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Ans. To The Q No. 1

Here,

$$f(x) = 2x - e^{-6x}$$

$$h = 0.2, \quad x_0 = 0.5$$

$$f'(x) = 2 + 6e^{-6x}$$

x	0.3	0.5	0.7
y	~~0.392~~ 0.435	0.95	1.385

a) The formula for forward difference,

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

$$f'(0.5) = \frac{f(0.5 + 0.2) - f(0.5)}{0.2}$$

$$f'(0.5) = \frac{f(0.7) - f(0.5)}{0.2}$$

$$= \frac{(2\times0.7 - e^{-6\times0.7}) - (2\times0.5 - e^{-6\times0.5})}{0.2}$$

$$\therefore f'(0.5) = 2.17397458$$

(Ans)

b) Central difference formula,

$$f'(x) = \frac{f(x+h) - f(x-h)}{2 \times h}$$

$$f'(0.5) = \frac{f(0.5+0.2) - f(0.5-0.2)}{2 \times 0.2}$$

$$f'(0.5) = \frac{f(0.7) - f(0.3)}{0.4}$$

$$= \frac{(2 \times 0.7 - e^{-6 \times 0.7}) - (2 \times 0.3 - e^{-6 \times 0.3})}{0.4}$$

$$\therefore f'(0.5) = 2.37575827 9$$

(Ans)

c) $f(x) = 2x - e^{-6x}$

$$f'(x) = 2 + 6e^{-6x}$$

Actual, $f'(2) = 2.000036865$

x	3	2.1	2.01	1	1.9	1.99	1.9999
y	5.99	4.19	4.019	1.99	3.79	3.979	3.9997

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Forward Difference Method,

$$f'(2) = \frac{f(2+h) - f(2)}{h}$$

h	Forward Difference	Truncation Error (Actual − Forward Diffenece)
1	2.000006129	5.0736×10^{-5}
0.1	2.000027722	9.143×10^{-6}
0.01	2.000035781	1.084×10^{-6}
0.0001	2.000036854	1.105×10^{-8}

Central Difference Method,

$$f'(2) = \frac{f(2+h) - f(2-h)}{2h}$$

h	Central Difference	Truncation Error
1	2.00.1239368	-1.202503×10^{-3}
0.1	2.000039117	-2.252×10^{-6}
0.01	2.000036887	-2.2×10^{-8}
0.0001	2.000036865	0

d) $f(x) = 2x - e^{-6x}$

$f'(x) = 2 + 6e^{-6x}$

Actual Value $= f'(0.2) = 2 + 6e^{-6\times0.2}$

Richardson Extrapolation Method,

$$D_h = \frac{f(x+h) - f(x-h)}{2h}$$

$$D_{0.5} = \frac{f(0.2+0.5) - f(0.2-0.5)}{2\times0.5}$$

$$= \frac{f(0.7) - f(-0.3)}{1}$$

$$= (2\times0.7 - e^{-6\times0.7}) - ((2\times-0.3) - e^{+6\times0.3})$$

$$= 8.034651888$$

$$D_{h/2} = \frac{f(x+h/2) - f(x-h/2)}{2\times\frac{h}{2}}$$

$$D_{0.25} = \frac{f(0.45) - f(-0.05)}{0.5}$$

$$= \frac{(2\times0.45 - e^{-6\times0.45}) - ((2\times-0.05 - e^{-6\times-0.05})}{0.5}$$

$$= 4.56530659$$

Truncation Error $=$ Actual Value $- \dfrac{2^r D_{h/2} - D_h}{2^r - 1}$

$$= 2 + 6e^{-6\times0.2} - \frac{(4\times4.56530659) - 8.034651888}{3}$$

$$= 0.39831$$

(Ans)

Ans. To The Q. No. 2

Here,

$$D_h^{(1)} = f'(x_0) - \frac{h^4}{480} f^{(5)}(x_0) + O(h^6)$$

a) $$D_h^{(1)} = f'(x_0) - \frac{f^5(x_0)\, h^4}{480} + O(h^6)$$

$$\Rightarrow D_{h/2}^{(1)} = f'(x_0) - \frac{f^5(x_0)}{480} \frac{h^4}{16} + O(h^6)$$

$$\Rightarrow 16 D_{h/2} = 16 f'(x_0) - \frac{f^5(x_0)}{480} h^4 + O(h^6)$$

$$16 D_{h/2} - D_h = 15 f'(x_0) - 0 + O(h^6)$$

$$\frac{16 D_{h/2} - D_h}{15} = f'(x_0) + O(h^6)$$

$$D_h^{(2)} = f'(x_0) + O(h^6)$$

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b) $D_h = f'(x_0) + \frac{f^3(x_0)}{3!} h^2 + \frac{f^5(x_0)}{5!} h^4 + O(h^6)$

$D_{h/3} = f'(x_0) + \frac{f^3(x_0)}{3!} \times \left(\frac{h^2}{9}\right) + \frac{f^5(x_0)}{5!} \times \left(\frac{h^4}{81}\right) + O(h^6)$

$= f'(x_0) + \frac{f^3(x_0)}{3!} \times \frac{h^2}{9} + \frac{f^5(x_0)}{5!} \frac{h^4}{81} + O(h^6)$

$9 D_{h/3} = 9 f'(x_0) + 0 + \frac{f^5(x_0)}{5!} \frac{h^4}{9} + O(h^6)$

$9 D_{h/3} - D_h = (9-1) f'(x_0) + \left(\frac{1}{9} - 1\right) \frac{f^5(x_0) h^4}{5!} + O(h^6)$

$\frac{9 D_{h/3} - D_h}{8} = \cancel{8} f'(x_0) + \left(-\frac{\cancel{8}}{9} \times \cancel{8}\right) \frac{f^5(x_0)}{5!} h^4 + O(h^6)$

$\therefore D_h^{(1)} = f'(x_0) - \frac{1}{9} \frac{f^5(x_0)}{5!} h^4 + O(h^6)$

c) from 'b'

$$D_h^{(1)} = f'(x_0) - \frac{1}{9}\frac{f^5(x_0)}{5!}h^4 + O(h^6)$$

Here Error Part of the expression,

$$-\frac{1}{9}\frac{f^5(x_0)}{5!}h^4 + O(h^6)$$

and Error Bound,

$$-\frac{1}{9}\frac{f^5(x_0)}{5!}h^4$$

$$= -\frac{f^5(x_0)}{1080}h^4$$

d) Here,
$f(x) = \ln(x)$, $x_0 = 1$, $h = 0.1$

from 'c' Upper 'Error' Bound

$$= \frac{f^5(x_0)}{1080}h^4$$

$$= -\frac{\cancel{25} \cdot \frac{24}{(1)^5}}{1080}(0.1)^4$$

$$= -2.22 \times 10^{-6}$$

(Ans)

$f'(x) = \frac{1}{x}$

$f''(x) = -\frac{1}{x^2}$

$f'''(x) = \frac{2}{x^3}$

$f^4(x) = -\frac{6}{x^4}$

$f^5(x) = \frac{24}{x^5}$