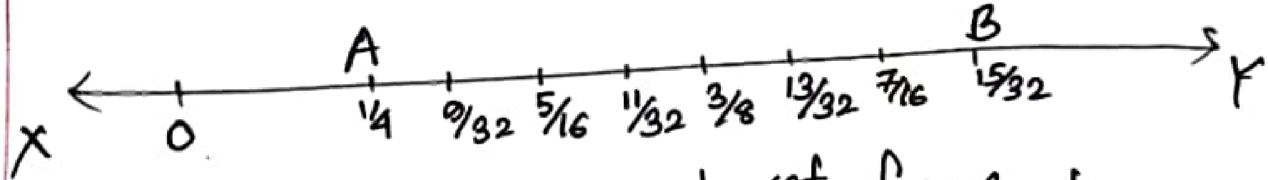
$(0.1011) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^4}\right) \times \frac{1}{2} = \frac{11}{32}$

$(0.1100) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^2}\right) \times \frac{1}{2} = \frac{3}{8}$

4 $(0.1101) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^4}\right) \times \frac{1}{2} = \frac{13}{32}$

$(0.1110) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3}\right) \times \frac{1}{2} = \frac{7}{16}$

$(0.1111) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}\right) \times \frac{1}{2} = \frac{15}{32}$



AB is equally spaced set, for $e = -1$

But the number line XY is not equally spaced.

Sol^n 2: $\beta=2, m=4, e_{min}=-1, e_{max}=2$

1. $\min|x| = (1.0000)_\beta \cdot \beta^e$

2 $= \beta^0 \times \beta^e$

$= \beta^e$

2. $|fl(x) - x| = \frac{1}{2} \times \beta^m \times \beta^e$

$\epsilon_M = \dfrac{\frac{1}{2} \times \beta^{-m} \times \beta^e}{\beta^e}$

2 $= \frac{1}{2}\beta^{-m}$

$= \frac{1}{2} \cdot \beta^{-4} = \frac{1}{2} \times \frac{1}{2^4}$

$\therefore \epsilon_M = \dfrac{1}{32}$

5. $\delta_{max} = \epsilon_M = \dfrac{|fl(x) - x|}{|x|}$

$\min |x| = (0.1 \ldots)_\beta \cdot \beta^{e}$

$= \beta^{-1} \times \beta^{e}$

$$= \frac{\frac{1}{2} \cdot \beta^{-m} \cdot \beta^{e}}{\beta^{-1} \times \beta^{e}}$$

$$= \frac{1}{2} \beta^{1-m}$$

2

$$= \frac{1}{2} \beta^{1-4}$$

$$= \frac{1}{2} \times \frac{1}{2^3}$$

$$= \frac{1}{16}$$

03. $f(x) = e^x - \sin x + x - 1$

Taylor expansion :

$$f(x) = f(x_0) + f'(x_0) * (x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \dots$$

5

$$\left[\begin{array}{l} f'(x) = e^x - \cos x + 1 \\ f''(x) = e^x + \sin x \end{array}\right. \qquad \& \quad x_0 = 0$$

$$\therefore f(x) = 0 + 1 \cdot x + \frac{1}{2!}x^2 + \dots$$

$$= x + \frac{1}{2}x^2 + \dots$$

$$P_2(x) = a_0 + a_1 x + a_2 x^2$$

Comparing,

$$a_0 = f(x_0) = 0$$

$$a_1 = f'(x_0) = 1$$

$$a_2 = \frac{f''(x_0)}{2!} = \frac{1}{2}$$

04 $f(x) = \tan x$

(a)

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \dots$$

$$f'(x) = \sec^2 x$$

$$f''(x) = 2 \sec x \frac{d}{dx} \sec x$$

$$= 2 \sec x (\sec x \tan x)$$

2

$$= 2 \sec^2 x \tan x$$

$$f'''(x) = 2 \left[\frac{d}{dx}\left((\sec^2 x)\right) \tan x + \frac{d}{dx}(\tan x) \sec^2 x \right]$$

$$\tan x$$

$$= 2 \left(2 \sec^2 x \tan x + \sec^2 x \sec^2 x \right)$$

$$= 2 \left(2 \sec^2 x \tan^2 x + \sec^4 x \right)$$

$$= 4 \sec^2 x \tan^2 x + \cancel{so}\ 2 \sec^4 x$$

$\therefore f(x) = 0 + 1\,x + \frac{0}{2!}x^2 + \frac{2}{3!}x^3 + \cdots$

$= x + \frac{1}{3}x^3 + \cdots$ [Taylor expansion of $\tan x$]

$P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

Comparing,

$a_0 = 0$, $a_1 = 1$, $a_2 = 0$, $a_3 = \frac{1}{3}$.

(b)

$$\text{Relative error} = \frac{|f(x) - P_3(x)|}{f(x)}$$

For, $x = \pi/4$,

$f(x) = \tan x$

$\Rightarrow f(\pi/4) = \tan \pi/4 = 1$

2 $P_3(x) = x + \frac{1}{3}x^3$

$\Rightarrow P_3(\pi/4) = \pi/4 + \frac{1}{3}(\pi/4)^3 = 0{\cdot}9469$

$\therefore$ relative error $= \frac{|1 - 0{\cdot}9469|}{1} = 0{\cdot}0531$

(c)

$$f(x) = P_3(x) + \frac{f^4 \xi}{4!} (x - x_0)^4$$

$$f''''(x) = -8 \sec^2 x \tan x + 24 \sec^4 x \tan x$$

5

Lagrange form of reminder $= \frac{f^4 \xi}{4!} (\pi/4 - 0)^4$

$$= \frac{80}{4!} (\pi/4)^4$$

$$= 1.268$$

$\xi \in [0, \pi/4]$

$\Rightarrow f^4(\xi = 0) = 0$

$\Rightarrow f^4(\xi = \pi/4) = 80$