Uncertainty quantification in an oil plume model.

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Introduction

Goal

Assess how input uncertainties of an oil plume model impact its outputs.

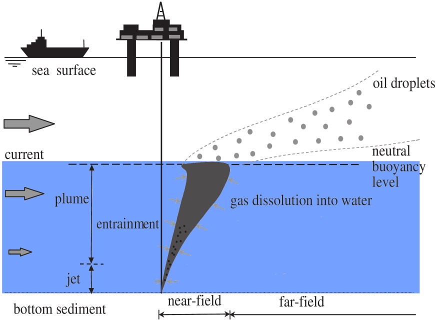

Figure: Modified from Li Zheng et al. 2003

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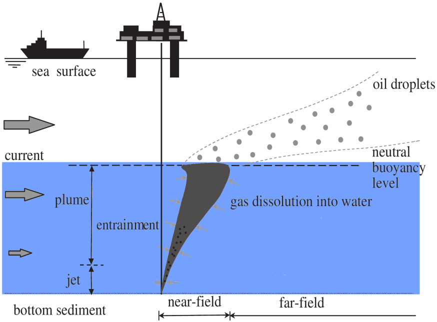
Important outputs from oil plume model

Trap height:
Plume dynamic regime ⇒ Advection and dispersion regime.
Gas mass fluxes:
Control the buoyancy force.



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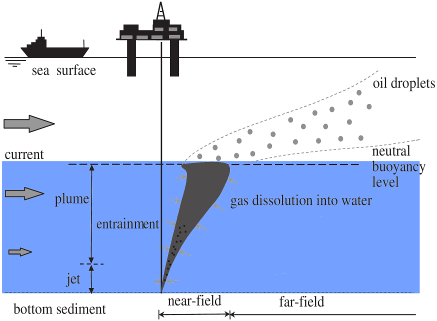
Possible uncertainties of model inputs

Entrainment coefficient: 0.06-0.116. (Bhaumik 2005) Gas to oil ratio (GOR): $1600 ft^3/barrel$, $2470 ft^3/barrel$, $3000 ft^3/barrel$. (Valentine et al. 2010; Reddy et al. 2012)
Other parameters: Oil droplet/gas bubble initial distribution, etc.



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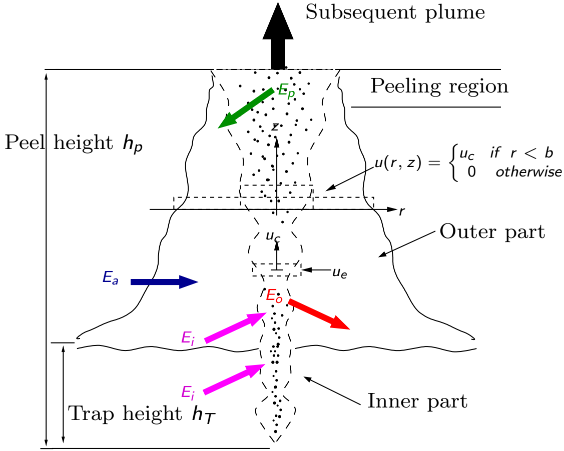
Integral plume model (Socolofsky et al. 2008)

Model descriptions

- Stratification dominated (*DWH*, Camilli et al. 2010, Socolofsky et al. 2011)
- Double-plume approach



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Self similarity assumption

$$Q(z) = 2\pi \int_0^\infty u(r,z)rdr = \pi b^2(z)u(z)$$

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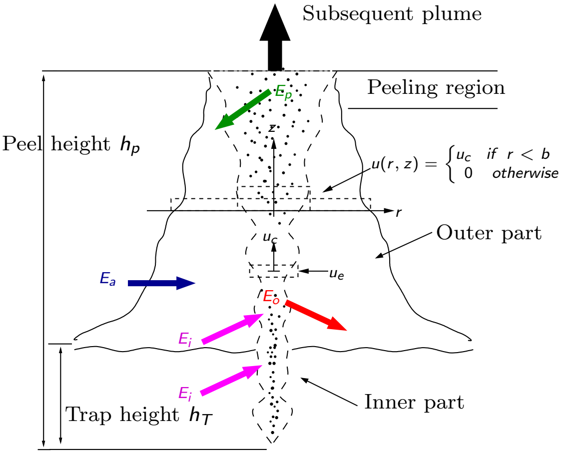
Variables have similar lateral profiles at different plume heights.



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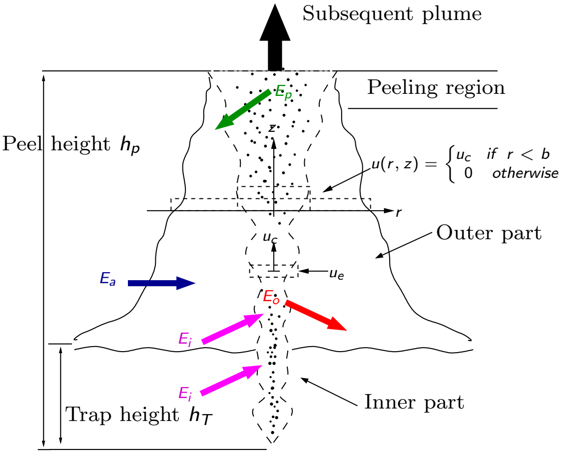
Entrainment hypothesis

$$u_e = \alpha u_c$$

The entrainment velocity u_e is proportional to the central velocity u_c.



Uncertainty Propagation

Main approach:

Construct the probability density function of the model output instead of focusing on single model run.

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Uncertainty inputs

Parameter	Distribution	Random variable
Entrainment coefficient	$U(0.06, 0.116)$	$\xi_1 \sim U(-1, 1)$
Entrainment ratio	$U(0.4, 0.6)$	$\xi_2 \sim U(-1, 1)$
Gas-to-oil ratio (bbl/ft^3)	$U(1400, 3000)$	$\xi_3 \sim U(-1, 1)$
95th percentile of the droplet size (D_{95}) (mm)	$U(1, 10)$	$\xi_4 \sim U(-1, 1)$
Droplet distribution spreading ratio	$U(1.5, 4)$	$\xi_5 \sim U(-1, 1)$

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Model outputs

Trap height, peel height and mass flux of different gas bubble sizes.

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Uncertainty propagation issue

Direct sampling of a 5-dimensional uncertainty space requires a large number of simulations and hinders operational decision making. Can we do better?

Emulator-type methods

Main idea

Indirect sample: Use a small ensemble to build a faithful proxy/surrogate/emulator for the model and use it to estimate the model statistics.

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Emulator methods

- ***Polynomial Chaos Expansion***: Series expansion in uncertain inputs.
  Various approaches to determine coefficients:
  Projection, Regression, Compressive Sensing.
- ***Gaussian Process Regression*** (non-polynomial approach).

Both techniques are ensemble based and we can build a faithful surrogate with as little as 50 realizations. Most importantly we can TEST the approximation properties.

Response surface in 1D: emulator-type methods v.s. model simulation

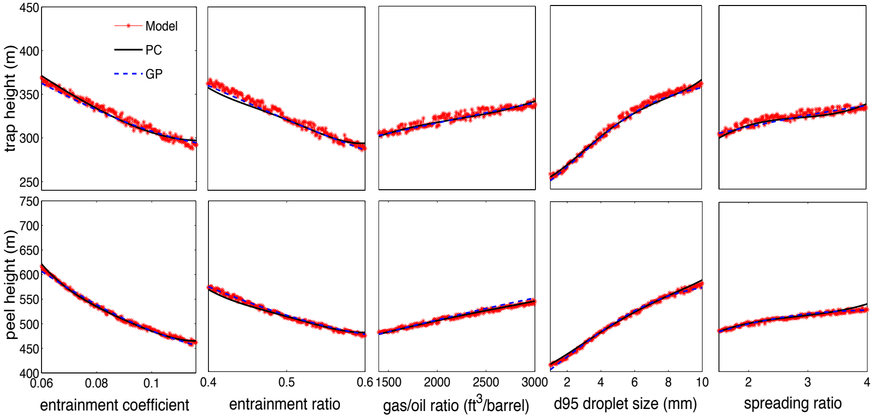
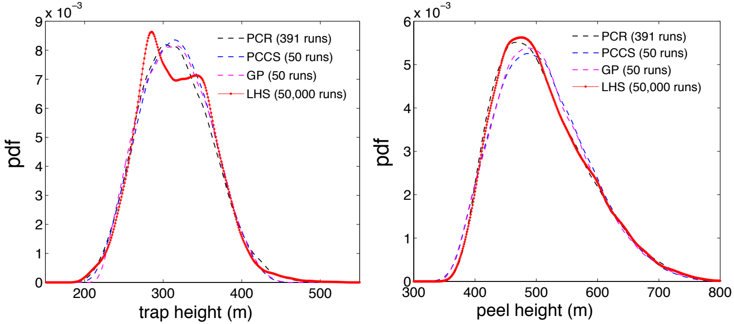


Figure: 1D comparision of emulator-type methods and model simulation (200 samples in each dimension). Red star is **direct sampling** from the model orthers are **indirect sampling** from emulator-type methods (PC in black curve and GP in blue dash).

PDF comparison

Numerical experiments:
- 50,000 Latin Hypercube Sampling to build reference statistics.
- Ensemble of 391 to build Polynomial Chaos series with regression.
- Faithful ensemble can be built using as little as 50 samples using compressed sensing or Gaussian Process techniques.



Trap height PDF

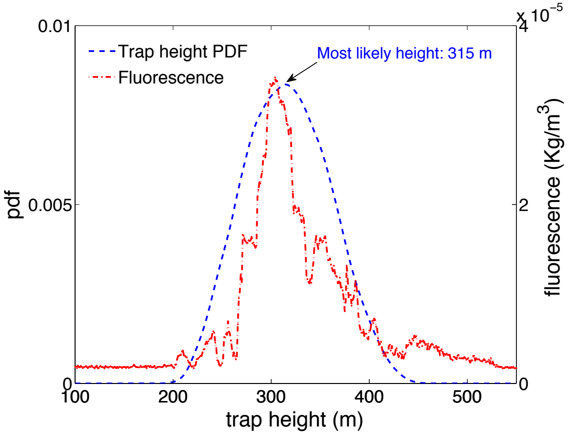


Figure: Trap height PDF produced by PC emulator with 100,000 samples, Fluorescence measurement, a proxy for oil concentration. Real computational cost: **50** instead of 100,000 model simualtions.

Gas mass flux PDF

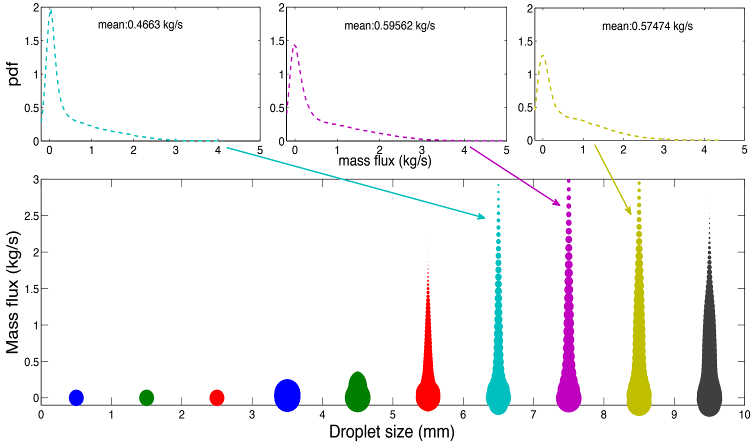


Figure: Mass flux PDF at the trap height for different gas bubble sizes.
Input for Lagrangian model - prediction with uncertainty.

Sensitivity index

Global sensitivity analysis aims to quantify the contribution of different random input variables to the model variability.

$$Sensitivity\ index = \frac{Variance\ of\ \xi_i}{Total\ Variance}$$

Parameter	Distribution	SI trap	SI peel
Entrainment coefficient	$U(0.06, 0.116)$	0.1981	0.2926
Entrainment ratio	$U(0.4, 0.6)$	0.1985	0.1466
Gas-to-oil ratio	$U(1400, 3000)$	0.0424	0.0706
95th percentile of the droplet size (D_{95})	$U(1, 10)$	0.5334	0.4917
Droplet distribution spreading ratio	$U(1.5, 4)$	0.0389	0.0384

Table: Total sensitivity indices associated with different random variables

Summary & future work

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- Propagating uncertainties through the plume model provides statistical characteristics of quantities of interest.
- The emulator-type methods agree well with the direct sampling method in the plume model.
- Sensitivity analysis points out the key parameter in the model

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Future work

- Propagating the output pdf from the plume model into a Lagrangian particle tracking model.
- Identify observational data to perform an inverse uncertainty propagation and to correct input uncertainties

Thank you ! Questions?

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