14

BRAC University (Department of Computer Science and Engineering)

CSE 330 (Numerical Methods) for Summer 2022 Semester

Quiz 3 (B)

Student ID: 1 9 1 0 1 0 7 7 **Full Marks: 15**

Name: Mohammad Shafkat Hosan **Duration: 25 minutes**

[No extra sheet will be provided. Write your answer to the questions in this answer script.]

1. [CO2] Consider the following linear system:
   $x_2+5x_3 = 5$
   $2x_1+4x_3 = 9$
   $2x_3 = 4$
   (a) ***(2 marks)*** Explain why the Gaussian elimination method fails to solve the system.
   (b) ***(3 marks)*** State how we can remove the problem and solve the system by Gaussian elimination method.
2. [CO2] Consider the following linear system:
   $2x_1+3x_2=5$
   $x_1+4x_2=11$
   (a) ***(3 marks)*** Construct the Frobenius matrix $F^{(1)}$ from this system.
   (b) ***(2 marks)*** Compute the unit lower triangular matrix L.
   (c) ***(5 marks)*** Now find the solution of the linear system using LU decomposition method. Use the unit lower triangular matrix found in the previous question.

Ans. To the Q.No. 1

a) From Eqn,

$$\left(\begin{array}{ccc|c} 0 & 1 & 5 & 5 \\ 2 & 0 & 4 & 9 \\ 0 & 0 & 2 & 4 \end{array}\right)$$

$m_{21} = \frac{a_{21}}{a_{11}} = \frac{2}{0}$ = Undifine

m_{21} is undifine. this matrix also don't form Upper or lower triangular matrix. So we can't use gaussian elimination

(b)

from a)

$$\left(\begin{array}{ccc|c} 0 & 1 & 5 & 5 \\ 2 & 0 & 4 & 9 \\ 0 & 0 & 2 & 4 \end{array}\right)$$

Swap Row 1 and 2

$$\left(\begin{array}{ccc|c} 2 & 0 & 4 & 9 \\ 0 & 1 & 5 & 5 \\ 0 & 0 & 2 & 4 \end{array}\right)$$

So,

$2x_3 = 4$

$x_2 + 5x_3 = 5$

$\Rightarrow x_2 + 5 \times 4 = 5 \Rightarrow x_2 = -15$

$\Rightarrow 5x_3 = 1$

$\Rightarrow x_3 = \frac{1}{5}$

$2x_1 + 4x_3 = 9$

$\Rightarrow 2x_1 + 4 \times 4 = 9$

$\Rightarrow 2x_1 = -7$

$\Rightarrow x_1 = -7/2$

Quiz 3 (B)

Student ID: 1910107 7

Name: Mohammad Shafkat Hosan

Full Marks: 15

Duration: 25 minutes

[No extra sheet will be provided. Write your answer to the questions in this answer script.]

1. [CO2] Consider the following linear system:
   $x_2+5x_3 = 5$
   $2x_1+4x_3 = 9$
   $2x_3 = 4$
   (a) ***(2 marks)*** Explain why the Gaussian elimination method fails to solve the system.
   (b) ***(3 marks)*** State how we can remove the problem and solve the system by Gaussian elimination method.
2. [CO2] Consider the following linear system:
   $2x_1+3x_2=5$
   $x_1+4x_2=11$
   (a) ***(3 marks)*** Construct the Frobenius matrix $F^{(1)}$ from this system.
   (b) ***(2 marks)*** Compute the unit lower triangular matrix L.
   (c) ***(5 marks)*** Now find the solution of the linear system using LU decomposition method. Use the unit lower triangular matrix found in the previous question.

Ans. To the Q.No. 2

a) $2x_1 + 3x_2 = 5$

$x_1 + 4x_2 = 11$

$$\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 1 & 4 & 11 \end{array}\right]$$

(A | b)

$$F^{(1)} = \begin{bmatrix} 1 & 0 \\ -m_{21} & 1 \end{bmatrix}$$

$$m_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{2}$$

$$m_{31} = \frac{a_{31}}{a_{11}}$$

$$= \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

b) $$L = \begin{bmatrix} 1 & 0 \\ m_{21} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$$

c) $U = E^2 \times A^2$

$$= \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 0 & 2.5 \end{bmatrix}$$

$Ly = b$

$$\begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$

$\therefore y_1 = 5$

$\frac{1}{2} y_1 + y_2 = 11 \Rightarrow \frac{1}{2} \times 5 + y_2 = 11$

$\Rightarrow y_2 = 11 - \frac{5}{2}$

$= \frac{17}{2}$

$Ux = y$

$$\begin{pmatrix} 2 & 3 \\ 0 & 2.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ \frac{17}{2} \end{pmatrix}$$

$2.5 x_2 = \frac{17}{2}$

$\Rightarrow x_2 = \frac{17}{2 \times 2.5} = \frac{17}{5}$

$2x_1 + 3x_2 = 5$

$\Rightarrow 2x_1 + 3.\frac{17}{5} = 5$

$\Rightarrow 2x_1 = -\frac{26}{5} \quad \therefore x_1 = -\frac{13}{5}$