(1) Test which of the following alternating series converge/diverge? Give reason for your answers.

   (a) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n^2}$

   (b) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n^{3/2}}$

   (c) $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \left(\dfrac{n}{10}\right)^n$

   (d) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}\ln n}{n}$

   (e) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}(\sqrt{n}+1)}{n+1}$

(2) Which of the following series converge absolutely and conditionally? Give reason for your answers.

   (a) $\sum\limits_{n=1}^{\infty} (-1)^{n+1}(0.1)^n$

   (b) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{\sqrt{n}}$

   (c) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n \sin n}{n^2}$

   (d) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n \ln n}{n - \ln n}$

   (e) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n\,(2n)!}{2^n\, n!\, n}$

   (f) $\sum\limits_{n=1}^{\infty} (-1)^n(\sqrt{n+\sqrt{n}} - \sqrt{n})$

(3) Estimate the magnitude of the error involved in using the sum of first four terms to approximate the sum of the entire series.

   (a) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n}$

   (b) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{10^n}$

(4) If $\sum\limits_{n=1}^{\infty} a_n$ and $\sum\limits_{n=1}^{\infty} b_n$ are convergent series of non-negative numbers, can any thing be said about $\sum\limits_{n=1}^{\infty} a_n b_n$? Give reason for your answer.

(5) Show that if $\sum\limits_{n=1}^{\infty} a_n$ converges, then $\sum\limits_{n=1}^{\infty} \left(\dfrac{a_n}{1 - a_n}\right)$ converges.