No.4: Uncertainty quantification in an oil plume model.

Shitao Wang, Mohamed Iskandarani, Ashwanth Srinivasan
Carlisle Thacker, J. Winokur and Omar Knio

Rosenstiel School of Marine and Atmospheric Science
Meteorology and Physical Oceanography

Introduction

Uncertainties

Flow rates: 5,000 barrels/day (first released report); 50,000-70,000 barrels/day (McNutt et al. 2012)
Gas to oil ratio (GOR): $1600ft^3/barrel$, $2470ft^3/barrel$, $3000ft^3/barrel$ (Valentine et al. 2010; Reddy et al. 2012)
Other parameters: Oil droplet/gas bubble initial distribution, entrainment coefficients, etc.

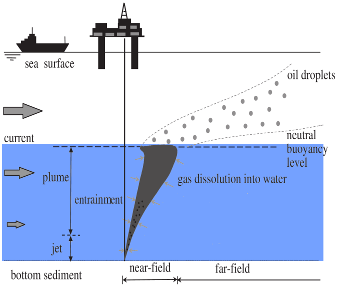

Figure : Modified from Li Zheng et al. 2003

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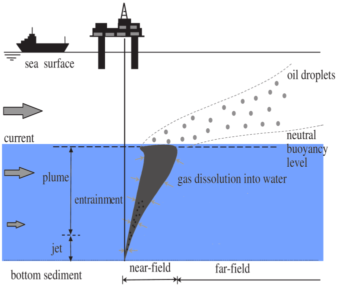

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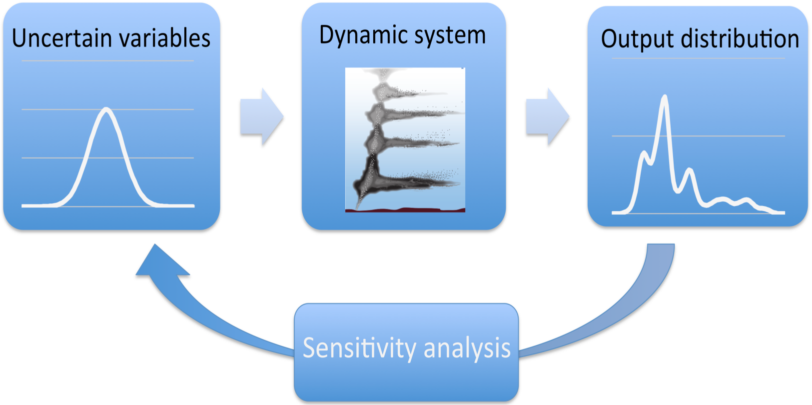
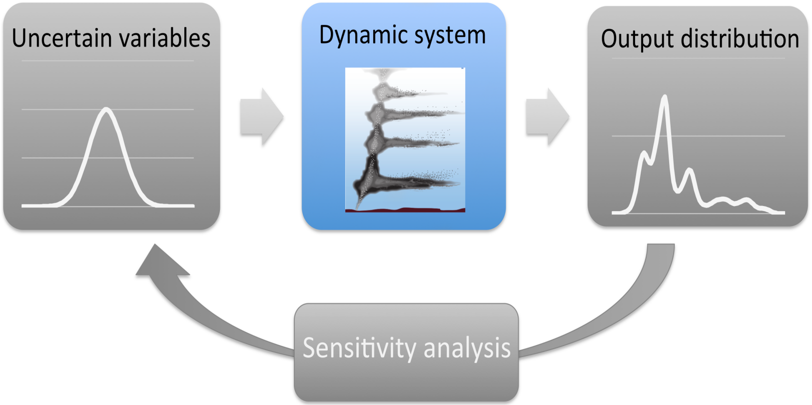
Goal

Assess how uncertainties of inputs of an oil plume model manifest in its outputs.

Uncertainty propagation



Uncertainty propagation: model & methods



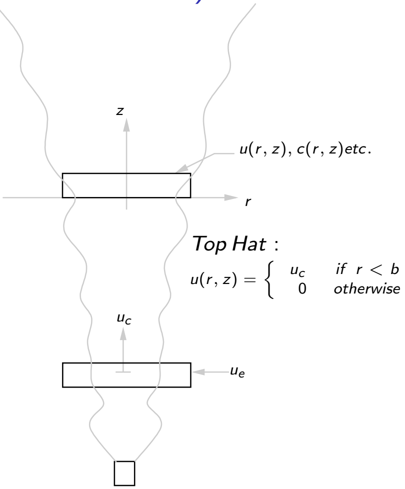
Integral plume model (Socolofsky et al. 2008)

Self similarity assumption

$$Q(z) = 2\pi \int_0^\infty u(r,z) r dr = \pi b^2(z) u(z)$$

$$M(z) = 2\pi \int_0^\infty u^2(r,z) r dr = \pi b^2 u^2$$

Variables have similar lateral profiles at different plume heights.



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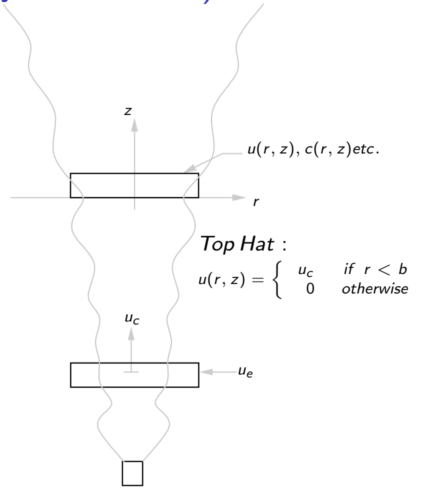
Entrainment hypothesis

$$u_e = \alpha u_c$$

The entrainment velocity u_e is proportional to the central velocity u_c.



Integral plume model (Socolofsky et al. 2008)

Model descriptions

- Stratification dominated (*DWH*, Camilli et al. 2010, Socolofsky et al. 2011)
- Double-plume approach

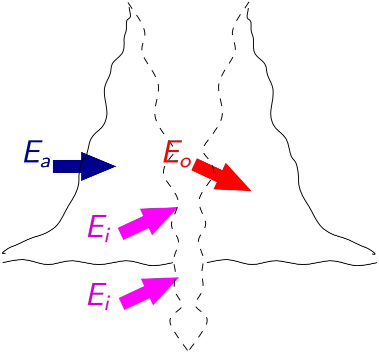


Figure : Entrainment processes using double-plume approach

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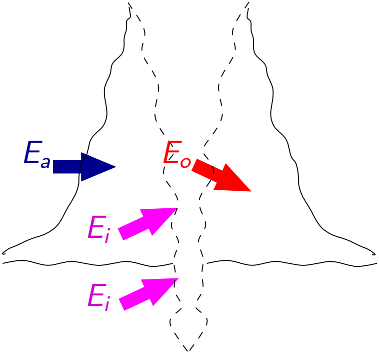
Primary processes

- Buoyancy force
- Turbulent entrainment
- Buoyant detrainment
- Dissolution of the gas bubbles

(Crounse et al. 2007)



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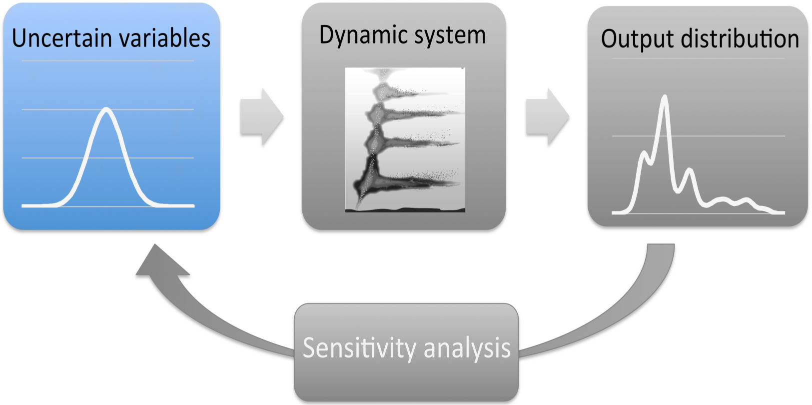
Uncertainty propagation: uncertain variables



Uncertainty parameters

Parameter	Distribution	Random variable
Entrainment coefficient	$U(0.06, 0.116)$	$\xi_1 \sim U(-1, 1)$
Entrainment ratio	$U(0.4, 0.6)$	$\xi_2 \sim U(-1, 1)$
Gas-to-oil ratio	$U(1600, 3000)$	$\xi_3 \sim U(-1, 1)$
95th percentile of the droplet/bubble size (D_{95})	$U(7, 10)$	$\xi_4 \sim U(-1, 1)$
Droplet distribution spreading ratio	$U(1.8, 4)$	$\xi_5 \sim U(-1, 1)$

Quantities of interest

Trap height, peel height and mass flux of different gas bubble sizes.

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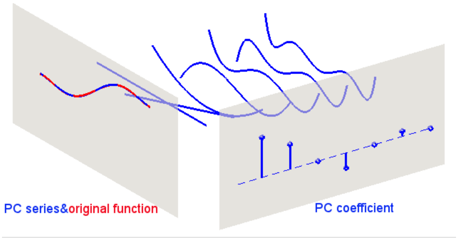
Uncertainty propagation

Monte-Carlo method requires over tens of thousands of simulations, how to speed up?

Polynomial Chaos Expansion (PCE)

Polynomial chaos (Wiener 1938)

$y(\xi) \approx y_P = \sum_{k=0}^{P} \hat{y}_k \psi_k(\xi)$ is a spectral type expansion in the uncertain variables ξ. Once the coefficients $\hat{y}_k$ are determined, the series can be analyzed for probabilistic and approximation information.



PC series with Legendre polynomials

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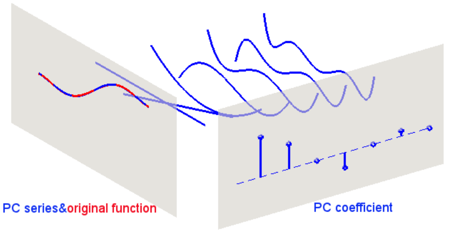

PC series with Legendre polynomials

Determine the coefficients

Non-intrusive Galerkin projection (integrals) in ξ-space:

$$\hat{\mathbf{y}}_\mathbf{k} = \frac{\langle y(\boldsymbol{\xi}), \psi_k(\boldsymbol{\xi})\rangle}{\langle \psi_k^2(\boldsymbol{\xi})\rangle} \approx \frac{\sum_{q=1}^{Q} y(\boldsymbol{\xi}_q)\psi_k(\boldsymbol{\xi}_q)\mathbf{w}_q}{\langle \psi_k^2(\boldsymbol{\xi})\rangle}$$

Gaussian Process Regression (GPR)

Gaussian Process Regression

$$\mathcal{M}(x) \approx \mathcal{M}^{(\mathcal{GP})}(x) = \beta^T \cdot f(x) + \mathcal{GP}(0, k(x, x'; \theta))$$

$\mathcal{GP}(0, k(x, x'; \theta))$ is a zero mean Gaussian process, $\beta^T \cdot f(x)$ is the mean value of the Gaussian process, β is a set of coefficients and $f(x)$ is a set of basis functions for the mean function.

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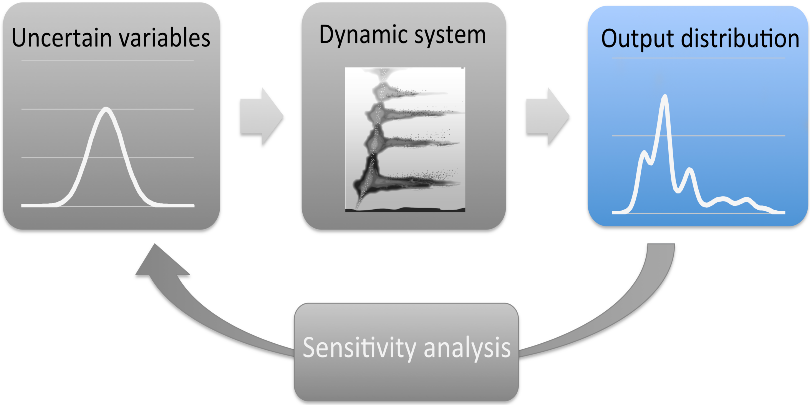
Determine the GPR emulator

The hyper-parameters, θ, in the covariance function $k(x, x'; \theta)$ and the β need to be estimated from a training set via maximum likelihood method.

Uncertainty propagation: output statistics



Probability density function (PDF) comparison

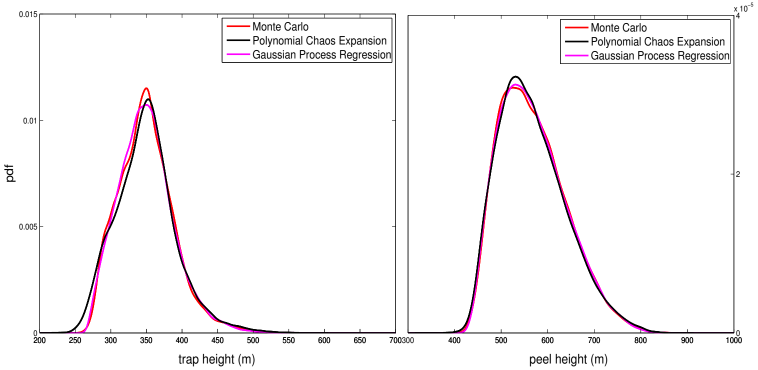


Figure : Monte-Carlo method needs 50,000 model simulations (about 30,000 CPU hours). Emulator-type methods need less than 1,000 model simulations, which decrease computation time by a factor of 50.

Fluorescence

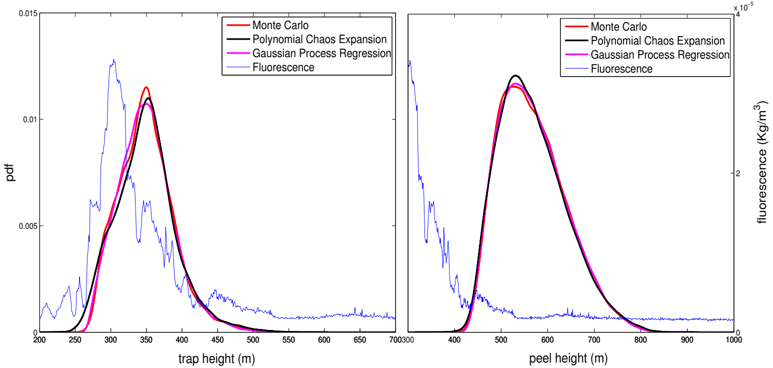


Figure : Fluorescence intensity (a proxy for oil concentration) in blue from the R/V Brooks McCall on May 30, 2010 (Socolofsky 2011).

Gas bubble distribution

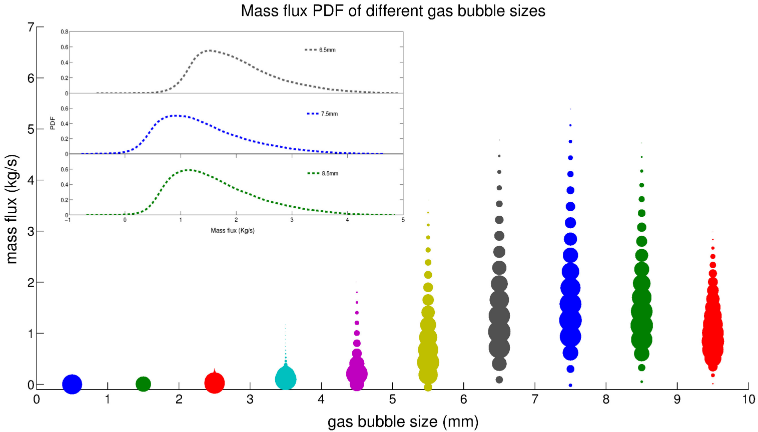
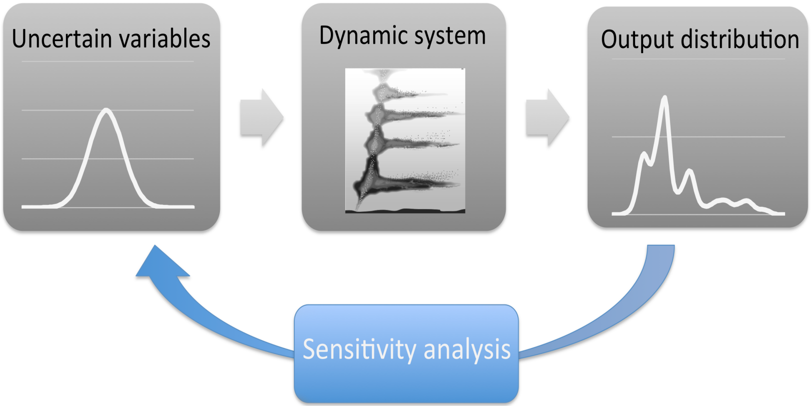


Figure : Mass flux PDF at the trap height for different gas bubble sizes.

Input for Lagrangian model - prediction with uncertainty

Uncertainty propagation: Global sensitivity analysis



Sensitivity index

Global sensitivity analysis aims to quantify the contribution of different random input variables to the model variability.

$$Sensitivity\ \ index = \frac{Variance\ \ of\ \ \xi_i}{Total\ \ Variance}$$

Parameter	Distribution	Sensitivity index
Entrainment coefficient	$U(0.06, 0.116)$	0.4191
Entrainment ratio	$U(0.4, 0.6)$	0.4394
Gas-to-oil ratio	$U(1600, 3000)$	0.1280
95th percentile of the droplet/bubble size (D_{95})	$U(7, 10)$	0.0699
Droplet distribution spreading ratio	$U(1.8, 4)$	0.0801

Table : Total sensitivity indices associated with different random variables

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Summary & future work

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- Propagating uncertainties through the plume model provides statistical characteristics of quantities of interest.
- The emulator-type methods agree very well with the Monte-Carlo set in the plume model.
- Sensitivity analysis points out the key parameter in the model

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- Sensitivity analysis points out the key parameter in the model

Future work

- Expand the input uncertain space to include additional sources of uncertainties.
- Identify observational data to perform an inverse uncertainty propagation and to correct input uncertainties

Thank you !

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