Machine Learning (CE 40477)

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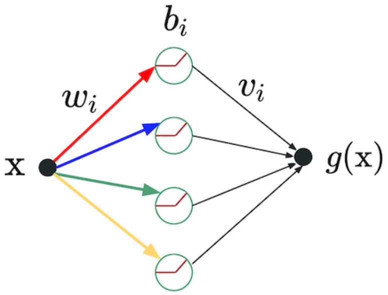
Why Deep Networks? (Deep Expressivity)

- While the universal approximation theorem states that a single-hidden-layer neural network can approximate any function to a certain level of accuracy, it does not specify how many hidden neurons are needed.
    - It states there exists an integer N and a set of parameters $\{v_i, w_i, b_i\}_{i=1}^{N}$
    - **Depth Separation Theorem:** For any integer $k > 0$, there exist neural networks with $\Theta(k^3)$ layers, $\Theta(1)$ nodes per layer, and $\Theta(1)$ distinct parameters which can't be approximated by networks with $O(k)$ layers unless they are exponentially large (they must possess $\Omega(2^k)$ nodes neurons).



Telgarsky. Benefits of depth in neural networks. PMLR (2016).

Intuition

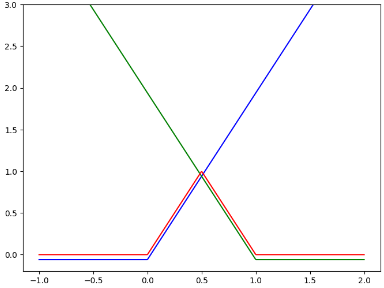
- Let's consider ReLU networks. Let $m(x)$ be the piecewise linear function as below:

$$m(x) = \begin{cases} 2x & 0 \le x \le 0.5 \\ 2-2x & 0.5 \le x \le 1 \\ 0 & o.w. \end{cases}$$

- This can be computed exactly by a two- or three-layer network (depending on your notational preferences), by the expression:

$$\sigma(2\sigma(x) - 4\sigma(x-0.5))$$

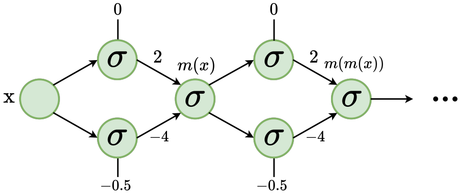
- Figure on the right shows it in terms of ReLU activation functions

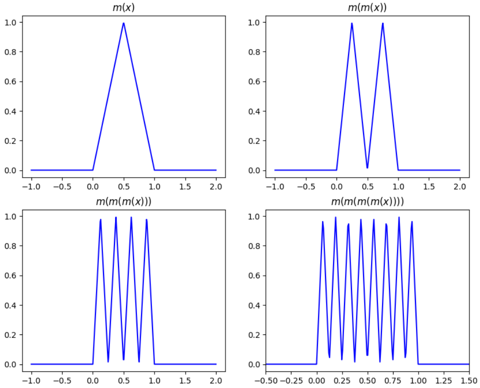


Function m is slightly offset for visual clarity

Intuition

- What do iterates of $m(x)$ look like?
- So $m^{(n)}(x)$ would have 2^n "teeth"
- We can represent this function with $3n+1$ nodes
- A shallow network requires many nodes to achieve this
- Depth increases the number of oscillations multiplicatively whereas width can only do so additively.





AlexNet [Krizhevsky, et al. NIPS (2012)]

Architecture:

[227 × 227 × 3] INPUT

[55 × 55 × 96] CONV1 + ReLU: 96 11 × 11 filters at stride 4, pad 0

[27 × 27 × 96] MAX POOL1: MAX POOL1: 3 × 3 filters at stride 2

[27 × 27 × 96] NORM1: Normalization layer

[27 × 27 × 256] CONV2 + ReLU: 256 5 × 5 filters at stride 1, pad 2

[13 × 13 × 256] MAX POOL2: 3 × 3 filters at stride 2

[13 × 13 × 256] NORM2: Normalization layer

[13 × 13 × 384] CONV3 + ReLU: 384 3 × 3 filters at stride 1, pad 1

[13 × 13 × 384] CONV4 + ReLU: 384 3 × 3 filters at stride 1, pad 1

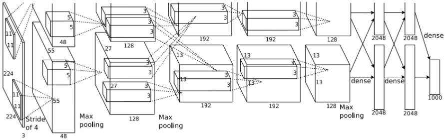
[13 × 13 × 256] CONV5 + ReLU: 256 3 × 3 filters at stride 1, pad 1

[6 × 6 × 256] MAX POOL3: 3 × 3 filters at stride 2

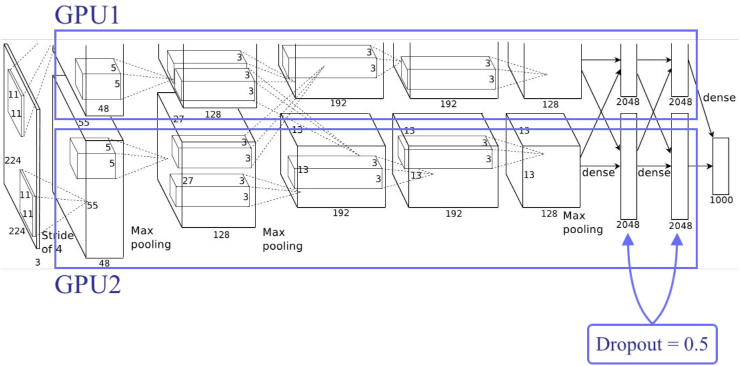
[4096] Fully Connected 6 + ReLU 4096 neurons

[4096] Fully Connected 7 + ReLU 4096 neurons

[1000] Fully Connected 8 1000 neurons (class scores)



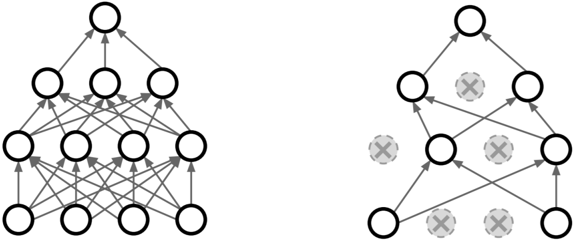
AlexNet At A Glance



- Training: 6 days on 2 NVIDIA GTX 580 GPUs (3 GB RAM)
- 8 layers / 60M parameters and 650K neurons
- Testing: Multi-crop
    - Classify different shifts of the image and vote over the lot!
    - Test-time data augmentation

Dropout

- In each forward pass, randomly deactivate some neurons.
  - → Probability of dropping is a hyperparameter ⇒ 0.5 is common
  - → Gradients are computed only for the weights and biases between active nodes.
  - → For the remaining, the gradient is just 0.



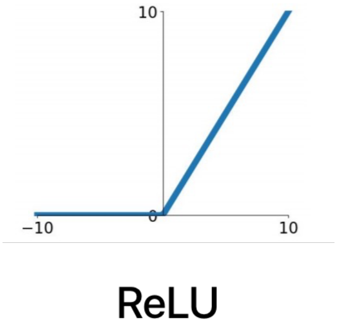
Srivastava et al., JMLR 2014

- Thus, a smaller network is trained for each sample.
  - → Smaller network provides a regularization effect.

ReLU

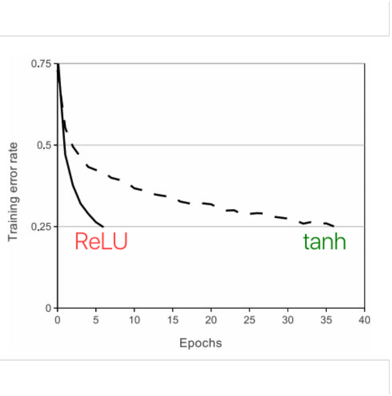
ReLU

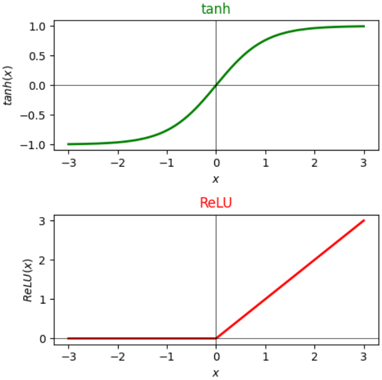
- Does not saturate (in the positive region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Actually more biologically plausible than sigmoid
- **Problems:**
  - → Not zero-centered output
  - → An annoyance:
    - ⋆ Hint: "what is the gradient when $x < 0$?"



ReLU

- ReLU trains faster than tanh.





Krizhevsky, et al., 2012

Local Response Normalisation

$$b_{x,y}^{i} = \frac{a_{x,y}^{i}}{\left(k + \alpha \sum_{j=\max(0, i-\frac{n}{2})}^{\min(N-1, i+\frac{n}{2})} \left(a_{x,y}^{j}\right)^{2}\right)^{\beta}}$$

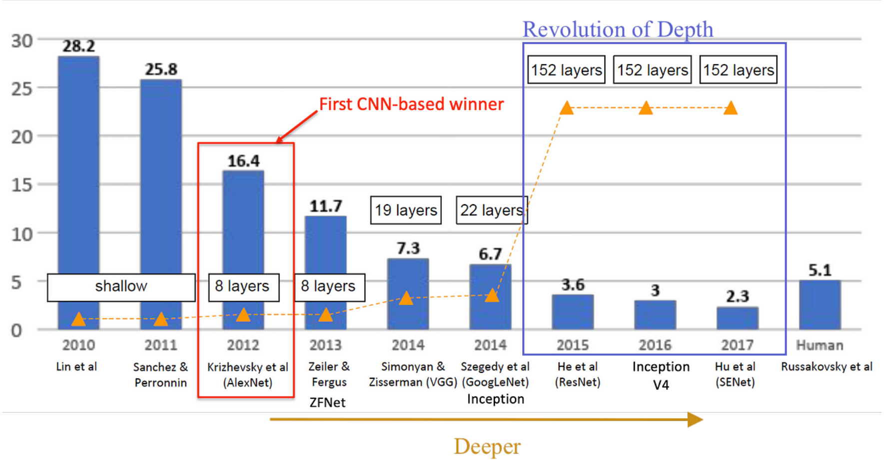
- Enhance the generalization of the model
- Encourages competition between adjacent features

  - $b_{x,y}^{i}$: Normalized activation at spatial location (x, y) in the i-th channel.
  - $a_{x,y}^{i}$: Original activation at the same location and channel.
  - N: Total number of channels.
  - n: Size of the local neighborhood (number of adjacent channels to consider for normalization).
  - k, α, β: Hyperparameters controlling the normalization.

Learning Magic In Alexnet

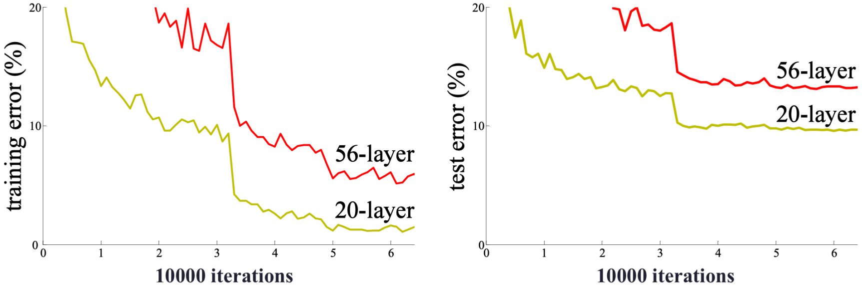
- First use of ReLU
  - → Made a large difference in convergence
- Dropout probability 0.5 (in FC layers only)
- Augmentations: translation, horizontal flip, altering the intensities of the RGB channels
- SGD with momentum of 0.9 and a mini-batch size of 128
- L2 weight decay 5e-4 ($l = l_{data} + \lambda||w||^2$)
- Learning rate: 0.01, decreased by 10 every time validation accuracy plateaus
- Evaluation metric: Validation accuracy
- **Final top-5 error: 18.2% with a single net, 15.4% using an ensemble of 7 networks**
  - → **Lowest prior error using conventional classifiers: > 25 %**

Revolution Of Depth



Challenge Of Making CNNs Deeper

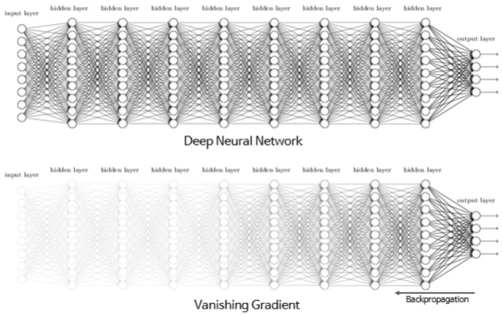
- What happens when we continue stacking deeper layers on a convolutional neural network?
- **Question**: What is strange about these training and test curves?



Training error (left) and test error (right) on CIFAR-10 with 20-layer and 56-layer "plain" networks

Challenge Of Making CNNs Deeper

- **Fact:** Deeper models have minotore representation power (more parameters) than shallower models.
- But **56-layer model performs worse** on both training and test error and it's not caused by overfitting!
- **Hypothesis:** The issue is related to optimization, deeper models are harder to optimize.



Vanishing Gradient

- Consider a deep neural network without residual blocks If we have the error signal $\bar{h}^{(T)}$. We then alternate between the following two backprop rules:

$$\bar{h}^{(t)} = \overline{z^{(t+1)}}\, w, \tag{1}$$

$$\overline{z^{(t)}} = \bar{h}^{(t)}\, \phi'\big(z^{(t)}\big). \tag{2}$$

- If we iterate the rules, we get:

$$\bar{h}^{(1)} = w^{T-1}\, \phi'\big(z^{(2)}\big) \cdots \phi'\big(z^{(T)}\big)\, \bar{h}^{(T)} \tag{3}$$

$$= \frac{\partial h^{(T)}}{\partial h^{(1)}}\, \bar{h}^{(T)}. \tag{4}$$

Vanishing Gradient

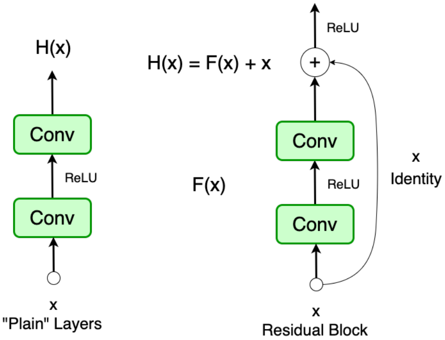
- Hence, $\bar{h}^{(1)}$ is a linear function of $\bar{h}^{(T)}$; the coefficient is the partial derivative $\dfrac{\partial h^{(T)}}{\partial h^{(1)}}$. If we make the simplifying assumption that the activation functions are linear, we get:

$$\frac{\partial h^{(T)}}{\partial h^{(1)}} = w^{T-1}, \tag{5}$$

- which can clearly explode or vanish unless w is very close to 1. For instance, if $w = 1.1$ and $T = 50$, then $\partial h^{(T)} / \partial h^{(1)} \approx 117.4$, whereas if $w = 0.9$ and $T = 50$, we get $\partial h^{(T)} / \partial h^{(1)} = 0.00515$.

Residual Block

- **Solution:** Use network layers to fit a residual mapping rather than directly fitting the desired underlying mapping.
- Identity mapping: $H(\mathbf{x}) = \mathbf{x}$ if $F(\mathbf{x}) = 0$



Use layers to fit residual F(x) = H(x) - x instead of H(x) directly

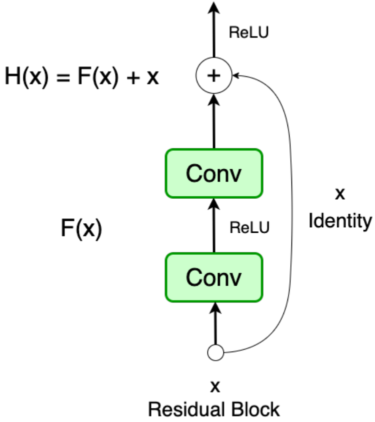
Backpropagation In Residual Block

- How exactly does the skip connection prevent gradient vanishing?
- **During backpropagation, there are two pathways.**

$$H(x) = F(x) + x$$

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial H} \times \frac{\partial H}{\partial x} = \frac{\partial L}{\partial H} \times \left(\frac{\partial F}{\partial x} + 1\right) = \frac{\partial L}{\partial H} \times \frac{\partial F}{\partial x} + \frac{\partial L}{\partial H}$$

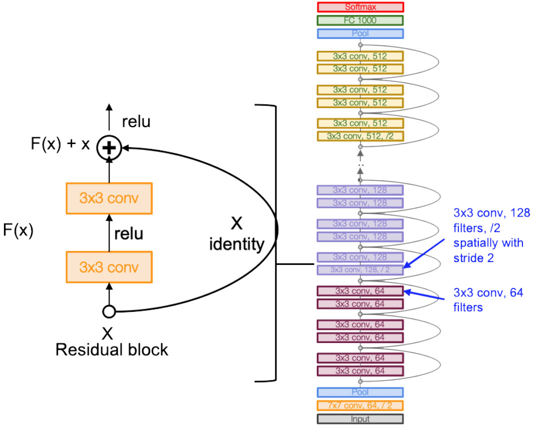
- If the gradient of Path 1 vanishes to 0, the gradient of Path 2 still remains non-zero (because this gradient does not encounter any weight layer)



ResNet [He, et al., (2015)]

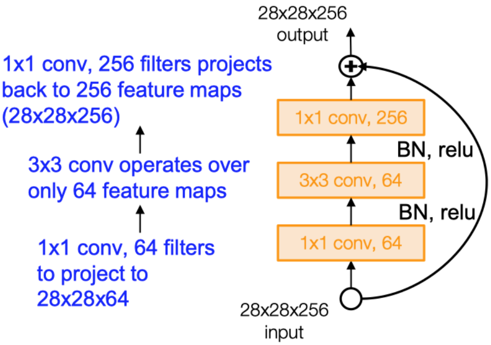
Architecture:

- Stack of residual blocks
- Doubling the filter count while halving spatial dimensions (via stride-2) at regular intervals in the network Reduce the activation volume by half.
- One FC layer at the end to output classes
- Total depths of 18, 34, 50, 101, or 152 layers for ImageNet



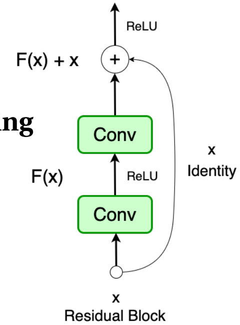
Deeper Residual Module (Bottleneck)

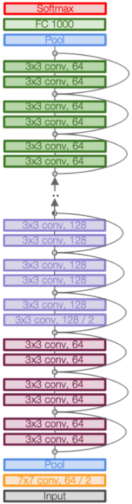
- For deeper networks (ResNet- 50+), use “bottleneck” layer instead, to improve efficiency.
- Bottleneck uses 1x1 convolution to:
  - Reduce number of paramters
  - Enable interaction between channels, creating new feature representations at each spatial location
  - Allow more layers to be added without a significant increase in computational cost or overfitting risk.



ResNet At A Glance

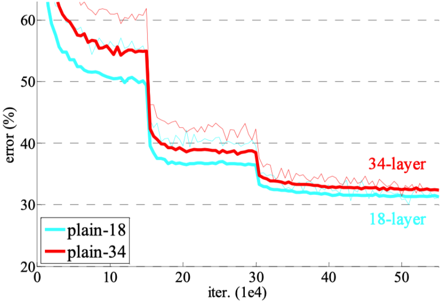
- Very deep networks using residual connections
- ResNet **introduces** the concept of skip connections (or residual connections) that allow the gradient to be directly backpropagated to earlier layers.
- **Skip connections helps overcoming gradient vanishing** problem, where the accuracy saturates and then degrades rapidly as the network depth increases.
- 152-layer model for ImageNet
- ILSVRC'15 classification winner (3.57 % top 5 error)
- Swept all classification and detection competitions in ILSVRC'15 and COCO'15!

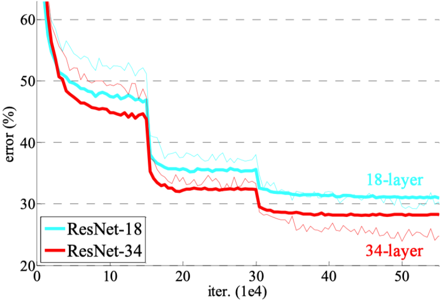




ResNet Experiments

- Able to train very deep networks without degrading (152 layers on ImageNet, 1202 on CIFAR)
- Deeper networks now achieve lower training error as expected.





Thin curves represent training error, and bold curves represent validation error [He, et al. 2015]

Training ResNet In Practice:

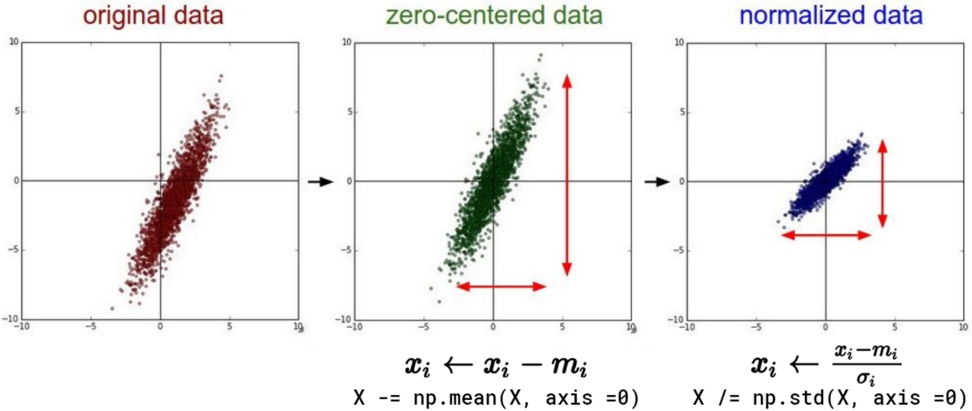
- Batch Normalization after every CONV layer
- Xavier initialization from He, et al.
- SGD + Momentum (0.9)
- Learning rate: 0.1, divided by 10 when validation error plateaus
- Mini-batch size 256
- Weight decay of 1e-5
- No dropout used

Key Points

- **AlexNet** showed that you can use CNNs to train Computer Vision models.
- **ResNet** showed us how to train extremely deep networks.
  - → Limited only by GPU & memory!
  - → Showed diminishing returns as networks got bigger (increasing the depth or size of the network doesn't proportionally improve its performance)
- After ResNet: CNNs were better than the human metric and focus shifted to other topics:
  - ★ Efficient Networks: **MobileNet, ShuffleNet**
  - ★ **Neural Architecture Search** can now automate architecture design

Normalizing The Data

- Assume $X_{n \times d}$ is data matrix, with each sample is represented in a row
- i is the index of dimension ($i = 1, \ldots d$)

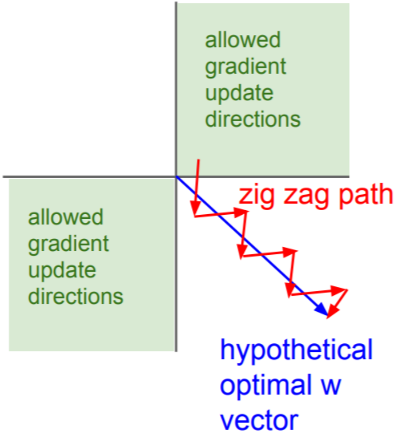


TLDR: In Practice For Images: Center Only

- Example: consider CIFAR-10 example with $[32, 32, 3]$ images
- 3 different approaches:
  1. Subtract the mean image (e.g. AlexNet)
     - mean image = $[32, 32, 3]$ array
  2. Subtract per-channel mean (e.g. VGGNet)
     - mean along each channel = 3 numbers
  3. Subtract per-channel mean and divide by per-channel std (e.g. ResNet)

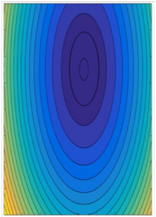
Why Zero-Mean The Input?

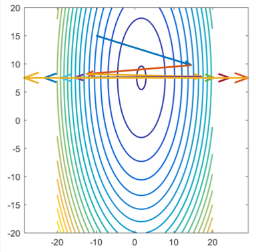
- Reminder: sigmoid
- Consider what happens when the input to a neuron is always positive
- What can we say about the gradients on w?
  - Always all positive or all negative
  - Leads to zig-zag dynamics in the gradient updates for the weights (in red) if the optimal w vector was the blue one
- This is also why zero-mean data is preferable.



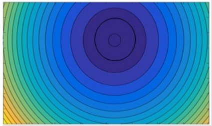
Why Normalize The Input?

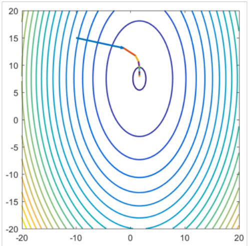
- Wide variation in the range of feature values
  - Leads to poor conditioning
- Results in noisy movement of gradient



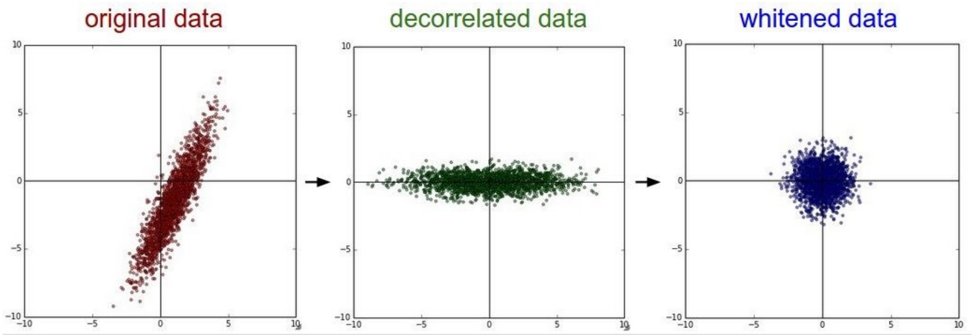


- Normalized data:
  - Enhances the geometry of the loss function
    - Mitigates issues of poor conditioning
  - Accelerates convergence during training





- While standardizing inputs is common practice, transformations like variance normalization, PCA, and whitening are used more selectively.

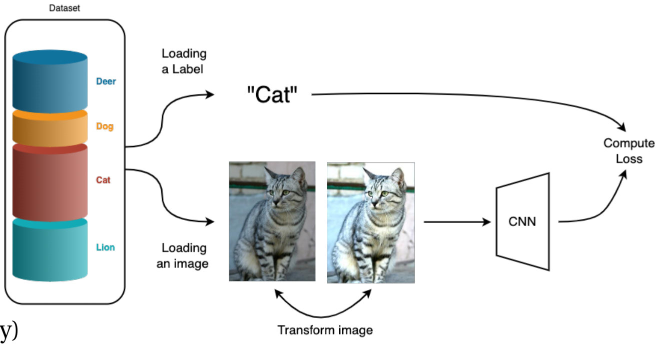


Data has diagonal covarianve matrix

Covarianve matrix is identity matrix

Motivation

- Getting more training data is often expensive
- But, we can often generate additional training examples from existing datasets.
- Transform each input data example in such a way that the label stays the same
- Benefits:
    - Reduces the risk of overfitting
    - Improves generalization
    - Acts as a regularization
- Examples of data augmentations
    - Translation
    - Flip (Horizontal/Vertical)
    - Rotation
    - Stretching
    - Shearing
    - Lens distortions, ... (going crazy)

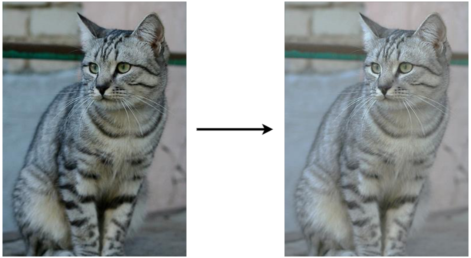


Augmentations In Action

- Simple
  - Randomize contrast and brightness levels
- Complex
  1. Apply PCA to all [R, G, B] pixels in training set
  2. Sample a “color offset” along principal component directions
  3. Add offset to all pixels of a training image

  (As seen in AlexNet, ResNet, etc)

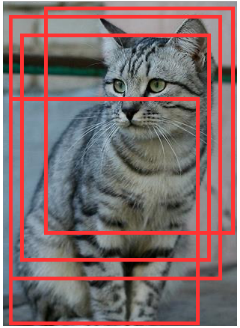
Color Jitter



Augmentations In Action

- **Training:** randomly sample crops and scales
  ResNet:
  1. Pick random L in range [256, 480]
  2. Resize training image, short side = L
  3. Sample random 224 x 224 patch
- **Testing:** average a fixed set of crops
  ResNet:
  1. Resize image at 5 scales: 224, 256, 384, 480, 640
  2. For each size, use 10 224 x 224 crops:
     4 corners + center + flips

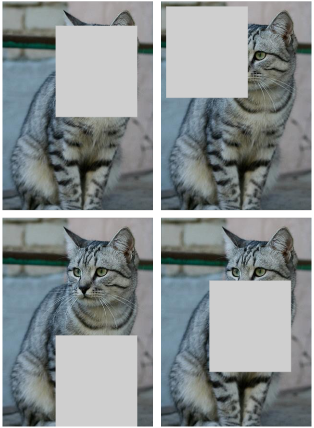
Random Crops and Scales



Augmentations In Action

- **Training:** Randomly set image regions to zero
- **Testing:** Use the full image
- Effective for small datasets like CIFAR, less common for large datasets like ImageNet

Cutout



DeVries and Taylor, "Improved Regularization of Convolutional Neural Networks with Cutout", arXiv 2017

Augmentation With Albumentations

```python
import albumentations as A
```

- Basic pixel-Level Augmentations
  - A.RandomBrightnessContrast(),A.HueSaturationValue(), A.GaussianBlur()
- Image Distortion and Warping
  - A.ElasticTransform(), A.GridDistortion(), A.OpticalDistortion()
- And many more:
  - Advanced Pixel-Level Augmentations, Cropping and Padding, Geometric Transformations, Composite Augmentations, etc.
- Albumentations can even be used to **augment bounding boxes for object detection tasks** (check this out!)
- Interactive demo: https://demo.albumentations.ai

Examples

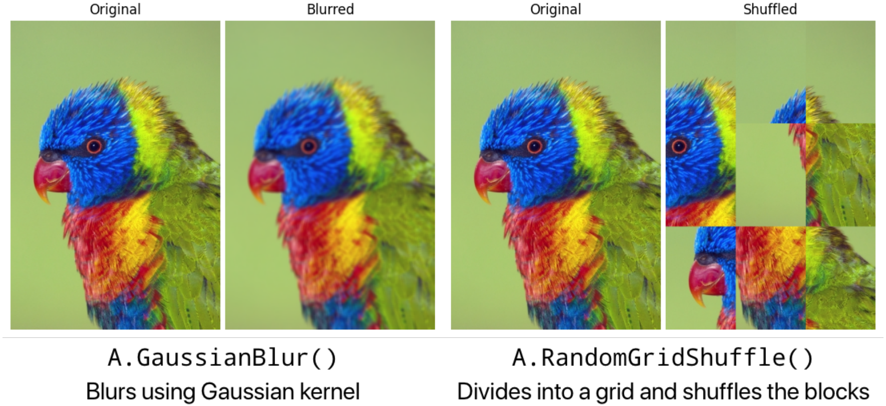


Image source: freepik.com

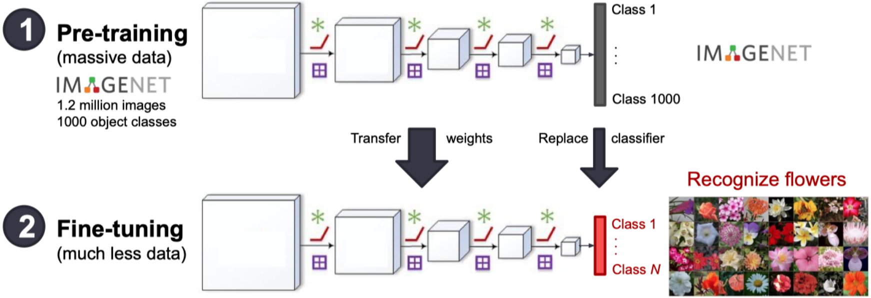
Motivation

- You need a lot of a data if you want to train/use CNNs
- Suppose you want to build an app to recognize flowers. . .

https://www.robots.ox.ac.uk/ vgg/data/flowers/102

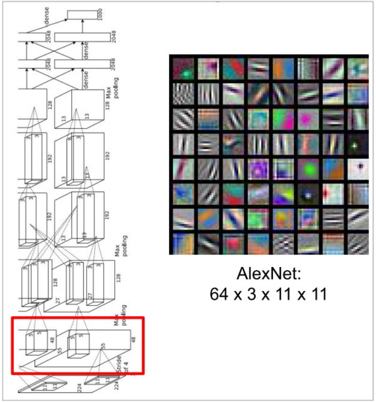
Transfer Learning

1. Pretrain on a large dataset (e.g. ImageNet)
2. Fine-tune it on a small target dataset (e.g. Flowers)

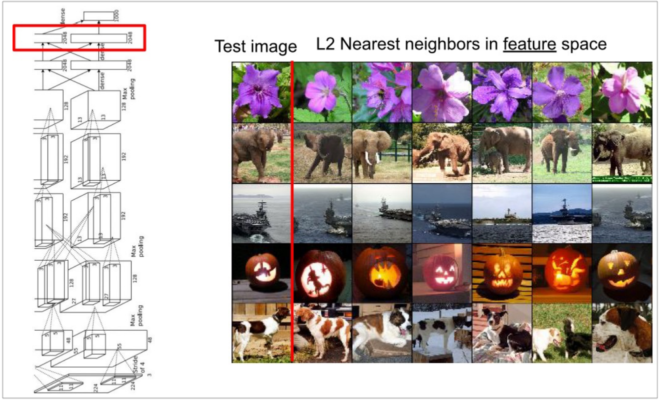


Transfer Learning: The idea

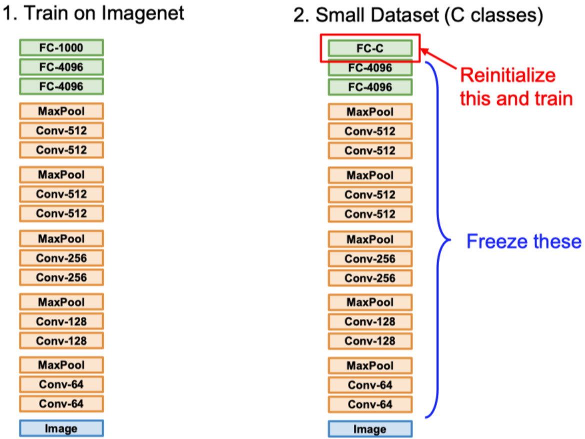
Earlier Layers -> more local features

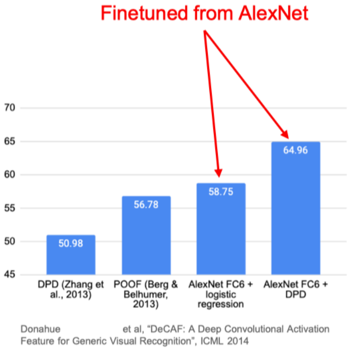


Final Layers -> more global features



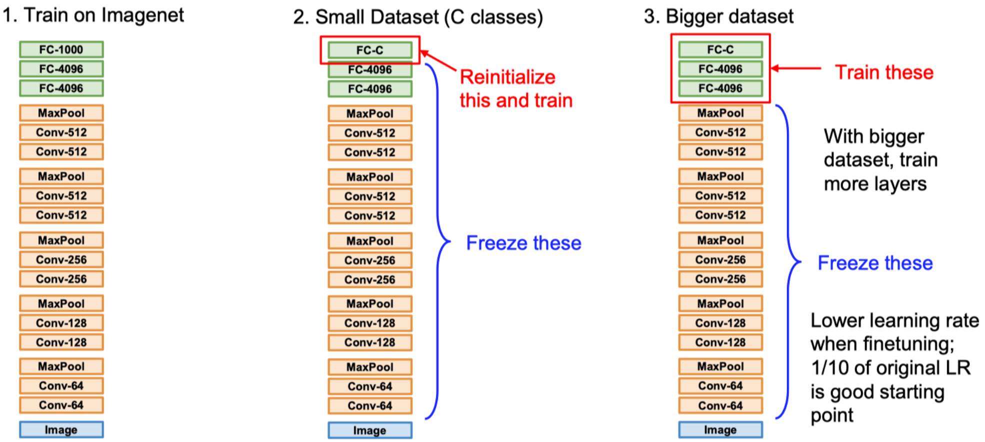
Transfer Learning With CNNs



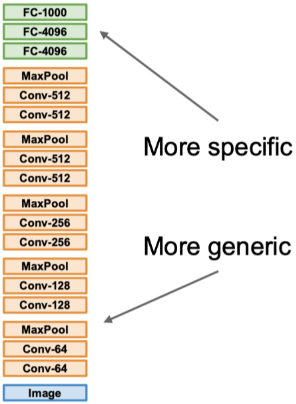


Donahue et al, (ICML 2014), Razavian et al, (CVPR Workshops 2014)

Transfer Learning With CNNs

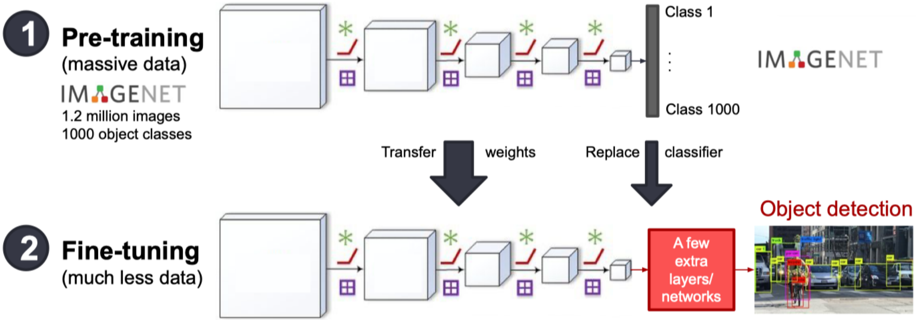


Transfer Learning With CNNs

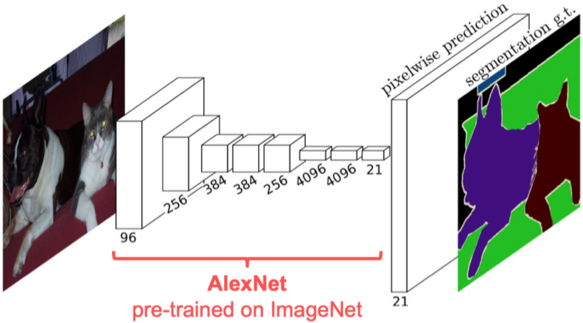


	very similar dataset	very different dataset
very little data	Use Linear Classifier on top layer	You're in trouble… Try linear classifier from different stages
quite a lot of data	Finetune a few layers	Finetune a larger number of layers or start from scratch!

Transfer Learning: Domain Shift



Example: Semantic Segmentation



Long, Shelhamer, Darrell, CVPR 2015

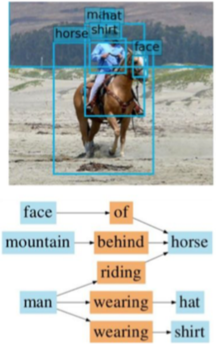
Transfer Learning With CNNs Is Pervasive. . .

Pose estimation

Image captioning

"two young girls are playing with lego toy."

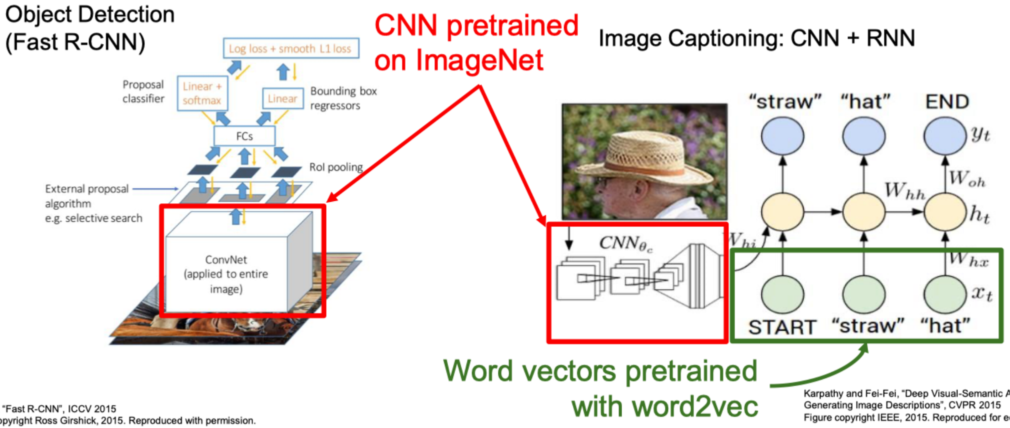
Scene graph prediction



. . .

Many, many more

Transfer Learning With CNNs Is Pervasive. . .

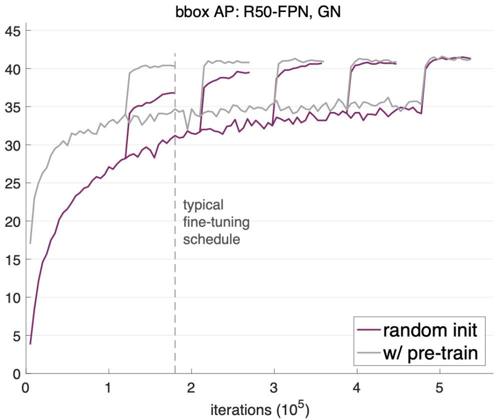


Girshick, "Fast R-CNN", ICCV 2015
Figure copyright Ross Girshick, 2015. Reproduced with permission.

Karpathy and Fei-Fei, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015
Figure copyright IEEE, 2015. Reproduced for educational purposes.

Transfer Learning Might Not Always Be Necessary!

- Training from scratch can perform as well as using a pretrained ImageNet model for object detection.
- But it requires 2-3 times longer to train.
- Collecting more data is better than finetuning on a related task



He et al (ICCV 2019)

Takehome Message

- Have some dataset of interest but it has fewer than ~ 1M images?
    1. Find a very large dataset that has similar data, train a big ConvNet there
    2. Transfer learn to your dataset
- Deep learning frameworks offer a "Model Zoo" of pretrained models so you don't need to train your own
    - **Pytorch:** https://github.com/pytorch/vision
    - **TensorFlow:** https://github.com/tensorflow/models

Slides by: Ali Bavafa

Any Questions?