

Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the part of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file.
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

1. A function is given by $f(x) = 2x - e^{-6x}$. Now answer the following:

- (3 marks) Approximate the derivative of $f(x)$ at $x_0 = 0.5$ with step size $h = 0.2$ using the forward difference method up to 5 significant figures.
- (3 marks) Approximate the derivative of $f(x)$ at $x_0 = 0.5$ with step size $h = 0.2$ using the central difference method up to 6 significant figures.
- (4+4 marks) Calculate the truncation error of $f(x)$ at $x_0 = 2$ using $h = 1, 0.1, 0.01, 0.0001$ in both of the above mentioned methods.
- (6 marks) Compute $D_{0.5}^{(1)}$ at $x_0 = 0.2$ using Richardson extrapolation method up to 6 significant figures and calculate the truncation error.

2. During the class, we derived in detail the first order Richardson extrapolated derivative, by using $h \rightarrow h/2$,

$$D_h^{(1)} \equiv f'(x_0) - \frac{h^4}{480} f^{(5)}(x_0) + \mathcal{O}(h^6) .$$

- (6 marks) Using $h \rightarrow h/2$, derive the expression for $D_h^{(2)}$ which is the second order Richardson extrapolation.
- (6 marks) Now starting from the definition of D_h and using $h \rightarrow h/3$, derive the expression for $D_h^{(1)}$.
- (2+2 marks) Now identify the Error Part of the expression found in the previous part, and also find the Error Bound of the expression found in the previous part.
- (4 marks) If $f(x) = \ln x$, $x_0 = 1$, $h = 0.1$, find the upper bound of error for $D_h^{(1)}$.