Ray Tracing

Last Time?

Quiz!

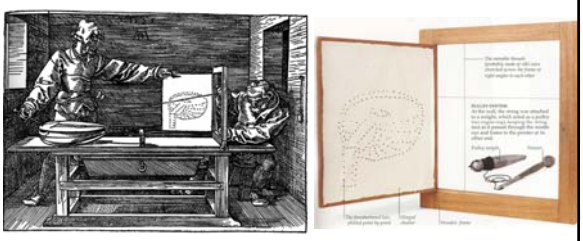
- Tuesday Oct 18th
- In class
- 1 page of notes

Today

- Ray Casting
  - Ray-Plane Intersection
  - Ray-Sphere Intersection
  - Point in Polygon
- Ray Tracing
  - Shadows
  - Reflection
  - Refraction
- Recursive Ray Tracing

Durer's Ray Casting Machine

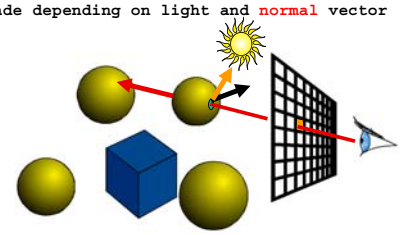
- Albrecht Durer, 16th century



Ray Casting

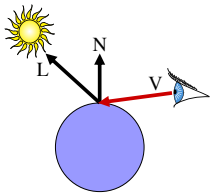
```
For every pixel
  Construct a ray from the eye
  For every object in the scene
    Find intersection with the ray
    Keep if closest
  Shade depending on light and normal vector
```

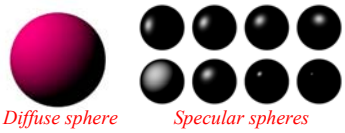
Finding the intersection and normal is the central part of ray casting



A Note on Shading

- Surface/Scene Characteristics:
  - surface normal
  - direction to light
  - viewpoint
- Material Properties
  - Diffuse (matte)
  - Specular (shiny)
  - …
- More later!

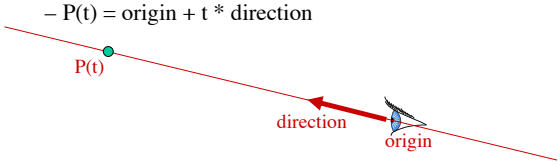




Diffuse sphere *Specular spheres*

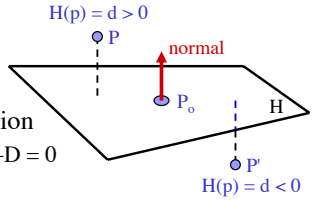
Ray Representation?

- Two vectors:
  - Origin
  - Direction (normalized is better)
- Parametric line (*explicit* representation)
  - P(t) = origin + t * direction



3D Plane Representation?

- Plane defined by
  - $P_o = (x,y,z)$
  - n = (A,B,C)
- Implicit plane equation
  - H(P) = Ax+By+Cz+D = 0
    = n·P + D = 0
- Point-Plane distance?
  - If n is normalized, distance to plane, d = H(P)
  - d is the *signed distance*!



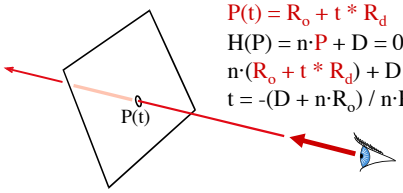
Explicit vs. Implicit?

- Ray equation is explicit $P(t) = R_o + t * R_d$
  - Parametric
  - Generates points
  - Hard to verify that a point is on the ray
- Plane equation is implicit $H(P) = n \cdot P + D = 0$
  - Solution of an equation
  - Does not generate points
  - Verifies that a point is on the plane
- Exercise: Explicit plane and implicit ray

Ray-Plane Intersection

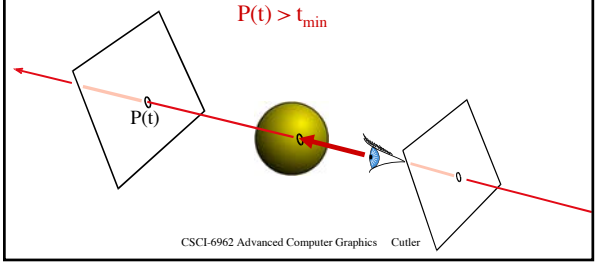
- Intersection means both are satisfied
- So, insert explicit equation of ray into implicit equation of plane & solve for t

$P(t) = R_o + t * R_d$
$H(P) = n \cdot P + D = 0$
$n \cdot (R_o + t * R_d) + D = 0$
$t = -(D + n \cdot R_o) / n \cdot R_d$



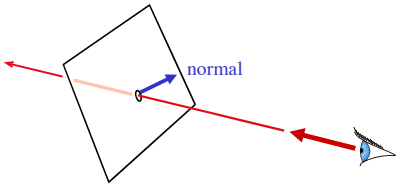
Additional Housekeeping

- Verify that intersection is closer than previous
  $P(t) < t_{current}$
- Verify that it is not out of range (behind eye)
  $P(t) > t_{min}$



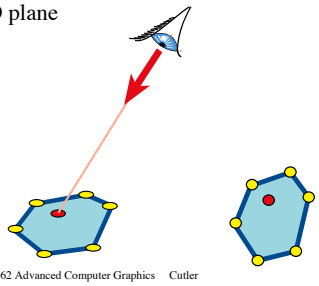
Normal

- For shading
  - diffuse: dot product between light and normal
- Normal is constant



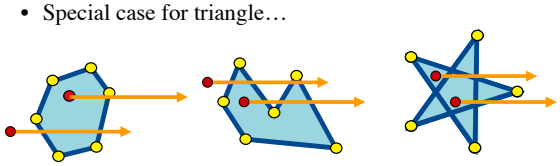
Ray-Polygon Intersection

- Ray-plane intersection
- Test if intersection is in the polygon
  - Solve in the 2D plane



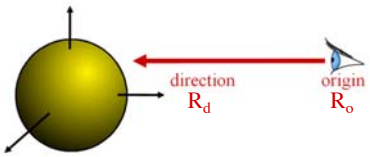
Point Inside/Outside Polygon

- Ray intersection definition:
  - Cast a ray in any direction
    - (axis-aligned is smarter)
  - Count intersections
  - If odd number, point is inside
- Works for concave and star-shaped
- Special case for triangle…



Sphere Representation?

- Implicit sphere equation
  - Assume centered at origin (easy to translate)
  - $H(P) = P \cdot P - r^2 = 0$



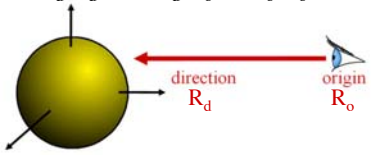
Ray-Sphere Intersection

- Insert explicit equation of ray into implicit equation of sphere & solve for t

$$P(t) = R_o + t*R_d \qquad H(P) = P \cdot P - r^2 = 0$$

$$(R_o + tR_d) \cdot (R_o + tR_d) - r^2 = 0$$

$$R_d \cdot R_d t^2 + 2R_d \cdot R_o t + R_o \cdot R_o - r^2 = 0$$



Ray-Sphere Intersection

- Quadratic: $at^2 + bt + c = 0$
  - $a = 1$ (remember, $\|R_d\| = 1$)
  - $b = 2R_d \cdot R_o$
  - $c = R_o \cdot R_o - r^2$
- with discriminant $d = \sqrt{b^2 - 4ac}$
- and solutions $t_{\pm} = \dfrac{-b \pm d}{2a}$

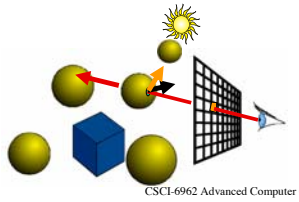
Questions?

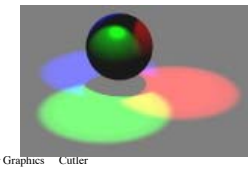
Today

- Ray Casting
  - Ray-Plane Intersection
  - Ray-Sphere Intersection
  - Point in Polygon
- Ray Tracing
  - Shadows
  - Reflection
  - Refraction
- Recursive Ray Tracing

How Can We Add Shadows?

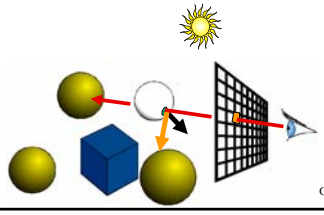
```
Find the point to be shaded
For every light,
  Construct ray from point to light
  For every object
    find intersection of ray with object
  If no objects between point and light
    Add contribution from light
```

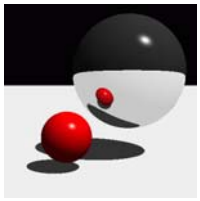




Mirror Reflection

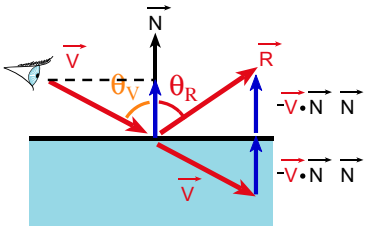
- Cast ray symmetric with respect to the normal
- Multiply by reflection coefficient (color)





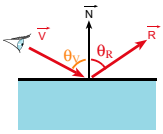
Reflection

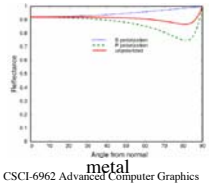
- Reflection angle = view angle
- $\mathbf{R} = \mathbf{V} - 2\,(\mathbf{V} \cdot \mathbf{N})\,\mathbf{N}$

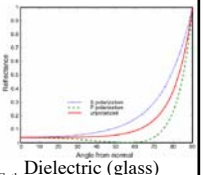


Amount of Reflection

- Traditional ray tracing (hack)
  - Constant `reflectionColor`
- More realistic:
  - Fresnel reflection term (more reflection at grazing angle)
  - Schlick's approximation: $R(\theta)=R_0+(1-R_0)(1-\cos\theta)^5$



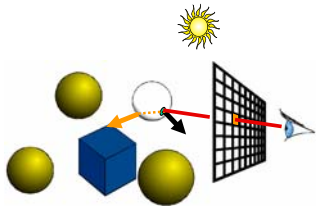




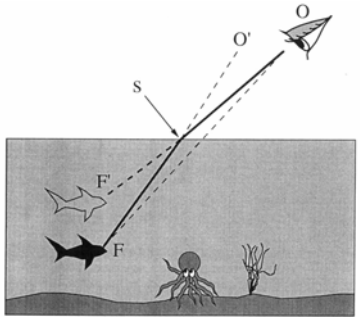
metal Dielectric (glass)

Transparency

- Cast ray in refracted direction
- Multiply by transparency coefficient (color)

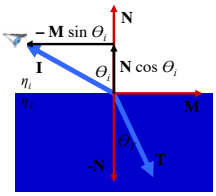


Qualitative Refraction



From "Color and Light in Nature" by Lynch and Livingston

Refraction



Snell-Descartes Law:

$\eta_I \sin \Theta_i = \eta_T \sin \Theta_T$

$$\frac{\sin \Theta_T}{\sin \Theta_i} = \frac{\eta_I}{\eta_T} = \eta_r$$

$$\mathbf{I} = \mathbf{N} \cos \Theta_i - \mathbf{M} \sin \Theta_i$$

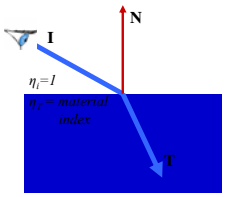
$$\mathbf{M} = (\mathbf{N} \cos \Theta_i - \mathbf{I}) / \sin \Theta_i$$

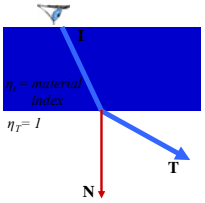
$$\begin{aligned}
\mathbf{T} &= -\mathbf{N} \cos \Theta_T + \mathbf{M} \sin \Theta_T \\
&= -\mathbf{N} \cos \Theta_T + (\mathbf{N} \cos \Theta_i - \mathbf{I}) \sin \Theta_T / \sin \Theta_i \\
&= -\mathbf{N} \cos \Theta_T + (\mathbf{N} \cos \Theta_i - \mathbf{I})\, \eta_r \\
&= [\, \eta_r \cos \Theta_i - \cos \Theta_T \,]\, \mathbf{N} - \eta_r \mathbf{I} \\
&= [\, \eta_r \cos \Theta_i - \sqrt{1 - \sin^2 \Theta_T} \,]\, \mathbf{N} - \eta_r \mathbf{I} \\
&= [\, \eta_r \cos \Theta_i - \sqrt{1 - \eta_r^2 \sin^2 \Theta_i} \,]\, \mathbf{N} - \eta_r \mathbf{I} \\
&= [\, \eta_r \cos \Theta_i - \sqrt{1 - \eta_r^2 (1 - \cos^2 \Theta_i)} \,]\, \mathbf{N} - \eta_r \mathbf{I} \\
&= [\, \eta_r (\mathbf{N} \cdot \mathbf{I}) - \sqrt{1 - \eta_r^2 (1 - (\mathbf{N} \cdot \mathbf{I})^2)} \,]\, \mathbf{N} - \eta_r \mathbf{I}
\end{aligned}$$

- **Total internal reflection when the square root is imaginary**
- **Don't forget to normalize!**

Refraction & the Sidedness of Objects

- Make sure you know whether you're entering or leaving the transmissive material:





- What about intersecting transparent objects?

Total Internal Reflection

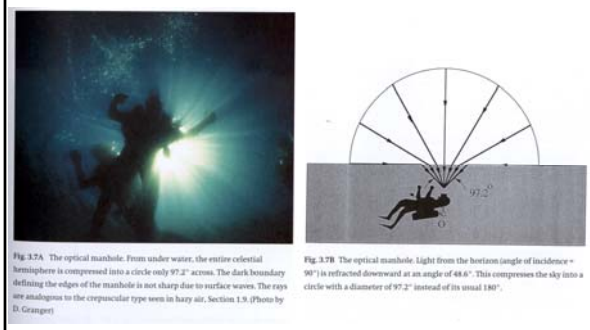


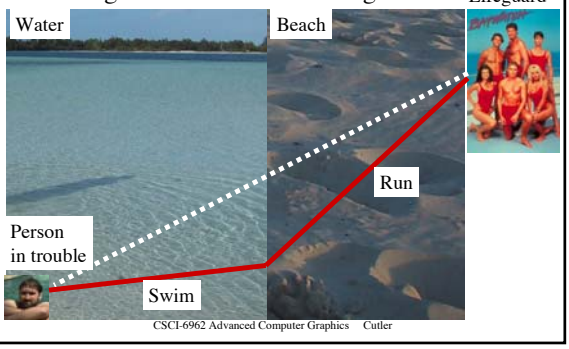
Fig. 3.7A The optical manhole. From under water, the entire celestial hemisphere is compressed into a circle only 97.2° across. The dark boundary defining the edges of the manhole is not sharp due to surface waves. The rays are analogous to the crepuscular type seen in hazy air. Section 1.9. (Photo by D. Granger)

Fig. 3.7B The optical manhole. Light from the horizon (angle of incidence = 90°) is refracted downward at an angle of 48.6°. This compresses the sky into a circle with a diameter of 97.2° instead of its usual 180°.

From "Color and Light in Nature" by Lynch and Livingston

Refraction and the Lifeguard Problem

- Running is faster than swimming



Questions?

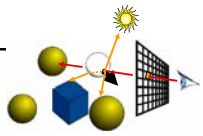
Today

- Ray Casting
  - Ray-Plane Intersection
  - Ray-Sphere Intersection
  - Point in Polygon
- Ray Tracing
  - Shadows
  - Reflection
  - Refraction
- Recursive Ray Tracing

Recap: Ray Tracing

```
trace ray
   Intersect all objects
   color = ambient term
   For every light
      cast shadow ray
      color += local shading term
   If mirror
      color += color_refl *
               trace reflected ray
   If transparent
      color += color_trans *
               trace transmitted ray
```

- ***Does it ever end?***



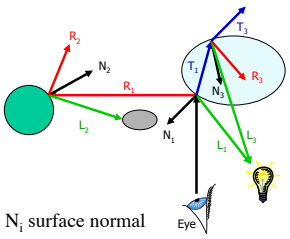
Stopping criteria:

- Recursion depth
  - Stop after a number of bounces
- Ray contribution
  - Stop if reflected / transmitted contribution becomes too small

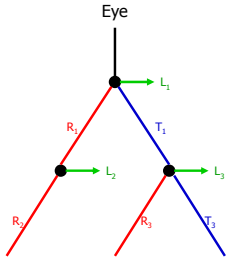
Recursion For Reflection

0 recursion | 1 recursion | 2 recursions

The Ray Tree



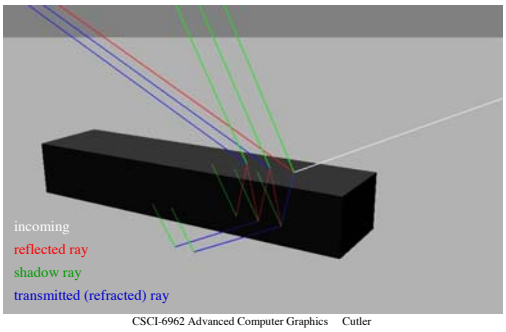
N_i surface normal
R_i reflected ray
L_i shadow ray
T_i transmitted (refracted) ray



Complexity?

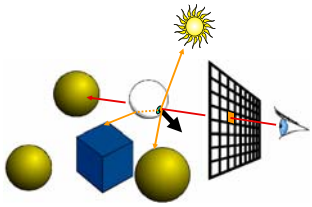
Ray Debugging

- Visualize the ray tree for single image pixel



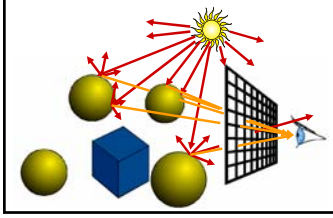
Does Ray Tracing Simulate Physics?

- Photons go from the light to the eye, not the other way
- What we do is *backward ray tracing*



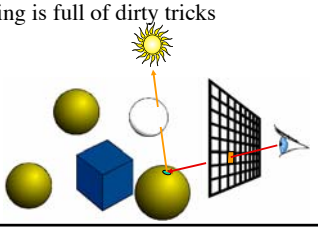
Forward Ray Tracing

- Start from the light source
  - But low probability to reach the eye
- What can we do about it?
  - Always send a ray to the eye…. still not efficient

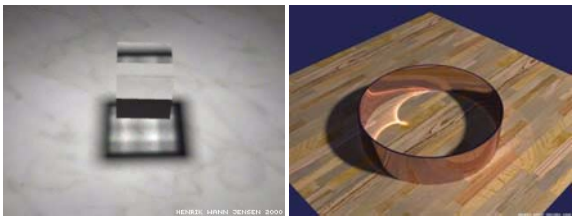


Transparent Shadows?

- What to do if ray to light source intersects a transparent object?
  - Pretend it's opaque?
  - multiply by transparency color? (ignores refraction & does not produce caustics)
- Ray Tracing is full of dirty tricks



Traditional Ray Tracing?

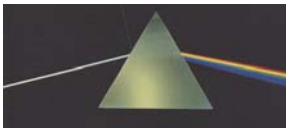


Images by Henrik Wann Jensen

- No, Refraction and complex reflection for illumination are not handled properly in traditional (backward) ray tracing

What makes a Rainbow?

- Refraction is wavelength-dependent
  - Refraction increases as the wavelength of light decreases
  - violet and blue experience more bending than orange and red
- Usually ignored in graphics
- Rainbow is caused by refraction + internal reflection + refraction

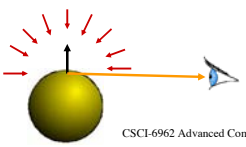


Pink Floyd, *The Dark Side of the Moon*

From "Color and Light in Nature" by Lynch and Livingstone

The Rendering Equation

- Clean mathematical framework for light-transport simulation
- At each point, outgoing light in one direction is the integral of incoming light in all directions multiplied by reflectance property



Questions?