

UVA CS 6316: Machine Learning

Lecture 18b: Random Forest / Boosting

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Course Content Plan →

Six major sections of this course

- Regression (supervised)
- Classification (supervised)
- Unsupervised models
- Learning theory
- Graphical models
- Reinforcement Learning

Y is a continuous

Y is a discrete

NO Y

About $f()$

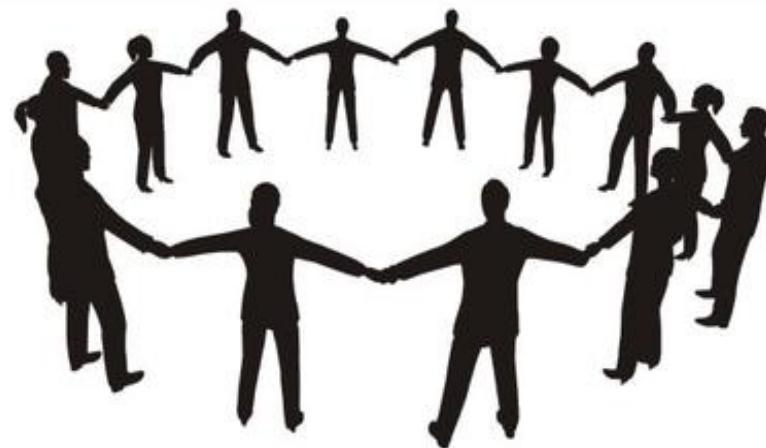
About interactions among X_1, \dots, X_p

Learn program to Interact with its environment

Three major sections for classification

- We can divide the large variety of classification approaches into **roughly three major types**
 1. Discriminative
 - directly estimate a decision rule/boundary
 - e.g., ~~support vector machine, decision tree, logistic regression,~~
~~e.g. neural networks (NN), deep NN~~
 2. Generative:
 - build a generative statistical model
 - e.g., ~~Bayesian networks, Naïve Bayes classifier~~
 3. ~~Instance based classifiers~~
 - Use observation directly (no models)
 - e.g. K nearest neighbors

Today: Ensemble



Today: Ensemble

- Framework of Ensemble:

- 1. Get a set of classifiers $f_1(x), f_2(x), f_3(x), \dots, f_B(x)$

They should be diverse.

- 2. Aggregate the classifiers (*properly*)

blend

Today: Ensemble



- Bagging / reduce Variance
- Bagged Decision Tree
- Random forests:
- Boosting / reduce bias , reduce Variance
 - Adaboost
 - Xgboost / Gradient Boosting
- Stacking

Bagging

- Bagging or *bootstrap aggregation*
 - a technique for reducing the variance of an estimated prediction function.
- For instance, for classification, a *committee of decision trees*
 - Each tree casts a vote for the predicted class.

Bootstrap

The basic idea:

randomly draw datasets *with replacement* (i.e. allows duplicates) from the training data, each samples *the same size as the original training set*

$f_1, f_2 \dots, f_B \quad \#|Tr|$
① change training set $\{s_1, s_2, \dots, s_B\}$

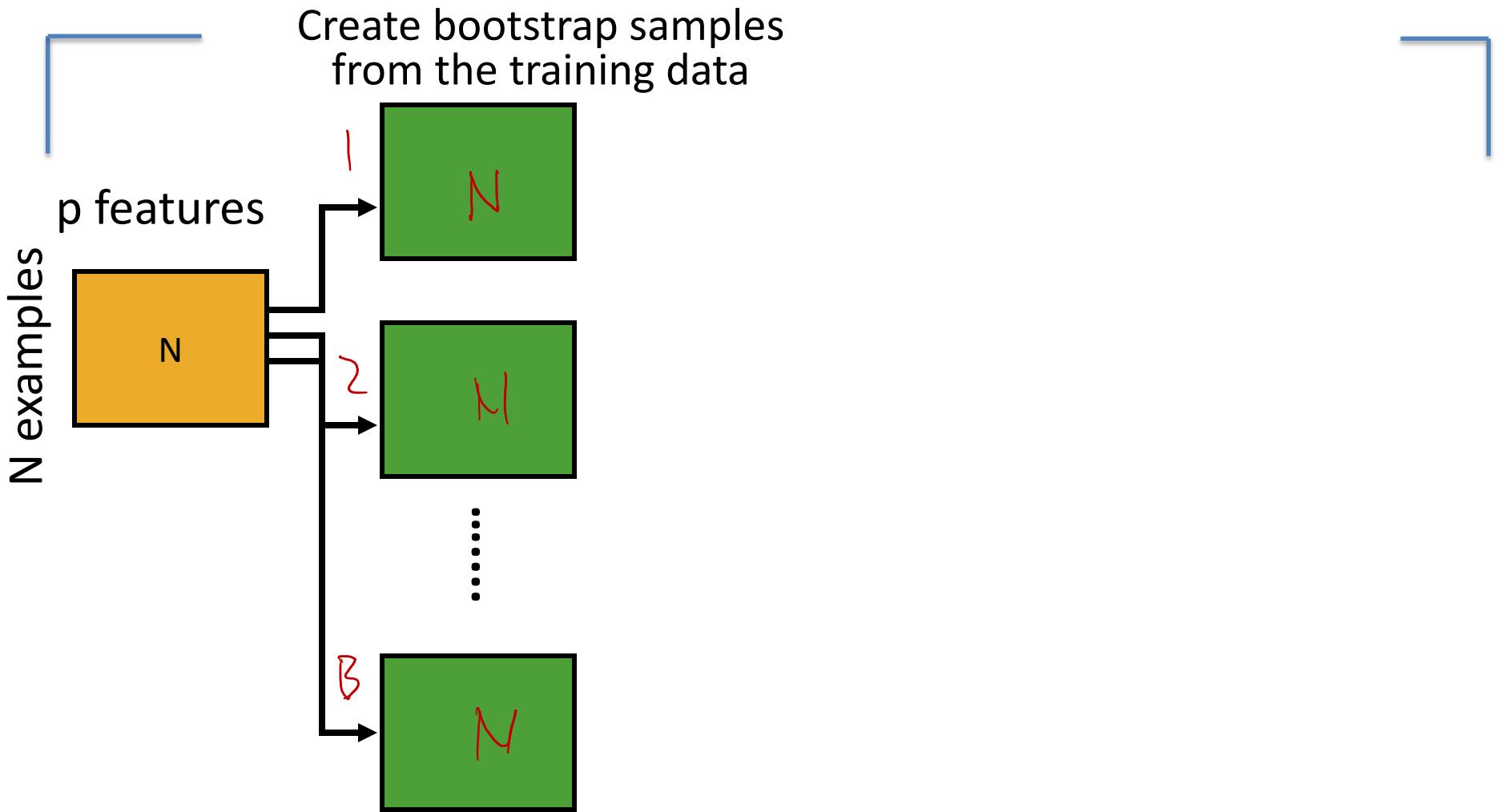
With vs Without Replacement

- 
- **Bootstrap with replacement** can keep the **sampling size the same as the original size** for every repeated sampling. The sampled data groups are independent on each other.

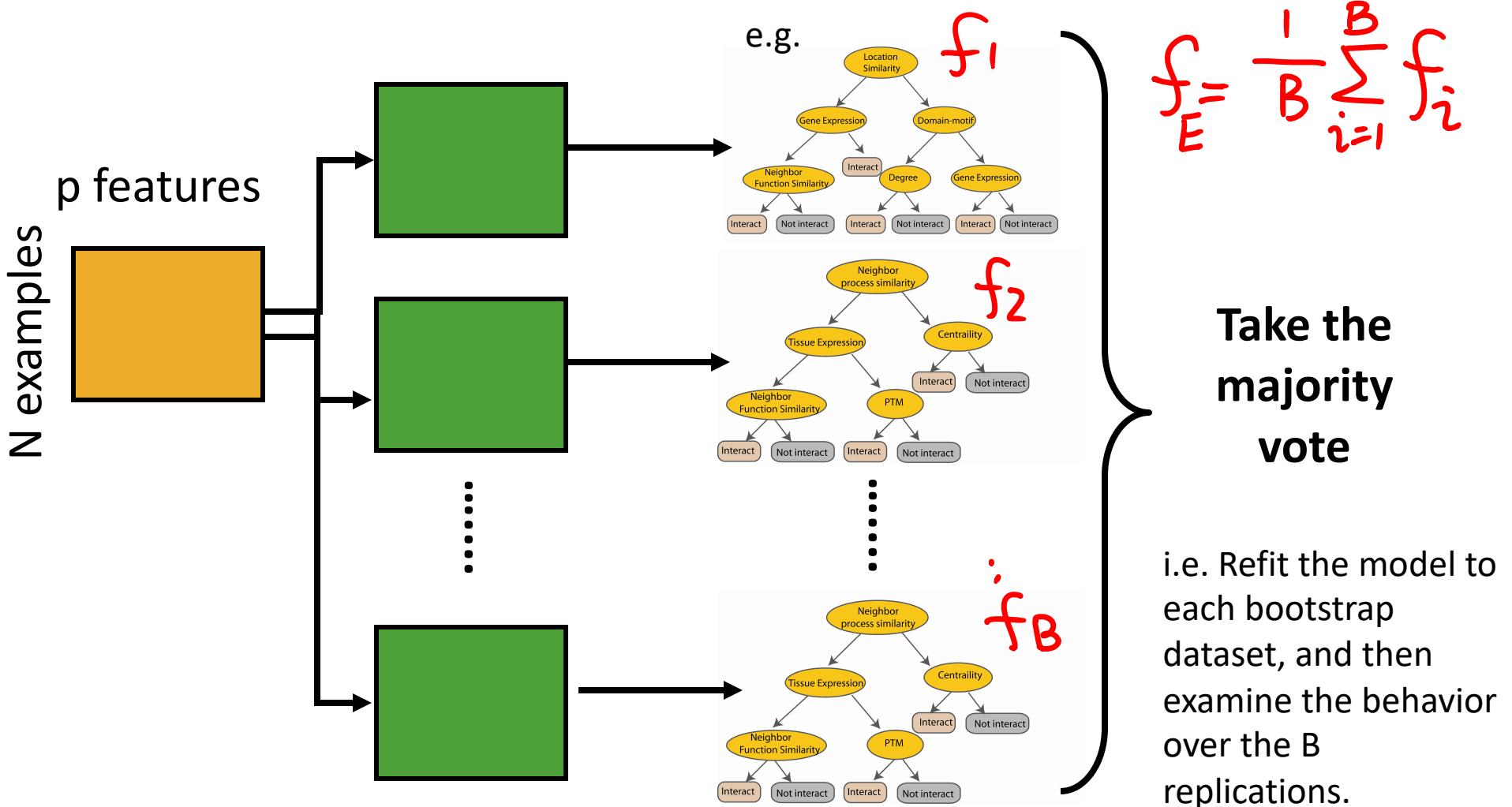
$$S_1, S_2, \dots, S_B$$

- **Bootstrap without replacement** cannot keep the sampling size the same as the original size for every repeated sampling. The sampled data groups are dependent on each other.

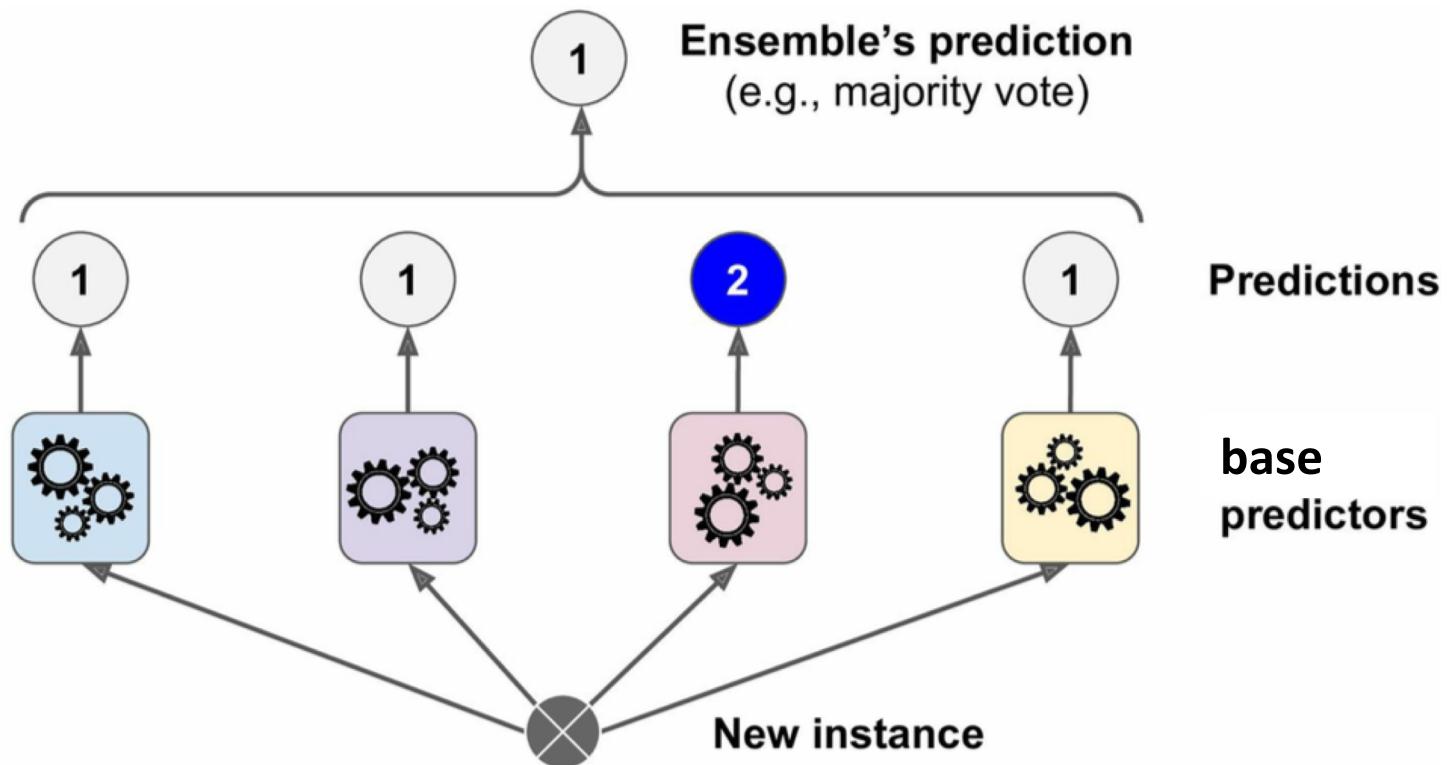
Bagging



Bagging of DT Classifiers



E.g., Predict by Hard voting



$$\text{Var}(f_i)$$

vs.

$$\text{Var}\left(\frac{1}{B} \sum_{i=1}^B f_i\right)$$

Decision Boundary Comparison

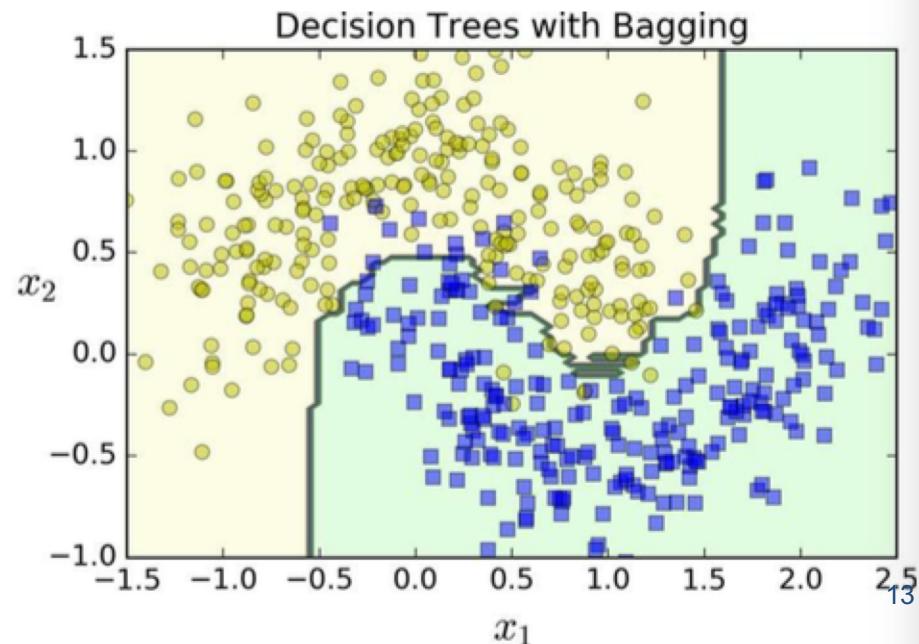
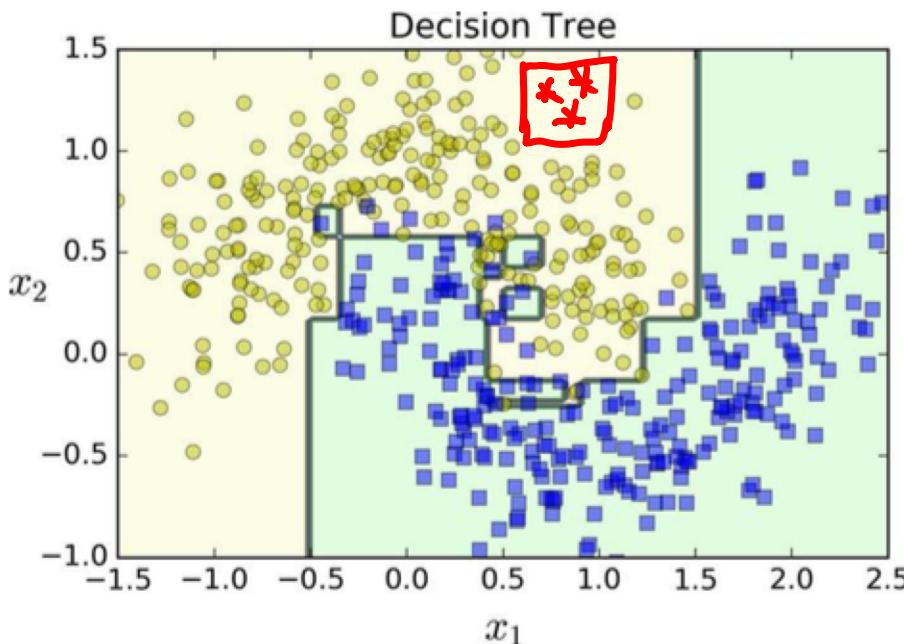
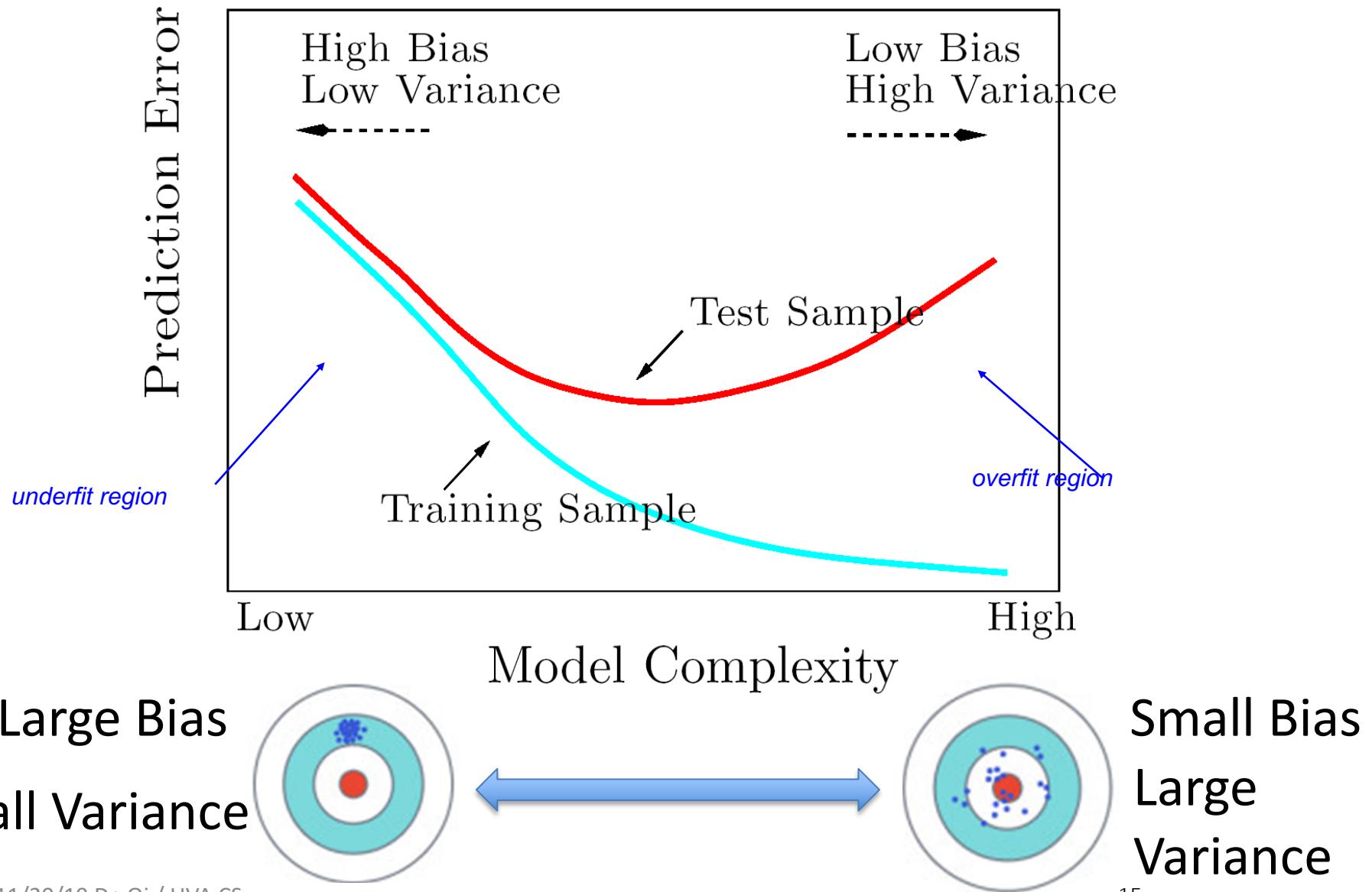


Figure 7-5. A single Decision Tree versus a bagging ensemble of 500 trees

Peculiarities of Bagging

- Model Instability is good when bagging
 - The more variable (unstable) the basic model is, the more improvement can potentially be obtained
 - Low-Variability methods (e.g. LDA) improve less than High-Variability methods (e.g. decision trees)

Recap: Bias-Variance Tradeoff / Model Selection



classifiers $f_1(x)$,

s_1

$f_2(x)$,

s_2

$f_3(x), \dots$

$f_B(x)$

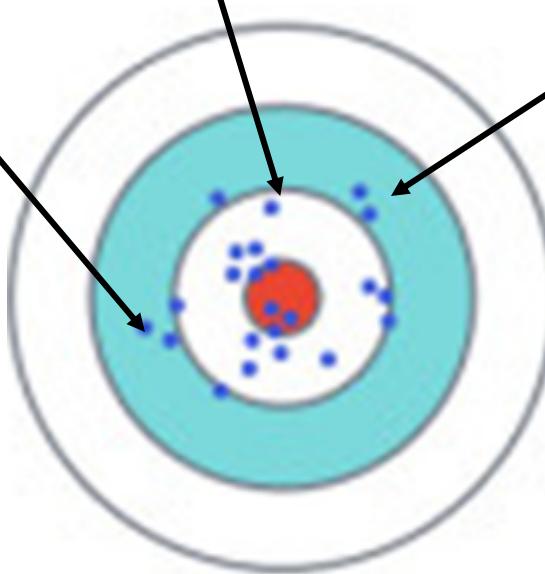
s_B

A complex model will have large variance.

We can average complex models to reduce variance.

If we average all the f_i , is it close to f^*

$$E[\hat{f}] = f^*$$



$$\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) + 2ab\text{Cov}(X, Y)$$

When $\text{Cov}(\hat{f}_i, \hat{f}_j) = 0$

$$Var(\hat{f}) = E((\hat{f} - \bar{f})^2) = Var\left(\frac{1}{B} \sum_{i=1}^B \hat{f}_i\right) = \frac{1}{B^2} \sum_{i=1}^B Var(\hat{f}_i) =$$

Base classifiers $f_1(x), f_2(x), f_3(x), \dots, f_B(x)$

$$Var(\hat{f}) = Var\left(\frac{1}{B} \sum_{i=1}^B \hat{f}_i\right)$$

$$= \frac{1}{B^2} \sum_{i=1}^B Var(f_i) \quad \text{when } \forall i, j, \text{Cov}(f_i, f_j) = 0$$

$$= \frac{1}{B} Var(f_i)$$

Because
 $|S_1| = |S_2| = \dots = |S_B|$

$Var(f_i) \approx Var(f_i)$

Bagging : an extreme case study using simulated data (with correlated features)

$N = 300$ training samples, $y \in \{0, 1\}$, x_1, x_2, \dots, x_5

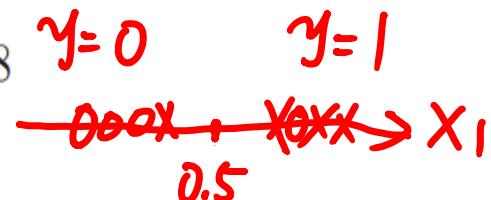
Y : Two classes and X : $p = 5$ features,

Each feature $N(0, 1)$ distribution and pairwise correlation .95

Response Y : generated according to:

$$\Pr(Y = 1|x_1 \leq 0.5) = 0.2$$

$$\Pr(Y = 1|x_1 > 0.5) = 0.8$$

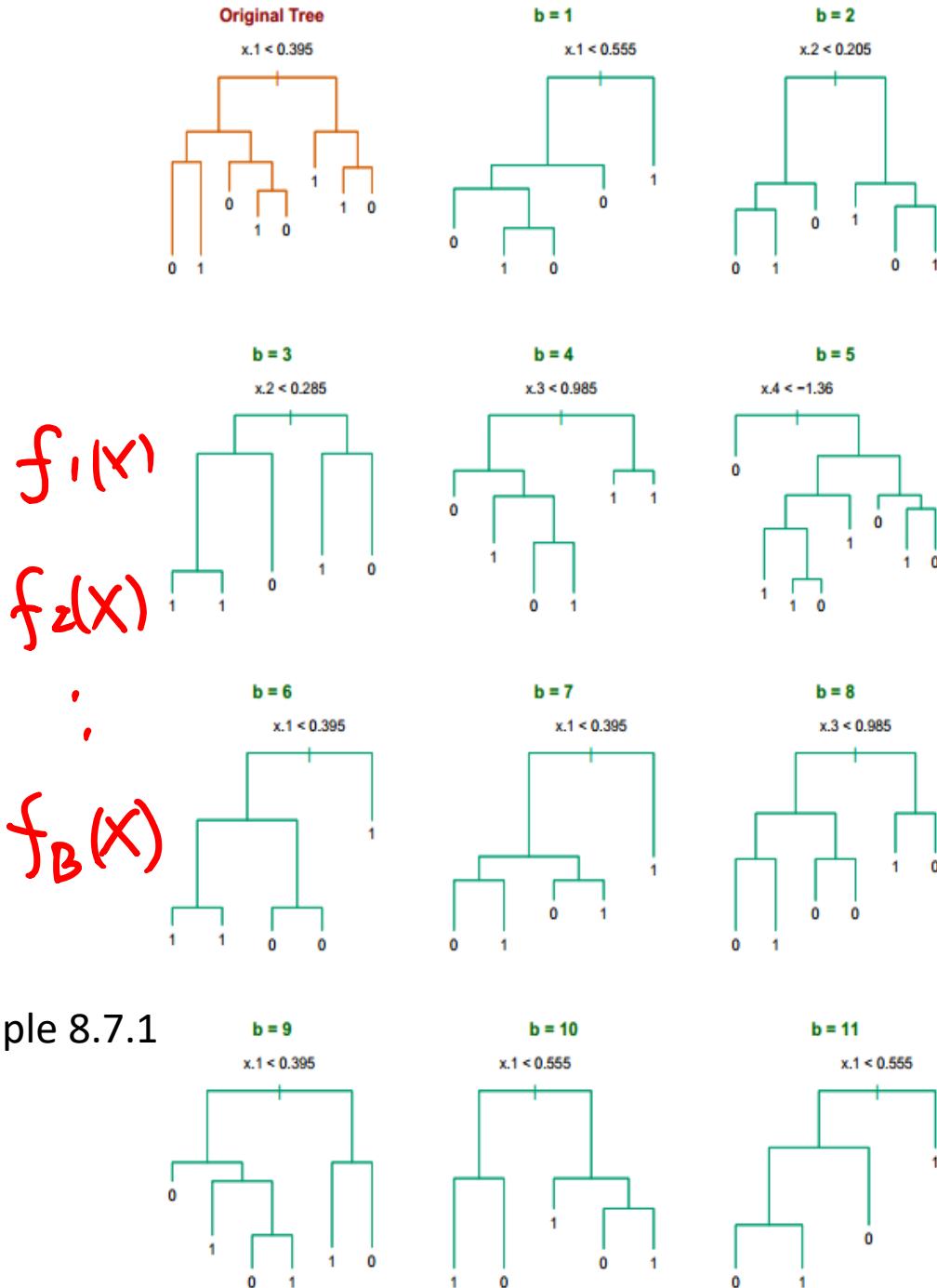


Test sample size of 2000

Fit classification trees to training set and bootstrap samples

$B = 200$

Notice the bootstrap trees look quite different from the original tree



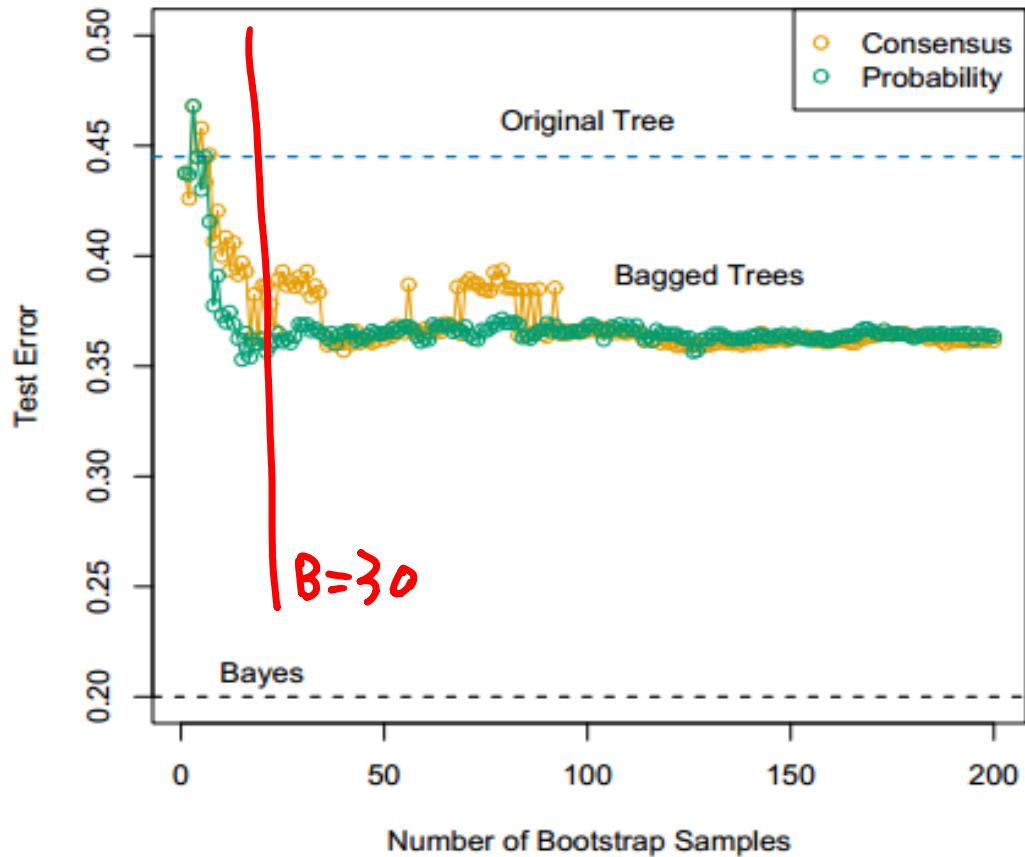
Five features highly correlated with each other

→ No clear difference with picking up which feature to split

→ Small changes in the training set will result in different tree

→ But these trees are actually quite similar wrt output classification

ESL book / Example 8.7.1



→ For $B > 30$, more trees do not improve the bagging results

→ Since the trees correlate highly to each other and give similar classifications

B

$$\text{Var}\left(\frac{1}{B} \sum f_i\right)$$

Consensus: Majority vote

Probability: Average distribution at terminal nodes

Bagging

- Slightly increases model complexity
 - Cannot help when greater enlargement of model diversity is needed
- Bagged trees are correlated
 - Use random forest to reduce correlation between trees

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(X, Y)$$

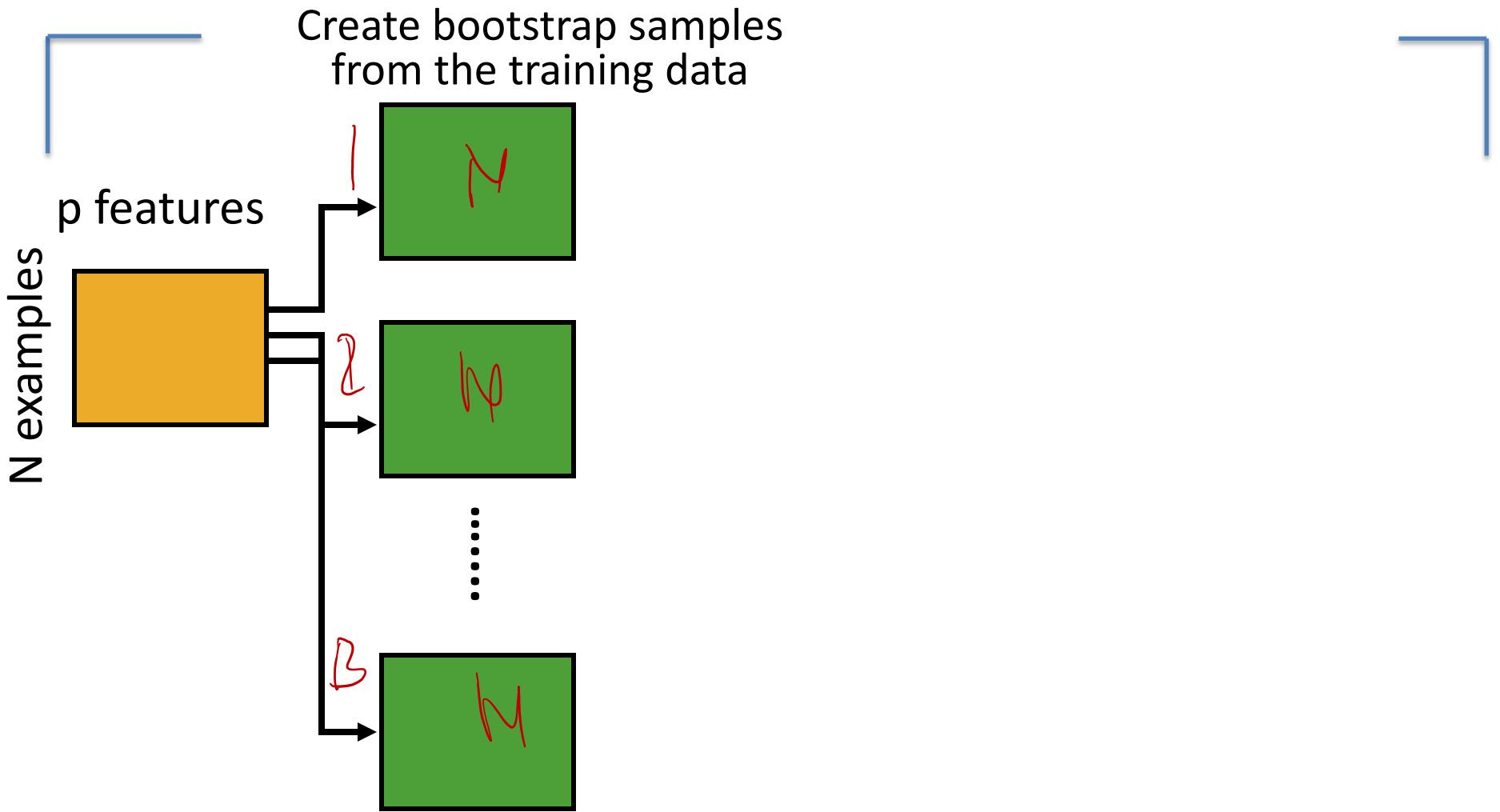
Today: Ensemble

- Bagging
 - Bagged Decision Tree
- Random forests:
 - Boosting
 - Adaboost
 - Xgboost
 - Stacking

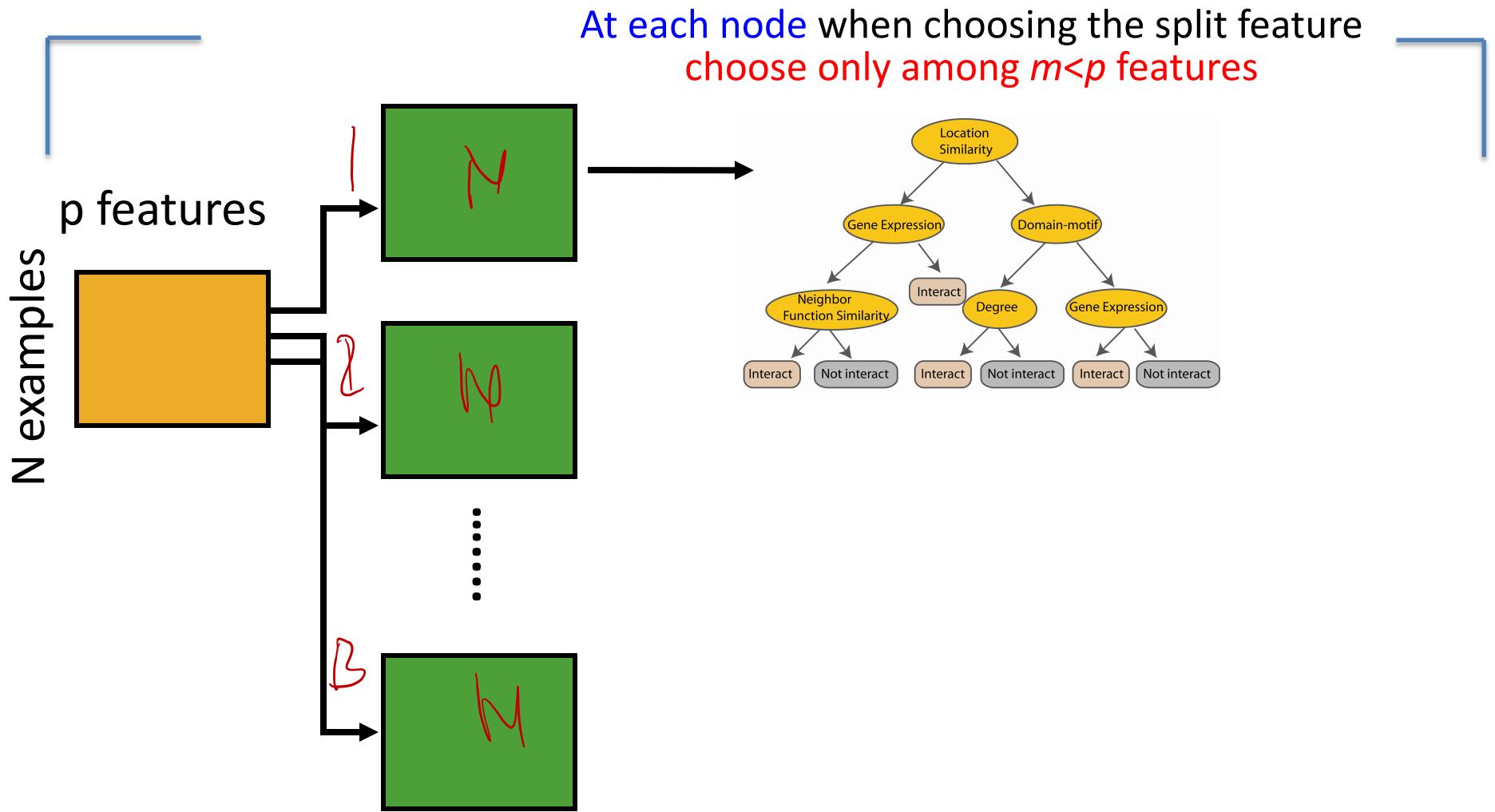
Random forest classifier

- Random forest classifier,
 - an extension to bagging
 - which *uses de-correlated trees.*

Random Forest Classifier

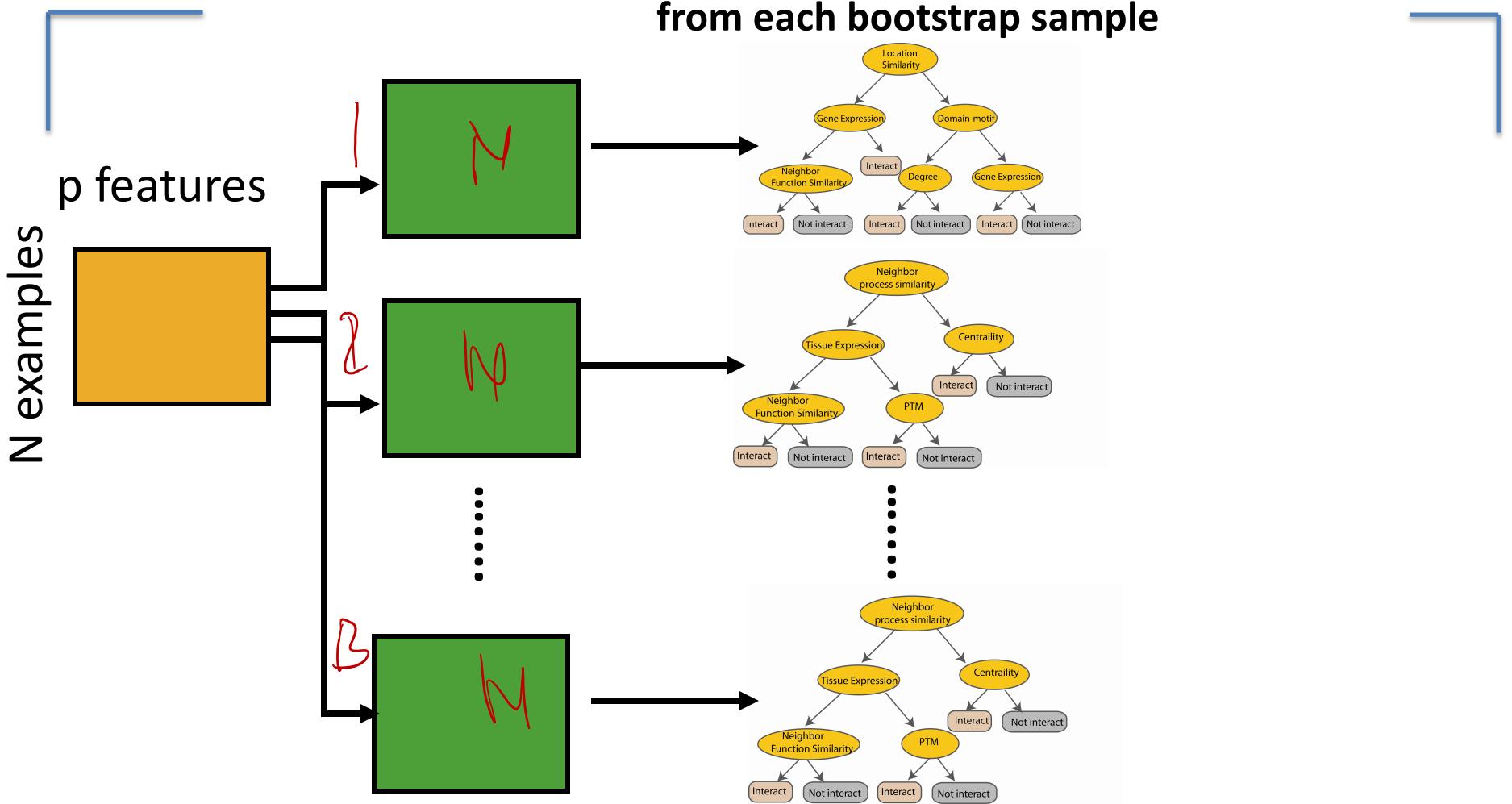


Random Forest Classifier

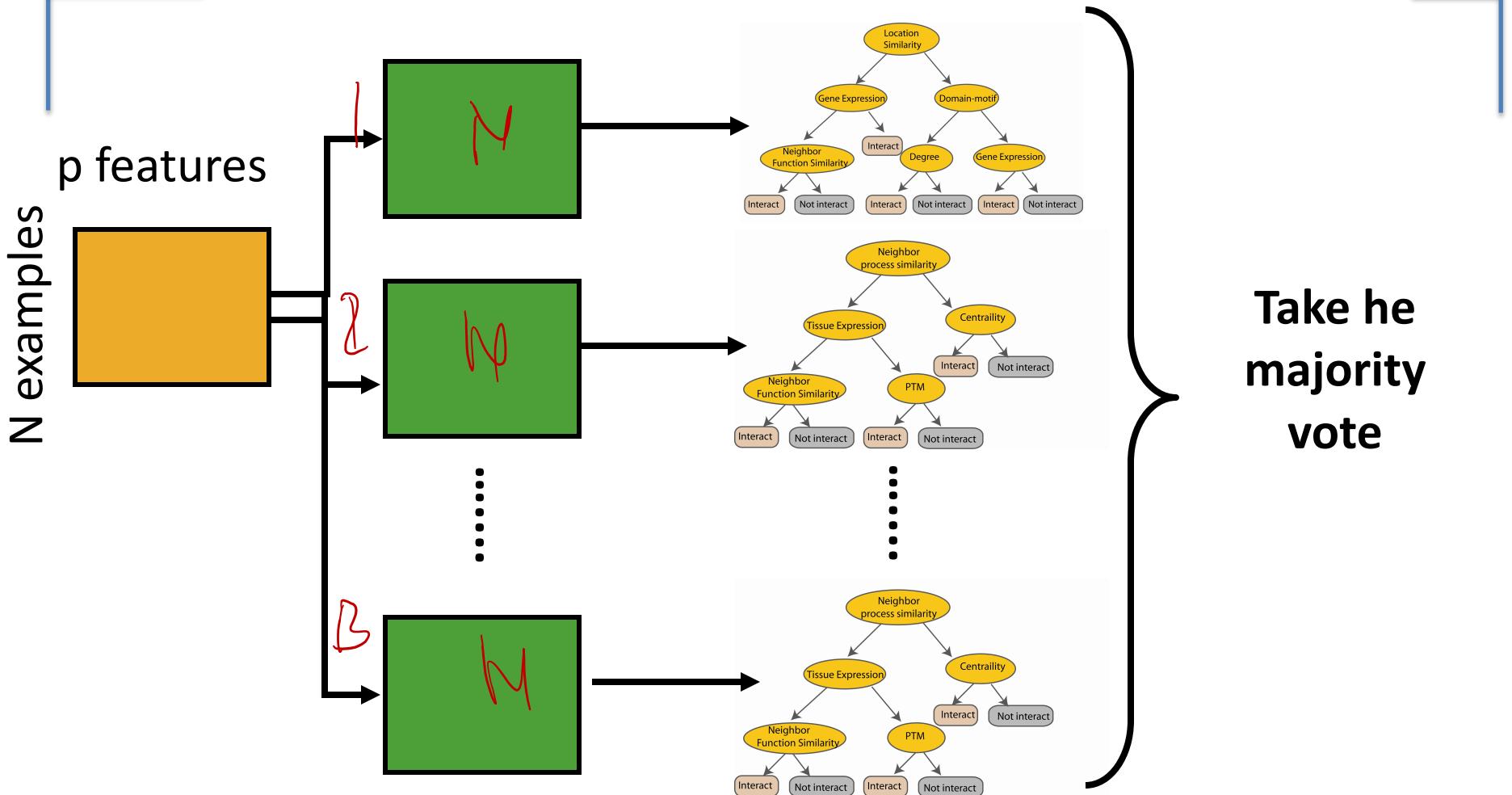


Random Forest Classifier

Create decision tree
from each bootstrap sample



Random Forest Classifier



Random Forests

For each of our B bootstrap samples

Form a tree in the following manner

i: Given p dimensions, **pick m of them**

ii: **Split only according to these m dimensions**

(we will NOT consider the other $p-m$ dimensions)

Repeat the above steps i & ii for each split

Note: we pick a different set of m dimensions for each split
on a single tree

$P = 0.06$

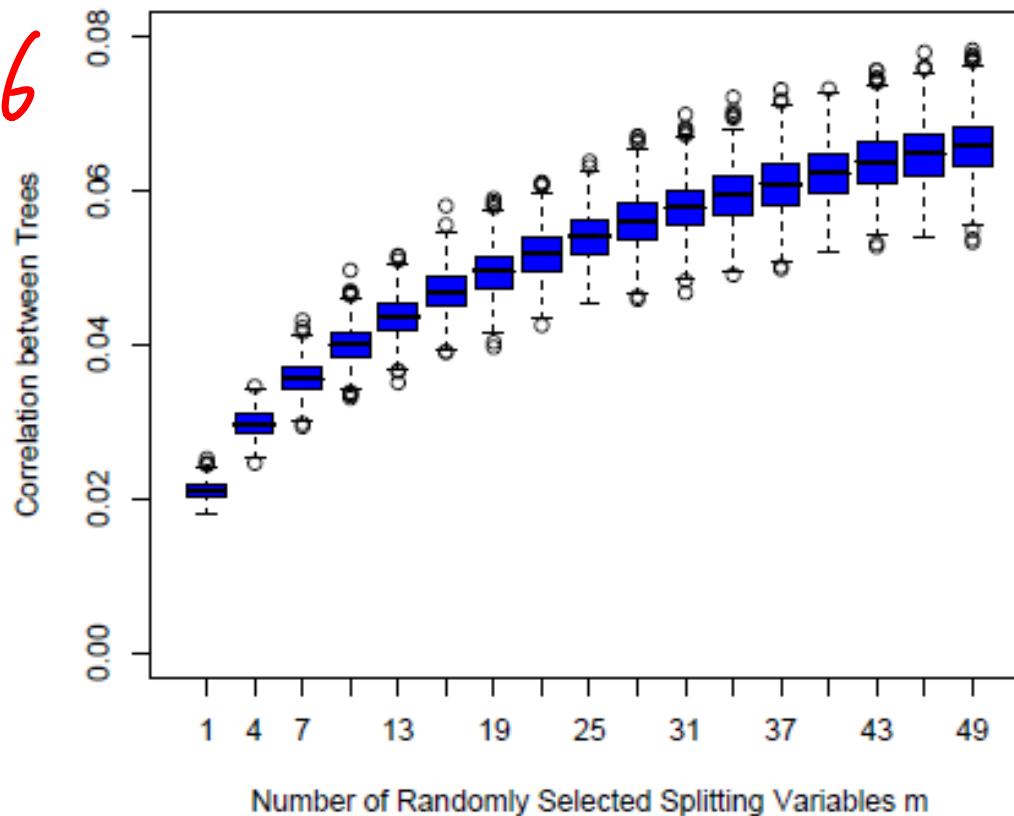


FIGURE 15.9. *Correlations between pairs of trees drawn by a random-forest regression algorithm, as a function of m . The boxplots represent the correlations at 600 randomly chosen prediction points x .*

Page 598-599 In ESL book

Random Forests

Random forest can be viewed as a refinement of bagging with a tweak of **decorrelating** the trees:

At each tree split, a random subset of m features out of all p features is drawn to be considered for splitting

Some guidelines provided by Breiman, but be careful to choose m based on specific problem:

- {
 - $m = p$ amounts to bagging
 - $m = p/3$ or $\log_2(p)$ for regression
 - $m = \sqrt{p}$ for classification

Why correlated trees are not ideal ?

Random Forests try to reduce correlation between the trees.

Why?

Why correlated trees are not ideal ?

Assuming each tree has variance σ^2

If trees are independently identically distributed, then average variance is σ^2/B

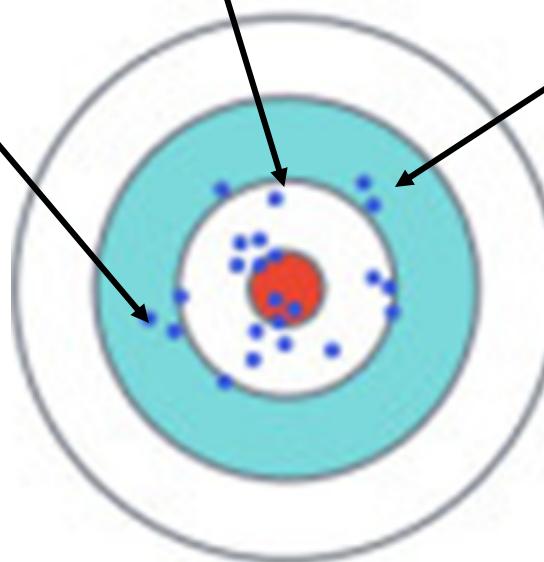
$$\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) + 2ab\text{Cov}(X, Y)$$

$$\text{Var}(\hat{f}) = E((\hat{f} - \bar{f})^2) = \text{Var}\left(\frac{1}{B} \sum \hat{f}_i\right) = \frac{1}{B^2} \sum \text{Var}(\hat{f}_i) = \frac{1}{B} \sigma^2$$

classifiers $f_1(x), f_2(x), f_3(x), \dots, f_B(x)$

A complex model will have large variance.

We can average complex models to reduce variance.



If we average all the f_i , is it close to f^*

$$E[\hat{f}] = f^*$$

Why correlated trees are not ideal ?

Assuming each tree has variance σ^2

If simply identically distributed, then average variance is

$$\rho_{ij} = \text{Corr}(f_i, f_j) = \rho \sigma^2 + \frac{1 - \rho}{B} \sigma^2$$

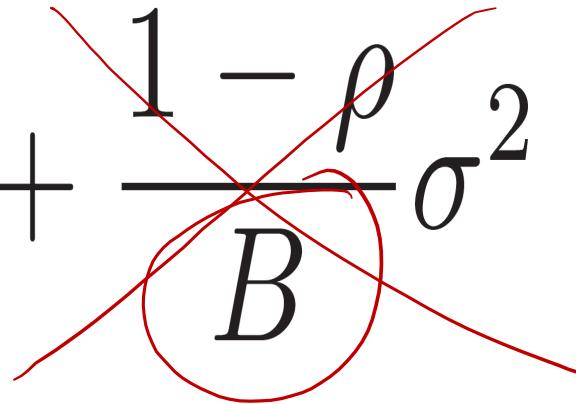
As $B \rightarrow \infty$, second term $\rightarrow 0$

Thus, the pairwise correlation always affects the variance

Why correlated trees are not ideal ?

Assuming each tree has variance σ^2

If simply identically distributed, then average variance is

$$\Rightarrow \rho\sigma^2 + \frac{1-\rho}{B}\sigma^2$$


RAM x B
trainTim x B
testTime x B

As $B \rightarrow \infty$, second term $\rightarrow 0$

Thus, the pairwise correlation always affects the variance

Why correlated trees are not ideal ?

How to deal?

If we reduce m (the number of dimensions we actually consider in each splitting),

then we reduce the pairwise tree correlation

Thus, variance will be reduced.

More about Random Forests

1. Construct subset $(x_1^*, y_1^*), \dots, (x_n^*, y_n^*)$ by sampling original training set with replacement.
2. Build tree-structured learners $h(x, \Theta_k)$, where at each node, **m predictors at random** are selected before finding the best split.
 - Gini Criterion.
 - No pruning.
3. Combine the predictions (average or majority vote) to get the final result.

Today: Ensemble

- Bagging
 - Bagged Decision Tree
 - Random forests:
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 - Adaboost
 - Xgboost
- Stacking



e.g. Ensembles in practice



Oct 2006 -
2009

Each rating/sample:

+ <user, movie, date of grade, grade>

Training set (100,480,507 ratings)

Qualifying set (2,817,131 ratings) → winner

- Training data is a set of users and ratings (1,2,3,4,5 stars) those users have given to movies.
- Predict what rating a user would give to any movie
- \$1 million prize for a 10% improvement over Netflix' s current method ($MSE = 0.9514$)

Ensemble in practice

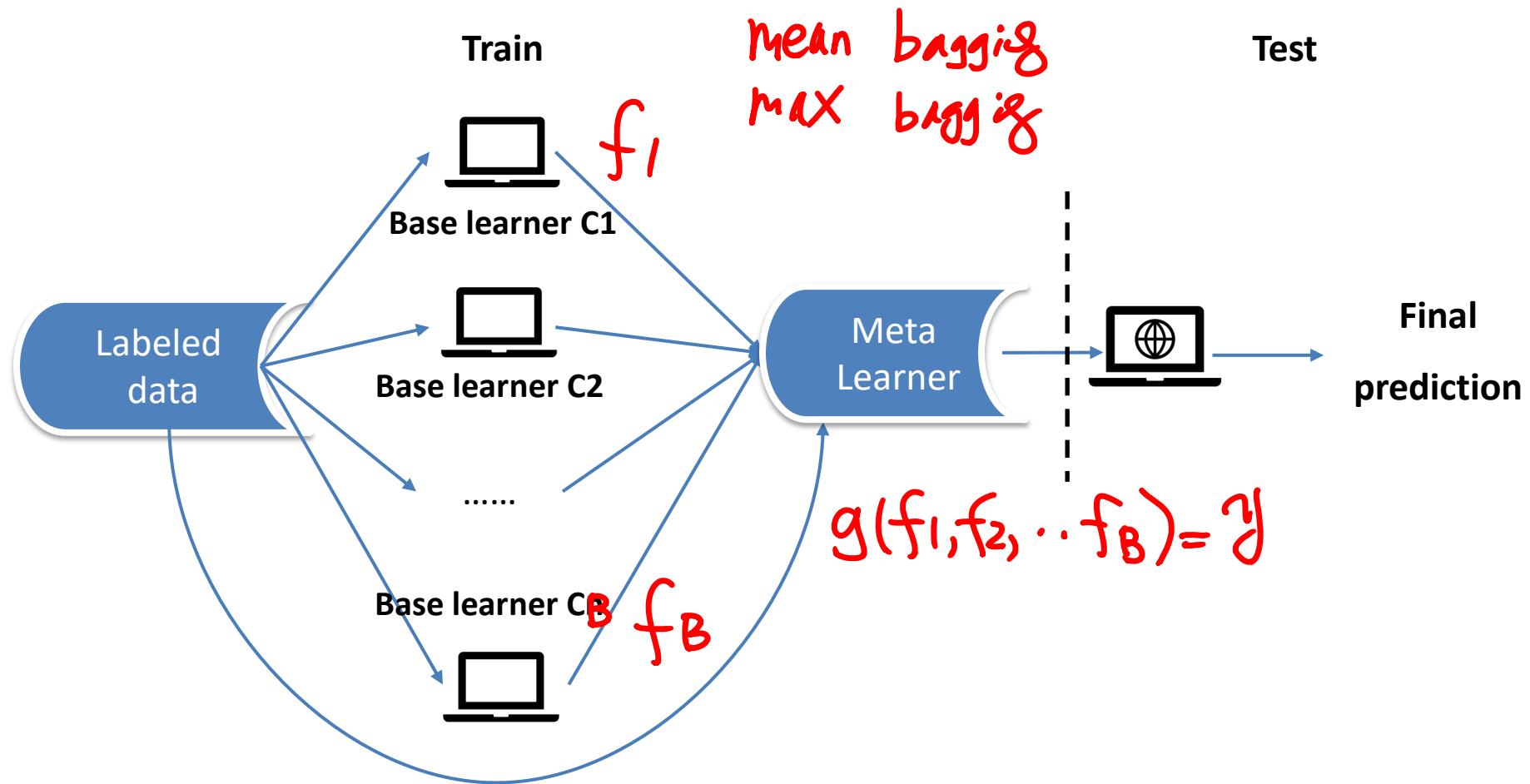
Team “Bellkor’s Pragmatic Chaos” defeated the team “ensemble” by submitting just 20 minutes earlier! → 1 million dollar !

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
<u>Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos</u>				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
3	Grand Prize Team	0.8582	9.90	2009-07-10 21:24:40
4	Opera Solutions and Vandelay United	0.8588	9.84	2009-07-10 01:12:31
5	Vandelay Industries!	0.8591	9.81	2009-07-10 00:32:20
6	PragmaticTheory	0.8594	9.77	2009-06-24 12:06:56
7	BellKor in BigChaos	0.8601	9.70	2009-05-13 08:14:09
8	Dace	0.8612	9.59	2009-07-24 17:18:43
9	Feeds2	0.8622	9.48	2009-07-12 13:11:51
10	BigChaos	0.8623	9.47	2009-04-07 12:33:59
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11

The ensemble team → blenders of multiple different methods

Stacking

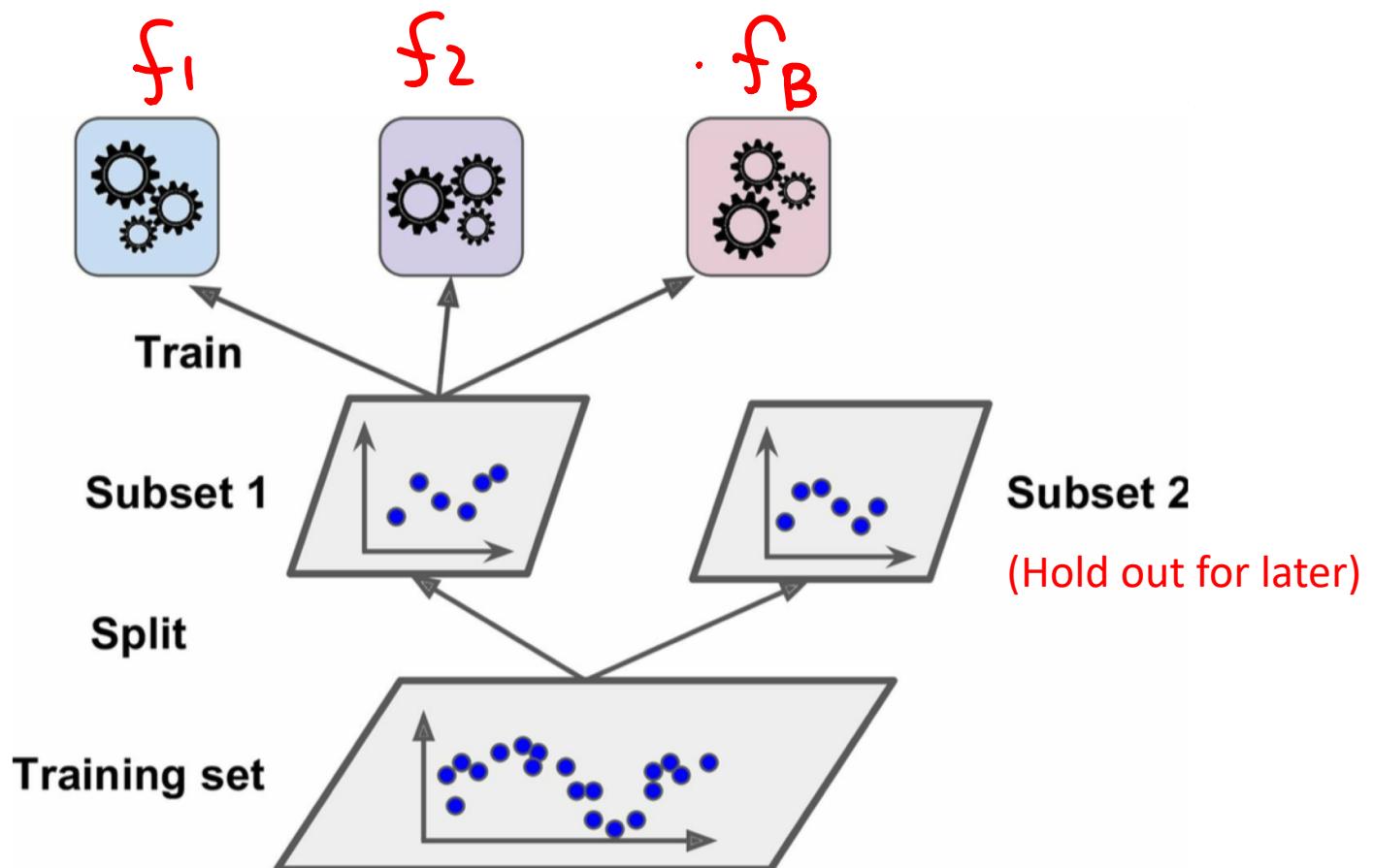
- Main Idea: Learn and combine multiple classifiers



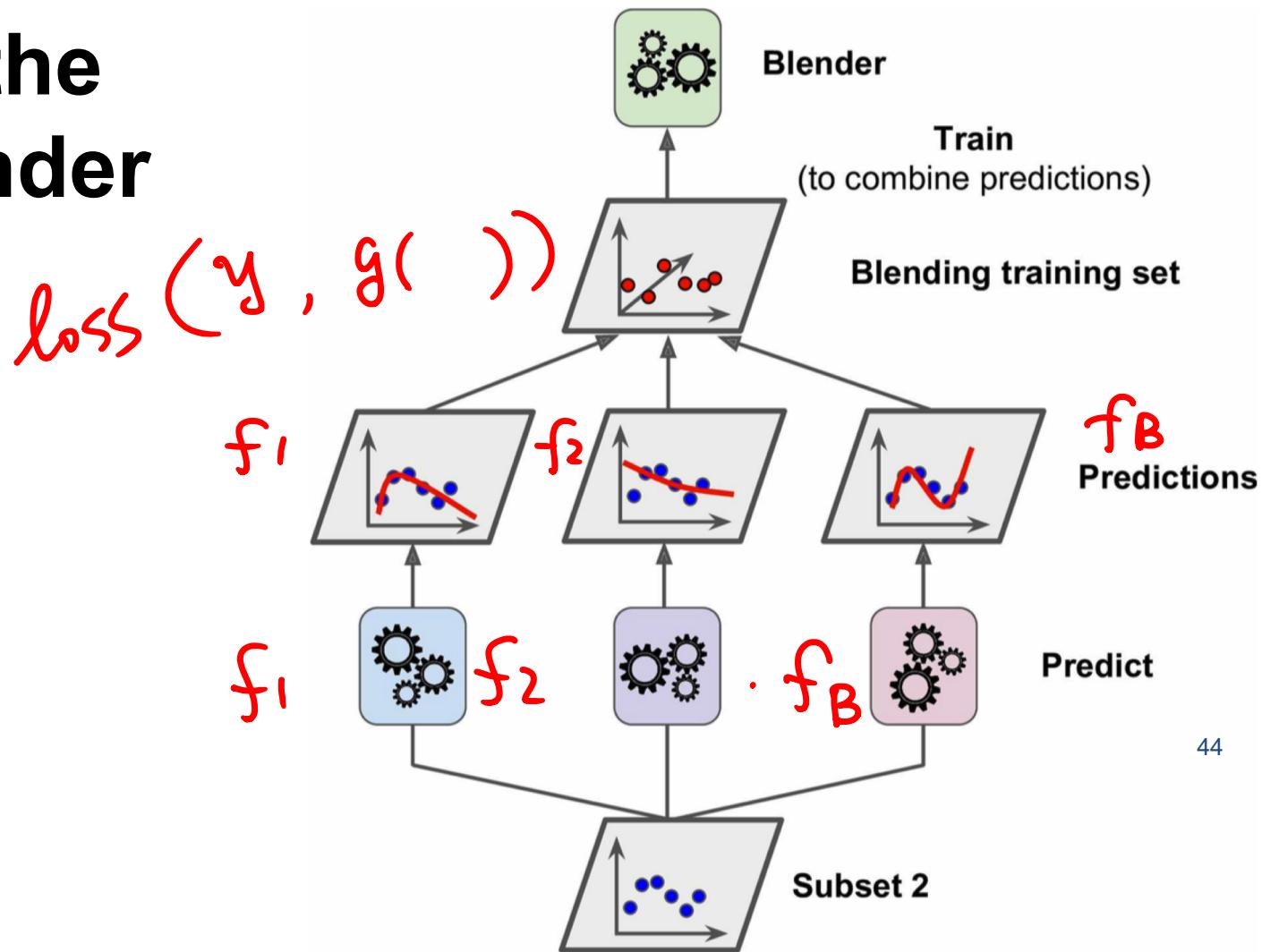
Generating Base and Meta Learners

- **Base model—efficiency, accuracy and diversity**
 - Sampling training examples
 - Sampling features
 - Using different learning models
 - **Meta learner**
 - Majority voting
 - Weighted averaging
 -
 - Higher level classifier — Supervised (e.g. Xgboost as blender)
- 
- Unsupervised

Training the base predictors



Training the meta blender



Today: Ensemble

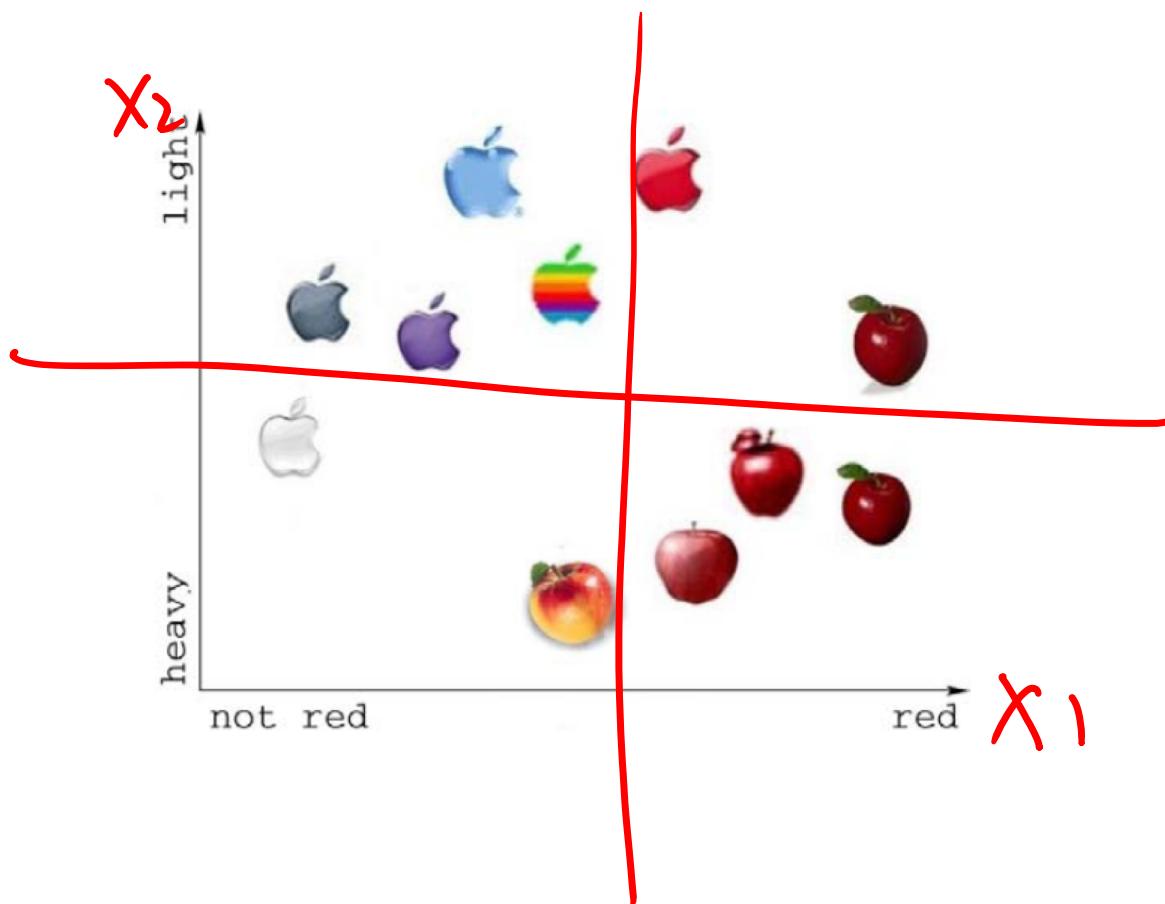
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Boosting Strategies

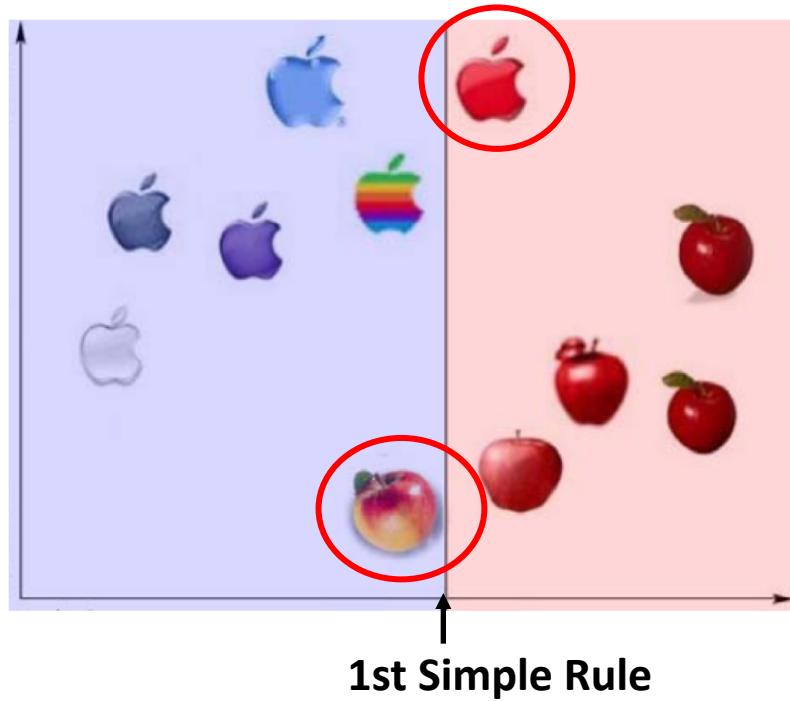
base learners : shallow DT
High bias

1. Have many rules (base classifiers) to **vote** on the decision
2. Sequentially train base classifiers that **corrects** mistakes of previous → focus on **hard** examples
3. Give higher **weight** to better rules

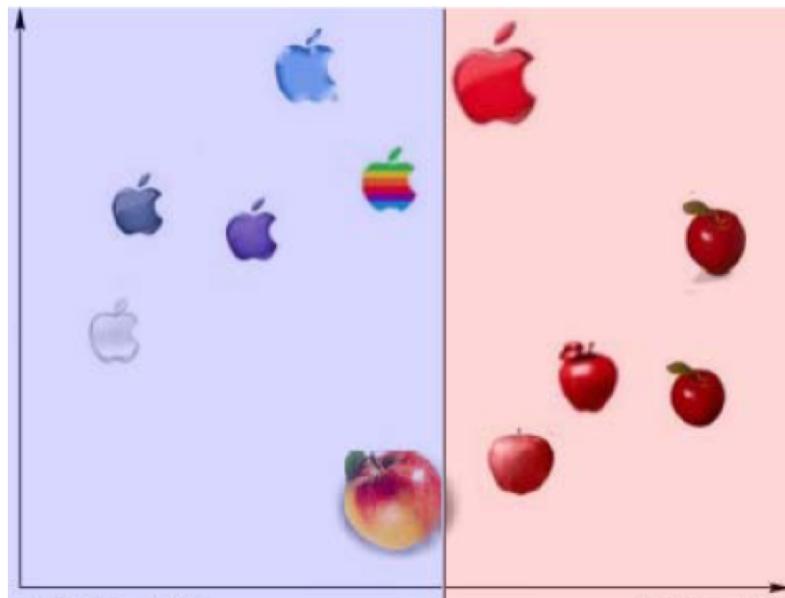
- Recognizing apples:
- (1) Collect a set of real apples and plastic apples
- (2) Observe some rules to tell them apart based on their characteristics



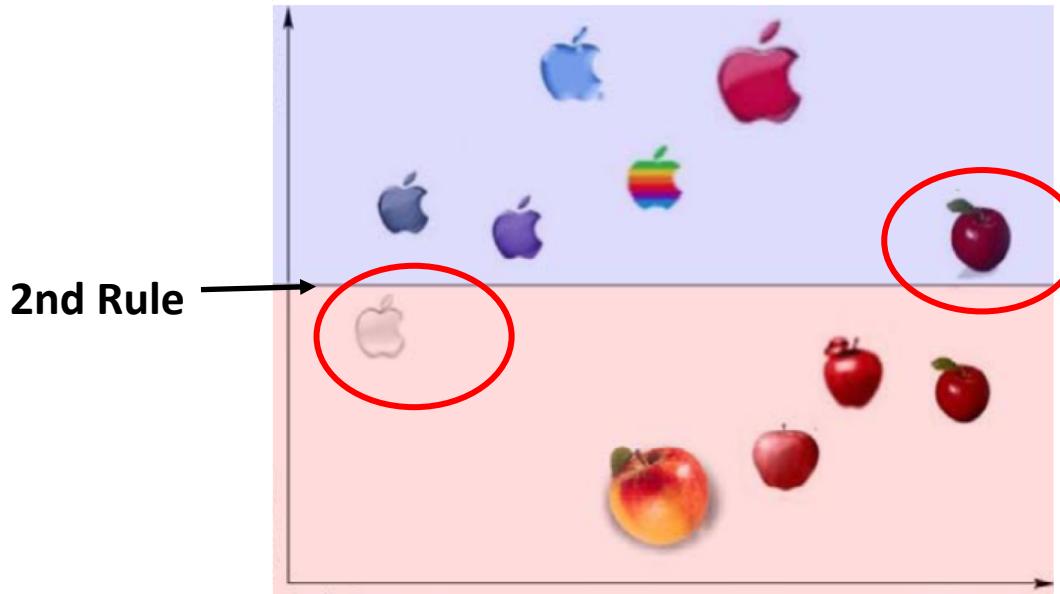
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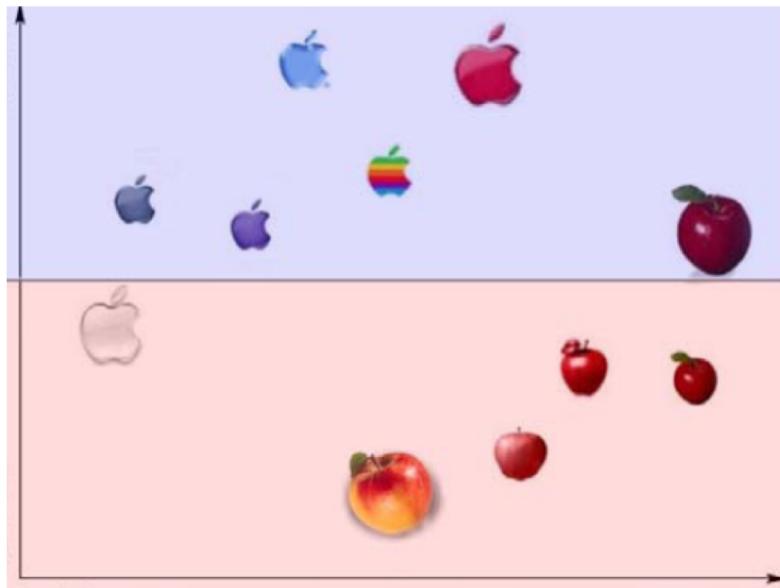
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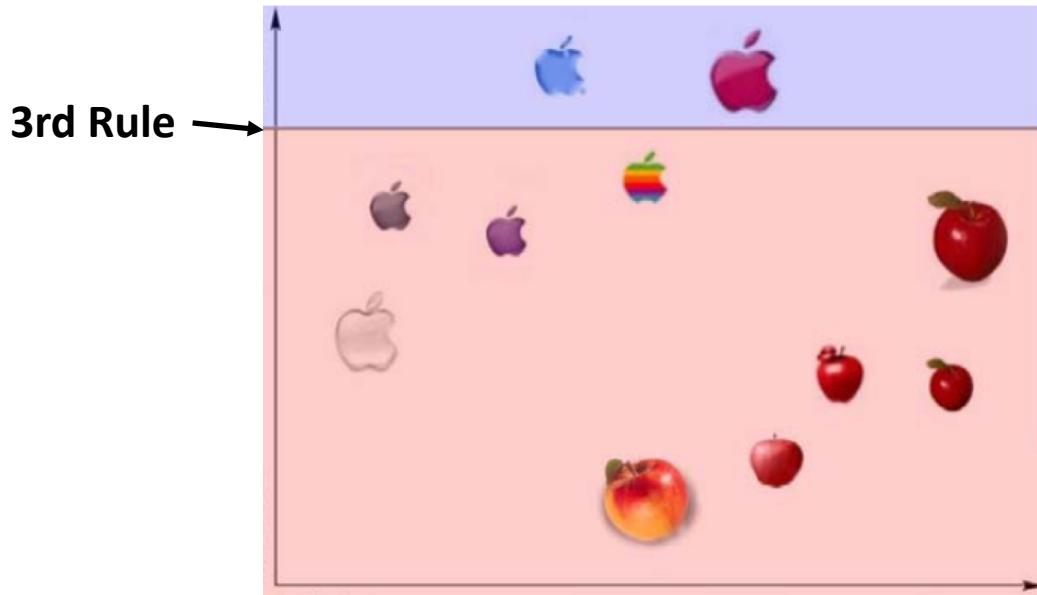
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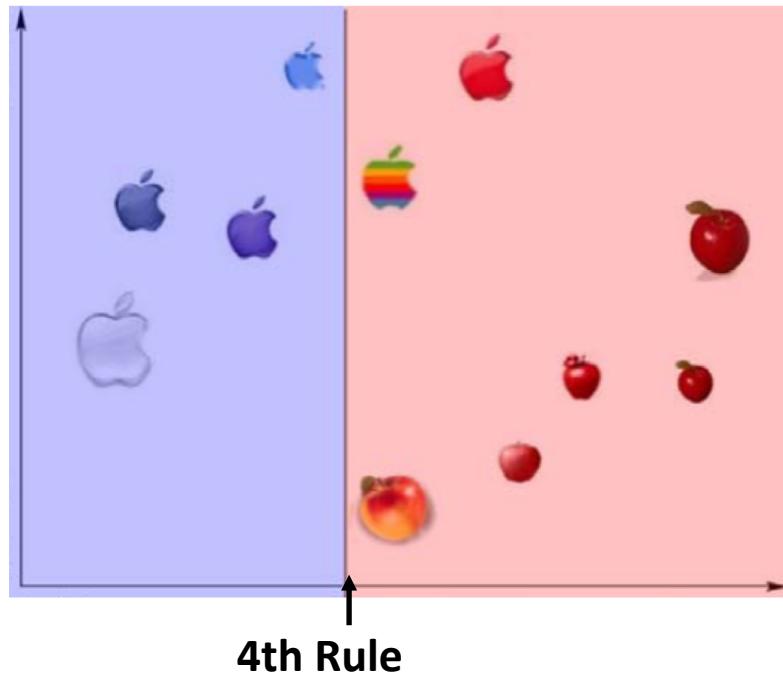
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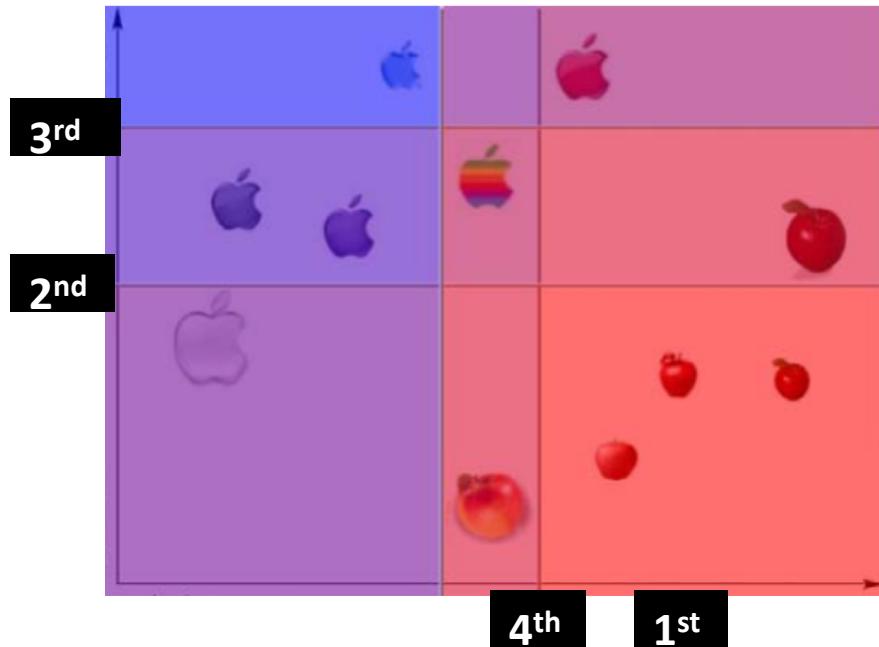


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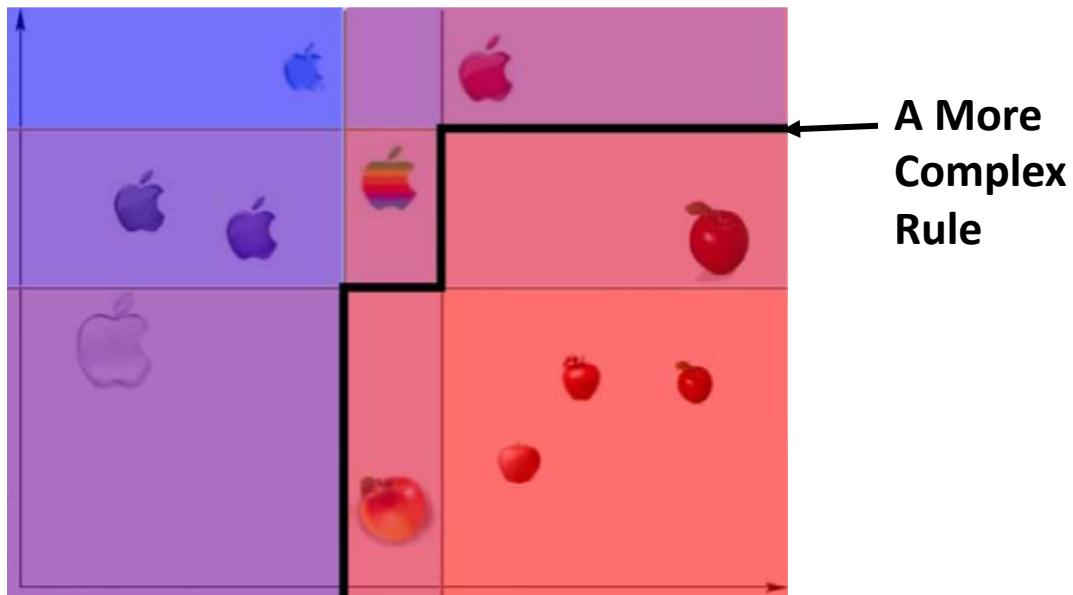


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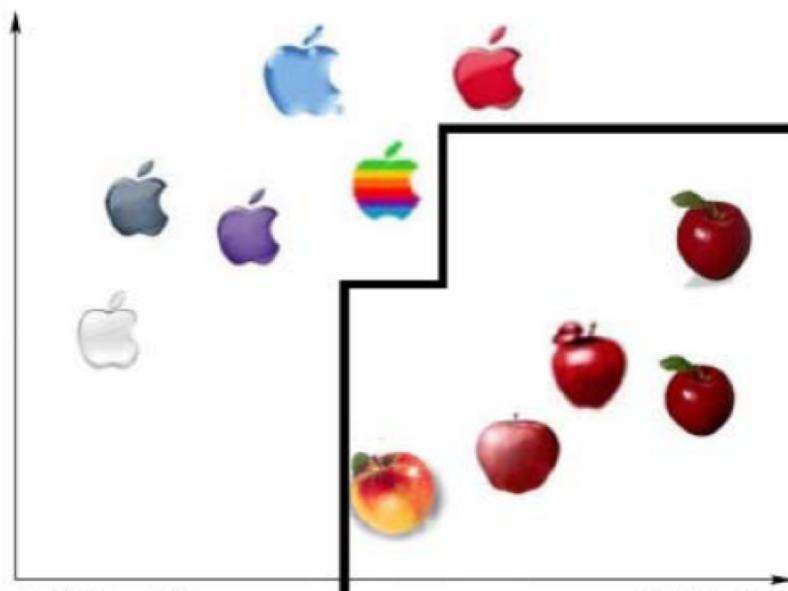
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Final Classifier is the additive combination of base rules:



Adaboost Algorithm (Proposed by Robert Schapire)

Training Data: $\mathbf{D} = \{(\mathbf{x}_i, y_i) | \mathbf{x}_i \in \mathcal{R}^n, y_i \in \{-1, 1\}, 1 \leq i \leq m\}$

Set uniform example weight $w_i, 1 \leq i \leq m$

For $t = 1$ to T iterations:

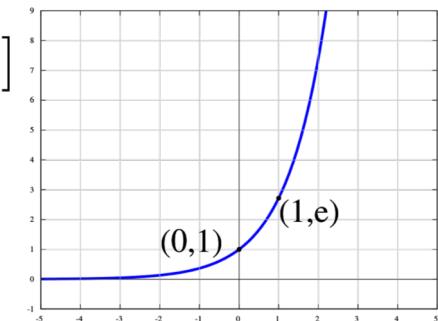
Select a base classifier: $h_t(\mathbf{x}_i) = \arg \min(\epsilon_t)$

$$\epsilon_t = \sum_{i=1}^m w_i [y_i \neq h_t(\mathbf{x}_i)]$$

Set classifier weight: $\alpha_t = \frac{1}{2} \ln \frac{1-\epsilon_t}{\epsilon_t}$

Update example weight: $w_i = w_i e^{-\alpha_t y_i h_t(\mathbf{x}_i)}$

Final Classifier: $\hat{f} = \text{sign} \left(\sum_{t=1}^T \alpha_t h_t(\mathbf{x}_i) \right)$



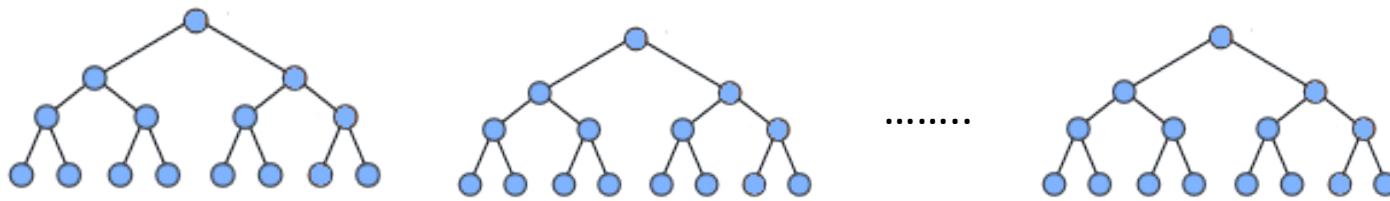
Boosting vs. Bagging

- Similar to bagging, boosting combines a weighted sum of many classifiers, thus **it reduces variance**.
- One key difference: unlike bagging, boosting fit the tree to the entire training set, and adaptively weight the examples.
- Boosting tries to do better at each iteration, thus **it reduces bias**.

XGBoost

- Additive tree model: add new trees that complement the already-built ones
- Response is the optimal linear combination of all decision trees
- Popular in Kaggle Competitions for efficiency and accuracy

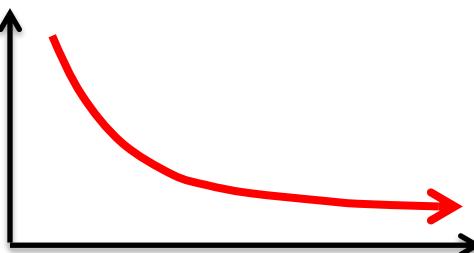
Additive tree model



More in 18c-
extraBoosting
Slides

Greedy Algorithm

Error



Number of Tree

References

- Prof. Tan, Steinbach, Kumar's "Introduction to Data Mining" slide
- ESLbook : Hastie, Trevor, et al. *The elements of statistical learning*. Vol. 2. No. 1. New York: Springer, 2009.
- Dr. Oznur Tastan's slides about RF and DT
- Dr. Rich's slides about boosting