

FIRST-ORDER (OGIC

forther retain

Ty. Vx. x +9 70 ~ x >5

existential quantifier universal

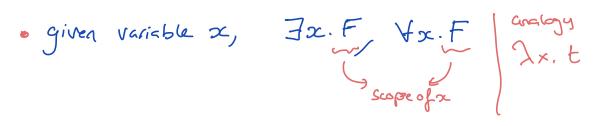
a language of FOL is L(C, F, R)Const functions relations

Basic term: either a constent (ce()) or a variable, oc, y, z, ...

Composite term: $f \in F$ $f(t_1, t_2, \dots, t_k)$ f hav a rify k

formular: afomic predicate $p(t_1, ..., t_n)$ $p \in R \quad \text{where } p \text{ has anty } n$ $e.g. \quad \text{older}(\alpha_1 y)$

· FINFZ OFIVE JOF



E.g.
$$\forall y . ((\forall x. p(x)) \Longrightarrow q(x,y))$$

frue

visible

a formula is closed iff it Joesn't have free variabler

ground formula — how no variables p(0,1)

G.g. $\forall x . \exists y . friend (x_1y)$ $\exists x . \forall y . friend (x_1y)$

Semantics

① universe U: a. non-empty set of objects U=Z U=Z U

U = { 0, 1,03

2) An interpretation I is a mapping from C, F, R to objects from U

I maps
$$ceC$$
 to U , i.e. $I(c)eU$

I maps feF to $I(f)eU^n \rightarrow U$

where f har arity n

I maps peR to $I(p) \subseteq U^k$

where p has arity k

E.g.
$$C = \{a, b, c\}$$

 $F = \{f\}$ binary
 $R = \{p\}$ terrary

interpretation
$$I(a) = 1$$
, $I(b) = 2$, $I(c) = 3$
 $I(f) = \{1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 1\}$
 $I(p) = \{(1,2,3), (3,2,1)\}$

a variable assignment 6 is amop from variables to elements of U

A formula F is SAT iff there is a structure S and 6 S.t. S,6 = F

Q.g.
$$\forall x.\exists y. p(x,y)$$

$$U = \mathbb{Z}$$

$$I(p) = \{(x,y) \mid x+y > 0\}$$

$$S = \{(u,T) \quad S \neq \forall x, \exists y. p(x,y)\}$$

Q.g.
$$\forall x. \left(\rho(x,x) \Rightarrow \exists y. \rho(x,y) \right)$$

$$U = Z$$

$$I(p) = \{(x_1y) \mid x + y > 0\}$$

FOL theories

a theory
$$T$$

$$0 \quad \text{I}_{T} = \left(C_{T}, F_{T}, R_{T}\right)$$

Ce.y.
$$\Sigma_{\tau} = (\phi, \phi, \xi_{\text{faller}})$$

axion:
$$\forall x, y$$
. teller $(x, y) \Rightarrow \forall x | \forall (y, x)$

$$U = \{A, B\}$$

$$T (faller) = \{CA, B\}\}$$

$$(U,T) \models axion$$

axions Az

$$\forall x. \quad x = x$$

$$\forall xyy \quad X = y \implies y = x$$

$$\forall x_1y_1 \neq x \quad X = y \land x = x \implies y = z$$

4.7.
$$U = \{0, 0\}$$

$$T(=) = \{(0,0), (0,0)\}$$

$$T(=) = \{(0,0), (0,0)\}$$

axiome
$$\forall \alpha. 7(x+1=0)$$

 $\forall \alpha. x+0=x$
 $\forall \alpha, x+1=y+1 \implies x=y$

$$\forall x_{i}y. \quad x + (y+1) = (x+y)+1$$

$$\forall x_{i}y. \quad x \cdot (y+1) = x \cdot y + x$$

$$\forall x, y. \quad x \cdot (y+1) = x \cdot y + x$$

I infinite set of axiom to define induction

Presburger arithmetic (throw away multiplication)

Theory of arrays

$$\forall \alpha_1 \cup_i ij \cdot i=j \Rightarrow \alpha(i=v)[j]=v$$