Thapar University, Patiala

School of Mathematics

UMA003: Mathematics-I, (Tutorial Sheet 01)

(1) Find the absolute maximum and absolute minimum values of each function on the given interval.

(a) $f(x) = \frac{2}{3}x - 5,\ [-2, -3]$ (b) $f(x) = 4 - x^2,\ [-3, 1]$

(c) $f(x) = -\sqrt{5 - x^2},\ [-\sqrt{5}, 0]$ (d) $f(x) = 2 - |x|,\ [-1, 3]$

(2) The function $f(x) = |x|$ has an absolute minimum at $x = 0$ even though f is not differentiable at $x = 0$. Is this consistent with First derivative theorem for local extreme values?

(3) If an odd function $f(x)$ has a local minimum value at $x = c$, can anything be said about the value of $f(x)$ at $x = -c$? What happens when function is even?

(4) Find the values of b and c, if Rolle's theorem is applicable to the function $f(x) = x^3 + bx^2 + cx,\ 1 \leq x \leq 2$ at $x = 4/3$.

(5) Prove that a cubic polynomial can have atmost three real roots.

(6) Show that at some instant during a two hour auto mobile trip the cars speedometer will equal the average speed for the trip.

(7) It took 14 second for the thermometer to rise from $-19°C$ to $100°C$ when it was taken from a freezer and placed in a boiling water. Show that somewhere along the way the mercury was rising at exactly $8.5°C$ per second.

(8) Show that $|\sin b - \sin a| \leq |b - a|$ for any numbers a and b.

(9) If f is differentiable on $[0, 1]$ and its derivative is never zero, then show that $f(0) \neq f(1)$.

(10) Graph the following curves by including local maxima, local minima, point of inflection and cusp(if any).

(a) $y = x^2 - 4x + 3$ (b) $y = x^3 - 3x + 3$ (c) $y = -2x^3 + 6x^2 - 3$ (d) $y = x^4 - 2x^2$

(e) $y = x^5 - 5x^4$ (f) $y = x^{\frac{1}{5}}$ (g) $y = x^{\frac{2}{3}}(\frac{5}{2} - x)$ (h) $y = x^{\frac{4}{5}}$ (i) $y = |x^2 - 1|$

(11) Graph the following functions including asymptotes and dominant terms.

(a) $y = \frac{1}{x-1}$ (b) $y = \frac{1}{x+1}$ (c) $y = \frac{1}{2x+4}$ (d) $y = \frac{2x^2+x-1}{x^2-1}$

(e) $y = \frac{x^2-1}{x}$ (f) $y = \frac{x^3+1}{x^2}$ (g) $y = \frac{x^4+1}{x^2}$ (h) $y = \frac{1}{x^2-1}$

(i) $y = -\frac{x^2-4}{x+1}$ (j) $y = \frac{x^2-x+1}{x+1}$ (k) $y = \frac{x^3+x-2}{x-x^2}$