BRAC University (Department of Computer Science and Engineering)

CSE 330 (Numerical Methods) for FALL 2022 Semester

Quiz 4(set A)

Student ID: **Full Marks: 20**

Section: **[CO3]**

Name: **Duration: 35 minutes**

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1. Consider the following function: $f(x) = xe^x - 1$.

(a) [6 marks] Find the solution of $f(x) = 0$ up to 5 iterations using Newton's method starting with $x0 = 1.5$. Keep up to four significant figures.

(b) [6 marks] Consider the fixed point function, $g(x) = (2x + 1)/\sqrt{(x + 1)}$. Show that to be super-linearly convergent, the root must satisfy $x_* = -3/2$.

(c) [4 Marks] For $f(x) = x^3 + x^2 - 4x - 4$ construct two different fixed point functions $g(x)$ such that $f(x) = 0$.

(d) [4 marks] Compute the convergence rate of each $g(x)$ obtained in the previous part(c) and state if the root is converging linearly, superlinearly or it is diverging.

1 (a) $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$; $x_0 = 1.5$

$f(x) = xe^x - 1 \Rightarrow f'(x) = xe^x + e^x = e^x(x+1)$

K	0	1	2	3	4	5
x_k	1.5	0.9893	0.6789	0.5766	0.5672	0.5671

(b) $g(x) = \frac{(2x+1)}{\sqrt{(x+1)}} \Rightarrow g'(x) = \frac{2x+3}{2(x+1)^{3/2}}$

$\Rightarrow g'(x) = 0 \Rightarrow \frac{2x+3}{2(x+1)^{3/2}} = 0 \Rightarrow 2x+3 = 0 \Rightarrow x = -\frac{3}{2}$

Answer

(c) $x^3 + x^2 - 4x - 4 = 0$

$\Rightarrow x^2(x+1) - 4(x+1) = 0 \Rightarrow x = -1, -2, 2 \rightarrow$ exact roots

$x^3 + x^2 - 4x - 4 = 0$

$\Rightarrow 4x = x^3 + x^2 - 4 \Rightarrow x = \frac{x^3+x^2-4}{4} = g(x)$ — (i)

$x^3 + x^2 - 4x - 4 = 0$

$\Rightarrow x(x^2+x-4) = 4$

$\Rightarrow x = \frac{4}{x^2+x-4} = g(x)$ — (ii)

(d) 1st Case:

$g'(x) = \frac{3x^2+2x}{4}$

$\lambda = |g'(x_*)| = \begin{cases} \frac{1}{4} ; x_* = -1 \rightarrow \text{linear convengence} \\ 2 ; x_* = -2 \\ 4 ; x_* = 2 \end{cases}$ divergence

2nd Case:

$g'(x) = \frac{-4(2x+1)}{(x^2+x-4)^2}$

$\lambda = |g'(x_*)| = \begin{cases} \frac{1}{4} ; x_* = -1 \rightarrow \text{linear convergence} \\ 3 ; x_* = -2 \\ 5 ; x_* = 2 \end{cases}$ divergence.

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CSE 330 (Numerical Methods) for FALL 2022 Semester

Quiz 4(set B)

Student ID: **Full Marks: 20**
Section: **[CO3]**

Name: **Duration: 35 minutes**

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1. Consider the following function: $f(x) = x^3 + 4x^2 - 10$.

(a) [6 marks] Find the solution of $f(x) = 0$ up to 5 iterations using Newton's method starting with $x0 = 2$. Keep up to six significant figures.

(b) [6 marks] Consider the fixed point function, $g(x) = (2x + 1)/ \sqrt{(x + 1)}$. Show that to be super-linearly convergent, the root must satisfy $x_* = -3/2$.

(c) [4 marks] For $f(x) = x^3 + x^2 - 25x - 25$ construct two different fixed point functions g(x) such that f(x)=0.

(d) [4 marks] Compute the convergence rate of each g(x) obtained in the previous part(c) and state if the root is converging linearly, superlinearly or it is diverging.

1 a $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

$f(x) = x^3 + 4x^2 - 10 \Rightarrow f'(x) = 3x^2 + 8x$; $x_0 = 2$

k	0	1	2	3	4	5
x_k	2	1.50000	1.37333	1.36526	1.36523	1.36523

Answer

b $g(x) = \frac{(2x+1)}{\sqrt{(x+1)}} \Rightarrow g'(x) = \frac{2x+3}{2(x+1)^{3/2}}$

to be superlinear: $g'(x) = 0$

$\Rightarrow \frac{2x+3}{2(x+1)^{3/2}} = 0 \Rightarrow 2x+3 = 0 \Rightarrow x = -\frac{3}{2}$

Answer

c $x^3 + x^2 - 25x - 25 = 0$

$\Rightarrow x^2 \cancel{-25}(x+1) - 25(x+1) = 0$

$\Rightarrow (x+1)(x+5)(x-5) = 0 \Rightarrow x = -1, -5, 5$ → exact roots

* $x^3 + x^2 - 25x - 25 = 0$

$\Rightarrow 25x = x^3 + x^2 - 25$

$\therefore x = \frac{x^3 + x^2 - 25}{25} = g(x)$ — (i)

* $x^3 + x^2 - 25x - 25 = 0$

$\Rightarrow x(x^2 + x - 25) = 25$

$\Rightarrow x = \frac{25}{x^2 + x - 25} = g(x)$ — (ii)

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(d) first Case:

$$g'(x) = \frac{3x^2+2x}{25}$$

$$\lambda = |g'(x_*)| = \begin{cases} \frac{1}{25} \longleftarrow x_* = -1 \longrightarrow \text{linearly converged} \\ \frac{65}{25} \longleftarrow x_* = -5 \\ \frac{85}{25} \longleftarrow x_* = 5 \end{cases} \quad \text{divergence}$$

Second Case:

$$g'(x) = \frac{-25(2x+1)}{(x^2+x-25)^2}$$

$$\lambda = |g'(x_*)| = \begin{cases} \frac{1}{25} \longleftarrow x_* = -1 \longrightarrow \text{linear convergence} \\ 9 \longleftarrow x_* = -5 \\ 11 \longleftarrow x_* = 5 \end{cases} \quad \text{divergence}$$