School of Mathematics

Thapar University, Patiala

UMA003: Mathematics-I, (Tutorial Sheet 06)

(1) For what values of x does the series converge absolutely and conditionally? Also, find the series radius of convergence and interval of convergence.

(a) $\sum_{n=1}^{\infty} \frac{nx^n}{n+2}$

(b) $\sum_{n=1}^{\infty} \frac{x^{2n+1}}{n!}$

(c) $\sum_{n=1}^{\infty} \frac{(2x+3)^{2n+1}}{n!}$

(d) $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$

(e) $\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$

(f) $\sum_{n=1}^{\infty} \frac{(x-\sqrt{2})^{2n+1}}{2^n}$

(2) Find the Taylor series generated by the following functions at $x = a$:

(a) $f(x) = x^3 - 2x + 4, \ a = 2$

(b) $f(x) = e^x \ at \ a = 2$

(c) $f(x) = \frac{x}{1-x}, \ a = 0$

(d) $f(x) = \frac{1}{x^2} \ at \ a = 1$

(3) Find Maclaurin series for the following functions:

(a) $f(x) = xe^x$

(b) $f(x) = x^2 \sin x$

(c) $f(x) = \cos^2 x$

(d) $f(x) = x \ln(1+2x)$

(4) For approximately what values of x, $\sin x$ can be replaced by $x - \frac{x^3}{3!}$ with an error of magnitude no greater than 5×10^{-4}? Give reason for your answer.

(5) When $0 \leq h \leq 0.01$, show that e^h may be replaced by $1 + h$ with an error of magnitude no greater than 0.6% of h. (use $e^{0.001} = 1.01$)

(6) Estimate the error in the approximation $\sinh x = x + \frac{x^3}{3!}$, when $|x| < 0.5$.