School of Mathematics
Thapar University, Patiala
UMA003, Tutorial Sheet 09, Multiple integrals

1. Evaluate the following double integral.

   (a) $\int_1^{10}\int_0^{1/y} ye^{xy}\ dx\ dy$ (b) $\int_0^1\int_0^{x^3} e^{y/x}\ dy\ dx$ (c) $\int_0^1\int_{\sqrt{y}}^{2-\sqrt{y}} xy\ dx\ dy$ (d) $\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}\ dx\ dy$

2. Sketch the region of integration and evaluate the double integrals by reversing the order of integration.

   (a) $\int_0^4\int_{-\sqrt{4-y}}^{(y-4)/2} dx\ dy$ (b) $\int_0^1\int_{x^2}^{x} \sqrt{x}\ dy\ dx$ (c) $\int_0^1\int_1^{e^x} dy\ dx$

3. Find the area of the region enclosed by the line $y = 2x + 4$ and the parabola $y = 4 - x^2$ in the xy-plane.

4. Find the area of the triangular region in the xy-plane that is bounded on the right by the parabola $y = x^2$, on the left by the line $x + y = 2$ and above the line $y = 4$.

5. Find the volume under the paraboloid $z = x^2 + y^2$ above the triangle enclosed by the lines $y = x$, $x = 0$ and $x + y = 2$ in the xy-plane.

6. Find the volume under the parabolic cylinder $z = x^2$ above the region enclosed by the parabola $y = 6 - x^2$ and the line $x = y$ in the xy-plane.

7. Change the cartesian integral into an equivalent Polar integral. Then evaluate the following polar integrals:

   (a) $\int_{-1}^{1}\int_0^{\sqrt{1-x^2}} dy\ dx$ (b) $\int_0^6\int_0^y x\ dxdy$ (c) $\int_0^{\ln 2}\int_0^{\sqrt{(\ln 2)^2-y^2}} e^{\sqrt{x^2+y^2}}\ dx\ dy$ (d) $\int_0^2\int_0^x y\ dy\ dx$

8. Find the area of the region cut from the first quadrant by the cardioid $r = 1 + \sin\theta$.

9. Find the area of the region common to the interiors of the cardioids $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.

10. Evaluate the following triple integrals:

    (a) $\int_0^1\int_0^\pi\int_0^\pi y\sin z\ dx\ dy\ dz$ (b) $\int_0^1\int_0^{2-x}\int_0^{2-x-y} dz\ dy\ dx$

Answer:

1. (a) $9e - 9$ (b) $(e-2)/2$ (c) $1/5$ (d) $8\ln 8 - 16 + e$

2. (a) $4/3$ (b) $4/35$ (c) $e - 2$

3. $4/3$ **4.** $37/6$ **5.** $4/3$ **6.** $125/4$

7. (a) $\pi/2$ (b) 36 (c) $\frac{\pi}{2}(2\ln 2 - 1)$ (d) $4/3$

8. $\frac{3\pi}{8} + 1$ **9.** $\frac{3\pi}{2} - 4$ **10.** (a) $\frac{\pi^3}{2}(1 - \cos 1)$ (b) $\frac{7}{6}$