Supporting Document

This document serves to explain the derivation process of the equation (24) in the manuscript.

Equation (23) is derived similar to the average gas pressure. Beucase the average gas density in (23) is used for the calculation of linepack energy in (24), to prove (23) is equivelant to prove the following equations:

$$\psi_{ij,k} = \frac{2}{3} V_{ij} \sum_{n \in \mathcal{N}} GCV_n \chi_{ij,k,n} \left(\rho_{i,k} + \rho_{i,k} - \frac{\rho_{i,k}\rho_{i,k}}{\rho_{i,k} + \rho_{i,k}} \right) \tag{1}$$

where $\rho_{i,k}$ and $\rho_{ij,k}$ are the gas densities at bus i and in pipeline ij at dispatch interval k, respectively; $\psi_{ij,k}$ is the linepack energy of pipeline ij at dispatch interval k; V_{ij} is the volume of pipeline ij; n and $\mathcal{N}$ are the index and set for gas component(s), respectively; GCV_n is the GCV of gas component n; $\chi_{ij,k,n}$ is the gas composition of component n.

As shown in Fig. x, let's consider a small segment of pipeline. In this small segment, we assume the gas density and gas pressure are uniform, denoted as $\rho_{ij,k}(x)$ and $p_{ij,k}(x)$, $x \in [0, L_{ij}]$, where L_{ij} is the length of the pipeline. We also assume the gas are fully mixed in the pipeline, which means the gas composition is also uniformed, i.e., $\chi_{ij,k,n}(x) = \chi_{ij,k,n}, \forall x \in [0, L_{ij}]$. Then, we can calculated the linepack energy in this segment (the notation of k and omitted):

$$d\psi_{ij}(x) = A_{ij} \cdot dx \cdot \rho_{ij}(x) \sum_{n \in \mathcal{N}} GCV_n \chi_n \tag{2}$$

where A_{ij} is the cross-sectional area of the pipeline.

According to the motion equation we have:

$$\rho_{ij} \frac{\partial p_{ij}}{\partial x} + \frac{8 f_{ij}}{\pi^2 D_{ij}^5} m_{ij}^2 = 0 \tag{3}$$

where f_{ij} and D_{ij} are the friction factor and diameter, respectively.

According to the ideal gas law, we have:

$$pV = \vartheta R^{gas} T \tag{4}$$

where p is the gas pressure, V is the volume, R^{gas} is the gas constant, T is the temperature, and ϑ is the amount of substance.

We can reformulated (4) and calculate the density of the gas mixture in the pipeline in standard temperature and pressure (STP) condition as:

$$\rho_{ij}^{stp} = m_{ij}/V_{ij} \tag{5}$$

$$= \sum_{n\in\mathcal{N}} \vartheta_{ij,n} M_n / V_{ij} \tag{6}$$

$$= \vartheta_{ij} \sum_{n\in\mathcal{N}} \chi_{ij,n} M_n / (\vartheta R^{gas} T^{stp} / p^{stp}) \tag{7}$$

$$= p^{stp} \sum_{n\in\mathcal{N}} \chi_{ij,n} M_n / (R^{gas} T^{stp}) \tag{8}$$

where p^{stp} and T^{stp} are the pressure and temperature at (STP) condition.

According to the gas state equation, we have:

$$p = \rho Z r T^{gas} \tag{9}$$

where Z is the compressibility factor; r is gas constant for gas mixture (since the gas composition in the pipeline is uniform, then the gas constant in the pipeline is also uniform); T^{gas} is the temperature of gas mixture.

Substitute (8) and (9) into (3), we have:

$$\rho_{ij} \frac{\partial \rho_{ij}}{\partial x} + \frac{8 f_{ij}}{\pi^2 D_{ij}^5 Z_{ij} r_{ij} T^{gas}} \left(\frac{p^{stp} \sum_{n\in\mathcal{N}} M_n}{R^{gas} T^{stp}} \right)^2 q_{ij}^2 = 0 \tag{10}$$

which can be simplified as:

$$\frac{\partial \rho_{ij}^2}{\partial x} + \Theta_{ij}^2 q_{ij}^2 = 0 \tag{11}$$

where $\Theta_{ij}^2 = \frac{8 f_{ij}}{\pi^2 D_{ij}^5 Z_{ij} r_{ij} T^{gas}} \left(\frac{p^{stp} \sum_{n\in\mathcal{N}} M_n}{R^{gas} T^{stp}} \right)^2$ is a constant; q_{ij} is the gas flow rate at STP condition.

In the quasi-dynamic gas flow model, we can use an average gas flow rate to represent the gas flow distribution in the pipeline. Then, substituting the boundary condition $\rho_{ij}|_{x=0} = \rho_i$ or $\rho_{ij}|_{x=L_{ij}} = \rho_j$, we can solve the function of $\rho_{ij}(x)$:

$$\rho_{ij}(x) = \sqrt{\int -\Theta_{ij}^2 q_{ij}^2 dx} \tag{12}$$

$$= \sqrt{\rho_i^2 - \Theta_{ij}^2 q_{ij}^2 x} \tag{13}$$

which is similar to Weymouth equation. Substitute it into (2), we have:

$$\psi_{ij} = \int_0^{L_{ij}} d\psi_{ij} \tag{14}$$

$$= A_{ij} \sum_{n \in \mathcal{N}} GCV_n \chi_n \int_0^{L_{ij}} \sqrt{\rho_i^2 - \Theta_{ij}^2 q_{ij}^2 x} dx \tag{15}$$

$$\overset{\iota=\rho_i^2-\Theta_{ij}^2 q_{ij}^2 x}{=} A_{ij} \sum_{n \in \mathcal{N}} GCV_n \int_{\rho_i^2}^{\rho_j^2} \iota^{\frac{1}{2}} d \frac{\rho_i^2 - \iota}{\Theta_{ij}^2 q_{ij}^2} \tag{16}$$

$$= -A_{ij} \sum_{n \in \mathcal{N}} GCV_n \frac{1}{\Theta_{ij}^2 q_{ij}^2} \int_{\rho_i^2}^{\rho_j^2} \iota^{\frac{1}{2}} d\iota \tag{17}$$

$$= -A_{ij} \sum_{n \in \mathcal{N}} GCV_n \frac{1}{\Theta_{ij}^2 q_{ij}^2} \left[\frac{2}{3} \iota^{\frac{3}{2}} \right]_{\rho_i^2}^{\rho_j^2} \tag{18}$$

$$= \frac{2}{3} A_{ij} \sum_{n \in \mathcal{N}} GCV_n \left(\rho_i + \rho_i - \frac{\rho_i \rho_i}{\rho_{i,k} + \rho_{i,k}} \right) \tag{19}$$

which proves (1).