

Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the part of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file.
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

1. Consider a set of four data points given below:

$$f(0) = 3, \quad f(4) = -2, \quad f(-1) = 2, \quad \text{and} \quad f(1) = 1.$$

Use the above data values to find the best fit polynomial of degree 2 by using the QR-decomposition method by answering the questions below step by step.

- (2.5 marks) Identify the matrix A and b . Also identify the linearly independent column vectors u_1 , u_2 and u_3 from the matrix A .
- (1+2+3 marks) Using Gram-Schmidt process construct the orthonormal column matrices (or vectors) q_1 , q_2 and q_3 from the linearly independent column vectors obtained in the previous part, and then write down the Q matrix.
- (3 marks) Now calculate the matrix elements of R , and write down the matrix R .
- (4.5 marks) Compute Rx and $Q^T b$, with $x = (a_0 \ a_1 \ a_2)^T$ being the coefficients of the polynomial $p_2(x)$, and finally find the expression for the polynomial $p_2(x)$.

2. Answer the following questions:

- (2 marks) Find the l_2 -norms of the vectors, $x = (1, 2, 3)^T$ and $y = (9, 11, 13)^T$.
- (6 marks) Show that the set,

$$S = \left\{ \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}^T, \frac{1}{\sqrt{30}} \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}^T, \frac{1}{\sqrt{24}} \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}^T \right\}$$

is orthonormal.

- (6 marks) Let $f(x) = \sin x$ and consider $x_0 = 4$, $x_1 = 9$ and $x_2 = -6$. For $f(x)$ take up to 3 decimal places after rounding. Now, Find the polynomial using Least Square Approximation for $n = 1$.