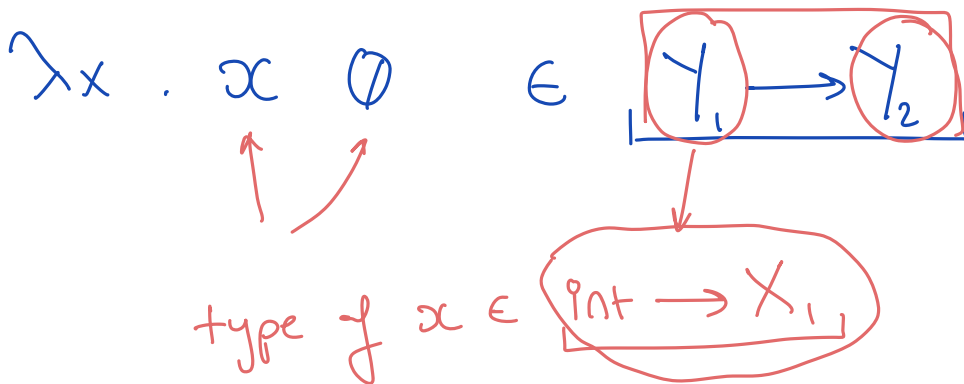


TYPE

Reconstruction

Inference



System
of constraints

$$\left\{ \begin{array}{l} \gamma_1 = \text{int} \rightarrow X_1 \\ \gamma_2 = X_1 \end{array} \right\}$$

$$C = \{S_i = T_i\}_{i \in [1, n]}$$

these are types
may contain variables
 X, Y, Z, \dots

E.g. $\text{int} \rightarrow X$

① How to solve constraints C

② How to construct C

Solving constraints

a type substitution σ

e.g. $\sigma = [X \mapsto T, Y \mapsto U, \dots]$

$$\text{dom}(\sigma) = \{X, Y, \dots\}$$

$$\text{range}(\sigma) = \{T, U, \dots\}$$

e.g. $\sigma = [X \mapsto \text{Bool}]$

$$\sigma(X \rightarrow X) = \text{Bool} \rightarrow \text{Bool}$$

$$\sigma(Y \rightarrow Y) = Y \rightarrow Y$$

A substitution σ unifies $S = T$ if $\sigma S = \sigma T$
Syntactically

σ unifies C if $\sigma S_i = \sigma T_i$

for all $S_i = T_i \in C$

Unification algorithm (Robinson's)

$\{int = Bool\}$

Goal: check if there are solutions
find the "best" possible solution

Def'n σ is more general than σ' ($\sigma \leq \sigma'$)
if $\sigma' = \gamma \circ \sigma$ for some substitution γ

Composition

$\gamma \circ \sigma =$

$X \mapsto \gamma T$ for each $X \mapsto T \in \sigma$

$X \mapsto T$ for each $X \mapsto T \in \gamma$
and $X \notin \text{dom}(\sigma)$

A **principal unifier** σ for C is s.t.
for all σ' unifying C , $\sigma \leq \sigma'$

σ is more general

e.g. $\{X = Y\}$

$\sigma = [X \mapsto Bool, Y \mapsto Bool]$

$\sigma X = \sigma Y$ ✓

$\sigma = [X \mapsto int, Y \mapsto int]$

$\sigma = [X \mapsto Y]$ principal unifier

$\sigma X = Y = \sigma Y$

e.g.

$$\{ \text{Nat} \rightarrow \text{Nat} = X \rightarrow Y \}$$

$$\sigma = [X \mapsto \text{Nat}, Y \mapsto \text{Nat}]$$

E.g. $\{ Y = \text{Nat} \rightarrow Y \}$ no solution

unify (C):

match C with

1. $\emptyset \rightarrow []$

2. $\{S = T\} \cup C' \rightarrow$

if $S = T$ (syntactically) then unify (C')

if S is of the form X and $X \notin \text{FV}(T)$

free variable

$$X = \dots T \dots$$

unify $([X \mapsto T]C') \circ [X \mapsto T]$

Symmetric cases

if T is of the form X and $X \notin \text{FV}(S)$

unify $([X \mapsto S]C') \circ [X \mapsto S]$

if S of the form $S_1 \rightarrow S_2$

T of the form $T_1 \rightarrow T_2$

unify $(C' \cup \{S_1 = T_1, S_2 = T_2\})$

else fail (no solution)

E.g.

$$\begin{array}{c} S = T \\ \nearrow \quad \searrow \\ (\text{Int} \rightarrow \text{Int}) \rightarrow \text{Int} = X \rightarrow \text{Int} \end{array}$$

case (3)

$$\boxed{\text{Int} \rightarrow \text{Int} = X}$$

$\text{Int} = \text{Int} \checkmark$

$$X \mapsto \text{Int} \rightarrow \text{Int}$$

$$\text{Int} \rightarrow (\text{Int} \rightarrow \text{Int}) = X \rightarrow \text{Int}$$

Typing relation

$$\Gamma \vdash t \in T$$

↑ typing context ↑ λ term ↑ type

$$\Gamma \vdash t \in T \mid C$$

E.g. $\emptyset \vdash (\lambda x:T. x) \emptyset : T' \mid \left\{ \begin{array}{l} T = T' \\ T' = \text{Nat} \end{array} \right\}$

$$\frac{\Gamma \vdash t_1 \in T \mid C \quad C' = C \cup \{T = \text{Nat}\}}{\Gamma \vdash \text{succ } t_1 : \text{Nat} \mid C'}$$

$$\Gamma \vdash t_1 \in T_1 \mid C_1 \quad \Gamma \vdash t_2 \in T_2 \mid C_2$$

$$C' = C_1 \cup C_2 \cup \{T_1 = T_2 \rightarrow X\}$$

$$\Gamma \vdash t_1, t_2 : X \mid C'$$

Context

$$\Gamma = x_1 \in T_1, x_2 \in T_2, x_3 \in T_3, \dots$$

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