School of Mathematics
Thapar University, Patiala,
UMA 003: Tutorial Sheet 08: Extreme Values and Saddle Points

1. Find the local maxima, local minima and saddle points of the functions.
   (i) $f(x, y) = x^2 + 3xy + 3y^2 - 6x + 3y - 6$
   (ii) $f(x, y) = 2xy - x^2 - 2y^2 + 3x + 4$
   (iii) $f(x, y) = x^3 + 3xy + y^3$
   (iv) $f(x, y) = \frac{1}{x} + xy + \frac{1}{y}$
   (v) $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$

2. Find the absolute maxima and minima of the functions on the given domains.
   (i) $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0$, $y = 2, y = 2x$ in the first quadrant.
   (ii) $f(x, y) = x^2 + y^2$ on the closed triangular plate bounded by the lines $x = 0$, $y = 0$, $y + 2x = 2$ in the first quadrant.
   (iii) $f(x, y) = x^2 + xy + y^2 - 6x + 2$ in the rectangular plate $0 \leq x \leq 5$, $-3 \leq y \leq 3$.
   (iv) $f(x, y) = 48xy - 32x^3 - 24y^2$ on the rectangular plate $0 \leq x \leq 1$, $0 \leq y \leq 1$.
   (v) $f(x, y) = (4x - y^2) \cos y$ on the rectangular plate $1 \leq x \leq 3$, $-\pi/4 \leq y \leq \pi/4$

3. A flat circular plate has the shape of the region $x^2 + y^2 \leq 1$. The plate, including the boundary where $x^2 + y^2 = 1$ is heated so that the temperature at the point (x, y) is $T(x, y) = x^2 + 2y^2 - x$. Find the temperature at the hottest and coldest points on the plate.

4. Find the absolute maximum and minimum values of $f(x, y) = x^2 - y^2 - 2x + 4y$ on the region R : the triangular region bounded below by the $x-$axis, above by the line $y = x + 2$ and on the right by the line $x = 2$.

5. Find the absolute maximum and minimum values of $f(x, y) = y^2 - xy - 3y + 2x$ on the region R : the square region enclosed by the lines $x = \pm 2$ and $y = \pm 2$.

Answers:

1. (i) local minimum (15, -8). (ii) local maximum (3, 3/2) . (iii) local maximum (-1,-1); saddle point (0,0). (iv) local minimum (1,1). (v) saddle point (0,0); local maximum (2,2).

2. (i) Maximum: (0,0); Minimum: (1,2). (ii) Maximum (0,4); Minimum (0,0). (iii) Maximum (0, -3); Minimum (4,-2). (iv) Maximum (1/2, 1/2); Minimum: (1,0) (v) Maximum (2,0); Minimum $(3, -\pi/4), (3, \pi/4)$, $(1, -\pi/4)$, $(1, \pi/4)$

3. Hottest: $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$; Coolest: $(\frac{1}{2}, 0)$.

4. Maximum: (-2,0); Minimum: (1,0)

5. Maximum: (2,-2); Minimum $(-2\frac{1}{2})$