

TYPE RECONSTRUCTION / INFERENCE

$$\boxed{(\lambda x: X. x \ \emptyset) : Y_1 \rightarrow Y_2}$$

Intuition \downarrow \downarrow \downarrow
 \square $\text{foo}(\square x, \square y)$

$$\left\{ \begin{array}{l} X = \text{int} \rightarrow X_1 \\ X = \perp \\ X_1 = \perp_2 \end{array} \right\} \text{ system of constraints}$$

a system of constraints is of the form

$$C = \{ S_i = T_i \}_{i \in [1, n]}$$

\uparrow \nearrow
types that may contain
variables X, Y, Z, \dots

a type substitution σ

e.g. $\sigma = [X \mapsto T, Y \mapsto U, \dots]$

$$\text{dom}(\sigma) \quad \text{range}(\sigma)$$

e.g. $\sigma = [X \mapsto \text{Bool}]$

$$\sigma(X \rightarrow X) = \text{Bool} \rightarrow \text{Bool}$$

$$\sigma(Y \rightarrow Y) = Y \rightarrow Y$$

A substitution σ unifies $S = T$ if $\sigma S = \sigma T$

σ unifies C if $\sigma S_i = \sigma T_i$ for all
 $S_i = T_i$ in C

Unification algorithm — Robinson

Goals: check if set of solutions is non-empty

find best possible solution

↳ all other solutions
can be derived from it

Def'n σ is more general than σ' ($\sigma \leq \sigma'$)
if $\sigma' = \gamma \circ \sigma$ for some substitution γ

↑
Composition

$\gamma \circ \sigma =$

$X \rightarrow \gamma T$ for each
 $X \rightarrow T \in \sigma$

$X \rightarrow T$ for each $X \rightarrow T \in \sigma$
and $X \notin \text{dom}(\sigma)$

A **principal unifier** σ for C is such that
for all σ' unifying C , $\sigma \leq \sigma'$

e.g.

$$\{X = Y\}$$

$$\sigma = [X \rightarrow \text{Bool}, Y \rightarrow \text{Bool}]$$

$$\sigma = [X \rightarrow \text{Int}, Y \rightarrow \text{Int}]$$

$$\sigma = [X \rightarrow Y] \text{ principal unifier}$$

$$\{\text{Nat} \rightarrow \text{Nat} = X \rightarrow Y\}$$

$$\sigma = [X \mapsto \text{Nat}, Y \mapsto \text{Nat}]$$

$$\{Y = \text{Nat} \rightarrow Y\} \text{ no solution}$$

$$\{\text{Int} = \text{Bool}\} \text{ no solution}$$

unify (C)

match C with

$\emptyset \rightarrow []$

$\{S = T\} \cup C' \rightarrow$

if $S = T$ (syntactically)

unify (C')

$\left\{ \begin{array}{l} x = 10 \\ y = 10 \\ x \rightarrow 10, y \rightarrow 10 \end{array} \right.$

else if S is of the form X and $X \notin FV(T)$
unify ($[X \rightarrow T]C'$) \circ $[X \rightarrow T]$
Symmetric \nearrow
composition \nearrow

else if T is of the form X and $X \notin FV(S)$
unify ($[X \rightarrow S]C'$) \circ $[X \rightarrow S]$

else if $S = S_1 \rightarrow S_2$ and $T = T_1 \rightarrow T_2$
unify ($C' \cup \{S_1 = T_1, S_2 = T_2\}$)

else fail (no solution)

e.g. Case 3

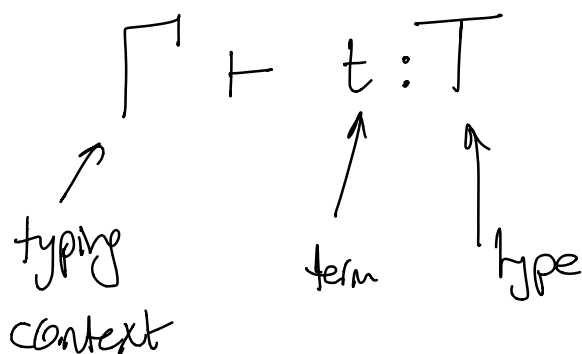
$$S = (\text{Int} \rightarrow \text{Int}) \rightarrow \text{Int}$$

$$T = X \rightarrow \text{Int}$$

$$\text{Int} = \text{Int}$$

$$X = \text{Int} \rightarrow \text{Int}$$

Typing relation



constraint typing relation

$$\Gamma \vdash t : T \mid C$$

assuming Γ , t is of type T whenever constraints C are satisfied.

e.g. $\emptyset \vdash (\lambda x:T. x) 0 : T' \mid \left\{ \begin{array}{l} T=T' \\ T'=\text{Nat} \end{array} \right\}$

$T=T' \ \& \ \text{Nat}$

$$\begin{array}{c}
 \Gamma \vdash t_1 : T \mid C \\
 C' = C \cup \{T = \text{Nat}\} \\
 \hline
 \Gamma \vdash \underline{\text{succ}} \underline{t_1} : \text{Nat} \mid C'
 \end{array}
 \quad T_{\text{succ}}$$

$$\begin{array}{c}
 \Gamma \vdash t_1 : T_1 \mid C_1 \\
 \Gamma \vdash t_2 : T_2 \mid C_2 \\
 C' = C_1 \cup C_2 \cup \{T_1 = T_2 \rightarrow X\} \\
 \hline
 \Gamma \vdash t_1 t_2 : X \mid C'
 \end{array}
 \quad T_{\text{APP}}$$