Thapar University, Patiala

School of Mathematics

UMA003: Mathematics-I,Tutorial Sheet 03

(1) Find a formula for the nth term of the following sequences:

(a) $1, -1, 1, -1, 1, ...$ (b) $1, 4, 9, 16, ...$

(c) $1, -4, 9, -16, 25, ...$ (d) $-3, -2, -1, 0, 1, ...$

(e) $1, 0, 1, 0, 1, ...$ (f) $0, 1, 1, 2, 2, 3, 3, 4, 4, ...$

(2) Determine which of the sequences are nondecreasing and bounded from above?

(a) $a_n = \frac{3n+1}{n+1}$ (b) $a_n = \frac{(2n+3)!}{(n+1)!}$

(c) $a_n = \frac{2^n 3^n}{n!}$ (d) $a_n = 2 - \frac{2}{n} - \frac{1}{2^n}$

(3) Is it true that a sequence $\{a_n\}$ of positive numbers must converge if it is bounded from above ? Give reason for your answer.

(4) Determine which of the sequences converge and which diverge? Give reason for your answer. Find the limit of each convergent sequence.

(a) $a_n = 1 - \frac{1}{n}$ (b) $a_n = n - \frac{1}{n}$

(c) $a_n = \frac{2^n - 1}{2^n}$ (d) $a_n = ((-1)^n + 1)\left(\frac{n+1}{n}\right)$

(e) $a_n = 2 + (0.1)^n$ (f) $a_n = \frac{2^n - 1}{2^n}$

(g) $a_n = \frac{\ln n}{\ln 2n}$ (h) $a_n = \left(\frac{3n+1}{3n-1}\right)^n$

(i) $a_n = \left(1 - \frac{1}{n^2}\right)^n$ (j) $a_n = \tan^{-1} n$

(k) $a_n = n - \sqrt{n^2 - n}$ (l) $a_n = \frac{1}{\sqrt{n^2-1} - \sqrt{n^2+n}}$

(5) Find the sum of the following series if converges.

(a) $1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + ... + (-1)^{n-1}\dfrac{1}{2^{n-1}} + ...$

(b) $\dfrac{1}{2 \times 3} + \dfrac{1}{3 \times 4} + \dfrac{1}{4 \times 5} + ... + \dfrac{1}{(n+1)(n+2)} + ...$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

(6) Find the value of b for which $1 + e^b + e^{2b} + e^{3b} + ... = 9$.

(7) For what values of r does the infinite series

$$1 + 2r + r^2 + 2r^3 + r^4 + 2r^5 + r^6 + ...$$

converge? Find the sum of the series when it converges.

(8) Give an example of two divergent infinite series whose term-by-term sum converges.