Machine Learning (CE 40717)

Fall 2024

Ali Sharifi-Zarchi

CE Department
Sharif University of Technology

November 16, 2024

Channels

Previously, we discussed 2D inputs; however, although images are inherently 2D, they need to be represented as **3D matrices** to display color information.

- Pixel values range from 0 to 255.
- We **cannot** represent all the colors in a picture with only one channel of numbers from 0 to 255.
- So we represent them in **3 channels**.

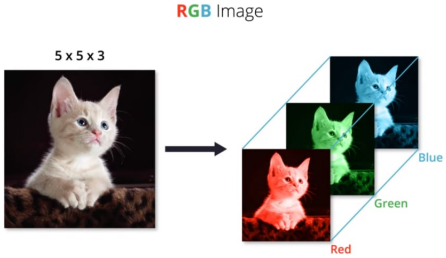


Figure adapted from Source

Channels

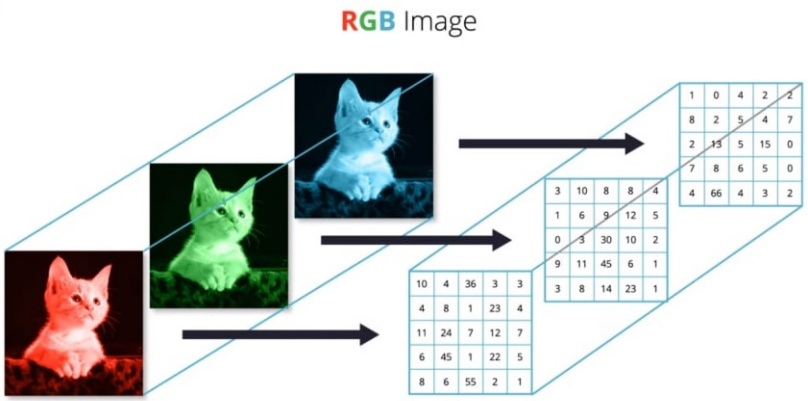


Figure adapted from Source

Let's take a closer look at the calculations

Channels

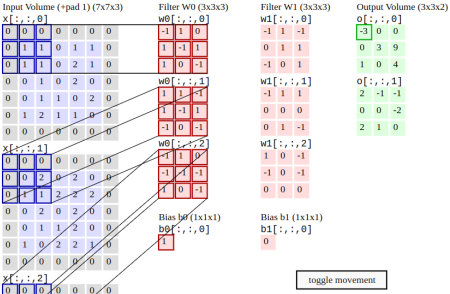


Figure adapted from [2]

Channels

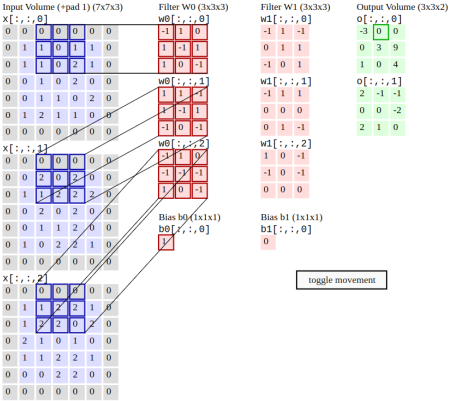


Figure adapted from [2]

Channels

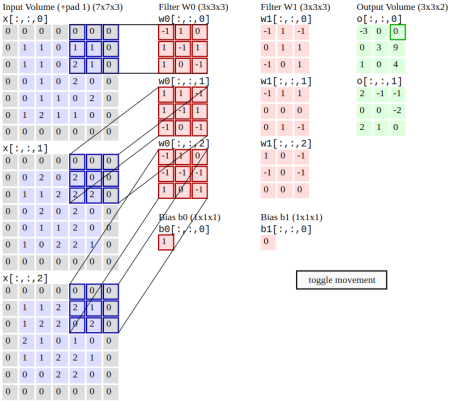


Figure adapted from [2]

Channels

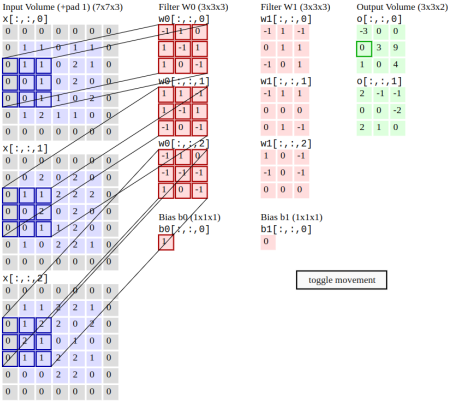


Figure adapted from [2]

Channels

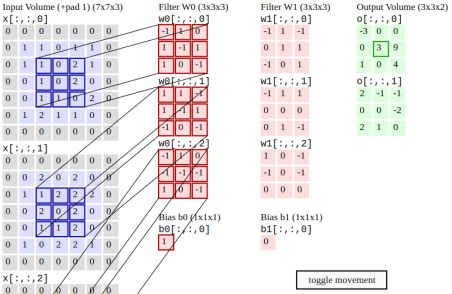


Figure adapted from [2]

Channels

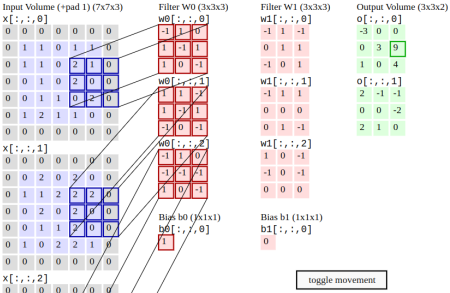


Figure adapted from [2]

Channels

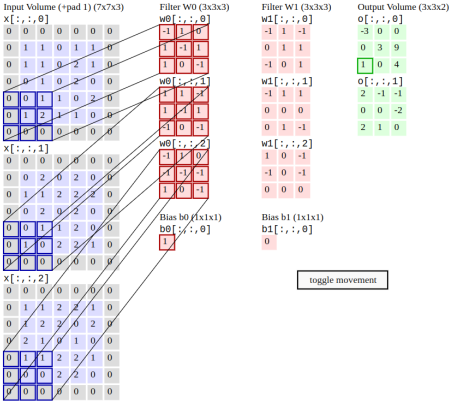


Figure adapted from [2]

Channels

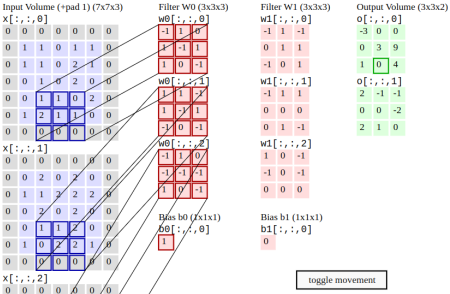


Figure adapted from [2]

Channels

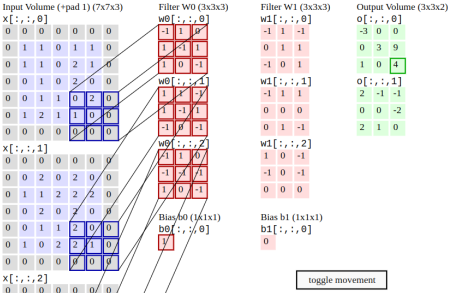


Figure adapted from [2]

Channels

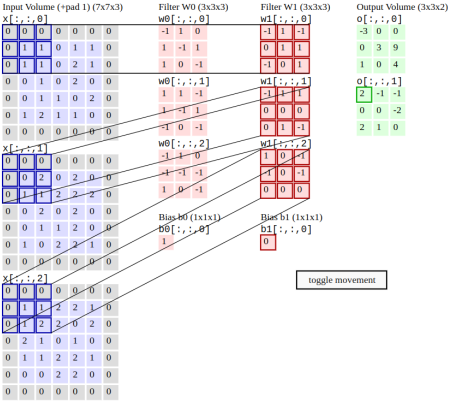


Figure adapted from [2]

Channels

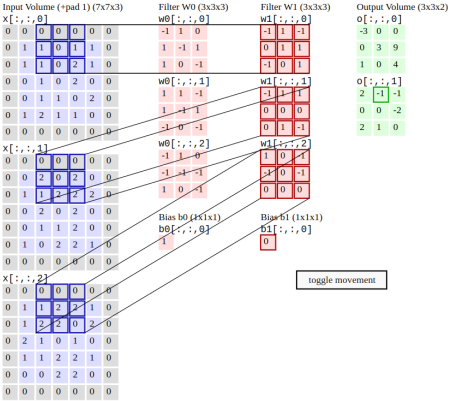


Figure adapted from [2]

Channels

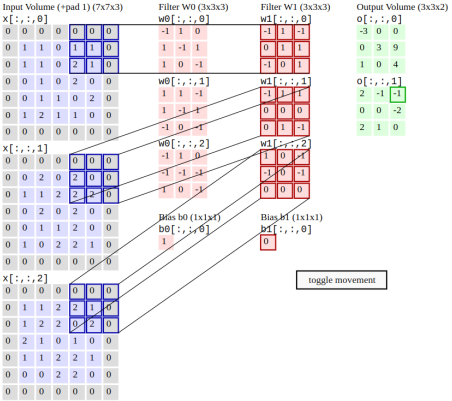


Figure adapted from [2]

Channels

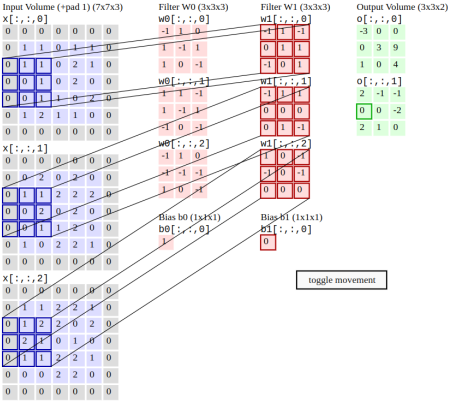


Figure adapted from [2]

Channels

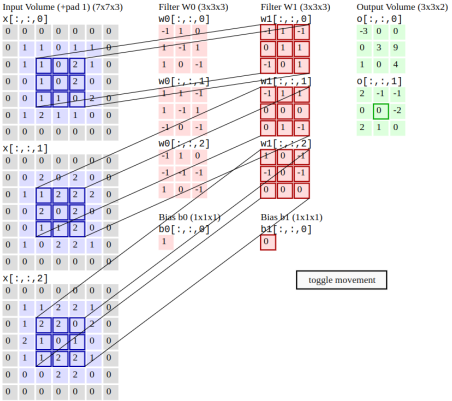


Figure adapted from [2]

Channels

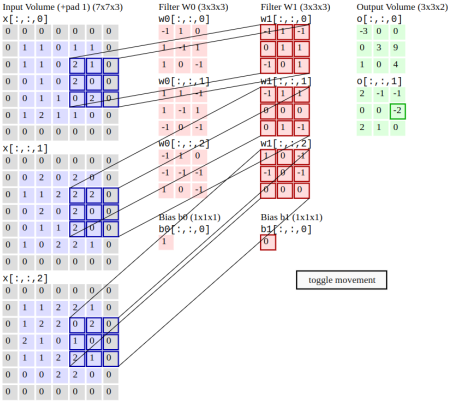


Figure adapted from [2]

Channels

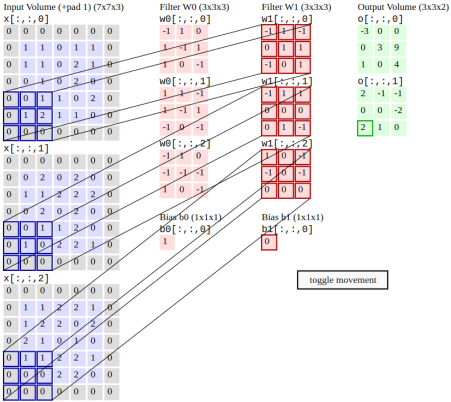


Figure adapted from [2]

Channels

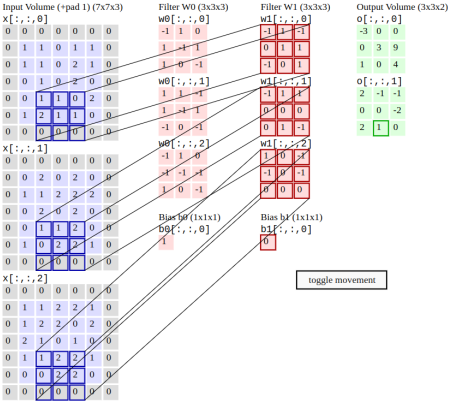


Figure adapted from [2]

Channels

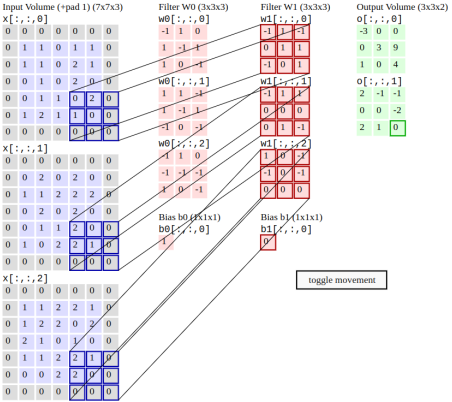


Figure adapted from [2]

Channels

- Each filter produces **one output channel**. By applying multiple filters, we can create multiple output channels, allowing each channel to learn **distinct** features.

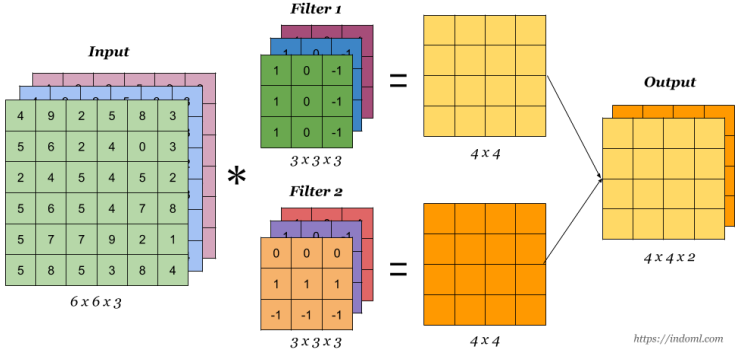


Figure adapted from Source

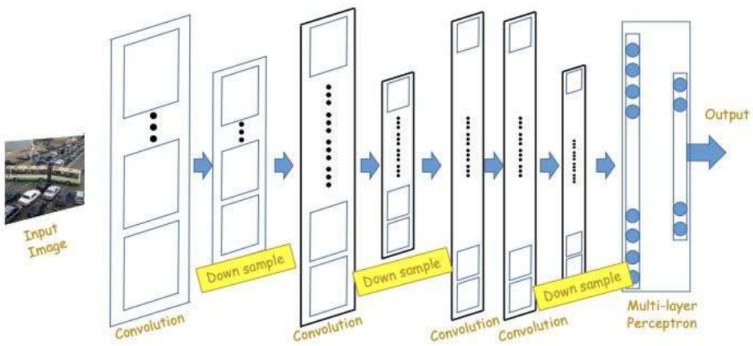
Review

Three Main Types of Layers

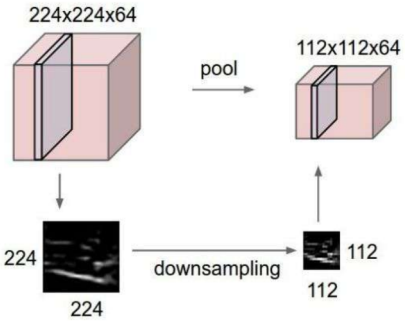
- **Convolutional Layer**
  - Neurons' outputs are connected to local regions in the input.
  - Applying the same filter across the entire image.
  - The CONV layer's parameters include a set of learnable filters.
- **Pooling Layer**
  - Performs a downsampling operation along the spatial dimensions.
- **Fully-Connected Layer**
  - Typically used in the final stages of the network for combining high-level features to make predictions.

Pooling

- Convolution and activation layers are often followed by pooling layers intermittently.
  - Often, they alternate with convolution, though this is not necessary.



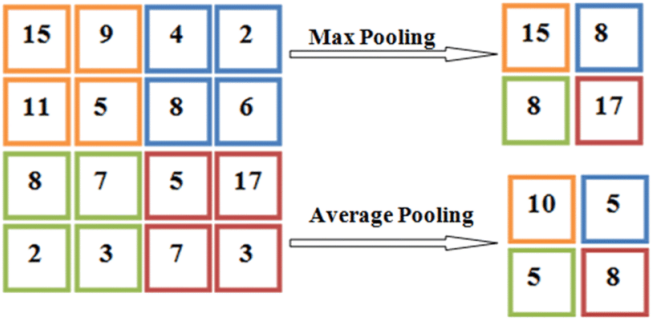
Pooling



- Reduces the **spatial size** of the representation.
  - To reduce the number of parameters and computational demands within the network.
  - To **reduce variance**.
- Enables the network to be invariant to small translations or distortions.
- Operates independently over each activation map.

Figure adapted from [2]

Pooling Type



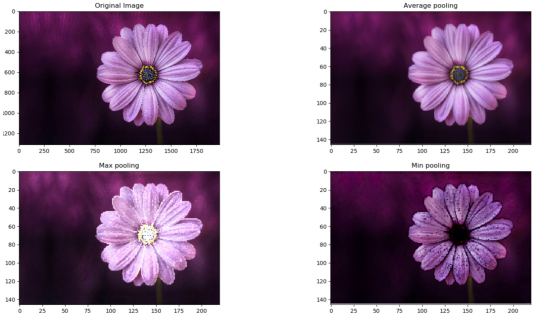
Two Primary Types of Pooling:

- **Max Pooling:** Selects the **maximum** value from each section of the feature map.
- **Min Pooling:** Selects the **minimum** value from each section of the feature map.
- **Average Pooling:** Calculates the **average** value for each section of the feature map.

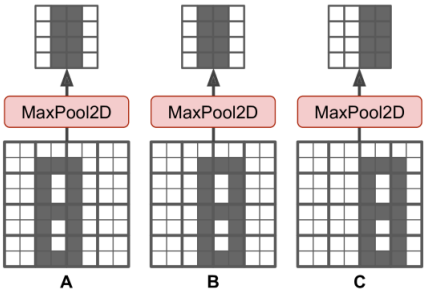
Figure adapted from source

Pooling Type

- Max Pooling works well **when the background is light**, and the object is dark.
- Min Pooling: performs well **when the background is dark**, and the object is light.
- Average Pooling: **smooths** the feature map.



Pooling Advantage



- The most common type of pooling layer.
- Provides invariance to small translations.

Parameter Setting

Question:
How does a pooling layer change the input image dimensions?

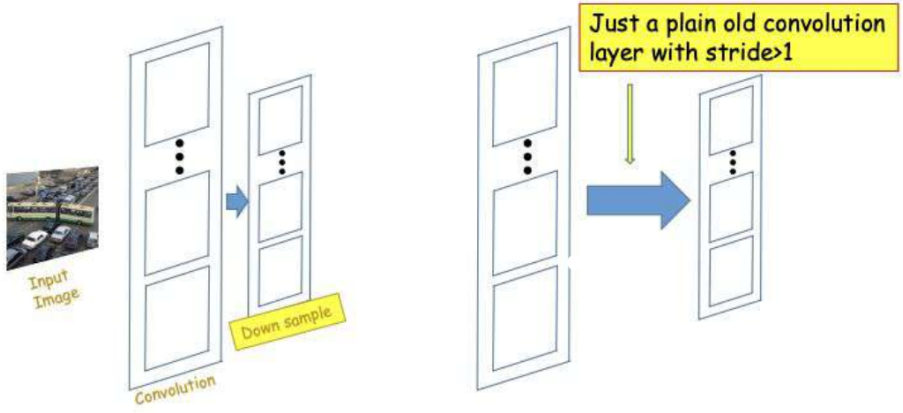
Answer:

- An $N \times N$ image compressed by a $P \times P$ pooling filter with stride S produces an output map with side length $\left\lceil \frac{(N-P)}{S} \right\rceil + 1$.

Parameter Setting

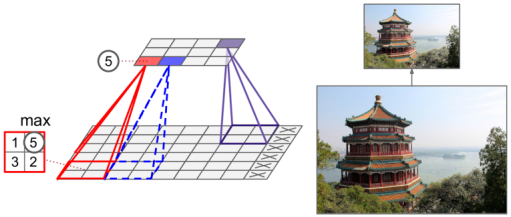
- Pooling takes in a volume of size $W_1 \times H_1 \times D_1$
- Requires two hyperparameters:
  - Their spatial extent F,
  - The stride S.
- Outputs a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = \frac{(W_1 - F)}{S} + 1$
  - $H_2 = \frac{(H_1 - F)}{S} + 1$
  - $D_2 = D_1$
- Introduces **zero parameters** since it computes a fixed function of the input.
- Using zero-padding for pooling layers is **uncommon**.

Alternative To Pooling



- Downsampling can be done by a simple convolution layer with stride larger than 1, Replacing the max pooling layer with a convolutional layer.

Pooling Summary



Max pooling layer (2 × 2 kernel, stride 2, no padding)

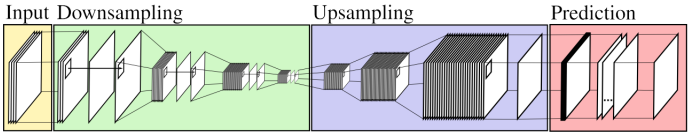
- The goal is to sub-sample the input to reduce:
  - Computational load
  - Memory usage
  - Number of parameters
  - Risk of overfitting

Question

- **What if we want to increase the dimensions?**

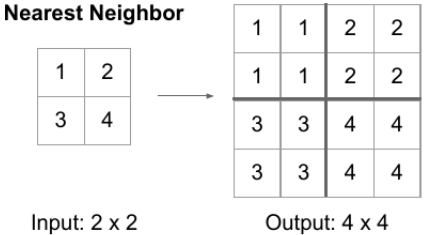
Up-sampling CNN

- Resizing feature maps is a **common** operation in neural networks, especially those used for image segmentation tasks.
- This architecture is commonly known as the **Encoder-Decoder** network.



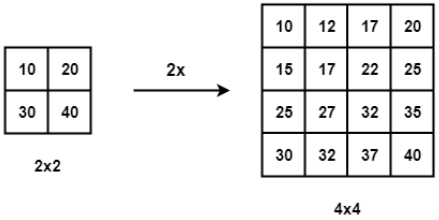
Nearest Neighbors

- **Nearest Neighbors**: Nearest Neighbors involves copying an input pixel value to the K-nearest neighboring pixels, with K based on the expected output.



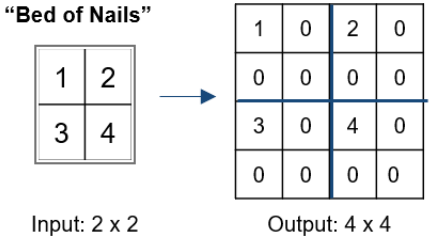
Bilinear Interpolation

- **Bilinear Interpolation**: In Bilinear Interpolation, the four nearest pixel values are used to compute a weighted average based on their distances, resulting in a smoothed output.



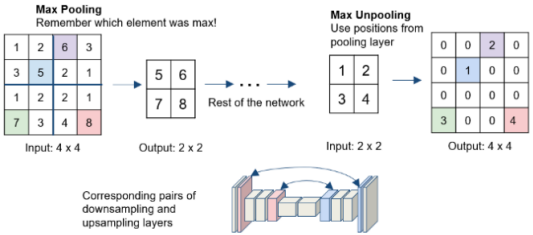
Bed Of Nails

- **Bed Of Nails**: In Bed of Nails, the input pixel value is copied to the corresponding position in the output image, with zeros filling the remaining positions.

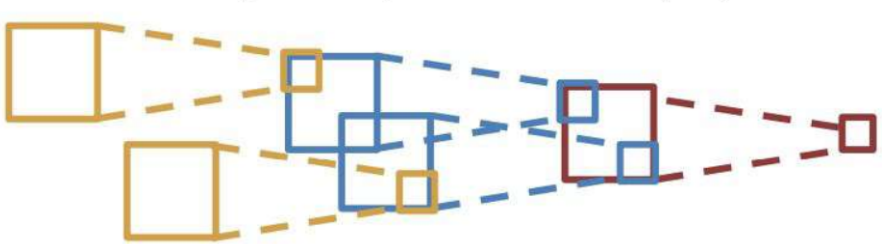


Max-Unpooling

- **Max-Unpooling**: In max-unpooling, the index of the **maximum value** is saved for each max-pooling layer during encoding. During decoding, the saved index is used to map the input pixel to its original position, with zeros filling all other positions.



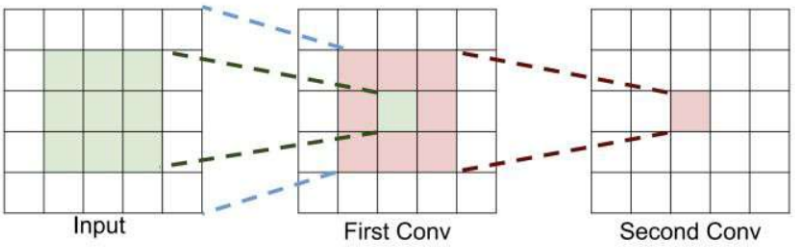
Receptive Field



- **Receptive Field:** How large is the region in the **input or previous layer** seen by a neuron on the n-th convolutional layer?
- In a convolution with kernel size K, each element in the next layer is based on a $K \times K$ **receptive field** from the previous layer.

Figure adapted from [3]

Receptive Field



- Units in the deeper layers can be **indirectly** connected to most of the input image.
- Each successive convolution adds $K-1$ to the receptive field size. With L layers, the receptive field size is $1+L\cdot(K-1)$.
- **Challenge**: For large images, many layers are required for each output to capture the entire image.
  - **Solution: Downsample** within the network using strides and pooling layers.

Power Of Small Filters

Assuming an input size of $H \times W \times C$, and convolutions are used with C filters to preserve depth (stride 1, with padding to maintain H and W dimensions).

one CONV with 7 x 7 filters
Number of weights
$= C \times (7 \times 7 \times C) = \mathbf{49}C^2$

three CONV with 3 x 3 filters
Number of weights
$= 3 \times C \times (3 \times 3 \times C) = \mathbf{27}C^2$

Both options achieve a receptive field of 7; however, using **multiple smaller filters** reduces the number of **parameters**, introduces more **nonlinearity**, and generally leads to a **more efficient**, expressive model.

Question

Question: In a convolutional neural network, each layer increases the receptive field size. Suppose a network has 3 convolutional layers, each with a kernel size of $K = 3$ and a stride of $S = 1$.

- Determine the receptive field size after each layer, beginning with an initial receptive field size of 1.
- How large is the receptive field after the third layer?
- Why is the growing receptive field important in deeper layers?

Answer

Answer:

- The receptive field size increases by $K-1$ with each layer.
  - After the 1st layer: $1+(3-1)=3$
  - After the 2nd layer: $3+(3-1)=5$
  - After the 3rd layer: $5+(3-1)=7$
- Consequently, the receptive field size after the third layer is 7.
- A larger receptive field enables neurons in deeper layers to capture more context from the input, crucial for recognizing higher-level patterns.

Inductive Bias In CNNs

Inductive Bias:
The assumptions a model uses to generalize from training data to new, unseen data.

Key Features of Inductive Bias in CNNs:

- **Weight Sharing:**
  A single filter is applied across various regions of the input, significantly decreasing the parameter count.
- **Locality:**
  CNNs utilize small filters (e.g., 3×3) that concentrate on local regions, aligning well with image data where local structures are significant.
- CNNs are more sample-efficient than FCNs due to their inductive biases.
- CNNs reduce sample complexity from $O(d^2)$ in fully connected networks (FCNs) to $O(\log^2 d)$.

Contributions

- **This slide was prepared with contributions from:**
  - Ali Aghayari

[1] M. Soleymani Baghshah, "Deep learning." Lecture slides.

[2] F.-F. Li, Y. Li, and R. Gao, "Cs231n: Deep learning for computer vision." Lecture slides.

[3] B. Raj, R. Singh, and B. Dhingra, "11-785: Introduction to deep learning." Lecture slides.

[4] M. Kellis, "6.874 computational systems biology: Deep learning in the life sciences." Lecture slides.

[5] H. Li, "Eleg 5491: Introduction to deep learning." Lecture slides.

[6] M. Elgendy, *Deep Learning for Vision Systems*. Manning Publications, 2020.

[7] DeepMind, "Deep learning for ai with geoffrey hinton, yoshua bengio, and yann lecun." YouTube video.