School of Mathematics
Thapar University, Patiala,
UMA 003: Tutorial Sheet 07: Partial Derivatives

1. Show that the following limits do not exist as $(x, y) \longrightarrow (0, 0)$.

   (i) $\lim \dfrac{x-2y}{x+y}$ (ii) $\lim \dfrac{xy^3}{x^2+y^6}$ (iii) $\lim \dfrac{xy^2}{x^2+y^4}$ (iv) $\lim \dfrac{xy^2}{x^2+y^2}$.

2. If $z = f(x, y)$, where $x = e^u \cos v$, $y = e^u \sin v$. Show that $x\dfrac{\partial f}{\partial v} + y\dfrac{\partial f}{\partial u} = e^{2u}\dfrac{\partial f}{\partial y}$

3. If $z = \log(u^2 + v)$; $u = e^{x+y^2}$, find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

4. If $H = f(x, y, z)$ where $x = u + v + w, y = vw + wu + uv$ and $z = uvw$, show that

$$u\frac{\partial H}{\partial u} + v\frac{\partial H}{\partial v} + w\frac{\partial H}{\partial w} = x\frac{\partial H}{\partial x} + 2y\frac{\partial H}{\partial y} + 3z\frac{\partial H}{\partial z}$$

5. Find the value of $\partial x/\partial z$ at the point $(1, -1, -3)$ if the equation $xz + y \ln x - x^2 + 4 = 0$ defines x as a function of the two independent variables y and z and the partial derivatives exists.

6. Find f_x, f_y, f_z for the functions: (i) $f(x, y, z) = x - \sqrt{y^2 + z^2}$ (ii) $f(x, y, z) = \sin^{-1}(xyz)$ (iii) $f(x, y, z) = e^{-(x^2+y^2+z^2)}$

7. Evaluate $\partial u/\partial x$, $\partial u/\partial y$ at the given point (x, y, z) for the following functions.

   (i) $u = \frac{p-q}{q-r}$, $p = x + y + z$, $q = x - y + z$, $r = x + y - z$; $(x, y, z) = (\sqrt{3}, 2, 1)$.

   (ii) $u = e^{qr} \sin^{-1} p$, $p = \sin x$, $q = z^2 \ln y$, $r = 1/z$; $(x, y, z) = (\pi/4, 1/2, -1/2)$

8. Find $\partial z/\partial u$ and $\partial z/\partial v$ when $u = \ln 2, v = 1$ if $z = 5 \tan^{-1} x$ and $x = e^u + \ln v$.

9. Find the directional derivative of the function $f(x, y, z) = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q has coordinates $(5, 0, 4)$. In what direction it will be maximum and what is its value? Ans. $\frac{4}{3}\sqrt{21}$, $2\sqrt{41}$.

10. Find the directional derivative of the function

    (i) $f(x, y, z) = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of $i + 2j + 2k$. Ans. $-11/3$

    (ii) $f(x, y, z) = x^2 + y^2 + 4xyz$ at $(1, -2, 2)$ in the direction of $2i - 2j + k$. Ans. $-14\frac{2}{3}$

    (iii) $f(x, y, z) = 4xz^3 - 3x^2yz^2$ at $(2, -1, 2)$ along z-axis. Ans. 144

11. The temperature at a point (x, y, z) in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at (1,1,2) desires to fly in such a direction that it will get warm as soon as possible. In what direction should it fly? Ans. $\frac{1}{3}(2i + 2j - k)$.

12. Find the directions in which the functions increase and decrease most rapidly at P_0. Then find the derivatives of the functions in these directions.

    (i) $f(x, y) = x^2y + e^{xy} \sin y$, $P_0(1, 0)$ (ii) $f(x, y, z) = \ln xy + \ln yz + \ln zx$, $P_0(1, 1, 1)$

    (iii)$f(x, y, z) = (x/y) - yz$, $P_0(4, 1, 1)$ (iv) $f(x, y, z) = \ln(x^2 + y^2 - 1) + y + 6z$, $P_0(1, 1, 0)$