

Machine Learning (CE 40477)

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1 Batch Normalization Fundamentals

2 Batch Normalization in Practice

3 MLPs as Universal Approximators

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1 Batch Normalization Fundamentals

Why Batch Normalization?

How Batch Normalization Works

2 Batch Normalization in Practice

③ MLPs as Universal Approximators

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1 Batch Normalization Fundamentals

Why Batch Normalization?

How Batch Normalization Works

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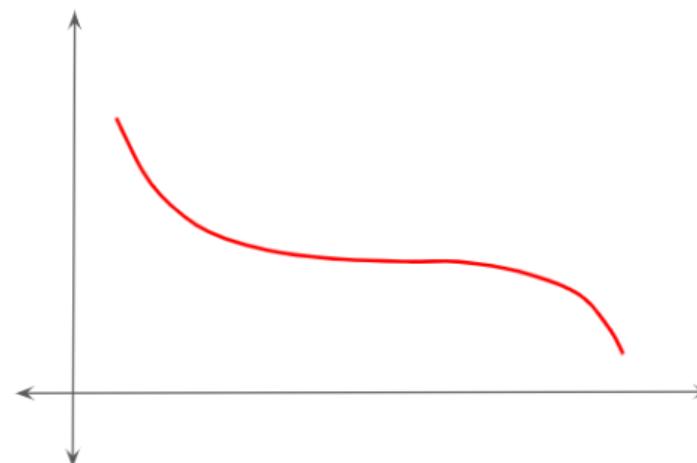
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Why Batch Normalization?

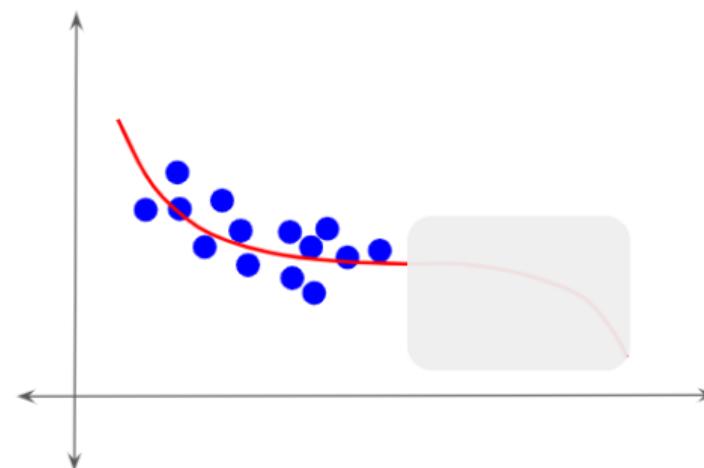
Problem: Internal Covariate Shift

- What does it mean, in simple terms?
- Let's say that we want to train a model and the ideal target output function that the model needs to learn is as below:



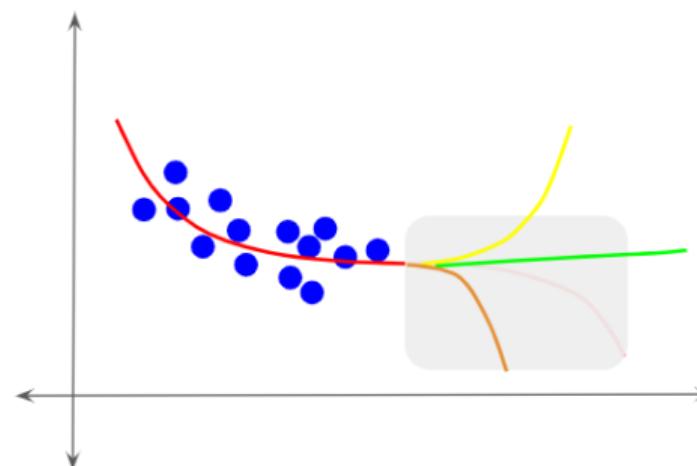
Problem: Internal Covariate Shift

- Suppose that the training data values input to the model cover only a part of the range of output values. Therefore, the model can only learn a subset of the target function.



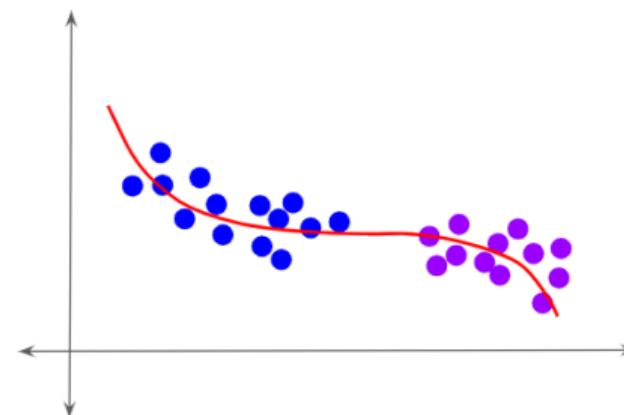
Problem: Internal Covariate Shift

- The model has no idea about the rest of the target curve. It could be anything.



Problem: Internal Covariate Shift

- Suppose we feed the model to the testing data as below.
- This has a very different distribution from the data that the model was initially trained with.
- The model cannot simply generalize its predictions for this new data.



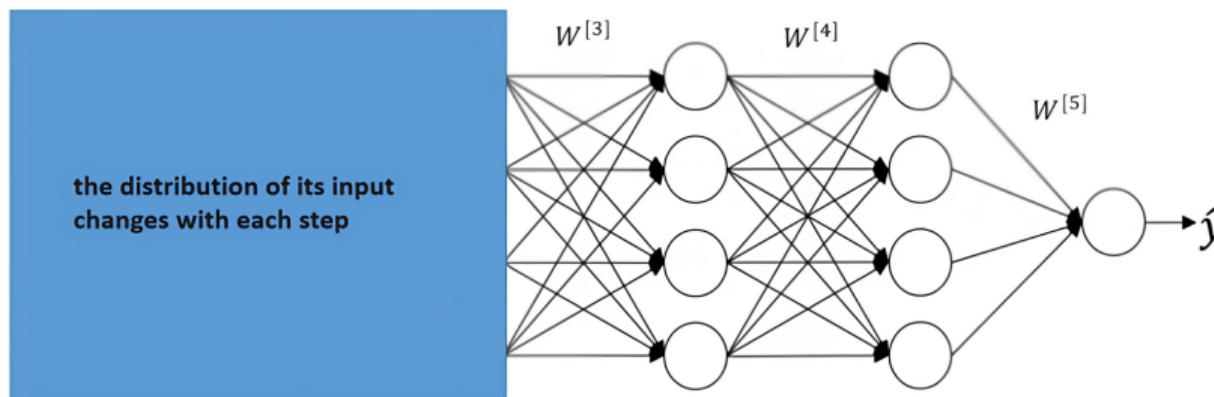
Problem: Internal Covariate Shift

- Covariate Shift occurs when **the model is fed data with a different distribution than what it was previously trained with**, even though that new data still conforms to the same target function.
- For the model to figure out how to adapt to this new data, it has to re-learn some of its target output functions.
- **This slows down the training process.**

Problem: Internal Covariate Shift

What happens inside Deep Network Layers?

- Consider what happens when training a deep network: As we update the weights of earlier layers, the data distribution in the deeper layer keeps shifting.
 - Deeper layers see new and varying patterns every time we update the weights in previous layers.



Problem: Internal Covariate Shift

What happens inside Deep Network Layers?

- The network must keep readjusting, which makes the learning process slower and more challenging.
- **In other words:** The network is constantly “re-learning” how to make predictions because the data it sees is never consistent.

Batch Normalization Solution

- **Goal:** Normalize inputs so that the mean is near 0 and the variance is close to 1.
- **How it helps:** Stabilizes learning, allowing for higher learning-rates and faster convergence.

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How Batch Normalization Works

Process Overview

- For each mini-batch during training, batch normalization normalizes the inputs to a layer by adjusting their mean and variance.

How Batch Normalization Works

Steps in Batch Normalization

① Compute the Mean and Variance

For a given mini-batch, compute the mean μ_B and variance σ_B^2 of the inputs:

$$\mu_B = \frac{1}{m} \sum_{i=1}^m x_i, \quad \sigma_B^2 = \frac{1}{m} \sum_{i=1}^m (x_i - \mu_B)^2$$

② Normalize the Inputs

Subtract the mean and divide by the standard deviation to get normalized activations:

$$\hat{x}_i = \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}$$

where ϵ is a small constant added for numerical stability.

How Batch Normalization Works (Continued)

Steps in Batch Normalization

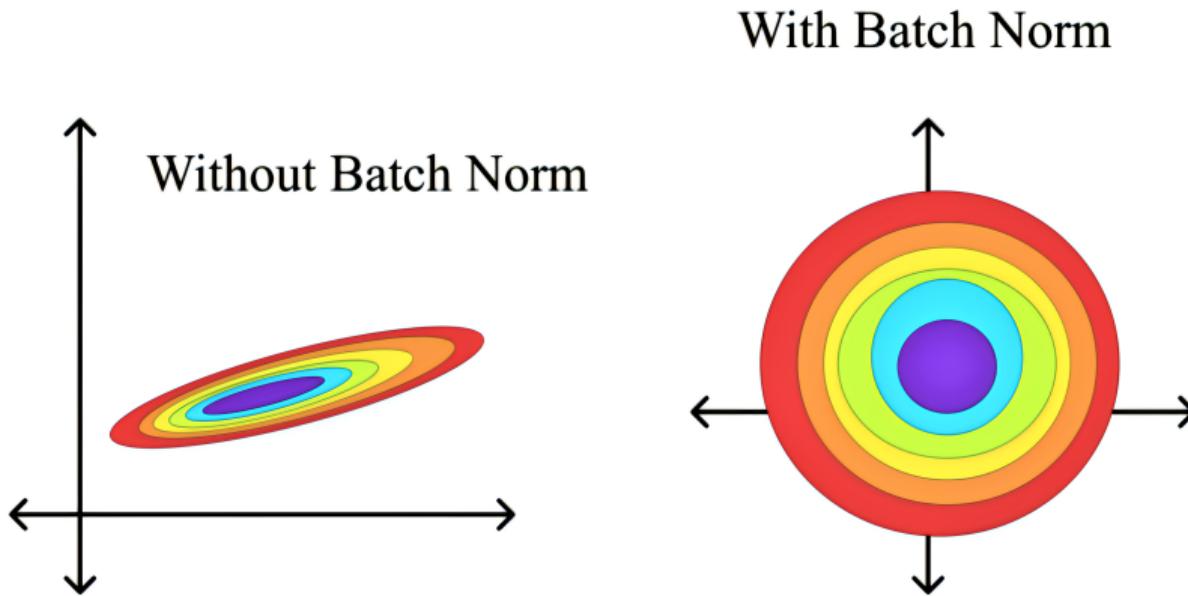
③ Scale and Shift

After normalization, introduce learnable parameters γ and β that allow the model to scale and shift the normalized output:

$$y_i = \gamma \hat{x}_i + \beta$$

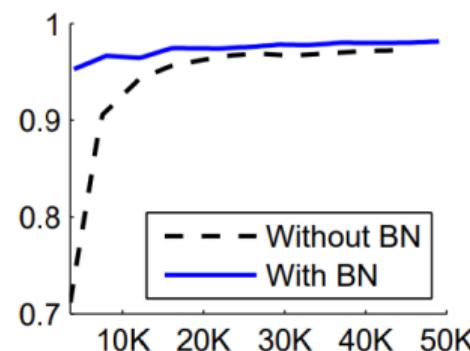
This allows the model to recover the original data distribution if necessary.

Smoothing the optimization space



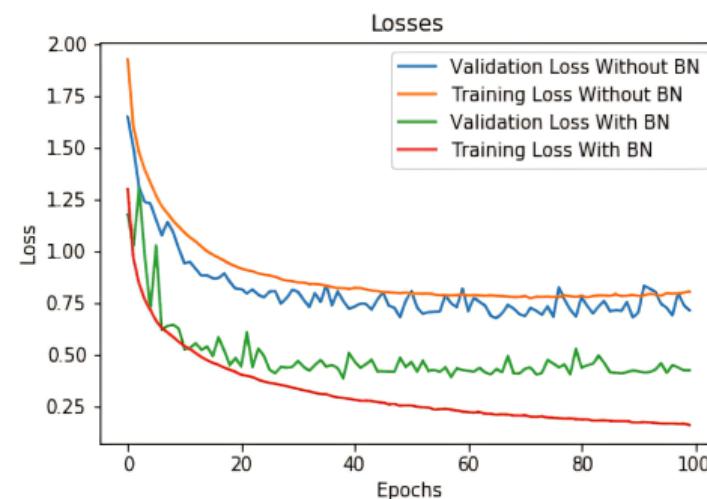
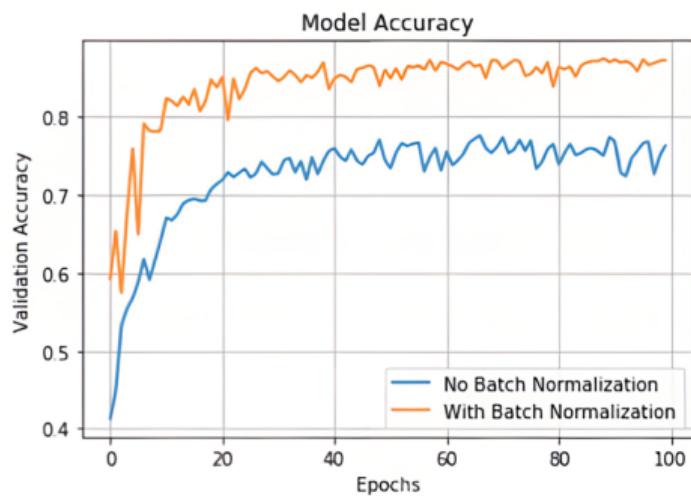
Effect of Batch Normalization

- Batch Normalization helps the network train faster and achieve higher accuracy.
- Batch Normalization **stabilizes the distribution** and reduces the internal covariate shift.

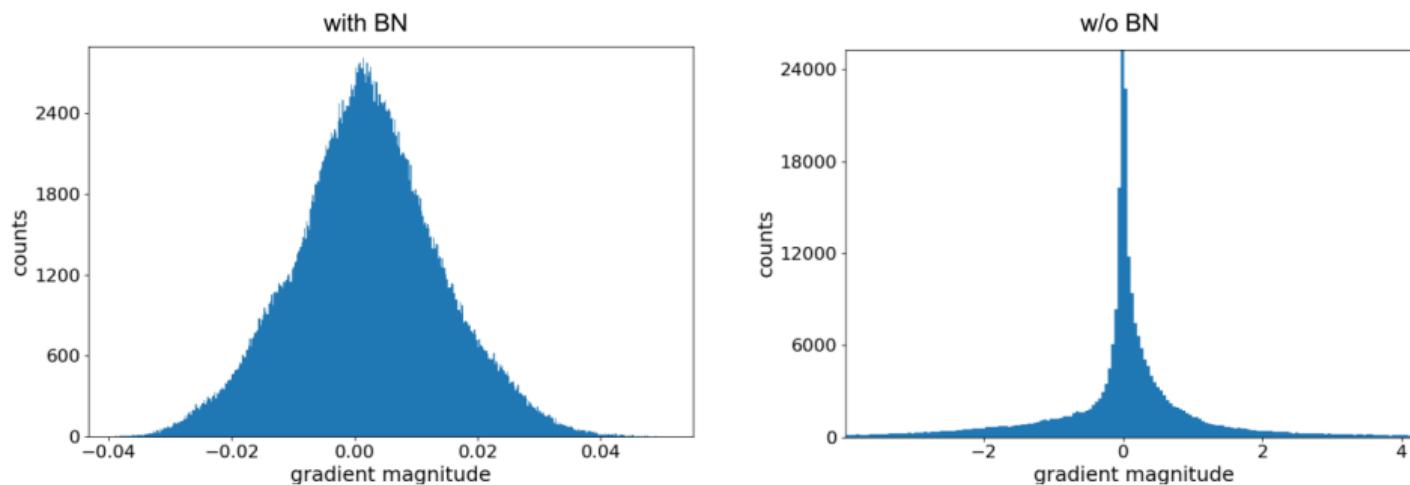


- The test accuracy of the MNIST network trained with and without Batch Normalization, vs the number of training steps.

Effect of Batch Normalization



Effect of Batch Normalization on Gradients



Effect of Batch Normalization on Gradients

The main benefit of batch normalization is that it reduces the dependency of gradient on the scale of input and parameters:

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\partial \mathcal{L}}{\partial y} \cdot \frac{\partial y}{\partial \hat{x}} \cdot \frac{\partial \hat{x}}{\partial x} \quad (1)$$

Where:

- $\frac{\partial y}{\partial \hat{x}} = \gamma$
- $\frac{\partial \hat{x}}{\partial x} = \frac{1}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}}$

Thus, the gradient becomes:

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\gamma}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \cdot \frac{\partial \mathcal{L}}{\partial y} \quad (2)$$

How Batch Normalization Smooths the Loss Landscape

The smoothing effect of batch normalization can be understood by observing how it constrains the gradient magnitudes. The expression shows that:

$$\frac{\partial \mathcal{L}}{\partial x} \text{ is scaled by } \frac{\gamma}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \quad (3)$$

This consistent scaling results in a smooth loss landscape because:

- It stabilizes gradient flow, ensuring controlled optimization step sizes.
- Reduces risk of large oscillations or abrupt changes in the loss landscape.
- Makes the optimization process less likely to be trapped in local minima or saddle points.

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Inference Mode, Pros, Cons & Implementation

Closing Takeaways on Batch Normalization

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Batch Normalization in Training vs Inference

Key Difference

- During **training**: Normalize using mini-batch statistics (mean and variance computed from current batch)
- During **inference**: Use running averages (exponential moving averages accumulated during training)
- **Why?** Inference typically uses single samples or small batches, so batch statistics are unreliable

Batch Normalization Pros

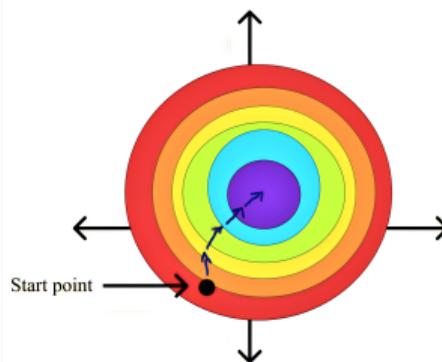
Pros

- **Faster Convergence:** Empirical results support that models with batch normalization converge faster and achieve higher accuracy, even with **higher learning rates**.
- **Reduced Sensitivity to Weight Initialization:** Mitigate the dependency on careful weight initialization.
- **Acts as Regularization:** Batch normalization can help reduce overfitting.
- **Reduces Vanishing/Exploding Gradients:** Maintains stable gradients throughout deep networks.

Batch Normalization Pros

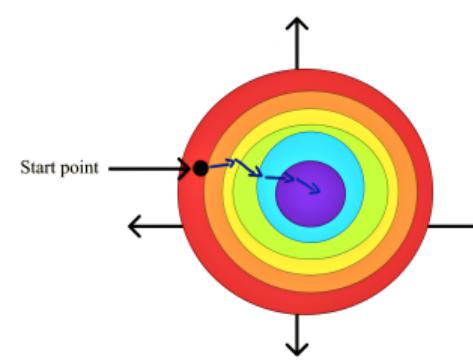
Why Using Batch Normalization Reduces Sensitivity to Weight Initialization?

With Batch Norm



(a)

With Batch Norm



(b)

Batch Normalization Pros

Why Does Batch Normalization Reduce Sensitivity to Weight Initialization?

- Batch normalization smooths the optimization landscape, reducing the dependency on initial weights.
- This allows the model to converge to a minimum efficiently, **regardless of where optimization begins.**

Batch Normalization Cons

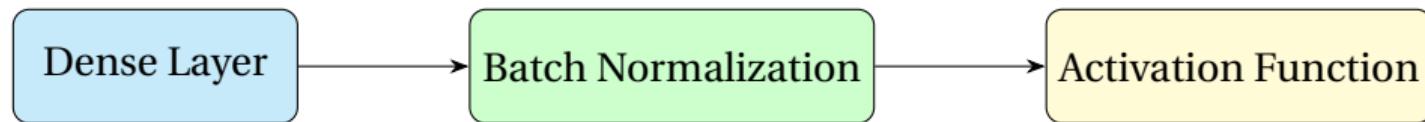
Cons

- **Batch Size Sensitivity:** Performance may depend on batch size. Very small batches potentially provide unstable statistics.
- **Computational Overhead:** Increases computational-overhead during training.
- **Behavior During Inference:** Switching from batch statistics to moving averages during inference may cause slight discrepancies.

Batch Normalization in Practice

Where to Apply

- **Typical Location:** Apply after the linear transformation (e.g., after a dense layer) and before the activation function.
- **Layer Placement:**



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Key Takeaways: Batch Normalization

- **Stabilization:** Reduces internal covariate shift and normalizes activations.
- **Acceleration:** Enables faster convergence with higher learning rates.
- **Regularization:** Acts as an implicit regularizer, reducing overfitting.
- **Scalability:** Facilitates training of deeper networks with less hyperparameter tuning.
- **Practical Impact:** One of the most important techniques in modern deep learning.

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Universal Approximation Theorem

Key Concept

- The Universal Approximation Theorem states that a feedforward neural network with:
 - A single hidden layer
 - Sufficient number of hidden neurons
 - Appropriate activation functions (e.g., sigmoid)

Can approximate any continuous function on a **compact subset** of \mathbb{R}^n to any desired accuracy.

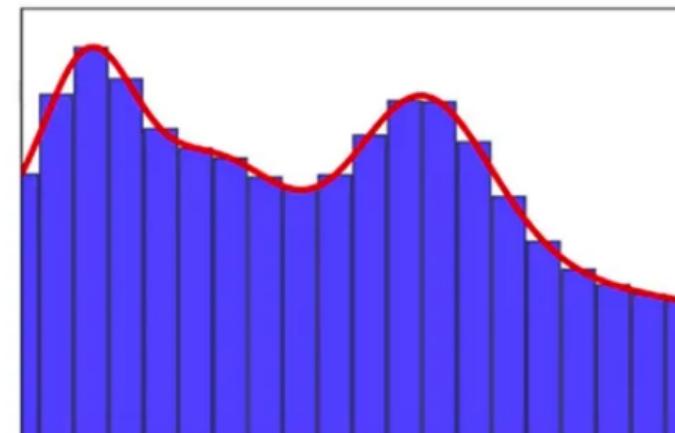
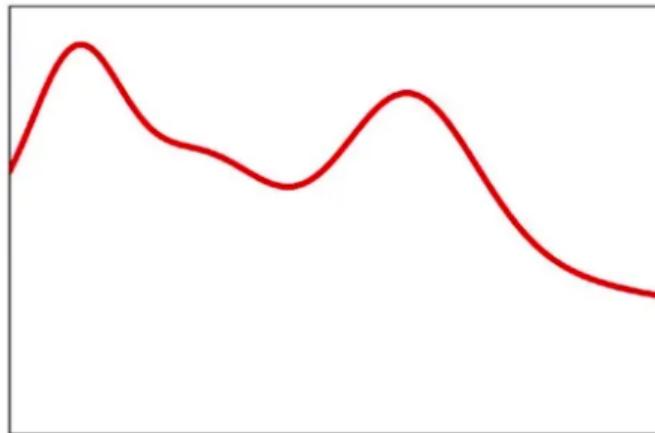
Understanding Compact Sets

What is a Compact Set?

- In the context of the Universal Approximation Theorem, approximation is guaranteed on a **compact subset** of \mathbb{R}^n .
- A set is compact if it is both:
 - **Bounded:** Enclosed within a finite space.
 - **Closed:** Contains all its boundary points.
- Compact sets ensure certain mathematical properties that enable reliable function approximation by the MLP within that region.

Breaking Down Complex Functions

- Idea: Complex functions can be decomposed into multiple smaller parts, each represented by a simpler function.
- By combining a series of simpler functions (like square pulses), the target function can be closely approximated.



MLPs as Universal Approximators

- By constructing a series of these Square Pulse functions, we can approximate any continuous function mapping from input to output.
- A simple 3-unit MLP with a summing output unit can generate a square pulse.
- Therefore, an MLP with enough units and the right configuration is a universal approximator!

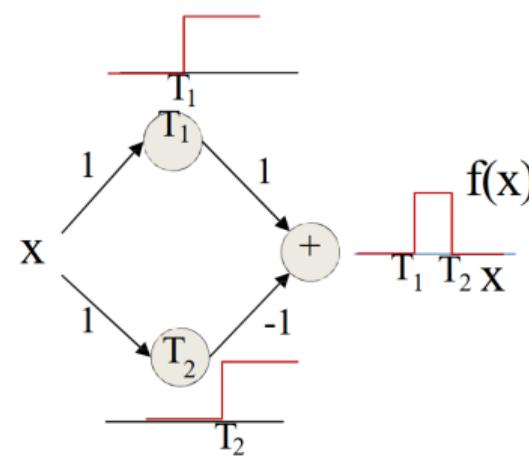


Figure adapted from Dr. Mahdieh Soleymani Baghshah, Deep Learning Course

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Contributions

These slides are authored by:

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Any Questions?