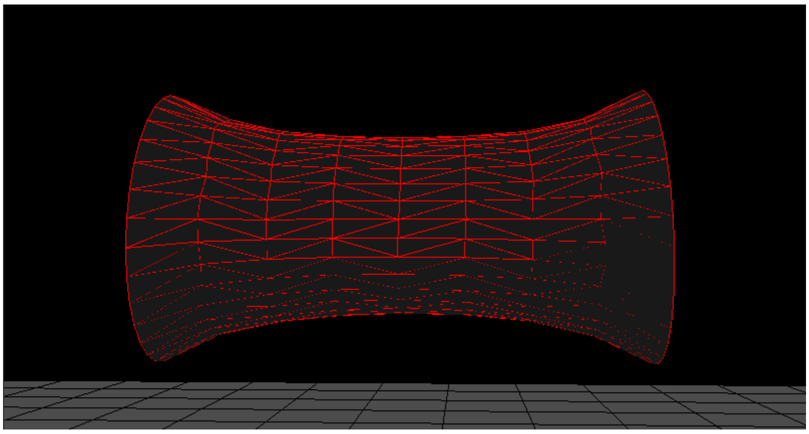
Assignment 3: Hyperelastic deformation



This coursework assignment is worth 20% of your total marks for the course. There is a total of 100 points in this assignment, which are split according to subtasks.

Deadline

The submission deadline for this assignment is 22 April 2022.

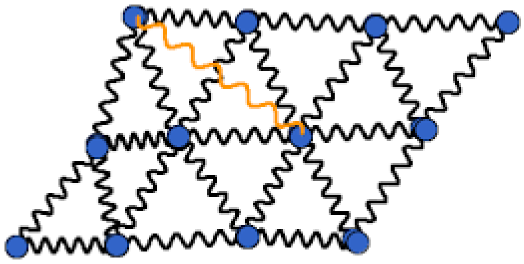
Overview of tasks and requirements

There are two parts to this assignment, theory and practical.

For *theory*, you are required to answer to a total of three questions with each one worth 5 points out of 100.

For the *practical*, you are asked to implement at least four hyperlastic constitutive models (i.e. via the 1st Piola Kirchhoff stress tensor $\mathbf{P}(\mathbf{F})$) and at least three explicit numerical integration schemes of your choice (e.g. symplectic euler etc.). In addition, you will be asked to provide a brief explanation for each choice of constitutive model and integration scheme, emphasising the advantages and disadvantages. It is strongly recommended that you refer to the lecture slides and the referenced literature for your explanations.

Theory (questions)



A mass-spring system (shown above) is described by the following equation:

$$\mathbf{f}_p = \left[k_s \left(\frac{||\mathbf{x}_q - \mathbf{x}_p||}{r} - 1 \right) + k_d \left(\frac{(\mathbf{v}_q - \mathbf{v}_p) \cdot (\mathbf{x}_q - \mathbf{x}_p)}{r||\mathbf{x}_q - \mathbf{x}_p||} \right) \right] \frac{\mathbf{x}_q - \mathbf{x}_p}{||\mathbf{x}_q - \mathbf{x}_p||} \tag{1}$$

1. What exactly does this equation describe? Give details about each term on the right-hand-side and state the *disadvantages* of using a mass-spring system compared to a hyperelastic deformation model using finite elements. [*5 points*]
2. What is the characteristic property of so-called 'hyperelastic' materials and how is this property related to the deformation gradient $\mathbf{F}$? [*5 points*]
3. Neohookean elasticity is one example of a material model that is defined using *isotropic* invariants I_1, I_2 and I_3. Discuss the advantages and disadvantages of defining a material model using these invariants versus using tensors/matrices (i.e. instead of working directly with $\mathbf{F}$ and other matrix quantities derived therefrom). [*5 points*]

Practical (implementation)

For this part, a working implementation of Tutorial 3 can serve a starting point. Thus, having a working implementation of Tutorial 3 will grant you *15 points* toward your assignments (i.e. it must run successfully with at-least one constituve model and numerical integration scheme).

The remainder of the assignment is as follows:

1. Implement a simulator that works on four (interchangeable) constitutive models (i.e. $\mathbf{P}(\mathbf{F}) = \ldots$). [*20 points*]
2. Write a short paragraph per constitutive model discussing its strengths and weaknesses. You may cross-reference multiple constitutive models in your discussion. [*20 points*]
3. Implement and discuss the properties of at-least three (interchangeable) explicit numerical integration schemes of your choice. [*15 points*]

NOTE: The term "interchangeable" simply means being able to be swapped-in and out via program arguments or C++ macros.

Extras

Students may attempt the following extra tasks to obtain extra marks for this assignment.

4. Implement an implicit time integration scheme for FEM using reference [1] as guidance (see Chapter 4.3). [*30 points*]
    1. You are permitted to skip task 3, if you choose to do this task.
5. Write a brief analysis of the contitutive models that you have implemented, evaluating properties such as average fraction of volume change $J = det(\mathbf{F})$ across elements, performance, force magnitude distribution over the entire domain (FEM mesh) etc. (be creative). [*15 marks*]
    1. You must complete task 1 and 2 in order to do this task.
6. Discuss the *disadvantages* of using a finite element solver when compared to a mass-spring system for simulating soft body deformation in 3D. Highlight some practical challenges and theoretical differences between the two approaches. [*5 points*]
7. Clarity of coding with adequate documentation. [*5 points*]

[1] - **SIGGRAPH 2012 Course** FEM Simulation of 3D Deformable Solids: A practitioner's guide to theory, discretization and model reduction. http://www.femdefo.org/

Submission

Submit your assignment to Moodle as a single compressed `.zip` file which contains the following

- The source files of your project, with adequate instructions about running the code.
    - If you have only modified `main.cpp` from the template provided (i.e. via Tutorial 3), then it is sufficient for you to submit only this file together with instructions on how to run (and build) your code.
- A `.txt`, `.pdf` or `.docx` file containing the answers to the theory part of the assignment.
- A `.txt`, `.pdf` or `.docx` file summarising what you have done. You may add any answers provided as part of the "Extras" in this file.