TYPE RECONSTRUCTION / INFERENCE

$$\boxed{(\lambda x : X \,.\, x\ \emptyset) : \boxed{Y_1 \to Y_2}}$$

Intuition

$\square\ foo(\square\ x, \square\ y)$

$$\left\{ \begin{array}{l} X = \underline{int \to X_1} \\ \underline{X = Y_1} \\ \underline{X_1 = Y_2} \end{array} \right\}$$ system of constraints

a system of constraints is of the form

$$C = \{ S_i = T_i \}_{i \in [1,n]}$$

types that may contain variables $X, Y, Z, \ldots$

a type substitution σ

e.g. $\sigma = [X \mapsto T,\ Y \mapsto U, \ldots]$

$dom(\sigma)$ $\quad range(\sigma)$

e.g. $\sigma = [X \mapsto Bool]$

$\sigma(X \to X) = Bool \to Bool$

$\sigma(Y \to Y) = Y \to Y$

A substitution σ unifier $S = T$ if $\sigma S = \sigma T$

σ unifier C if $\sigma S_i = \sigma T_i$ for all

$S_i = T_i$ in C

Unification algorithm — Robinson

Goals: check if set of solutions is non-empty

find best possible solution

└ all other solutions can be derived from it

Def'n σ is more general than σ' $(\sigma \leq \sigma')$

if $\sigma' = \gamma \circ \sigma$ for some substitution γ

Composition

$\gamma \circ \sigma =$

$X \to \gamma T$ for each $X \to T \in \sigma$

$X \to T$ for each $X \to T \in \gamma$ and $X \notin dom(\sigma)$

A principal unifier σ for C is such that for all σ' unifying C, $\sigma \leq \sigma'$

e.g.

$\{X = Y\}$

$\sigma = [X \mapsto Bool, Y \mapsto Bool]$

$\sigma = [X \mapsto Int, Y \mapsto Int]$

$\sigma = [X \mapsto Y]$ principal unifier

$\{Nat \to Nat = X \to Y\}$

$\sigma = [X \mapsto Nat, Y \mapsto Nat]$

$\{Y = Nat \to Y\}$ no solutions

$\{Int = Bool\}$ no solution

unify (C)

match C with

| $\emptyset \longrightarrow [\,]$

| $\{S = T\} \cup C' \longrightarrow$

if $S = T$ (syntactically)

unify(C')

variables in T

else if S is of the form X and $X \notin FV(T)$

unify($[X \mapsto T]C'$) $\circ$ $[X \longrightarrow T]$

composition

Symmetric

else if T is of the form X and $X \notin FV(S)$

unify($[X \mapsto S]C'$) $\circ$ $[X \longrightarrow S]$

else if $S = S_1 \longrightarrow S_2$ and $T = T_1 \longrightarrow T_2$

unify($C' \cup \{S_1 = T_1, S_2 = T_2\}$)

else fail (no solution)

$x = 10$

$y = 10$

$x \to 10, y \to 10$

e.g. <u>Case 3</u>

$$S = (Int \to Int) \to Int$$

$$T = X \to Int$$

$$Int = Int$$

$$X = Int \to Int$$

Typing relation

$$\Gamma \vdash t : T$$

typing context

term

type

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constraint typing relation

$$\Gamma \vdash t : T \mid C$$

assuming Γ, t is of type T whenever constraints C are satisfied.

e.g. $\emptyset \vdash (\lambda x : T .\ x)\ 0 : T' \mid \{ T = T', T' = Nat \}$

$T = T' = Nat$

$$\frac{\Gamma \vdash t_1 : T \mid C \qquad C' = C \cup \{T = Nat\}}{\Gamma \vdash \underline{succ}\ \underline{t_1} : Nat \mid C'}\ T_{succ}$$

$$\frac{\Gamma \vdash t_1 : T_1 \mid C_1 \qquad \Gamma \vdash t_2 : T_2 \mid C_2 \qquad C' = C_1 \cup C_2 \cup \boxed{\{T_1 = T_2 \rightarrow X\}}}{\Gamma \vdash t_1\ t_2 : X \mid C'}\ T_{App}$$