Machine Learning (CE 40717)

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How Do Humans See?

How can we enable computers to **see**?

Figure adapted from [4]

What Computers **See**: Images As Numbers

An image is just a matrix of numbers .i.e., $1080 \times 1080 \times 3$ for an RGB image.
Question: is this Lincoln? Washington? Jefferson? Obama?
How can the computer answer this question?

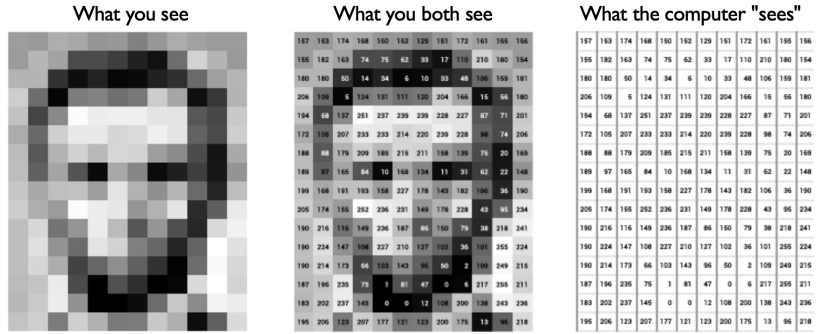


Figure adapted from [4]

What Is Computer Vision?

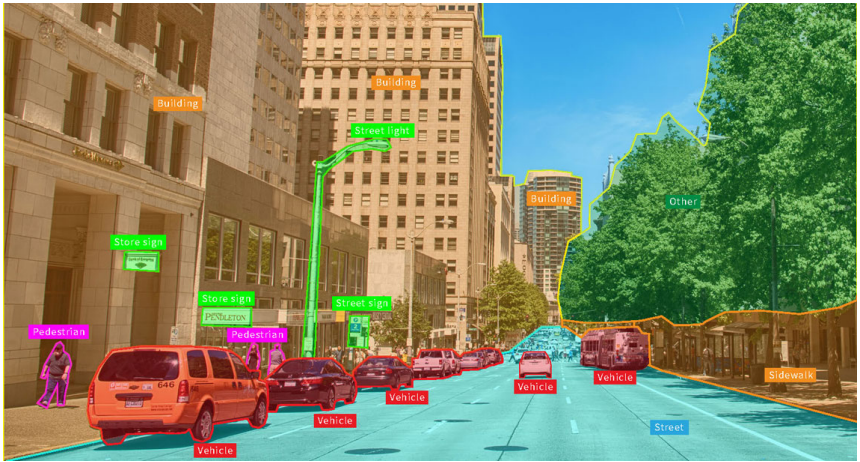


Figure adapted from source

What Is Computer Vision?

Noisy image

Denoised image

Figure adapted from source

What Is Computer Vision?

Hand-crafted Features: Before Deep Learning Features

Two types of features used for action recognition.

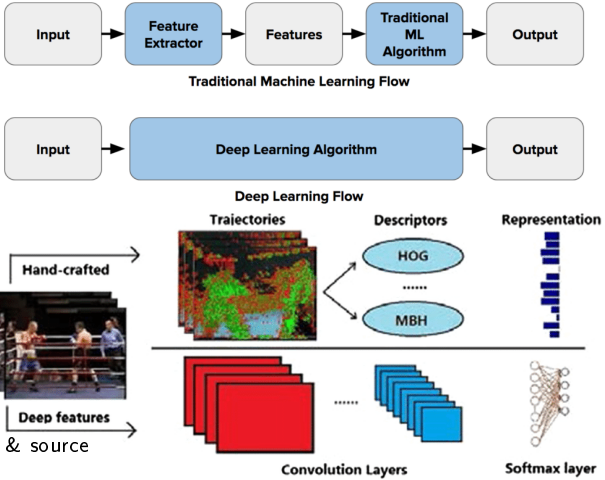


Figure adapted from source & source

Hand-crafted Features: Before Deep Learning Features

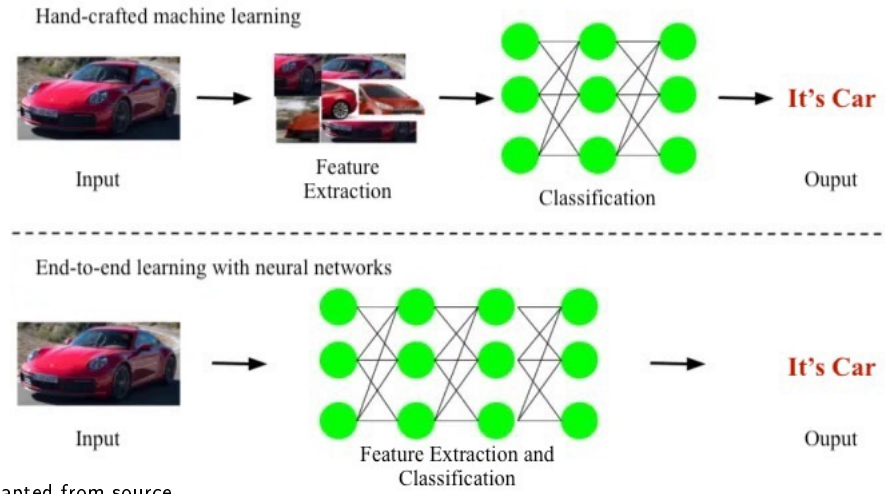


Figure adapted from source

Image Features

Fusion of multi-scale bag of deep visual words features of chest X-ray images to detect COVID-19 infection

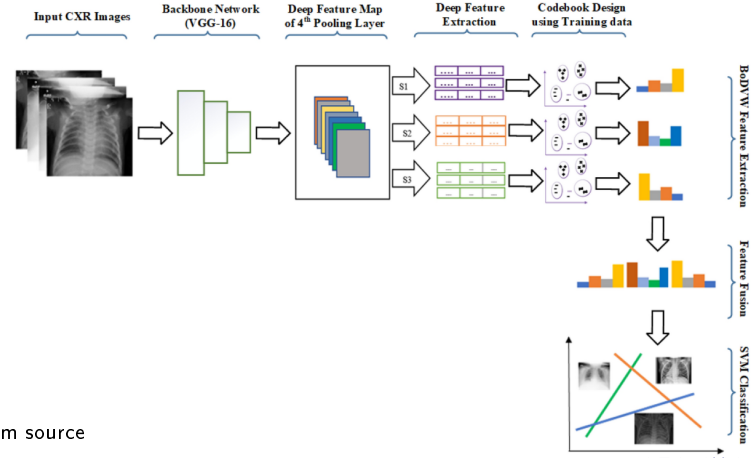


Figure adapted from source

Effect Of DL

Comparison between Deep Learning and Traditional Models

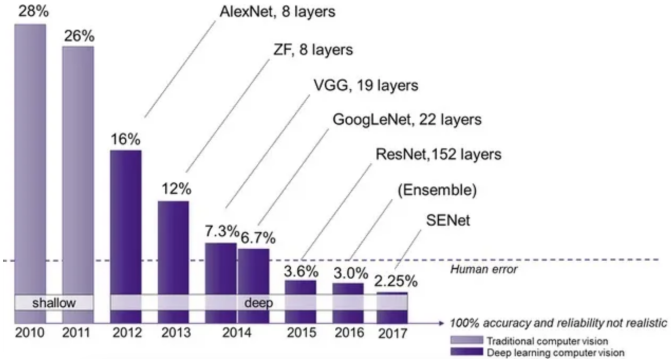


Figure adapted from source

FCNs On Images

- What we've been using?
  - **Dense** Vector Multiplication

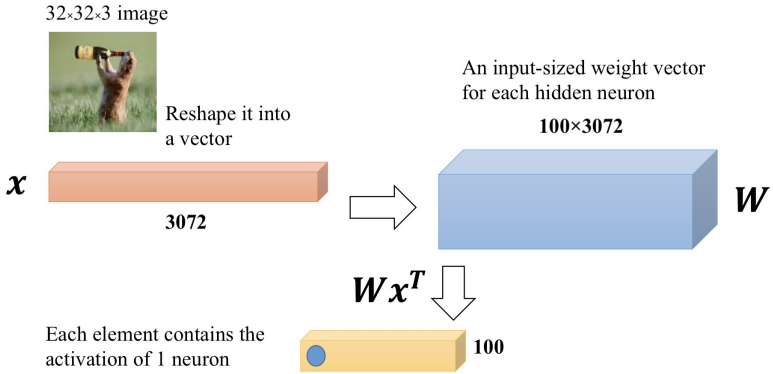


Figure adapted from [3]

Fully Connected Layers

- Neurons in a single layer operate independently and **do not share connections**, even for inputs.
- To be shift-invariant, **many samples** (in various time or locations) must be shown to them.
- **Regular Neural Nets don't scale well to full images**.
  - Parameters would **add up** quickly!
  - Full connectivity is wasteful and the huge number of parameters would quickly **lead to overfitting**.

Solution?

What can we do?

Invariance VS. Equivariance

- **Invariance:** The output **remains the same** when the input undergoes a transformation.
- **Equivariance:** The output **varies predictably** when the input undergoes a transformation.

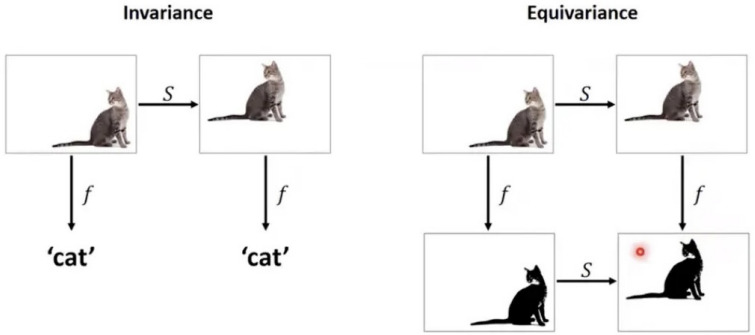
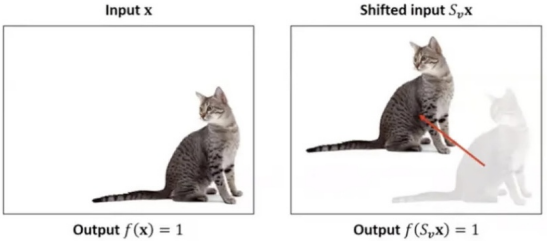


Figure adapted from source

Shift-invariance



- *Cat detector* $f : \mathbb{R}^d \to \mathbb{R}$
- *Shift operator* $S_v : \mathbb{R}^d \to \mathbb{R}^d$ shifting the image by vector v

Shift invariance: $f(\mathrm{x}) = f(S_v\mathrm{x})$

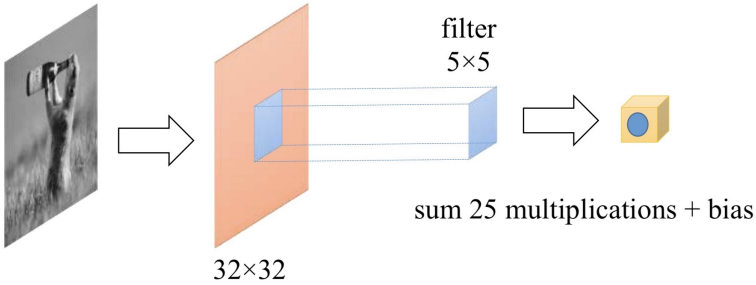
Question: Will an MLP that recognizes the left image as a cat also recognize the shifted image on the right as a cat?

A Problem

- In many problems the **location** of a pattern is not important.
  - Only the **presence** of the pattern matters.
- Traditional MLPs are **sensitive** to pattern location.
  - Moving it by one component results in an entirely different input that the MLP won't recognize.
- **Requirement:** Network must be **shift invariant**.

Convolution On Images

Convolution (Refresher)



Matrix input preserving **spatial structure**

Figure adapted from [3]

Fully Connected Or Convolution

Question: Can I just do classification on an flatten image (e.g., a 1080x1080x3 image) with fully connected layers?

Fully Connected Or Convolution

Question: Can I just do classification on an flatten image (e.g., a 1080x1080x3 image) with fully connected layers?

Answer: No, using fully connected layers on such a large image is inefficient due to:

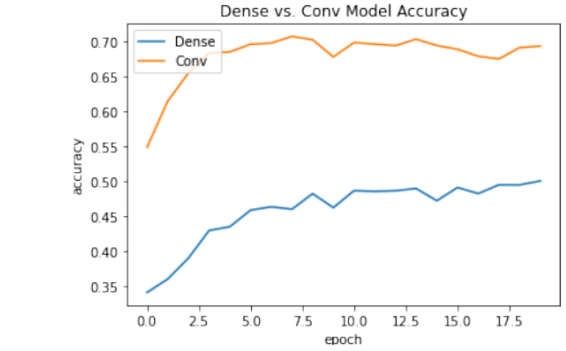
- Loss of spatial structure
- High parameter count

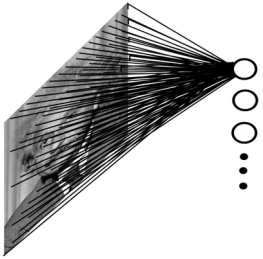
Fully Connected Or Convolution

Why Use Convolution:

- **Exploit spatial structure:** Convolution sees local patterns by using filters over small patches of the image.
- **Reduce parameters:** By sharing weights across the image, convolution reduce the number of parameters.
- **Build hierarchical features:** Convolution starts with small patterns, then combines them to recognize more complex patters.

Model Architecture	# Params	Test Accuracy
CNN	**2M**	**67.8%**
FCN	9M	50.6%





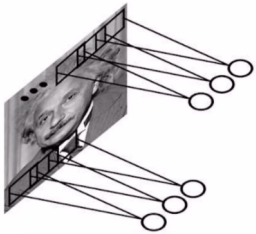


Figure adapted from source

What Is CNN?

- It is a class of deep learning.
- Convolutional neural network (ConvNet's or CNNs) is one of the main categories to do image recognition, image classifications, object detection, recognition of faces, etc.
- It is **similar to the basic neural network**. CNNs also have learnable parameters like weights and biases, similar to neural networks.
- CNN is heavily used in **computer vision**.
- There are **3 basic components** to define CNN:
  - The Convolution Layer
  - The Pooling Layer
  - The Output Layer or Fully Connected Layer

Architecture Of CNN

The basic idea of Convolutional Neural Networks (CNNs) is similar to Backpropagation Neural Networks (BPNNs) but differs in implementation.

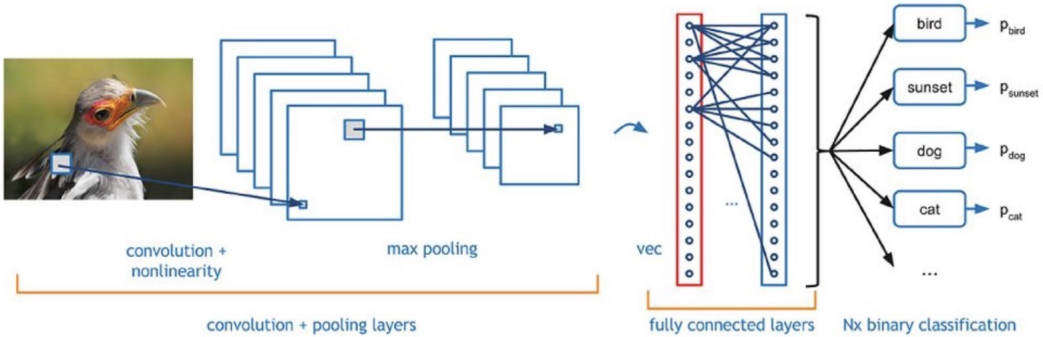


Figure adapted from source

After vectorized (vec), the 2D arranged inputs become ID vectors. Then the network is just like a BPNN (Back propagation neural networks)

The Basic Structure

- Alternating **Convolution (C)** and **subsampling layer (S)**
- Subsampling allows flexible positioning of features.

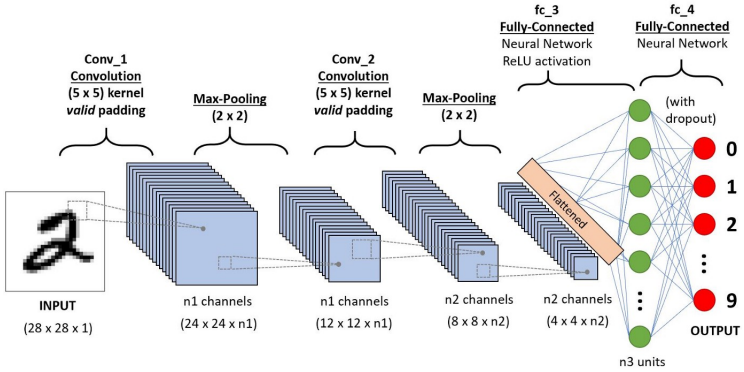


Figure adapted from source

The Basic Structure

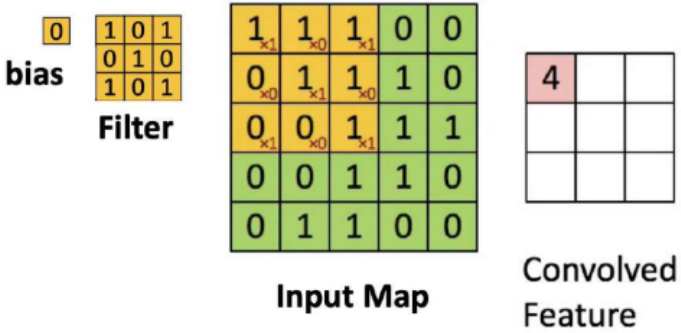
Three Main Types of Layers

- **Convolutional Layer**
  - Output of neurons are connected to **local regions** in the input.
  - Applying the **same filter** across the entire image.
  - CONV layer's parameters consist of a set of **learnable filters**.
- **Pooling Layer**
  - Performs a **downsampling** operation along the spatial dimensions.
- **Fully-Connected Layer**
  - Typically used in the final stages of the network to combine high-level features and **make predictions**.

CNN VS. FCN

- **Fully Connected Networks (FCNs):**
  - High number of parameters, leading to **overfitting on large inputs** (e.g., images).
  - Lack of spatial awareness, making it **sensitive to the exact positioning** of patterns.
  - Suitable for structured data but **inefficient for image processing**.
- **Convolutional Neural Networks (CNN):**
  - Uses filters to process only small parts of the input at a time (**locality**).
  - Weight sharing and local connectivity **reduce the number of parameters**.
  - Can recognize patterns independent of their location within the image (**shift invariance**).
  - **Efficient** for image, video, and speech data.

What Is A Convolve

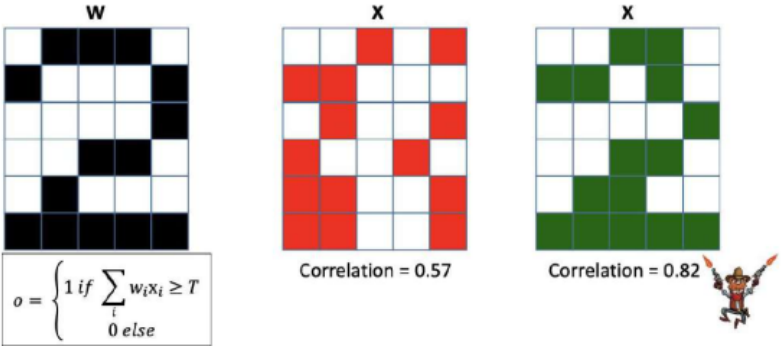


Scanning an image with a "filter"

- A filter is really **just a perceptron**, with weights and a bias.
- At each location, the filter is **multiplied component-wise** with the underlying map values, and the products are summed along with the bias.

Figure adapted from [3]

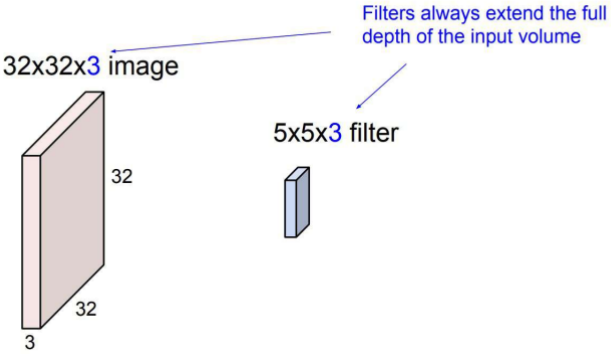
Weights Showing Correlation



- The **weights** of the filter **represent the appearance** of the number "2".
- The green has a higher correlation with the filter compared to the red.
- The green pattern is more likely to represent the number "2".

Figure adapted from [1]

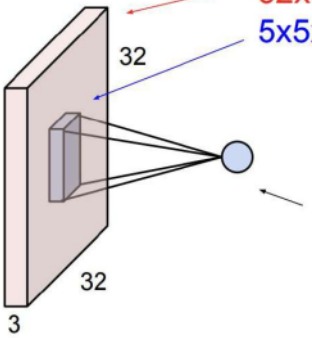
What Is Convolution



- **Convolve:** Slide over the image spatially, computing dot products.
- This allows us to **preserve the spatial structure** of the input.

Figure adapted from [2]

What Is The Output



- It's simply a neuron with local connectivity!

Figure adapted from [2]

Convolution Process

- Consider this as the filter we are going to use:

1	0	1
0	1	0
1	0	1

Convolution Process

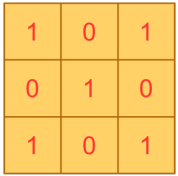
- This is how we calculate the convolutional layer's output:

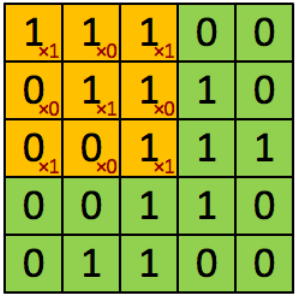
$$\text{Convolved Feature}(i,j) = (I * K)(i,j) = \sum_{a=0}^{k_h-1} \sum_{b=0}^{k_w-1} I(i+a, j+b) K(a,b)$$

I: Input Image
K: Our Kernel
k_h and k_w: The height and width of the Kernel

Convolution Process

- Convoluting a 5x5x1 image with a 3x3x1 kernel to get a 3x3x1 convolved feature.





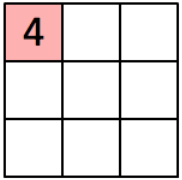


Figure adapted from [3]

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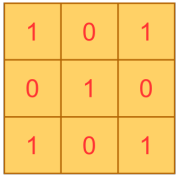
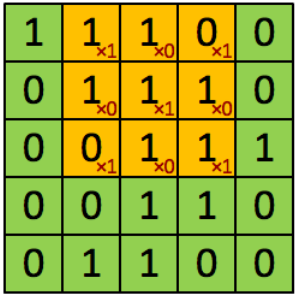
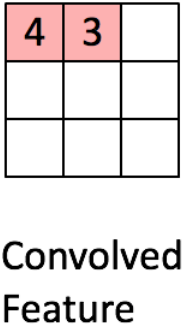
- Convoluting a 5x5x1 image with a 3x3x1 kernel to get a 3x3x1 convolved feature.







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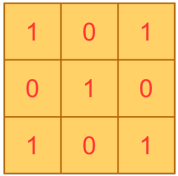

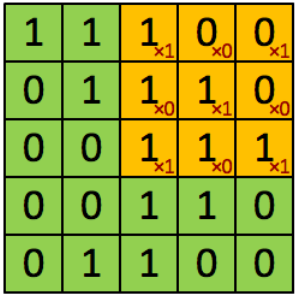

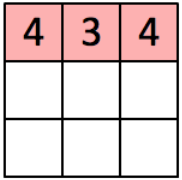

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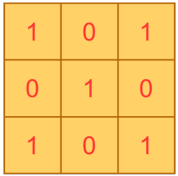

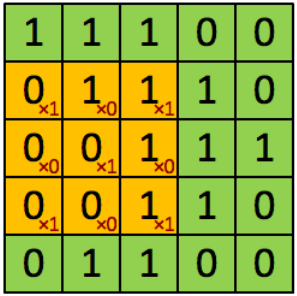

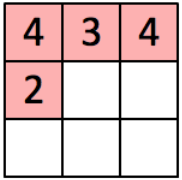

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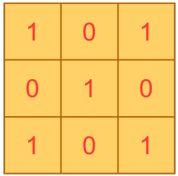
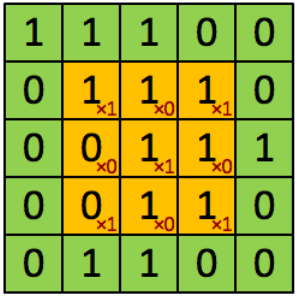
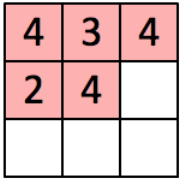




Image



Convolved Feature

Figure adapted from [3]

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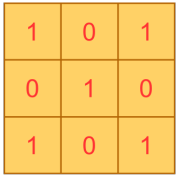

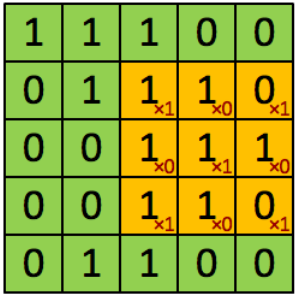

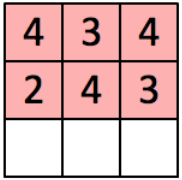

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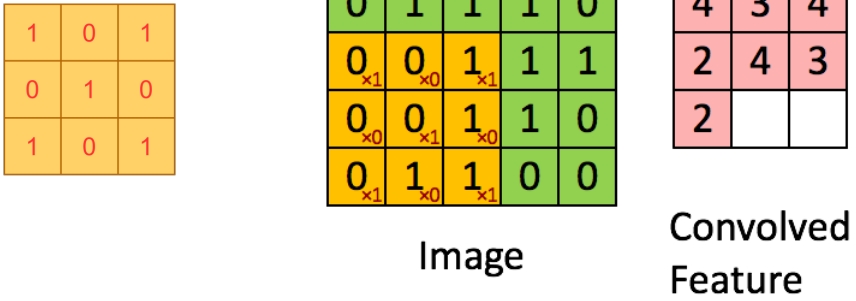

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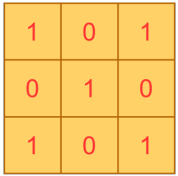
- Convoluting a 5x5x1 image with a 3x3x1 kernel to get a 3x3x1 convolved feature.



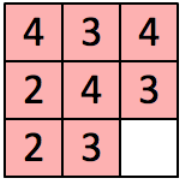
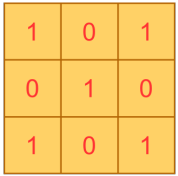
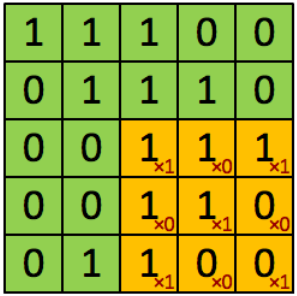


Figure adapted from [3]

Convolution Process

- Convoluting a 5x5x1 image with a 3x3x1 kernel to get a 3x3x1 convolved feature.





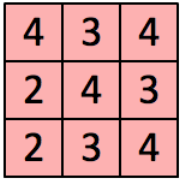
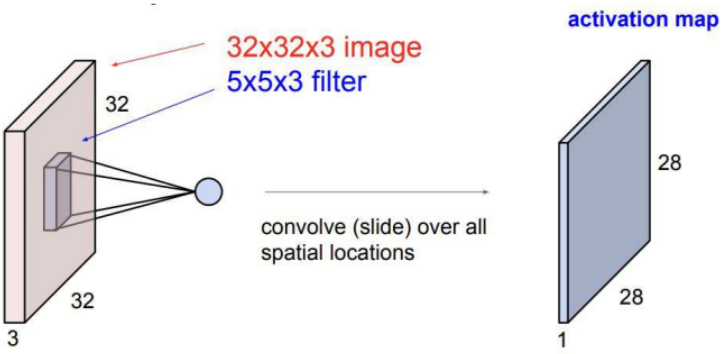


Figure adapted from [3]

To Summarize

- We apply the same filter on different regions of the input.
- Convolutional filters learn to make **decisions based on local spatial input**, which is an important attribute when working with images.
- Uses a lot **fewer parameters** compared to Fully Connected Layers.

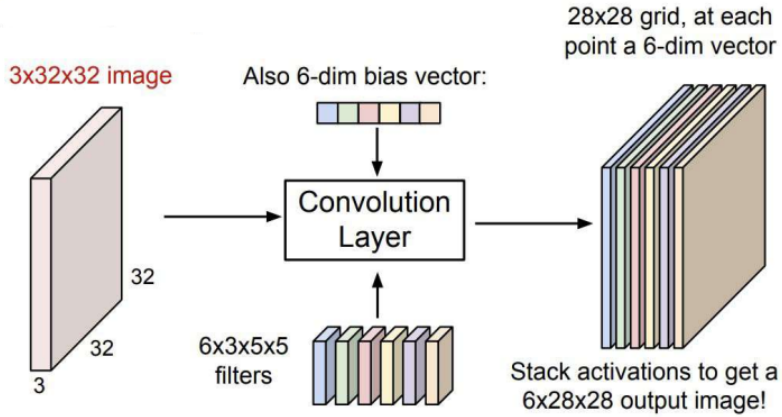
Convolution Outlook



- If we consider the **image** to be of size $n \times m \times b$ and the **filter** to be of size $n' \times m' \times b$, we will have an **activation map** of size $(n-n'+1) \times (m-m'+1) \times 1$ for each filter.

Figure adapted from [2]

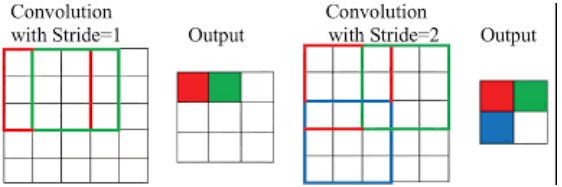
Convolution Outlook



- Multiple filters can be applied, each with its own bias, to extract different features from the input.

Figure adapted from [2]

What Is Stride



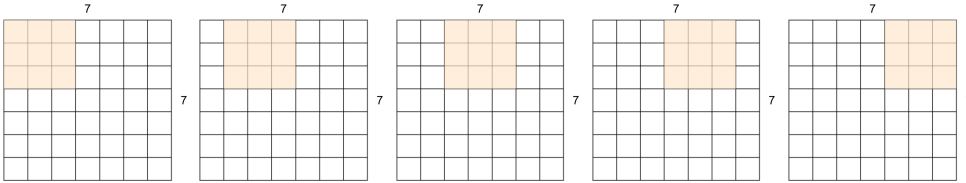
- The scans of the individual filters may **advance by more than one** pixel at a time.
  - The **stride** can be greater than 1.
  - Effectively increasing the granularity of the scan.
    - **Saves computation**, sometimes at the risk of **losing information**.
- This results in a **reduction** in the size of the **resulting maps**.
  - They will shrink by a factor equal to the stride.
- This can happen at **any layer**.

Figure adapted from [3]

Stride-1

Closer look

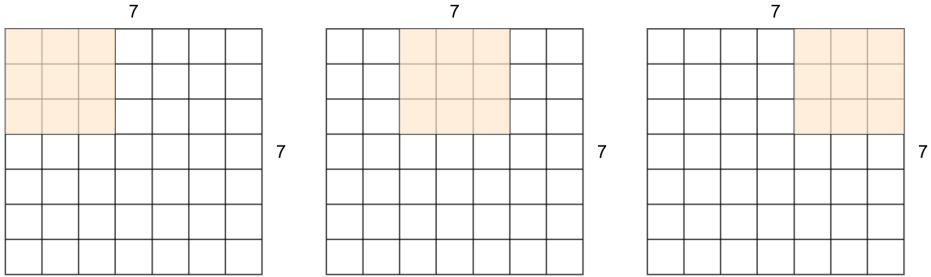
- 7x7 input with 3x3 filter
- This was a Stride 1 filter
- => Outputs 5x5



Strides-2

Now let's use Stride 2

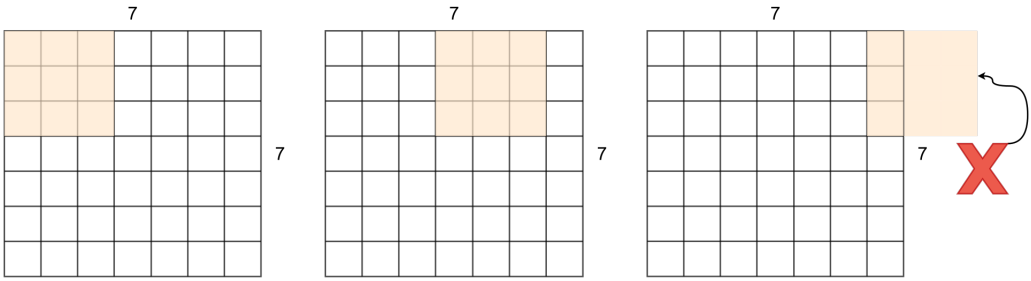
- 7x7 input with 3x3 filter
- This was a Stride 2 filter
- => Outputs 3x3



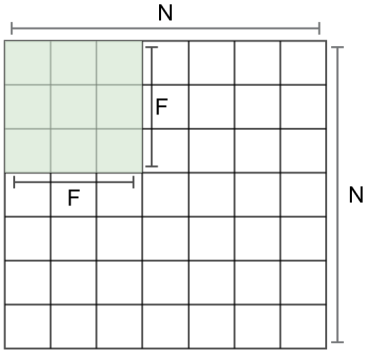
Strides-3

What about Stride 3?

- 7x7 input with 3x3 filter



Strides Formula



$$OutputSize = \frac{(N-F)}{\textbf{Stride}} + 1$$

$N = 7, F = 3 \Rightarrow$

- Stride 1: $(7-3)/1+1 = 5$
- Stride 2: $(7-3)/2+1 = 3$
- **Stride 3:** $(7-3)/3+1 = 2.33$!!!

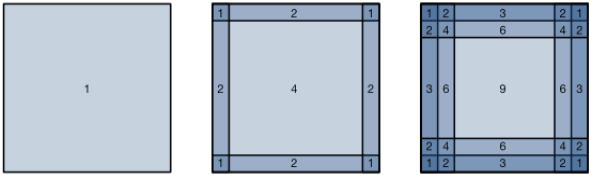
Figure adapted from [2]

A problem?

What could cause problems?

Problem-1

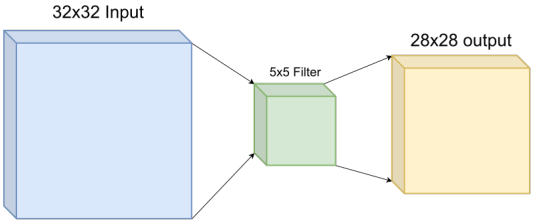
- **The borders don't get enough attention.**
- Outputs shrink!



Pixel utilization for convolutions of size 1×1, 2×2, and 3×3 respectively.

Problem-2

- Borders don't get enough attention.
- **Outputs shrink!**



32x32 input shrinks to 28x28 output.

(information loss occurs)

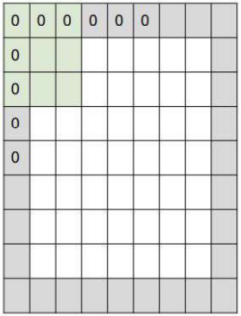
Solution?

What is the solution?

Padding

Padding: the secret ingredient to keep strides in line!

Padding



In practice, it is common to zero-pad the border.

Recall: The original formula for convolution without padding:

$$\frac{N-F}{\text{stride}}+1$$

Now, we should adjust the formula considering padding P:

$$\frac{N+2P-F}{\text{stride}}+1$$

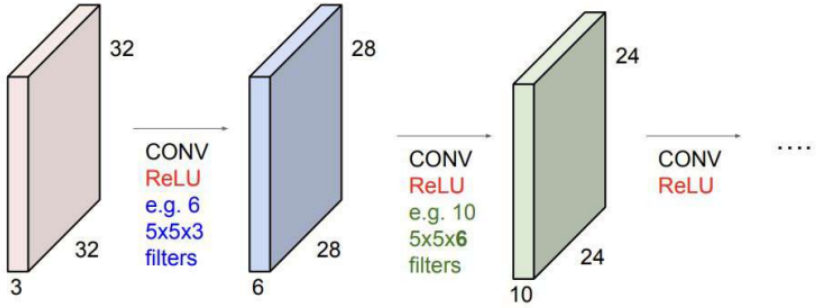
Figure adapted from [2]

Padding

- Zero-padding is used not only for stride > 1, but also **to prevent a reduction** in output size even when $S = 1$.
- For stride > 1, zero padding is adjusted to ensure that the size of the convolved output is: $\left\lceil \frac{N}{S} \right\rceil$

  This is achieved by zero padding the image with: $P = S\left\lceil \frac{N}{S} \right\rceil - N$
- For an F width filter:
  - **Odd** F: Pad on both left and right with $\frac{F-1}{2}$ columns of zeros.
  - **Even** F: Pad one side with $\frac{F}{2}$ columns of zeros, and the other with $\frac{F}{2} - 1$ columns of zeros.
  - The resulting image is width: $N + F - 1$
- The top & bottom zero padding follows the same rules to maintain map height after convolution.

Convnet



- **ConvNet:** a sequence of convolution layers **interspersed with activation functions.**

Figure adapted from [2]

Convnet

Question: What do convolutional filters at different levels of a ConvNet learn?

Convnet

Question: What do convolutional filters at different levels of a ConvNet learn?

Answer:

- Filters in the **early layers** typically detect **simple features** such as edges, textures, and basic shapes.
- As we move **deeper** into the network, the filters learn **more complex and abstract features**, such as specific parts of objects (e.g., a nose, eyes, or other high-level patterns).

Convolution Output

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyper-parameters:
  - Number of filters K
  - Their spatial extent F
  - The stride S
  - The amount of zero padding P
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = \left(\frac{W_1 - F + 2P}{S}\right) + 1$
  - $H_2 = \left(\frac{H_1 - F + 2P}{S}\right) + 1$
  - $D_2 = K$

Parameter Setting

- With parameter sharing, convolution introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.

Common settings:

- K = powers of 2 (e.g., 32, 64, ...)
- $F = 3$, $S = 1$, $P = 1$
- $F = 5$, $S = 1$, $P = 2$
- $F = 5$, $S = 2$, $P =$ adjusted accordingly.
- $F = 1$, $S = 1$, $P = 0$

Example

Question: Given an input volume of 32x32x3, we apply 10 filters, each of size 5x5, with a stride of 1 and padding of 2.

- What is the output volume size of this convolutional layer?
- How many parameters are required for this convolutional layer?

Example

Output Volume Size:

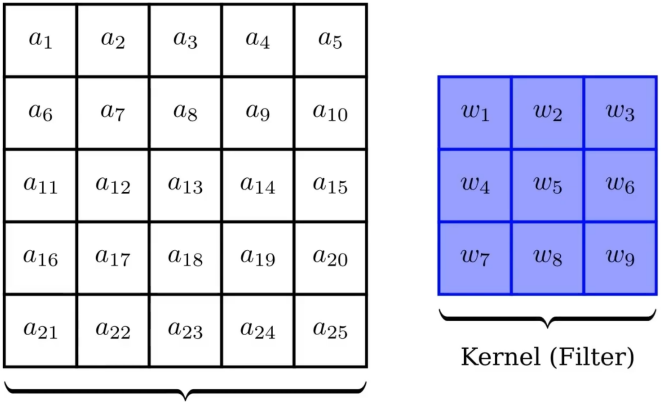
$$\left(\frac{32+2\times 2-5}{1}\right)+1=32 \quad \text{(spatial dimensions)}, \quad 32\times 32\times 10$$

Number of Parameters:

Each filter parameters: $5\times 5\times 3+1=76$ *(+1 for bias)*.
Therefore, total parameters: $76\times 10=\mathbf{760}$.

Example Overview

- we will tackle this problem with an example using stride = 2 and padding = 0.

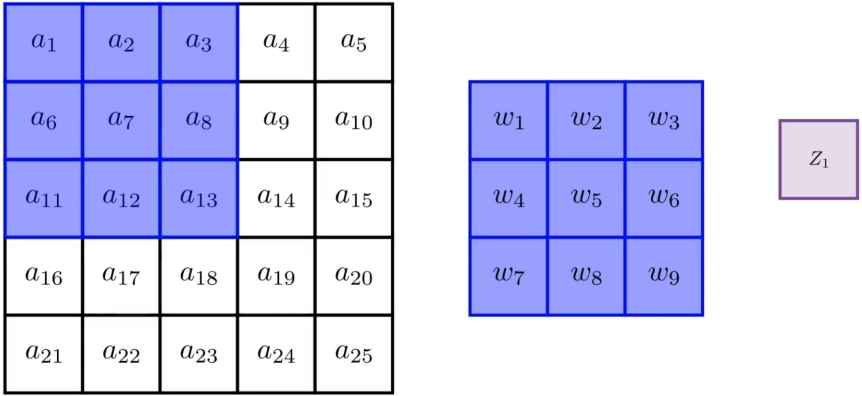


Figures adapted from [7]

Convolution

Recall: convolution process

Convolution



Convolution

a_1	a_2	a_3	a_4	a_5
a_6	a_7	a_8	a_9	a_{10}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}

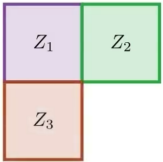
w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

z_1	z_2

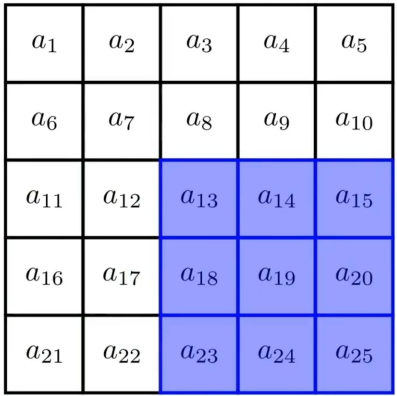
Convolution

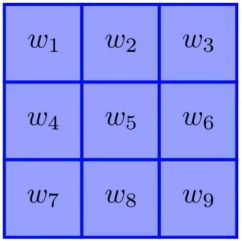
a_1	a_2	a_3	a_4	a_5
a_6	a_7	a_8	a_9	a_{10}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9



Convolution





Another view

$$z_1 = w_1 \times a_1 + w_2 \times a_2 + w_3 \times a_3 + w_4 \times a_6 + + \cdots + w_9 \times a_{13}$$

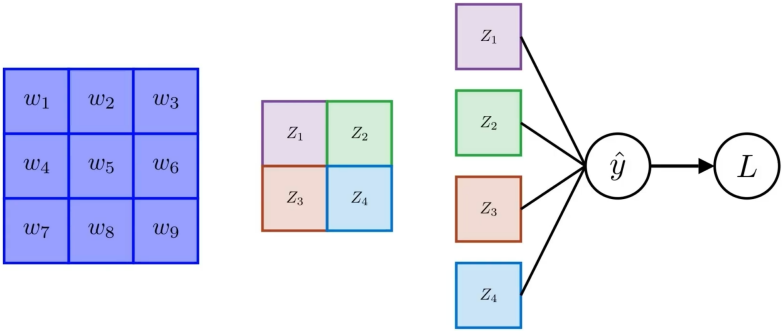
$$z_2 = w_1 \times a_3 + w_2 \times a_4 + w_3 \times a_5 + w_4 \times a_8 + \cdots + w_9 \times a_{15}$$

$$z_3 = w_1 \times a_{11} + w_2 \times a_{12} + w_3 \times a_{13} + w_4 \times a_{16} + \cdots + w_9 \times a_{23}$$

$$z_4 = w_1 \times a_{13} + w_2 \times a_{14} + w_3 \times a_{15} + w_4 \times a_{18} + \cdots + w_9 \times a_{25}$$

Whole Network

- For easier computation, we will assume one layer of convolution and a perceptron as our whole network.
- **Recall**: $w_i^* = w_i - \alpha \times \frac{\partial L}{\partial w_i}$



Gradient

- We can easily calculate the gradients:

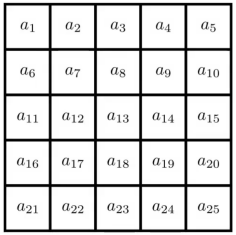
$$\begin{aligned}\frac{\partial L}{\partial w_1} &= \frac{\partial z_1}{\partial w_1}\frac{\partial L}{\partial z_1} + \frac{\partial z_2}{\partial w_1}\frac{\partial L}{\partial z_2} + \frac{\partial z_3}{\partial w_1}\frac{\partial L}{\partial z_3} + \frac{\partial z_4}{\partial w_1}\frac{\partial L}{\partial z_4} \\ &= a_1\frac{\partial L}{\partial z_1} + a_3\frac{\partial L}{\partial z_2} + a_{11}\frac{\partial L}{\partial z_3} + a_{13}\frac{\partial L}{\partial z_4}\end{aligned}$$

Gradient

Do the same for each w

$$\frac{\partial L}{\partial w_1} = a_1 \frac{\partial L}{\partial z_1} + a_3 \frac{\partial L}{\partial z_2} + a_{11} \frac{\partial L}{\partial z_3} + a_{13} \frac{\partial L}{\partial z_4}$$
$$\frac{\partial L}{\partial w_2} = a_2 \frac{\partial L}{\partial z_1} + a_4 \frac{\partial L}{\partial z_2} + a_{12} \frac{\partial L}{\partial z_3} + a_{14} \frac{\partial L}{\partial z_4}$$
$$\frac{\partial L}{\partial w_3} = a_3 \frac{\partial L}{\partial z_1} + a_5 \frac{\partial L}{\partial z_2} + a_{13} \frac{\partial L}{\partial z_3} + a_{15} \frac{\partial L}{\partial z_4}$$
$$\frac{\partial L}{\partial w_4} = a_6 \frac{\partial L}{\partial z_1} + a_8 \frac{\partial L}{\partial z_2} + a_{16} \frac{\partial L}{\partial z_3} + a_{18} \frac{\partial L}{\partial z_4}$$
$$\frac{\partial L}{\partial w_5} = a_7 \frac{\partial L}{\partial z_1} + a_9 \frac{\partial L}{\partial z_2} + a_{17} \frac{\partial L}{\partial z_3} + a_{19} \frac{\partial L}{\partial z_4}$$
$$\dots$$

Overview



$$\begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial L}{\partial z_1} & \frac{\partial L}{\partial z_2} \\ \frac{\partial L}{\partial z_3} & \frac{\partial L}{\partial z_4} \end{bmatrix}$$

$$\begin{aligned}
\frac{\partial L}{\partial w_1} &= a_1 \frac{\partial L}{\partial z_1} + a_3 \frac{\partial L}{\partial z_2} + a_{11} \frac{\partial L}{\partial z_3} + a_{13} \frac{\partial L}{\partial z_4} \\
\frac{\partial L}{\partial w_2} &= a_2 \frac{\partial L}{\partial z_1} + a_4 \frac{\partial L}{\partial z_2} + a_{12} \frac{\partial L}{\partial z_3} + a_{14} \frac{\partial L}{\partial z_4} \\
\frac{\partial L}{\partial w_3} &= a_3 \frac{\partial L}{\partial z_1} + a_5 \frac{\partial L}{\partial z_2} + a_{13} \frac{\partial L}{\partial z_3} + a_{15} \frac{\partial L}{\partial z_4} \\
\frac{\partial L}{\partial w_4} &= a_6 \frac{\partial L}{\partial z_1} + a_8 \frac{\partial L}{\partial z_2} + a_{16} \frac{\partial L}{\partial z_3} + a_{18} \frac{\partial L}{\partial z_4} \\
\frac{\partial L}{\partial w_5} &= a_7 \frac{\partial L}{\partial z_1} + a_9 \frac{\partial L}{\partial z_2} + a_{17} \frac{\partial L}{\partial z_3} + a_{19} \frac{\partial L}{\partial z_4} \\
\frac{\partial L}{\partial w_6} &= a_8 \frac{\partial L}{\partial z_1} + a_{10} \frac{\partial L}{\partial z_2} + a_{18} \frac{\partial L}{\partial z_3} + a_{20} \frac{\partial L}{\partial z_4} \\
\frac{\partial L}{\partial w_7} &= a_{11} \frac{\partial L}{\partial z_1} + a_{13} \frac{\partial L}{\partial z_2} + a_{21} \frac{\partial L}{\partial z_3} + a_{23} \frac{\partial L}{\partial z_4} \\
\frac{\partial L}{\partial w_8} &= a_{12} \frac{\partial L}{\partial z_1} + a_{14} \frac{\partial L}{\partial z_2} + a_{22} \frac{\partial L}{\partial z_3} + a_{24} \frac{\partial L}{\partial z_4} \\
\frac{\partial L}{\partial w_9} &= a_{13} \frac{\partial L}{\partial z_1} + a_{15} \frac{\partial L}{\partial z_2} + a_{23} \frac{\partial L}{\partial z_3} + a_{25} \frac{\partial L}{\partial z_4}
\end{aligned}$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_6 & a_7 & a_8 \\ a_{11} & a_{12} & a_{13} \end{bmatrix} \times \frac{\partial L}{\partial z_1} + \begin{bmatrix} a_3 & a_4 & a_5 \\ a_8 & a_9 & a_{10} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \times \frac{\partial L}{\partial z_2} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{16} & a_{17} & a_{18} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \times \frac{\partial L}{\partial z_3} + \begin{bmatrix} a_{13} & a_{14} & a_{15} \\ a_{18} & a_{19} & a_{20} \\ a_{23} & a_{24} & a_{25} \end{bmatrix} \times \frac{\partial L}{\partial z_4} = \begin{bmatrix} \frac{\partial L}{\partial w_1} & \frac{\partial L}{\partial w_2} & \frac{\partial L}{\partial w_3} \\ \frac{\partial L}{\partial w_4} & \frac{\partial L}{\partial w_5} & \frac{\partial L}{\partial w_6} \\ \frac{\partial L}{\partial w_7} & \frac{\partial L}{\partial w_8} & \frac{\partial L}{\partial w_9} \end{bmatrix}$$

Result

$$\begin{bmatrix} w_1^* & w_2^* & w_3^* \\ w_4^* & w_5^* & w_6^* \\ w_7^* & w_8^* & w_9^* \end{bmatrix} = \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix} - \alpha \times \begin{bmatrix} \frac{\partial L}{\partial w_1} & \frac{\partial L}{\partial w_2} & \frac{\partial L}{\partial w_3} \\ \frac{\partial L}{\partial w_4} & \frac{\partial L}{\partial w_5} & \frac{\partial L}{\partial w_6} \\ \frac{\partial L}{\partial w_7} & \frac{\partial L}{\partial w_8} & \frac{\partial L}{\partial w_9} \end{bmatrix}$$

$$w_i^* = w_i - \alpha \times \frac{\partial L}{\partial w_i}$$

What's next?

Proceed with gradient descent.

So Far

- The highly structured nature of image data.
- Local correlations are important.
- CNNs have local and shared connections.
- Strides & padding.

Up Next

Learn more details about CNNs!

Contributions

- **This slide has been prepared thanks to:**
  - Ali Aghayari