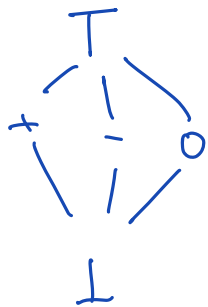


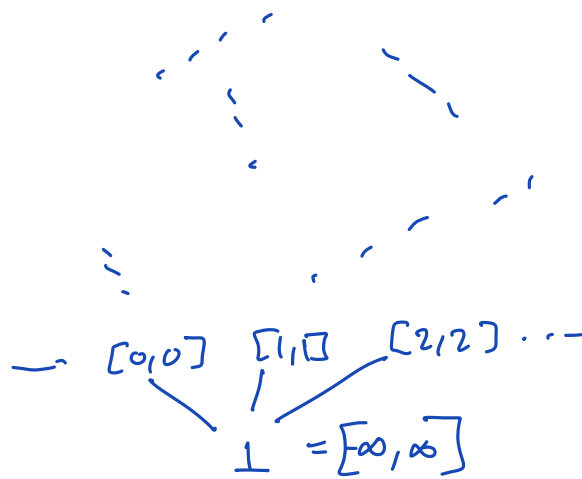
Numerical Abstract Domains

Signs



Interval domain

$$T = [-\infty, \infty]$$

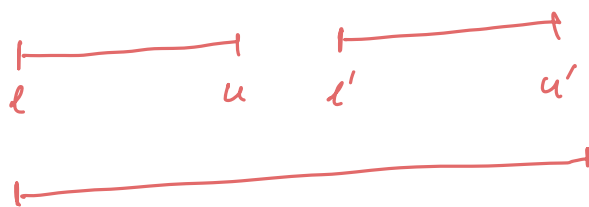


$$[l, u] \sqsubseteq [l', u']$$



$$[l, u] \sqcup [l', u']$$

$$= [\min(l, l'), \max(u, u')]$$



$$\alpha : 2^{\mathbb{Z}} \longrightarrow I$$

$$\alpha(S) = \begin{cases} \perp & \text{if } S = \emptyset \text{ then } \perp \text{ (empty interval)} \\ [\min(S), \max(S)] & \text{if } S \text{ is finite} \\ [\inf(S), \sup(S)] & \text{if } S \text{ is infinite} \end{cases}$$

$$\gamma([l, u]) = \{c \mid l \leq c \leq u\}$$

corner cases for inf/sup

$$2^{\mathbb{Z}} \xrightleftharpoons[\gamma]{\alpha} I$$

$$[l, u] + [l', u'] = [l + l', u + u']$$

$$[l, u] * [l', u'] = [\min(B), \max(B)]$$

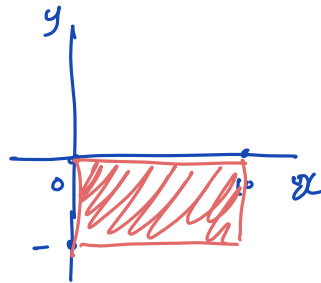
$$B = \{l * l', l * u', u * l', u * u'\}$$

$$[-1, 1] * [-1, -1] = [1, -1] \times$$

$$= [-1, 1]$$

Vars \longrightarrow Intervals

$$[x \mapsto [0, 10], y \mapsto [-1, 0] \dots]$$



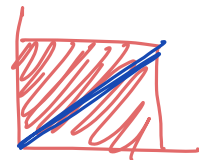
$$[x \mapsto [0, 10], y \mapsto T, z \mapsto T]$$

$$y := x$$

$$[x \mapsto [0, 10], y \mapsto [0, 10], z \mapsto T]$$

$$z := x - y$$

$$[x \mapsto [0, 10], y \mapsto [0, 10], z \mapsto [-10, 10]]$$



non-relational
domain

$$x = 1$$

$$[-\infty, \infty]$$

$$[1, 1]$$

$$[1, 2] \rightarrow [1, 3] \dots [1, 100]$$

$$x \geq 100$$

while ($x < 100$)

$$[1, 1]$$

$$[1, 2]$$

$$x = x + 1$$

$$[2, 2]$$

$$[2, 3]$$

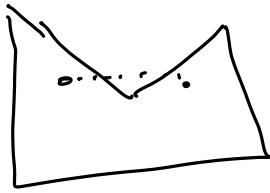
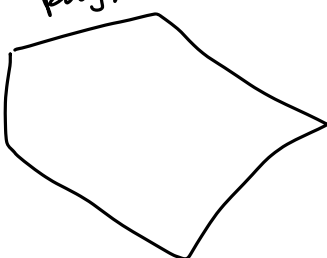
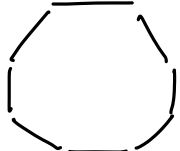
$$[100, 100]$$

Widening

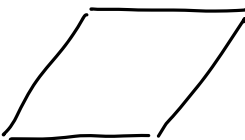
Intervals

Octagons

Polyhedra



Zonotopes



1-dimensional zonotope

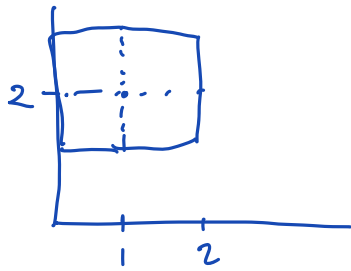
$$c_0 + c_1 \epsilon \quad \text{where } \epsilon \in [-1, 1]$$

$$[c_0 - c_1, c_0 + c_1]$$

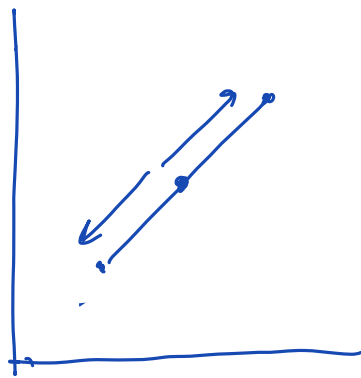
ϵ 's are generators

2D zonotope

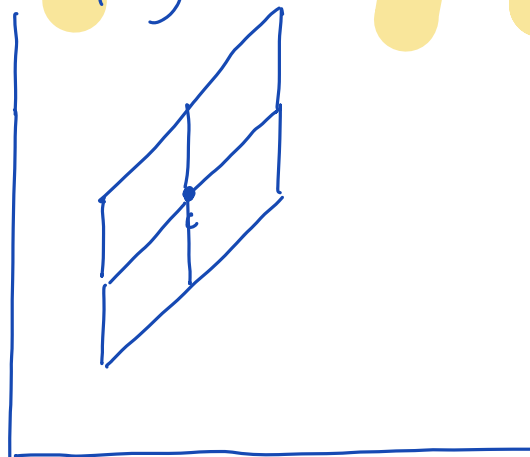
$$(1 + \varepsilon_1, 2 + \varepsilon_2) \quad \text{where } \varepsilon_1, \varepsilon_2 \in [-1, 1]$$



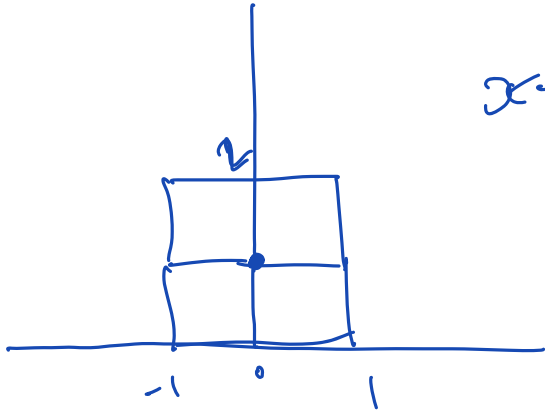
e.g. $(2 + \varepsilon_1, 2 + \varepsilon_1) \quad \varepsilon_1 \in [-1, 1]$



e.g. $(2 + \varepsilon_1, 3 + \varepsilon_1 + \varepsilon_2)$

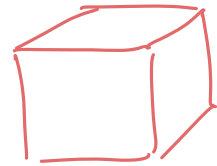


e.g. $(0 + \varepsilon_1, 1 + \varepsilon_2) \rightsquigarrow (1 + \varepsilon_1 + \varepsilon_2)$
 $= [-1, 3]$



$$x+y = [-1, 3]$$

$$(1 + 10\varepsilon_1, \varepsilon_2, \varepsilon_3)$$



$$y := x$$

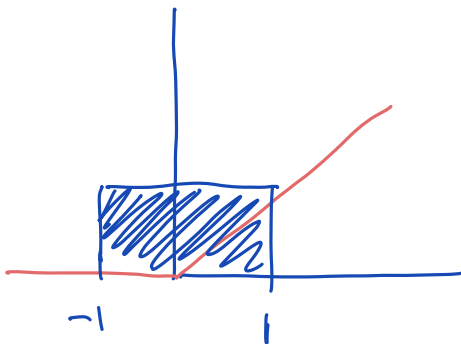
$$(1 + 10\varepsilon_1, 1 + 10\varepsilon_1, \varepsilon_3)$$



$$z := x - y$$

$$(1 + 10\varepsilon_1, 1 + 10\varepsilon_1, 0)$$

intervals



zonotope

