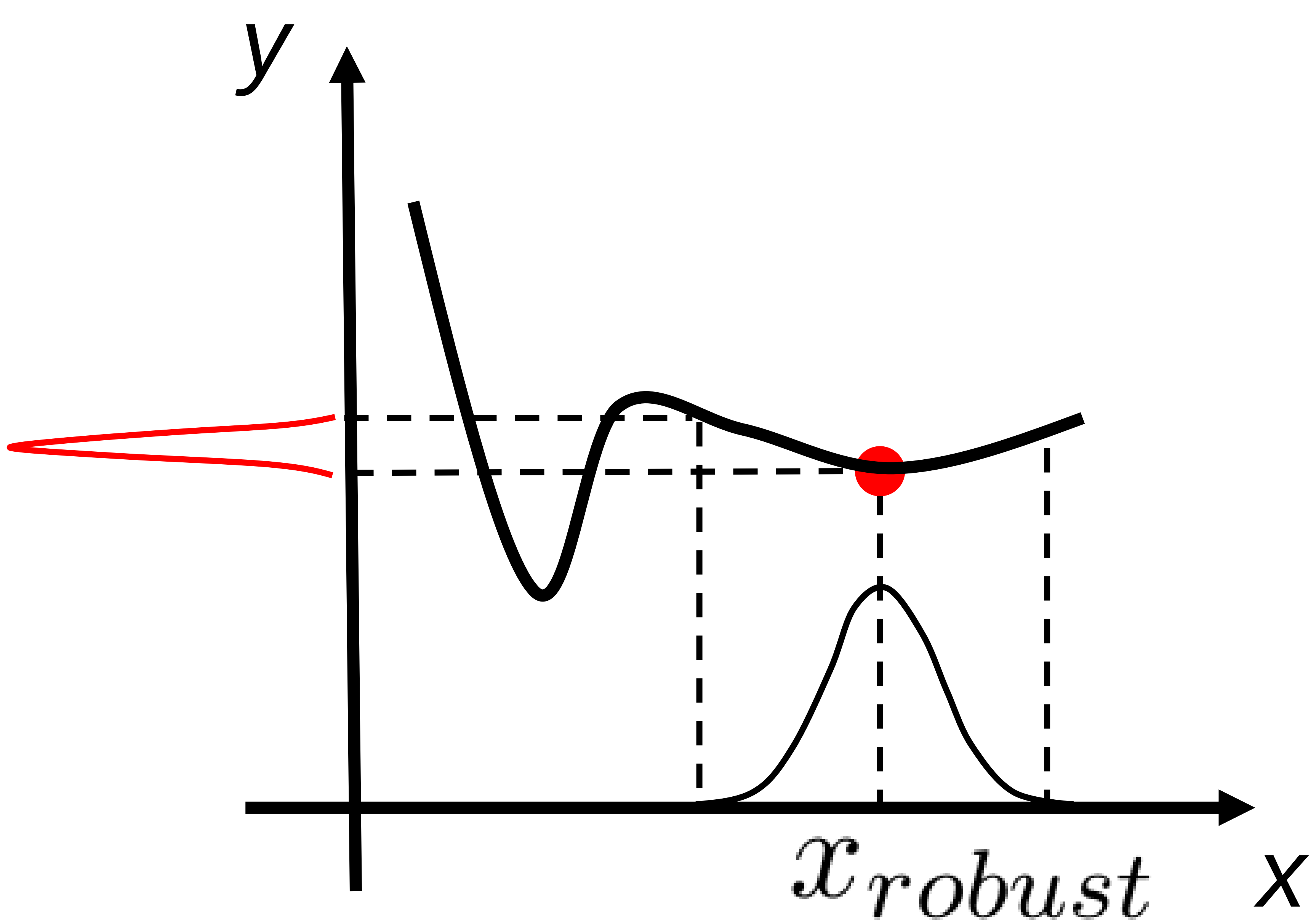
Efficient Robust Design for Thermoacoustic Instability Analysis: A Gaussian Process Approach

S. Guo, C. F. Silva, W. Polifke



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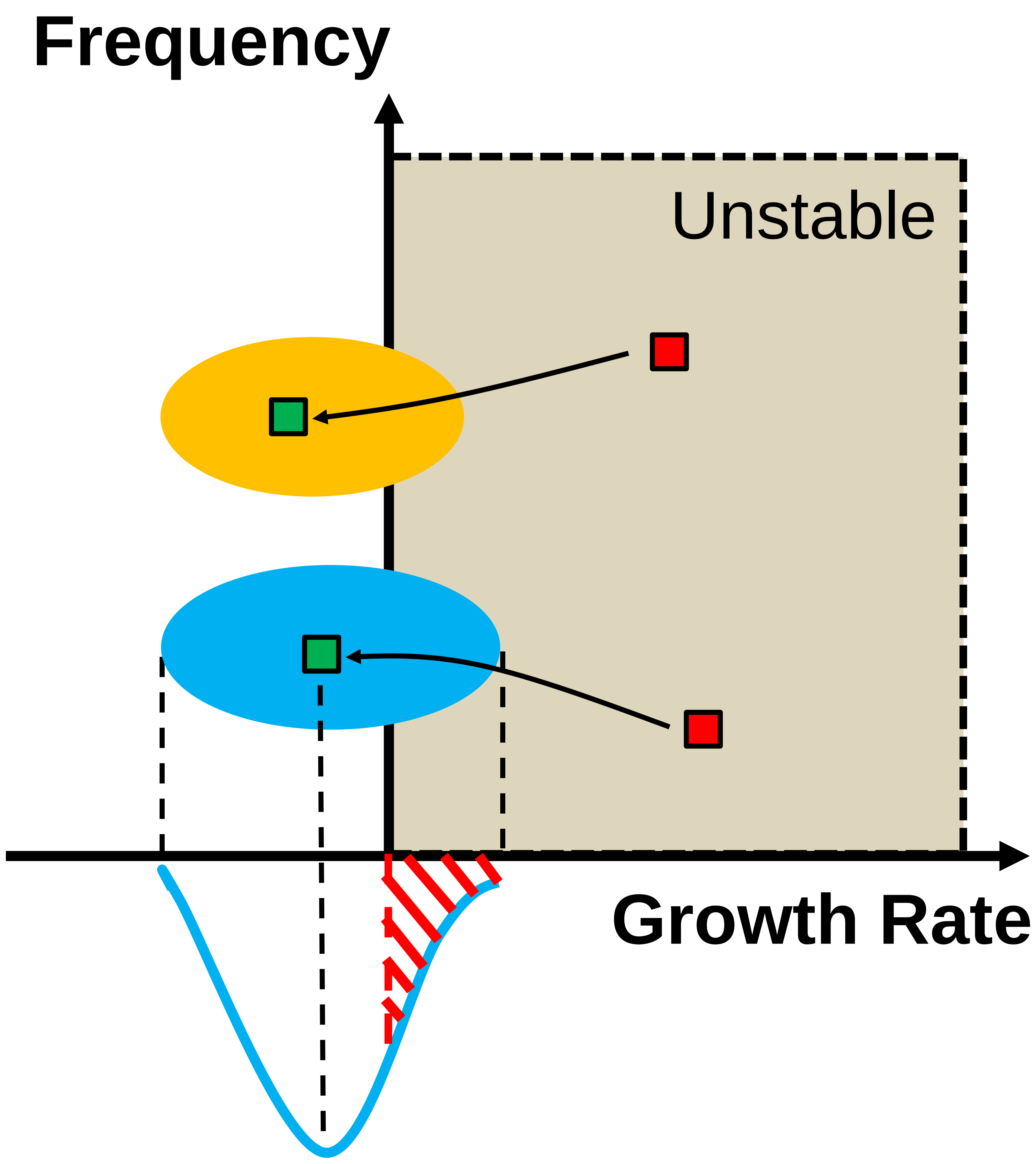
Robust analysis takes into account input uncertainties to realize risk-free thermoacoustic design



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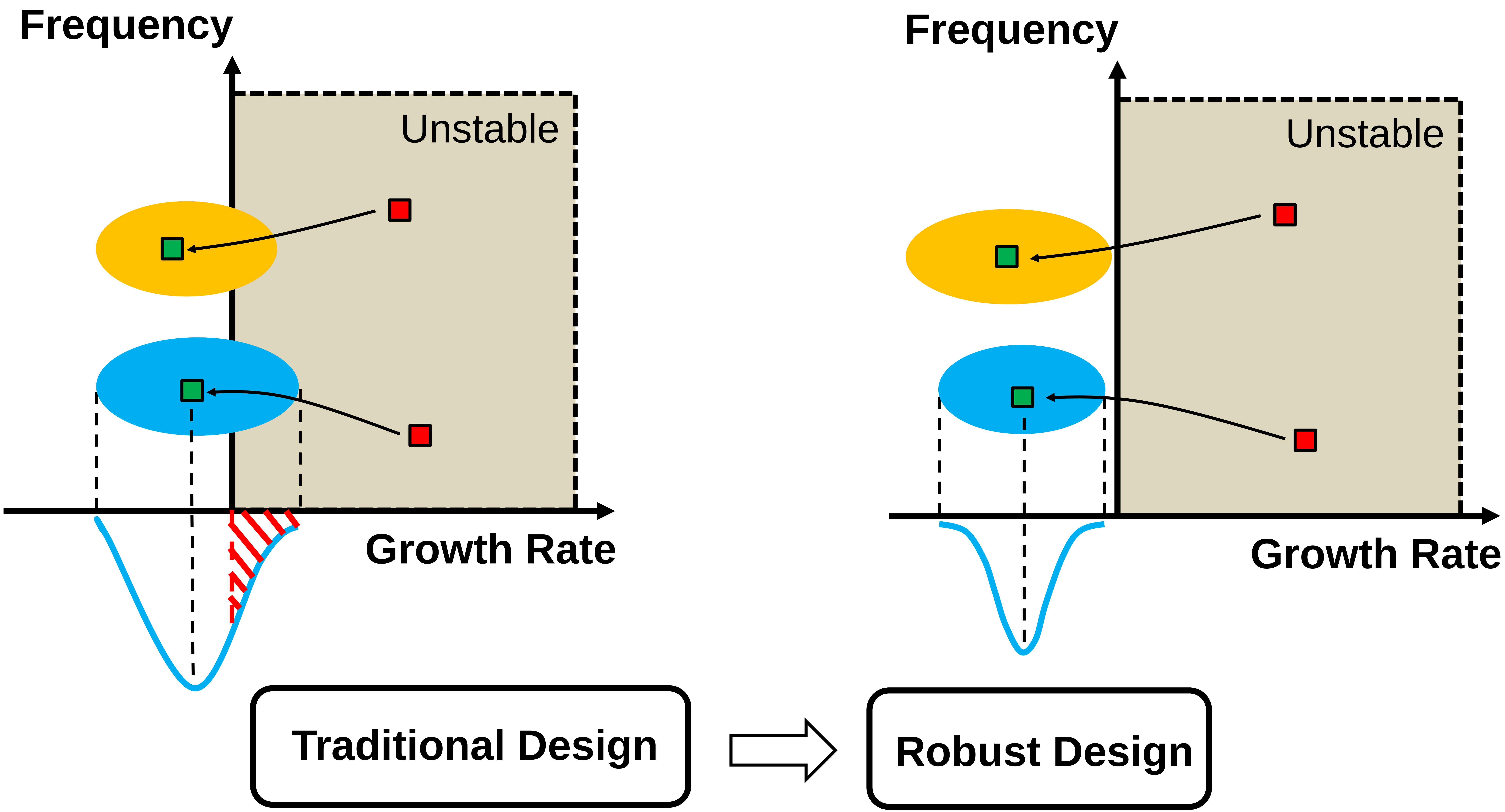
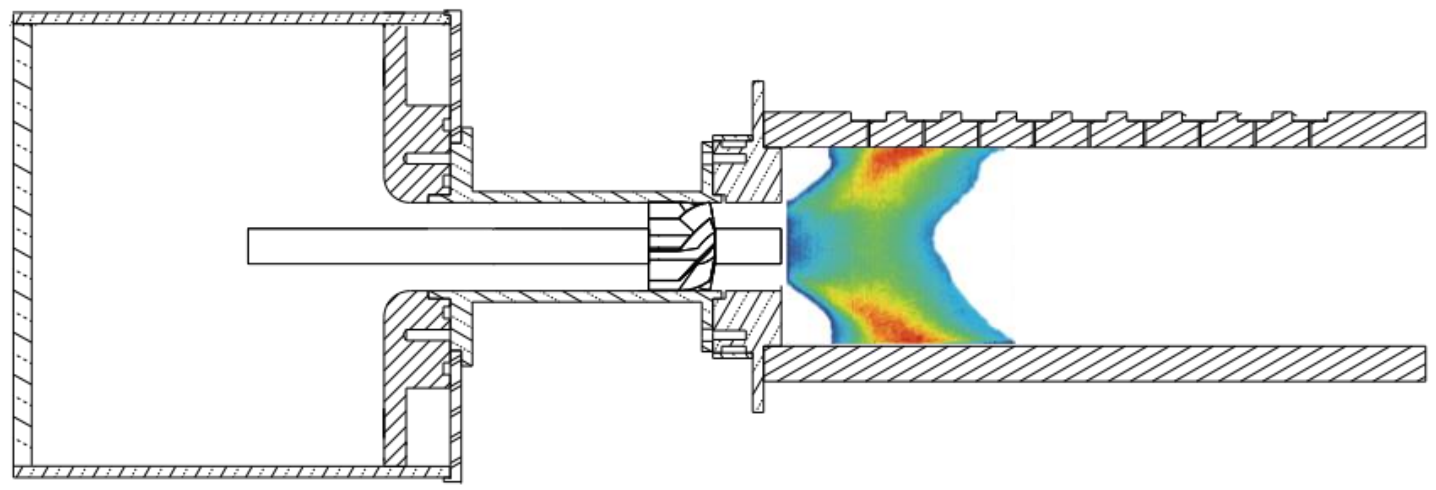


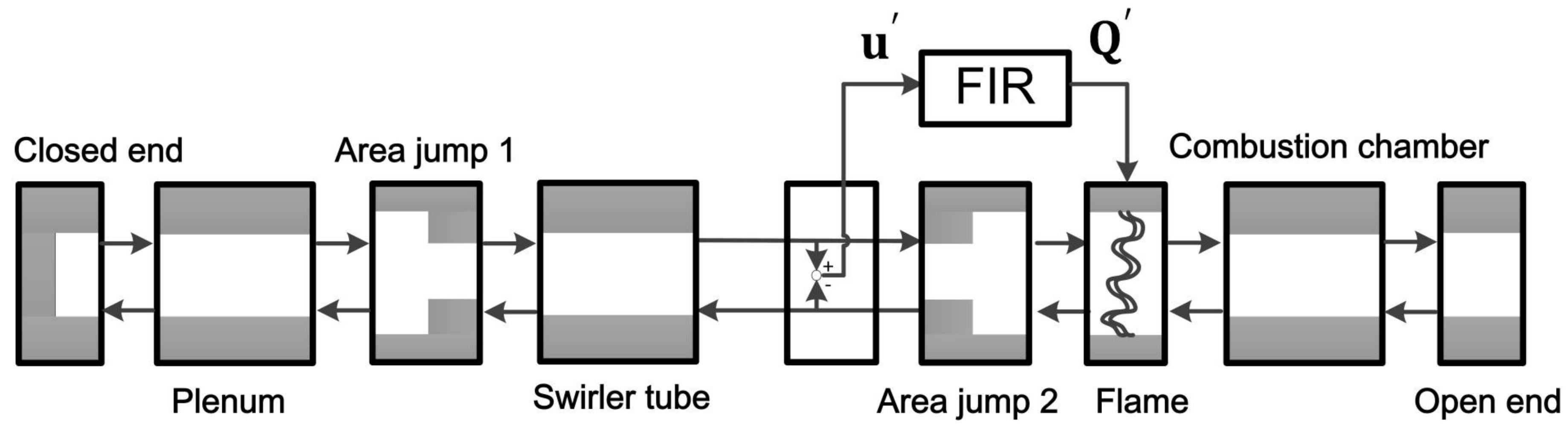
Presentation overview

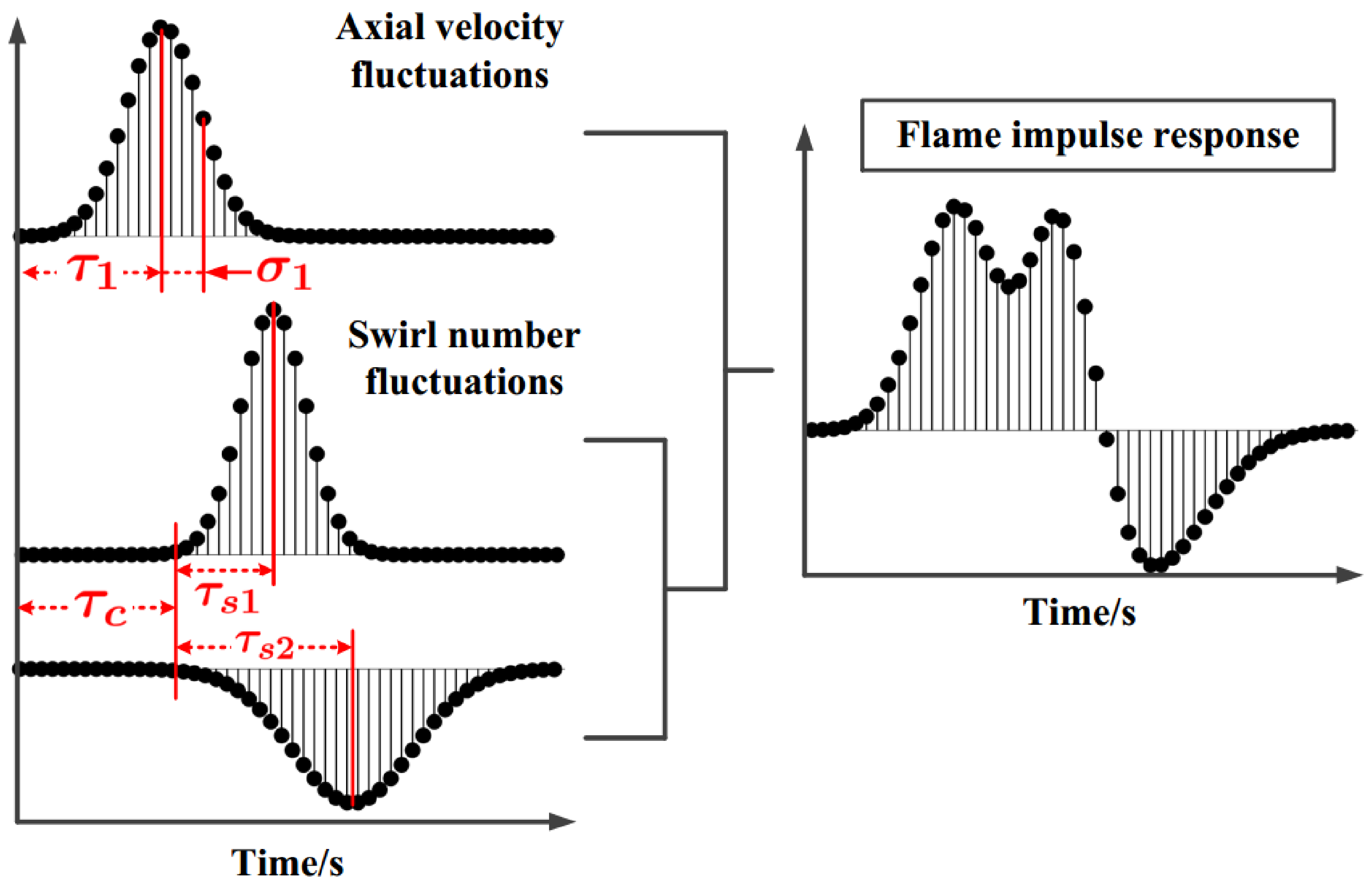
- Motivation
- Thermoacoustic problem settings
- Robust design tasks
  - Risk Analysis
  - Gaussian Process
  - Ideal Control Design
  - Realistic Control Design
  - Tolerance Design

  → Risk Diagram
- Conclusions

Thermoacoustic settings: solver, flame model, mode specification







Distributed Time Lag Model [1]

[1] Komarek, T., Polifke, W., 2010, *J Eng Gas Turbines Power.*

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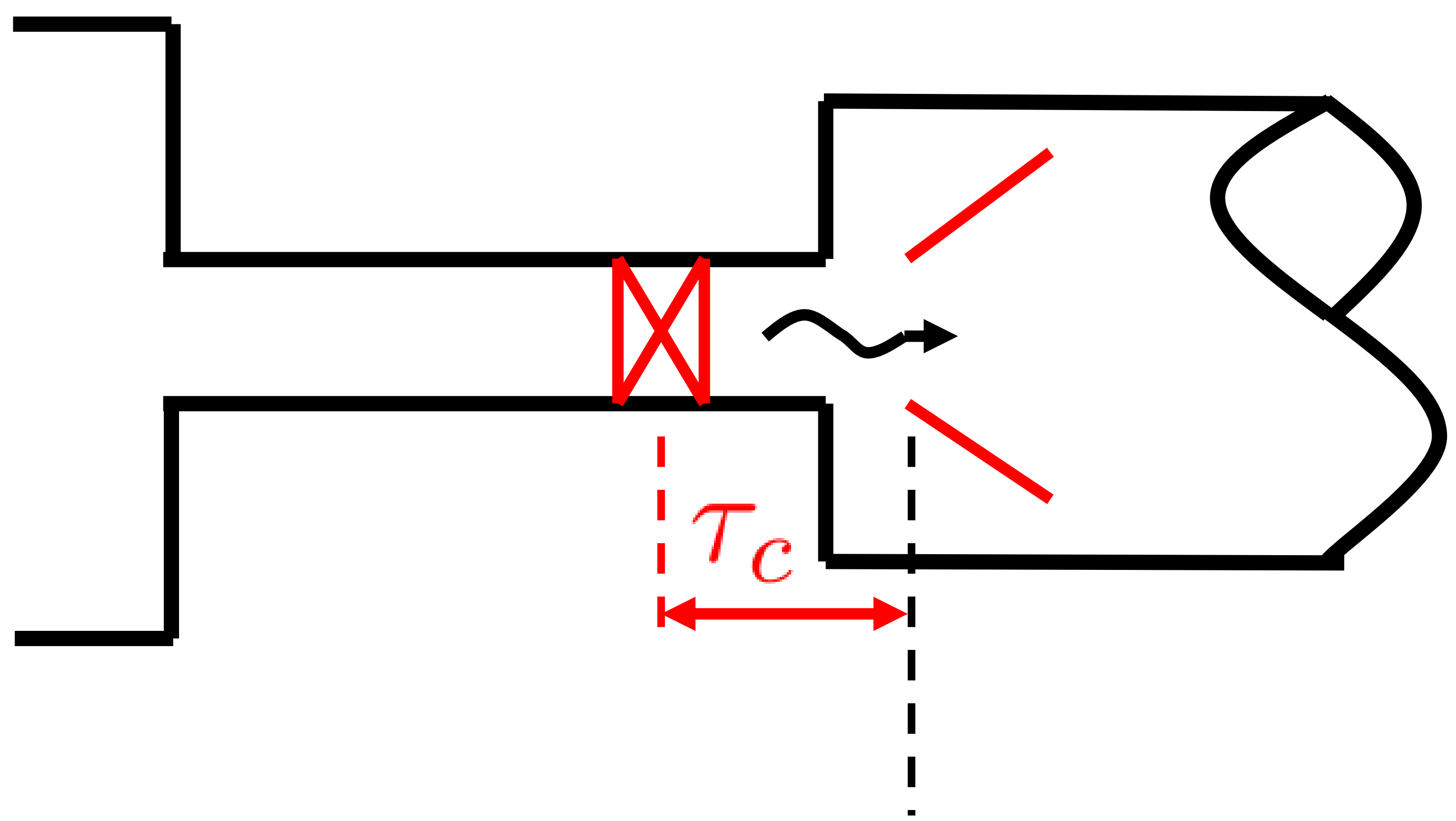

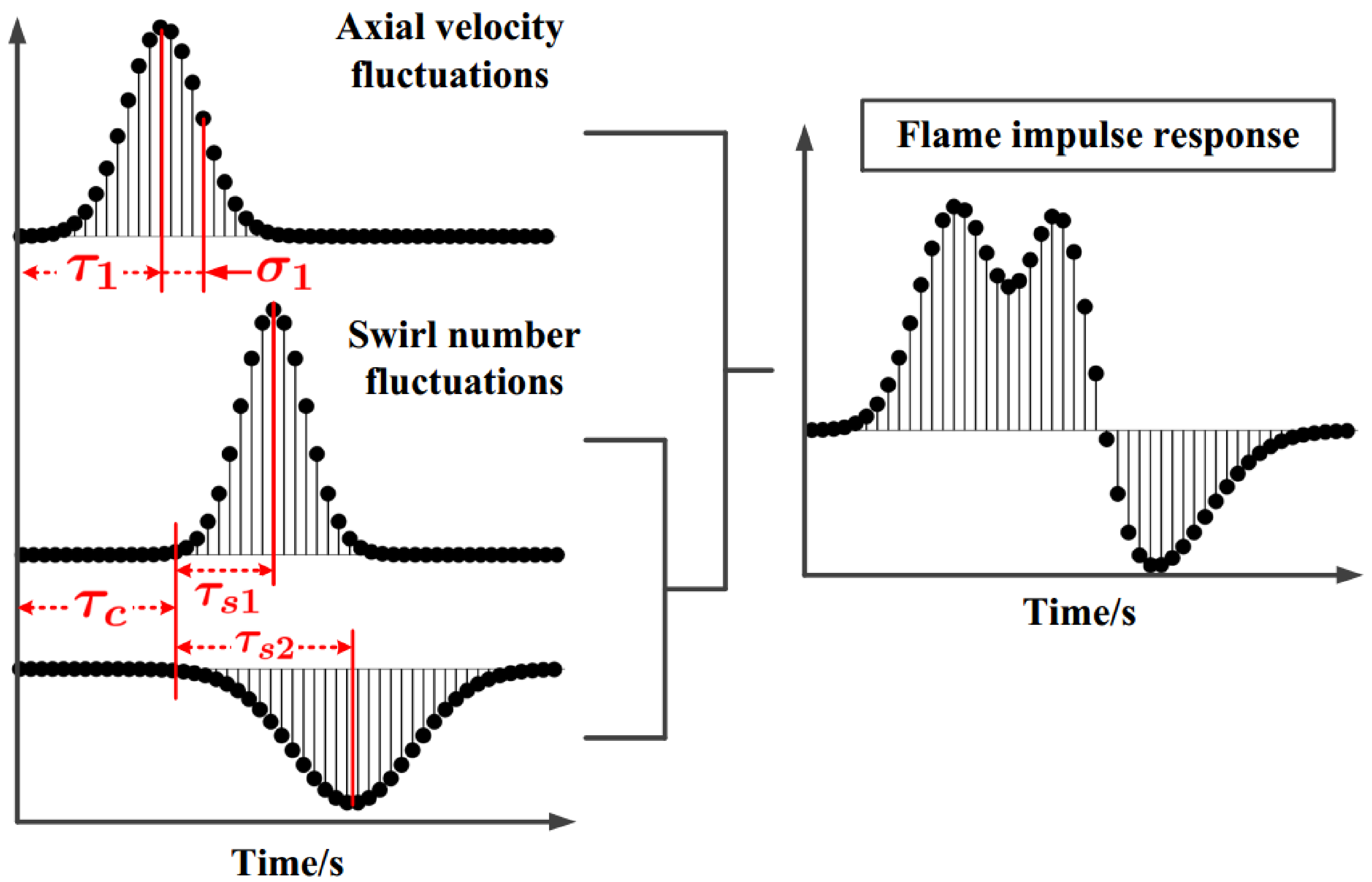

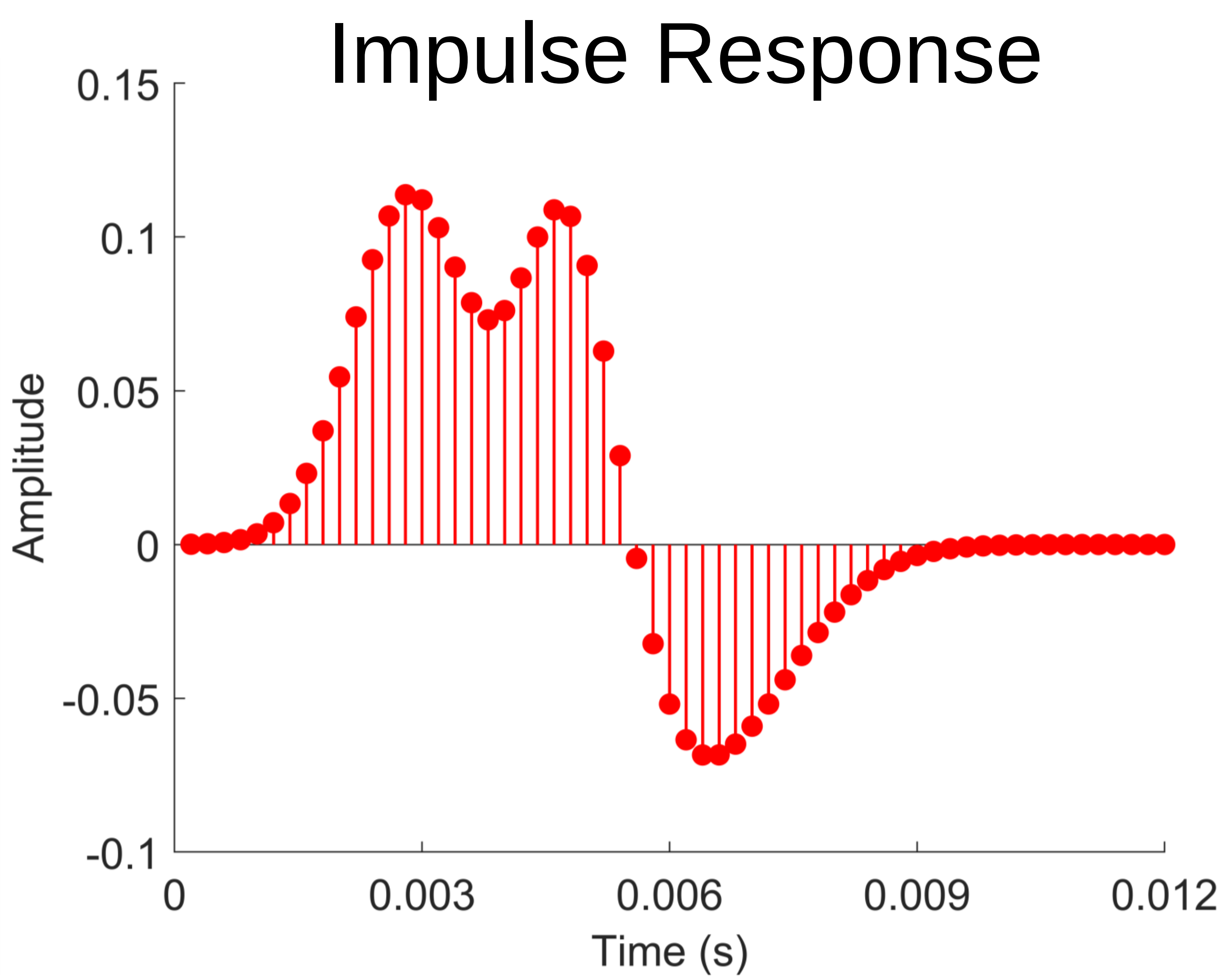

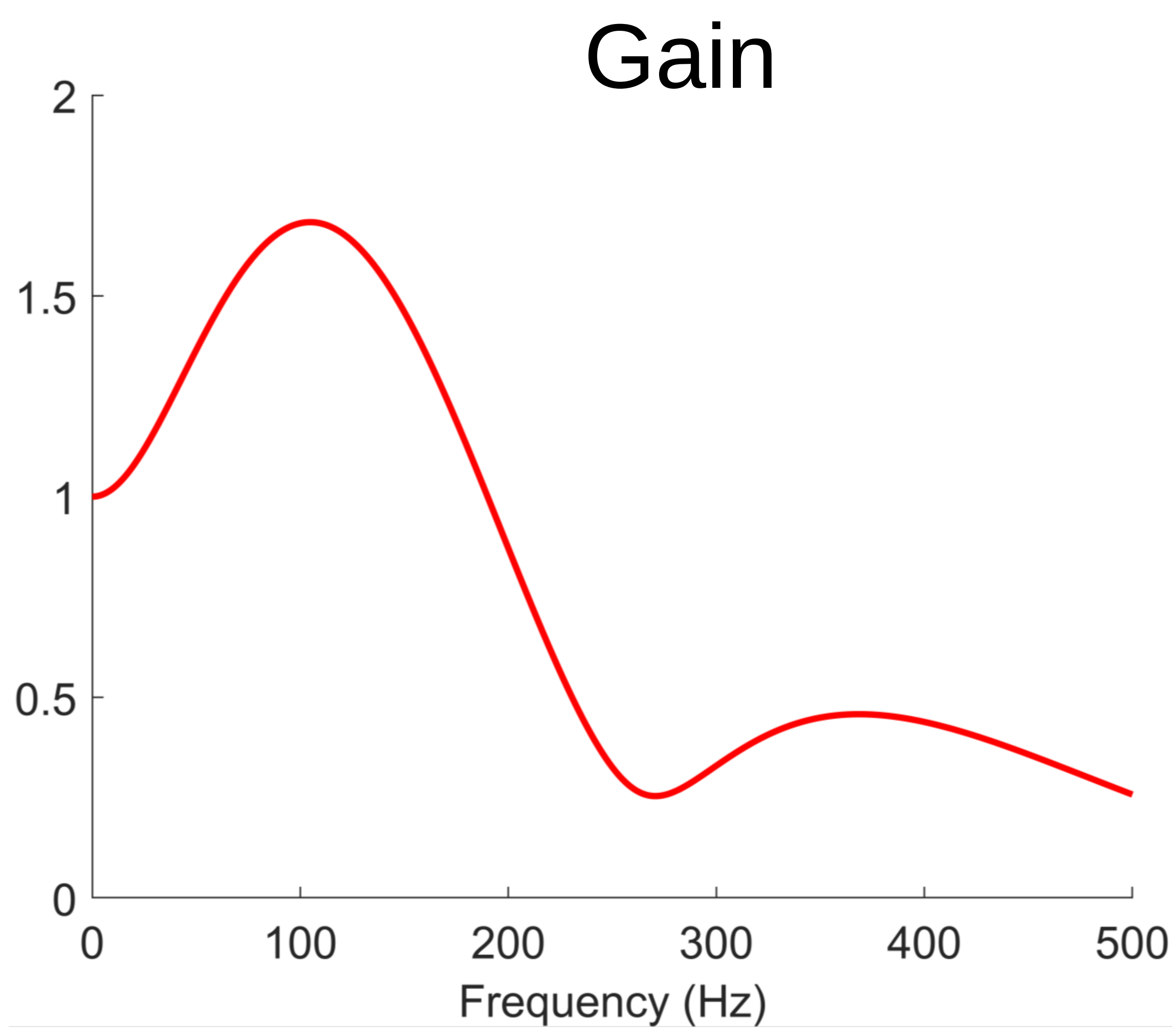

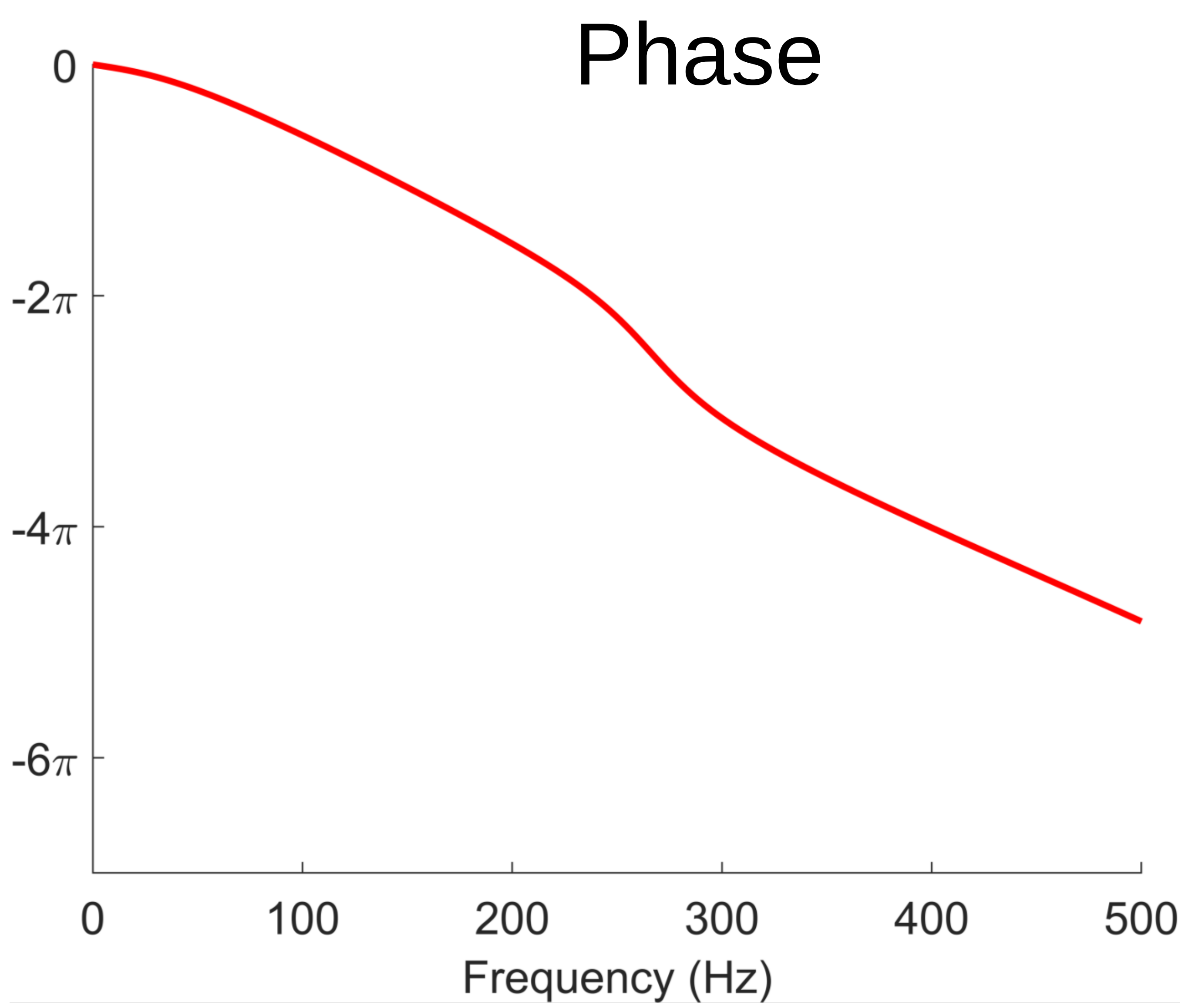

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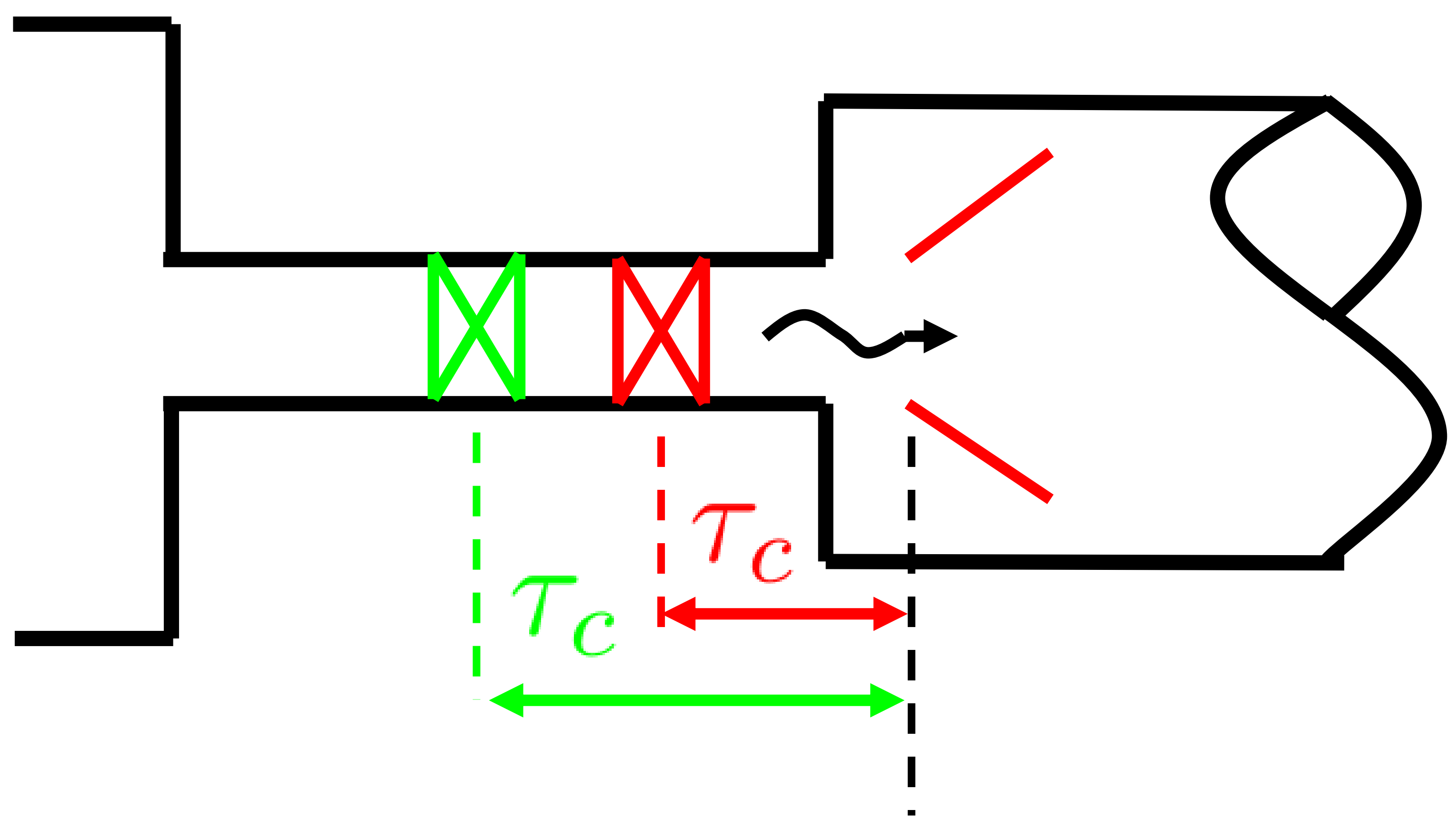

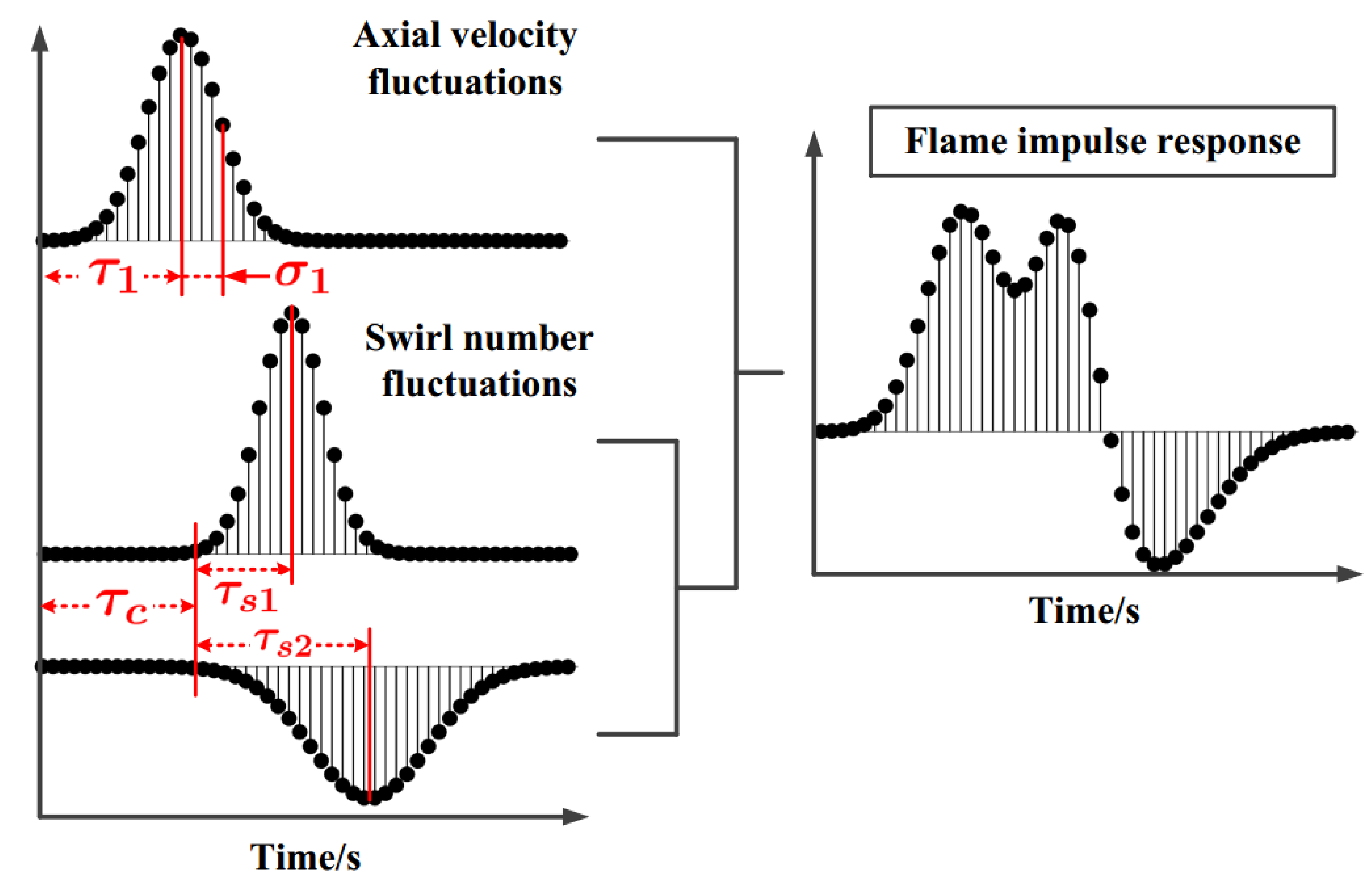

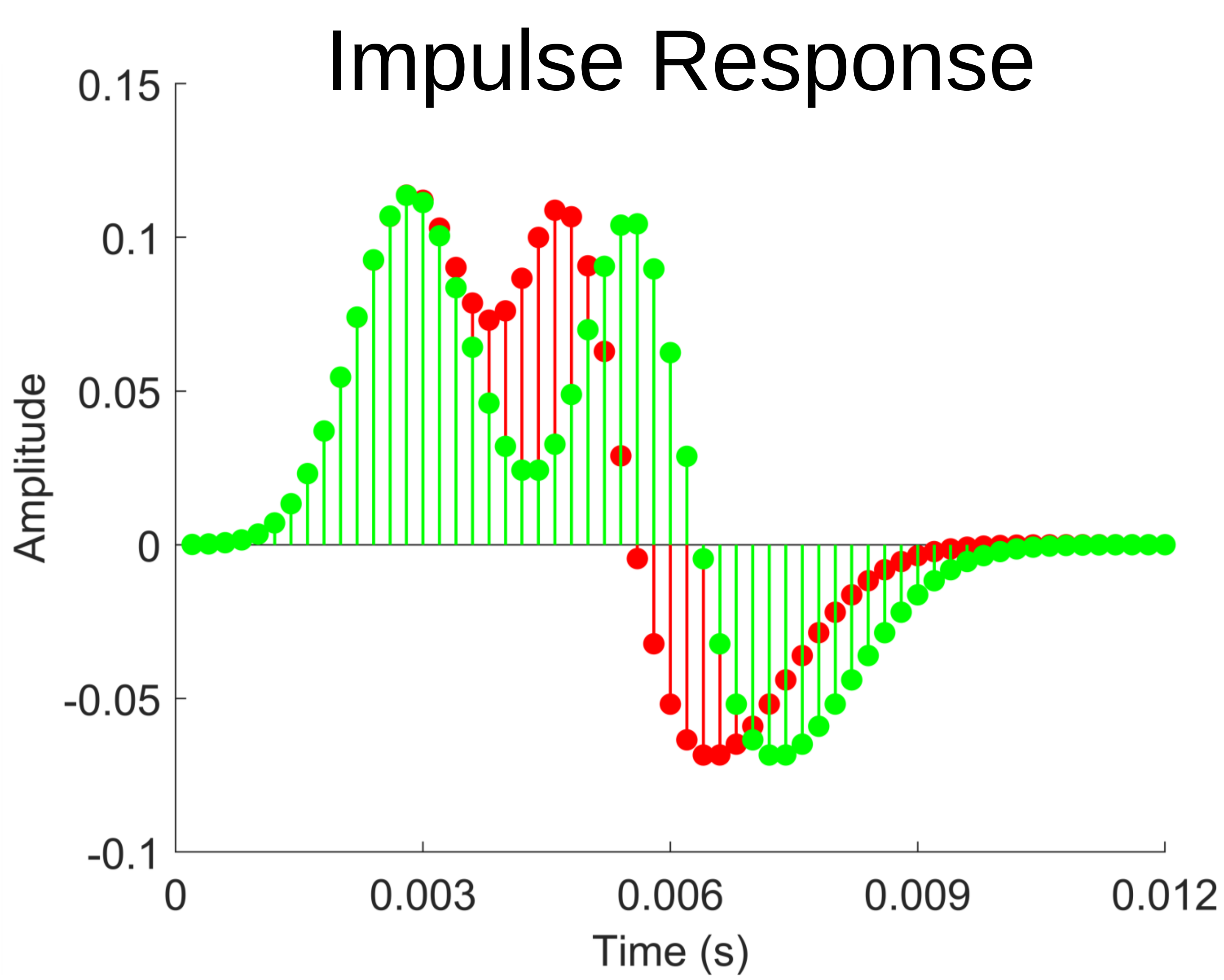

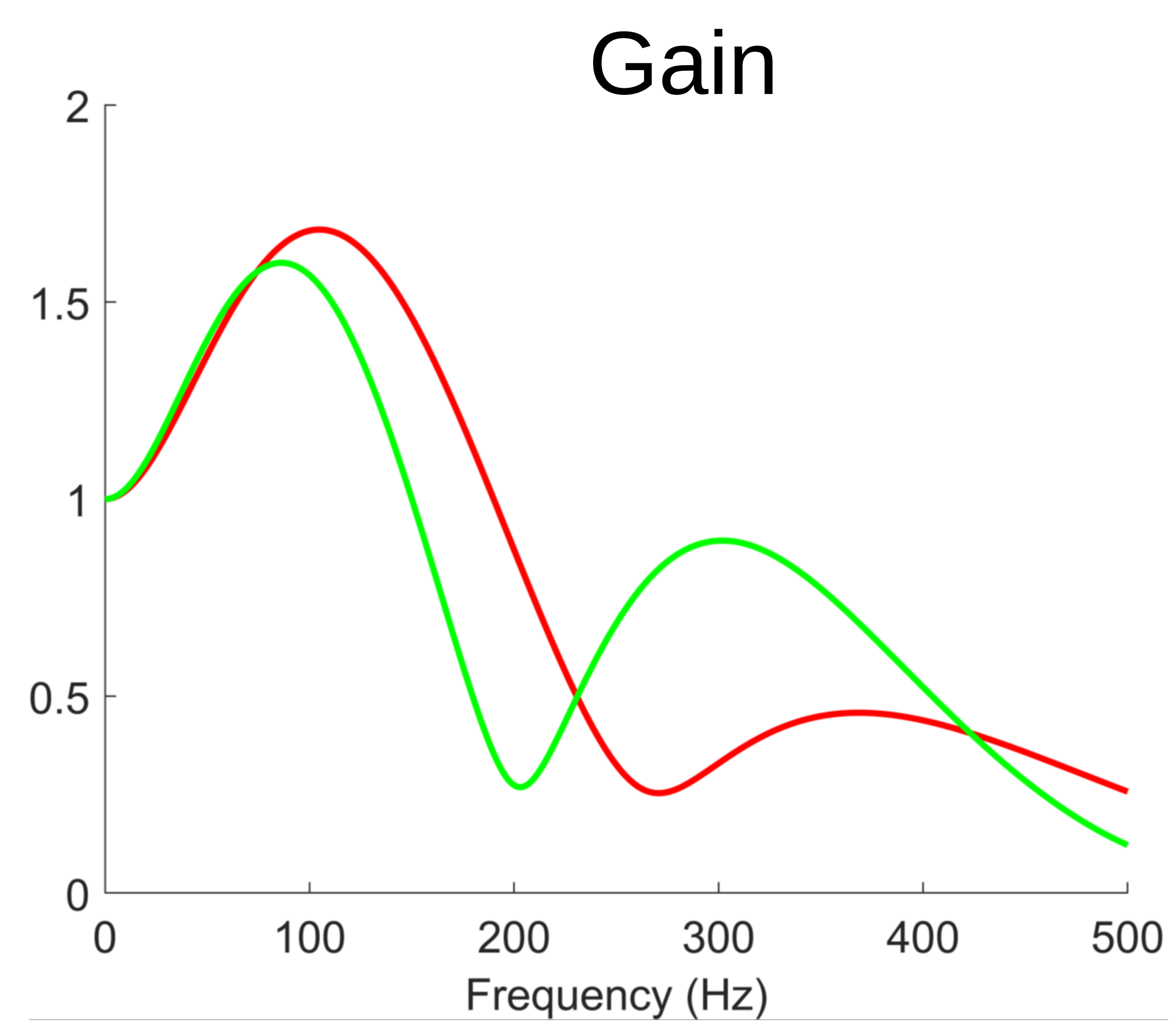

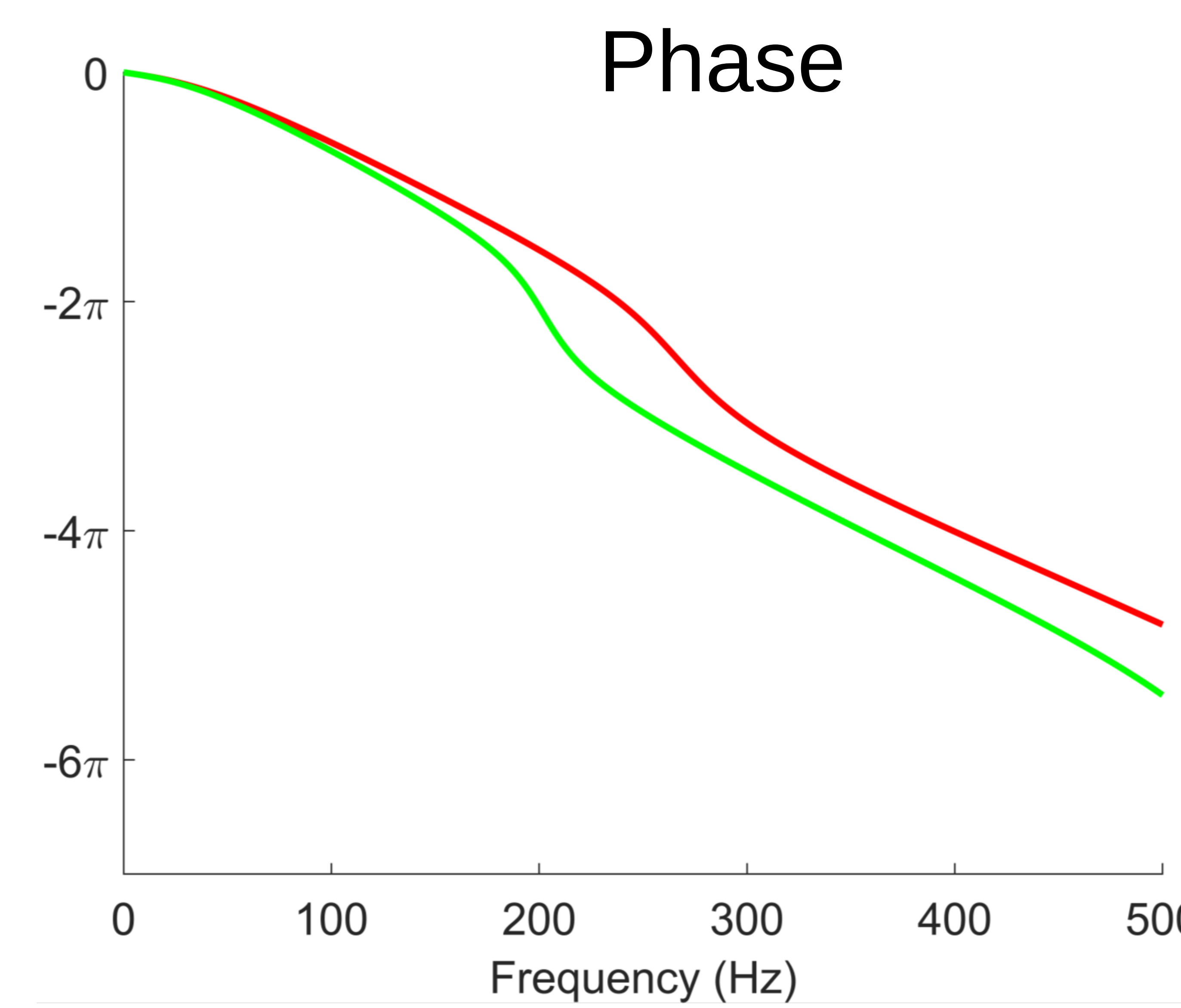

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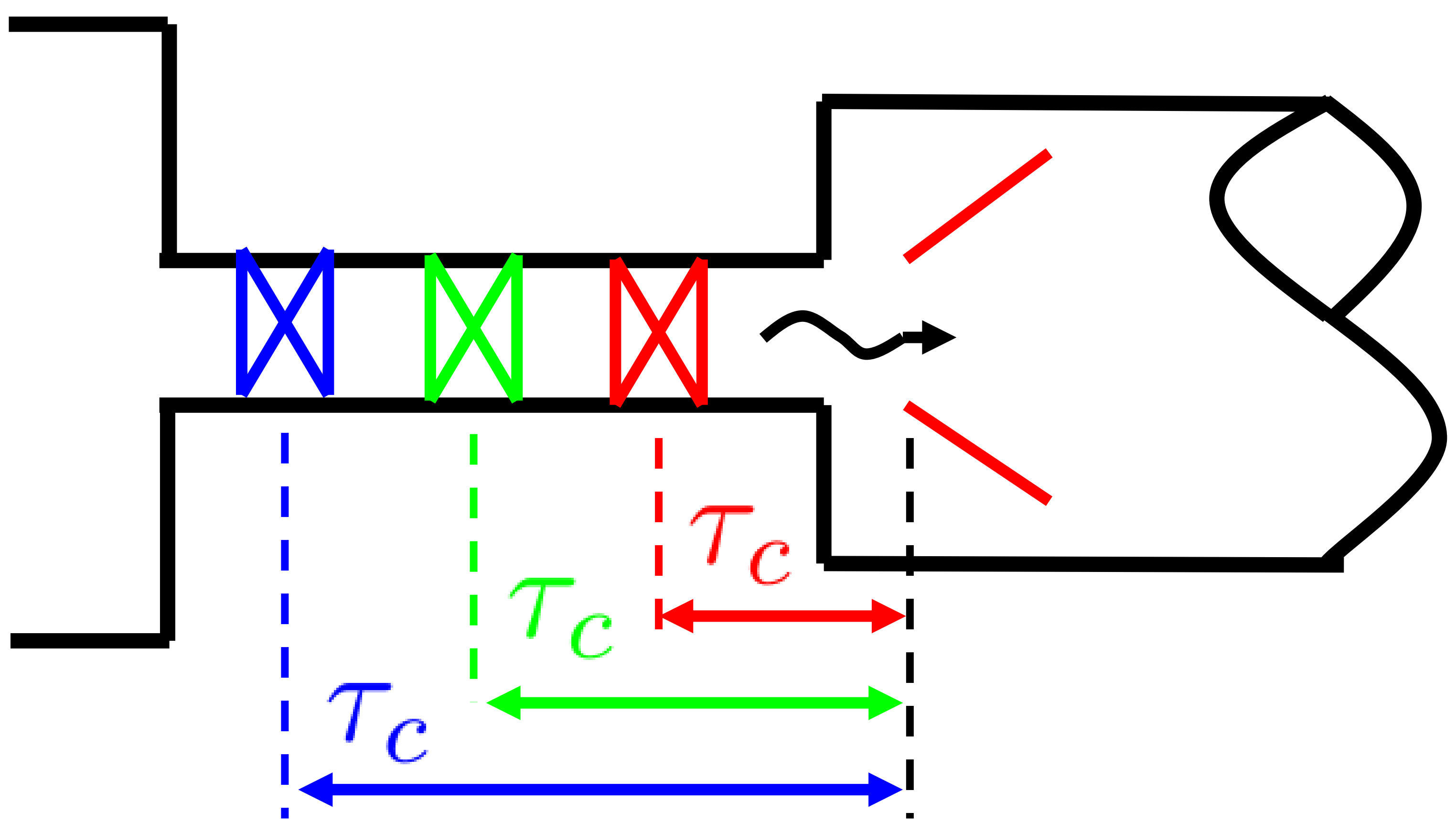

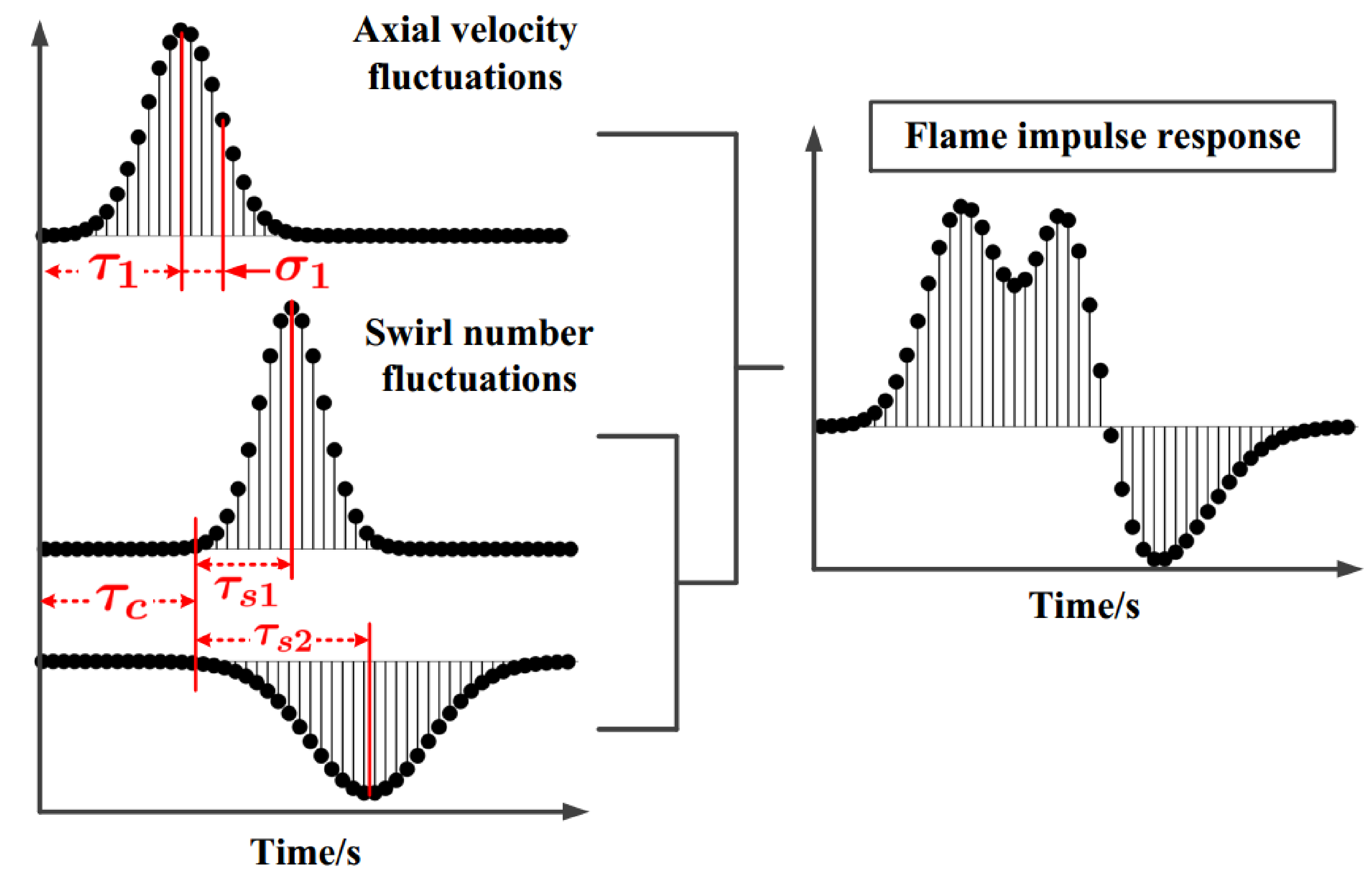

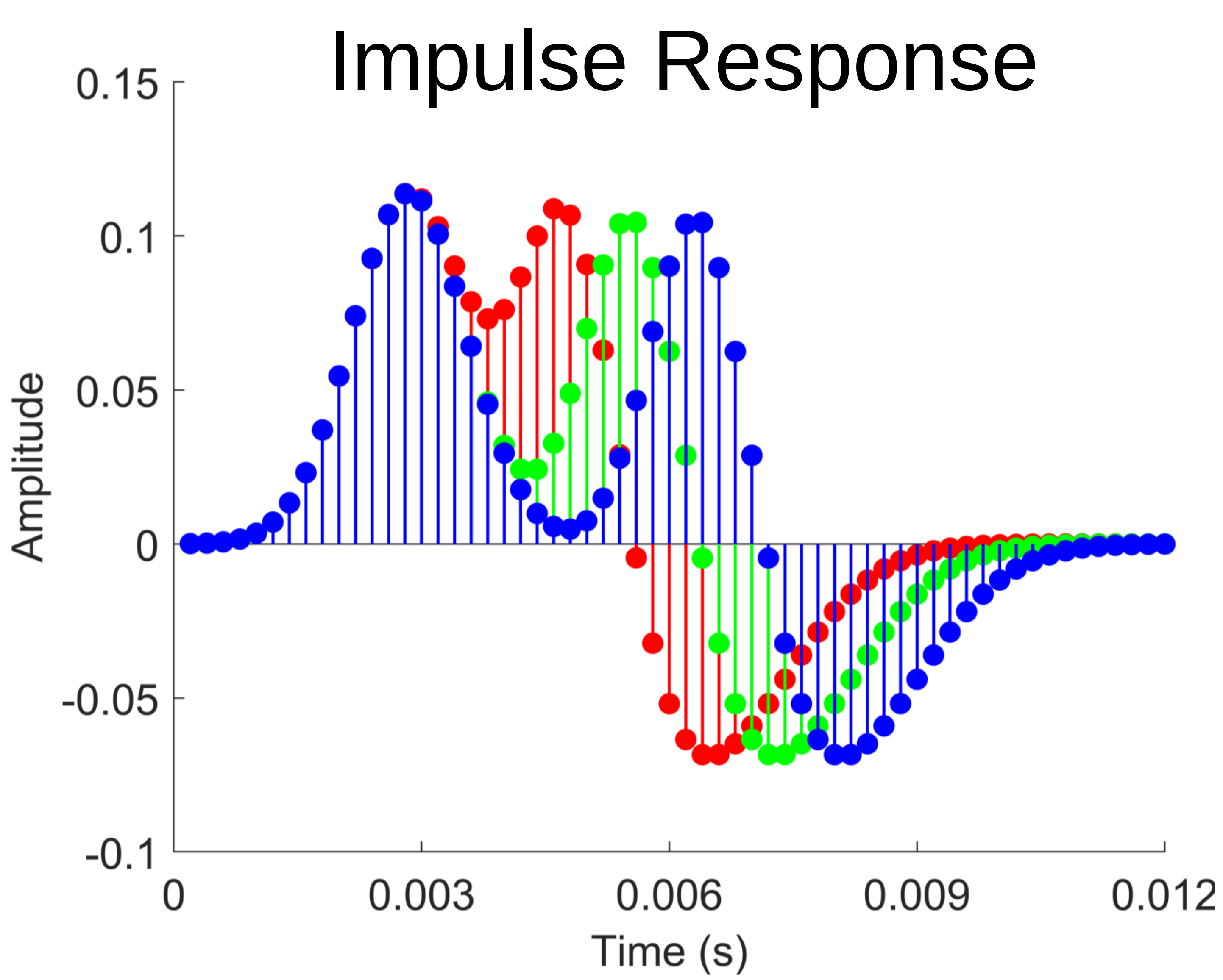

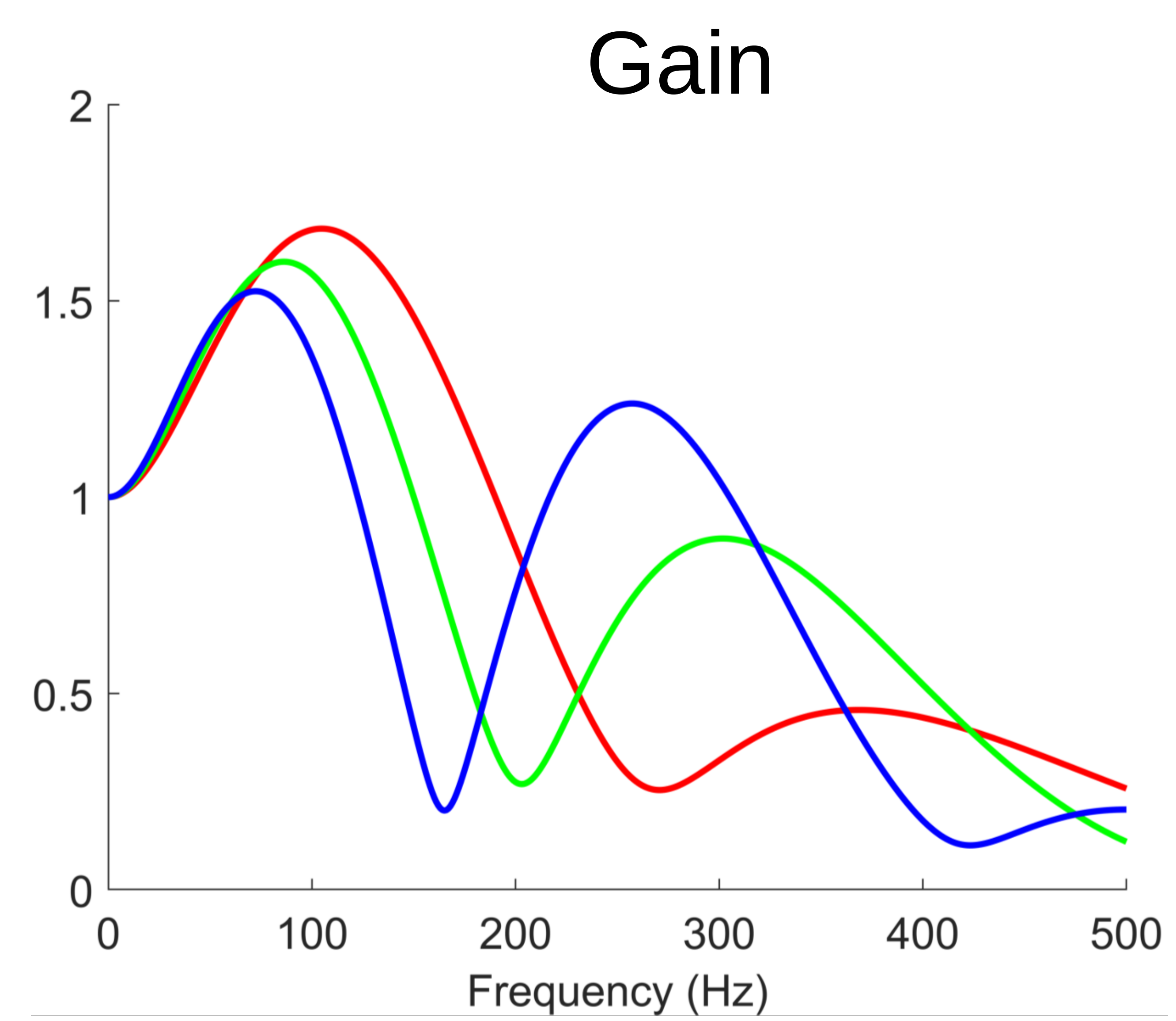

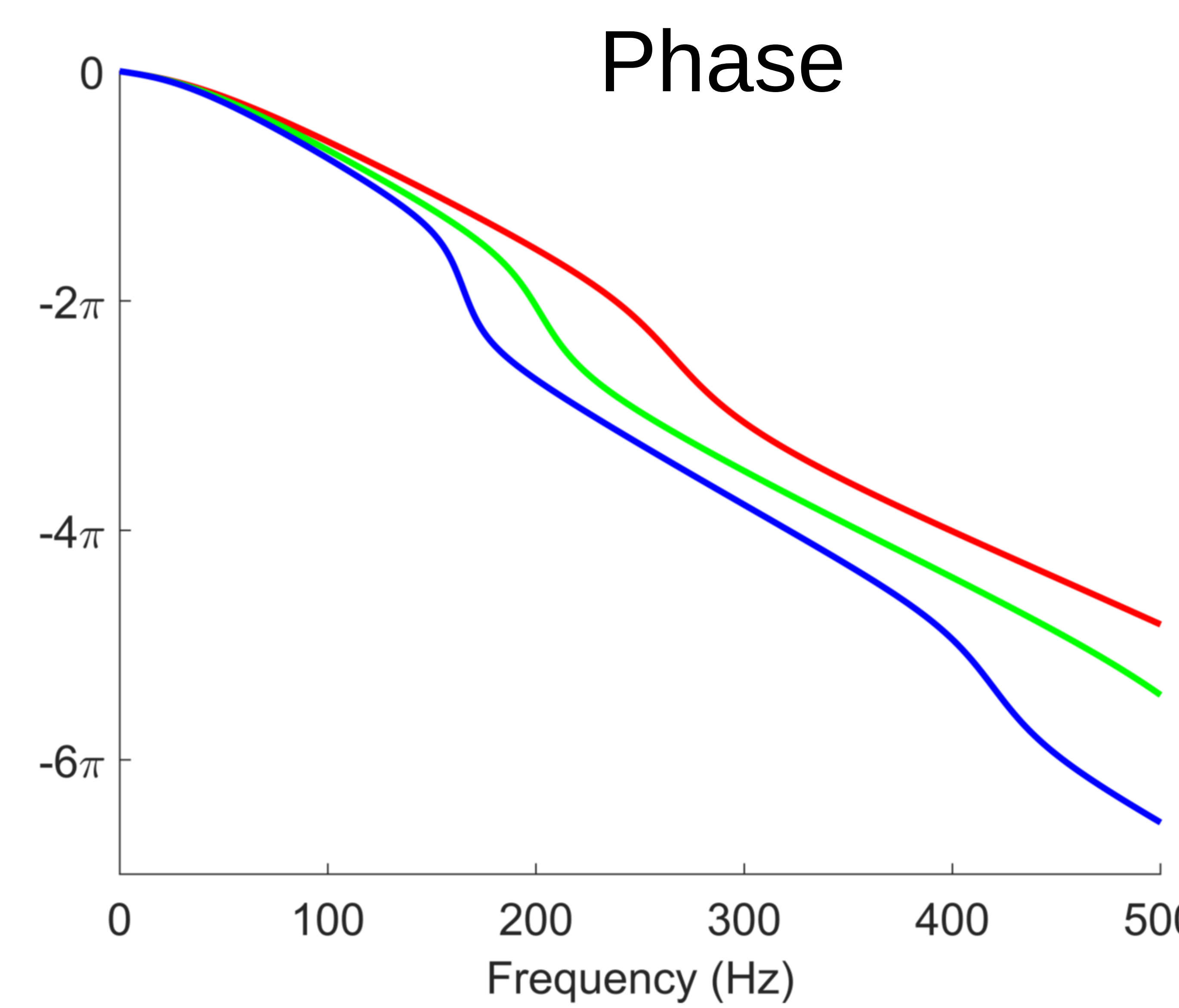

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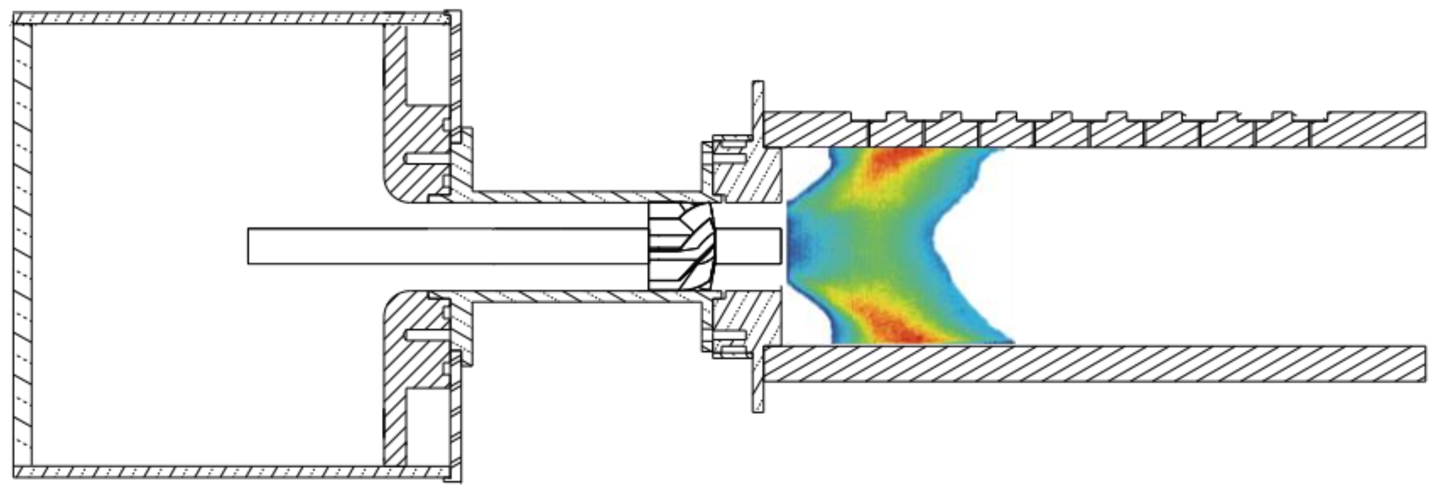

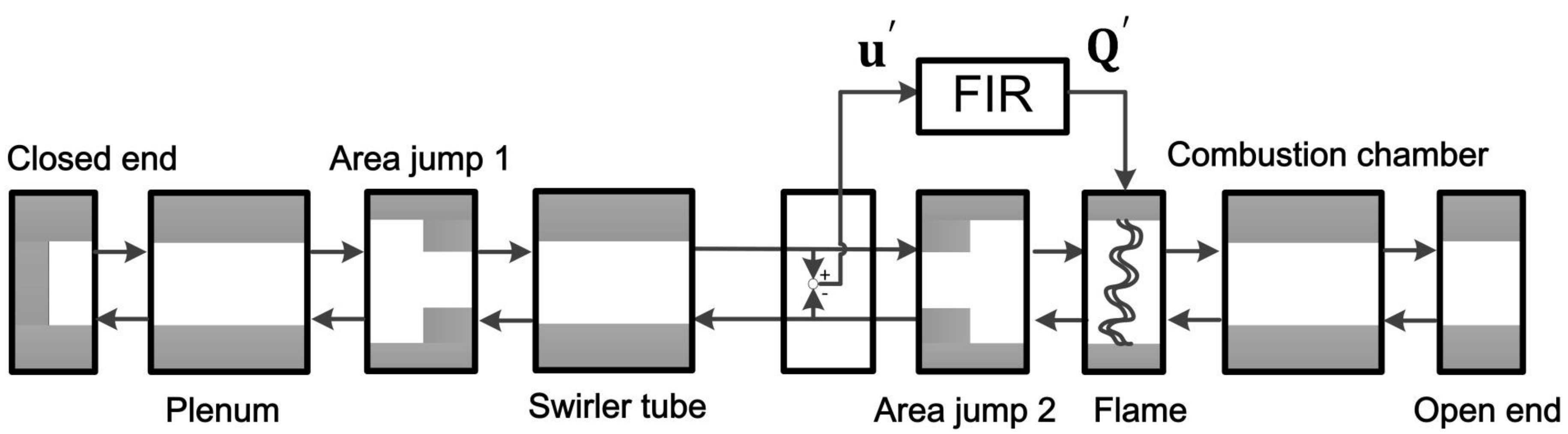

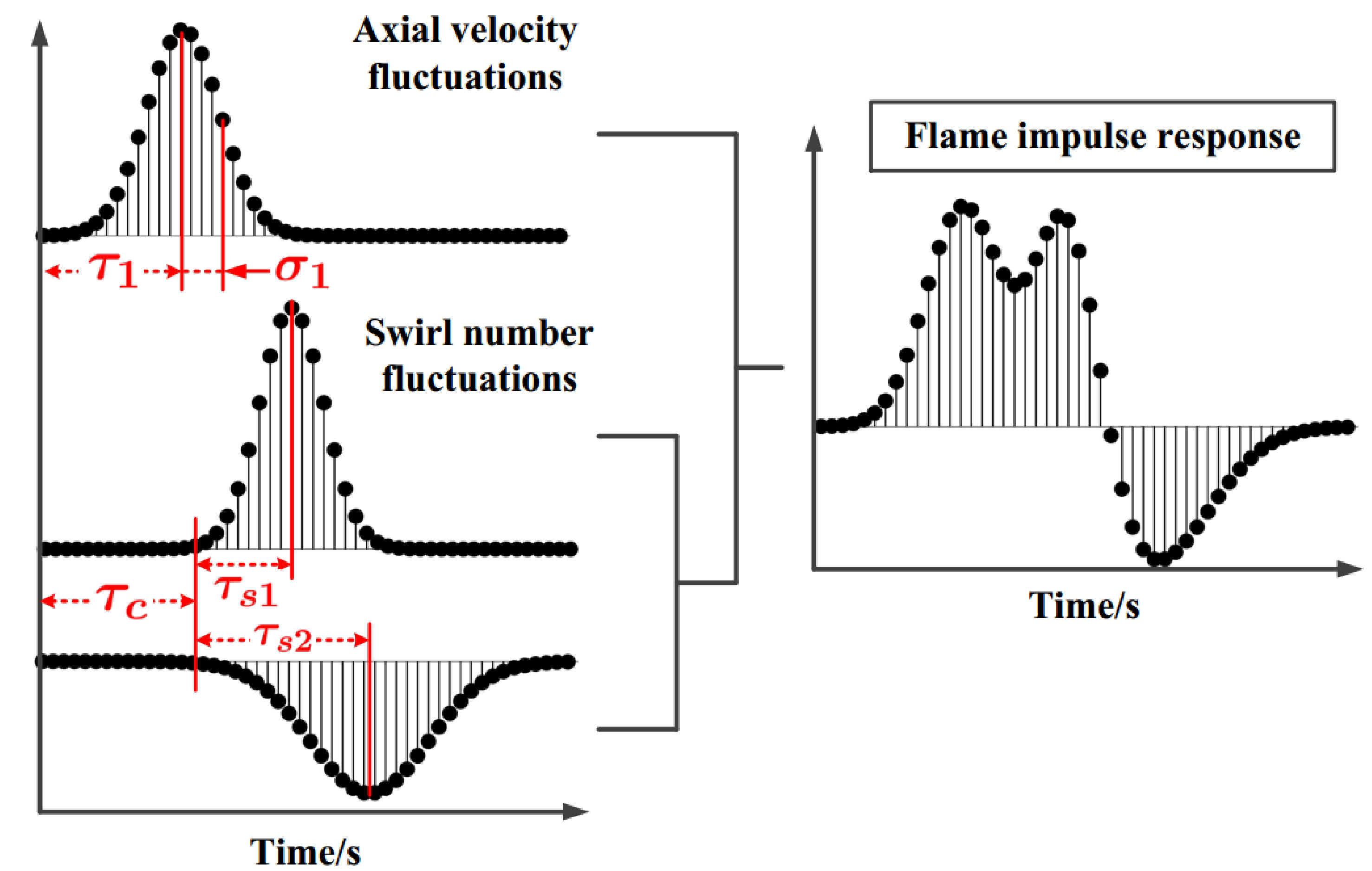

Distributed Time Lag Model [1]

Parameters		Nominal	Range
Flame (units: ms)	τ_1	$\tau_1^0 = 2.85$	$0.9\tau_1^0 \sim 1.1\tau_1^0$
	σ_1	$\sigma_1^0 = 0.7$	$0.9\sigma_1^0 \sim 1.1\sigma_1^0$
	τ_{s1}	$\tau_{s1}^0 = 1.8$	$0.9\tau_{s1}^0 \sim 1.1\tau_{s1}^0$
	τ_{s2}	$\tau_{s2}^0 = 3.3$	$0.9\tau_{s2}^0 \sim 1.1\tau_{s2}^0$

[1] Komarek, T., Polifke, W., 2010, *J Eng Gas Turbines Power.*

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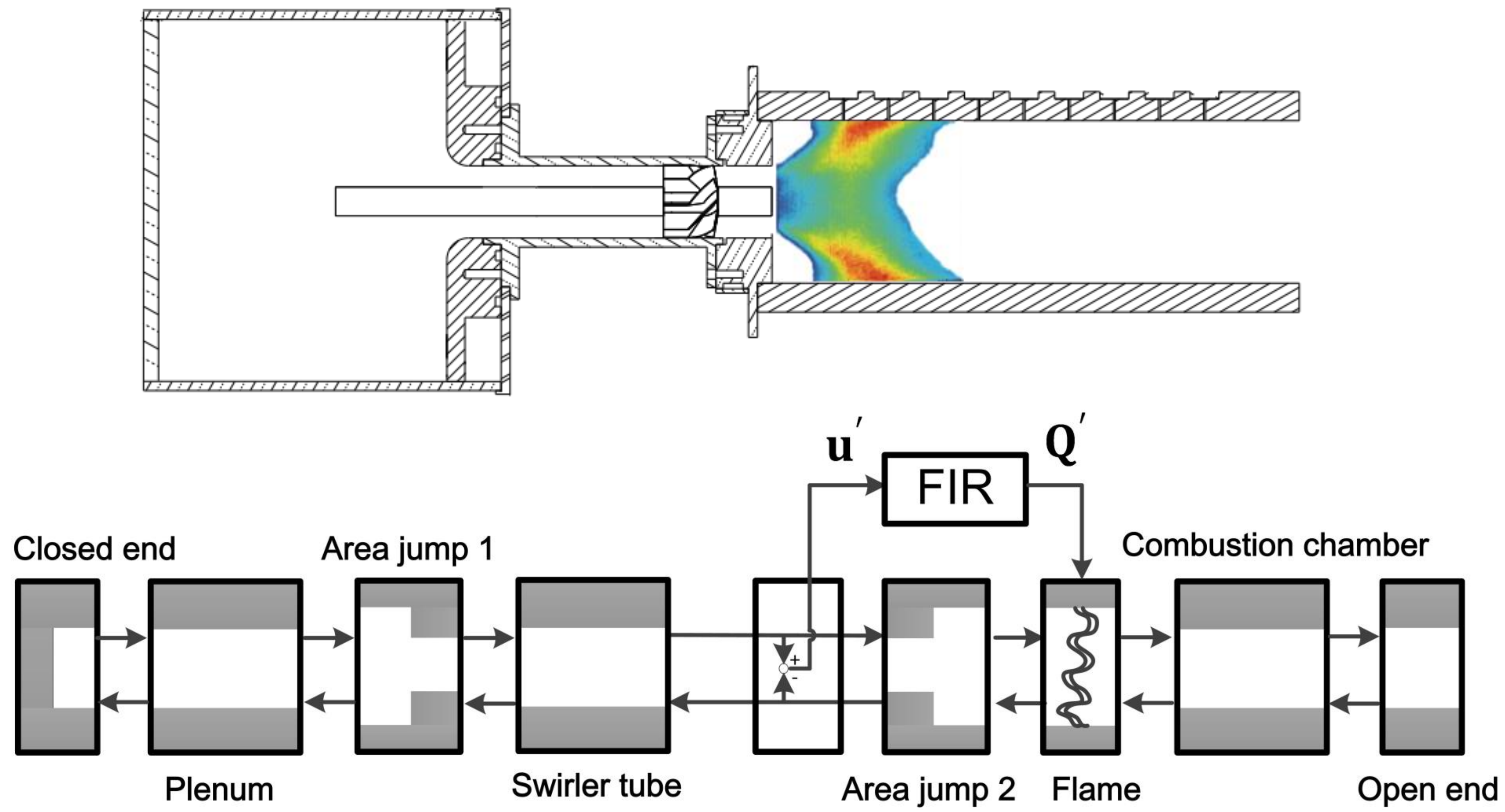

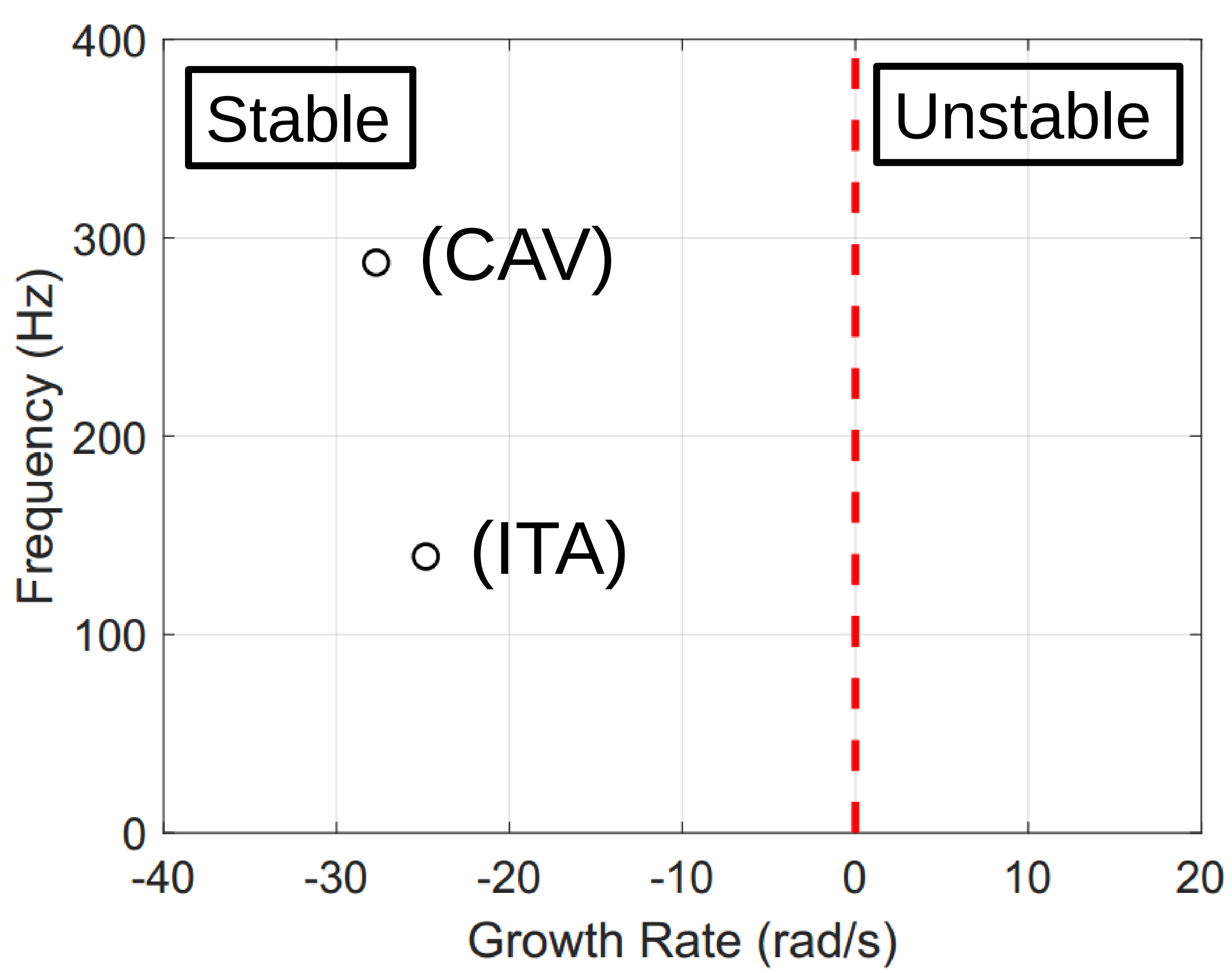

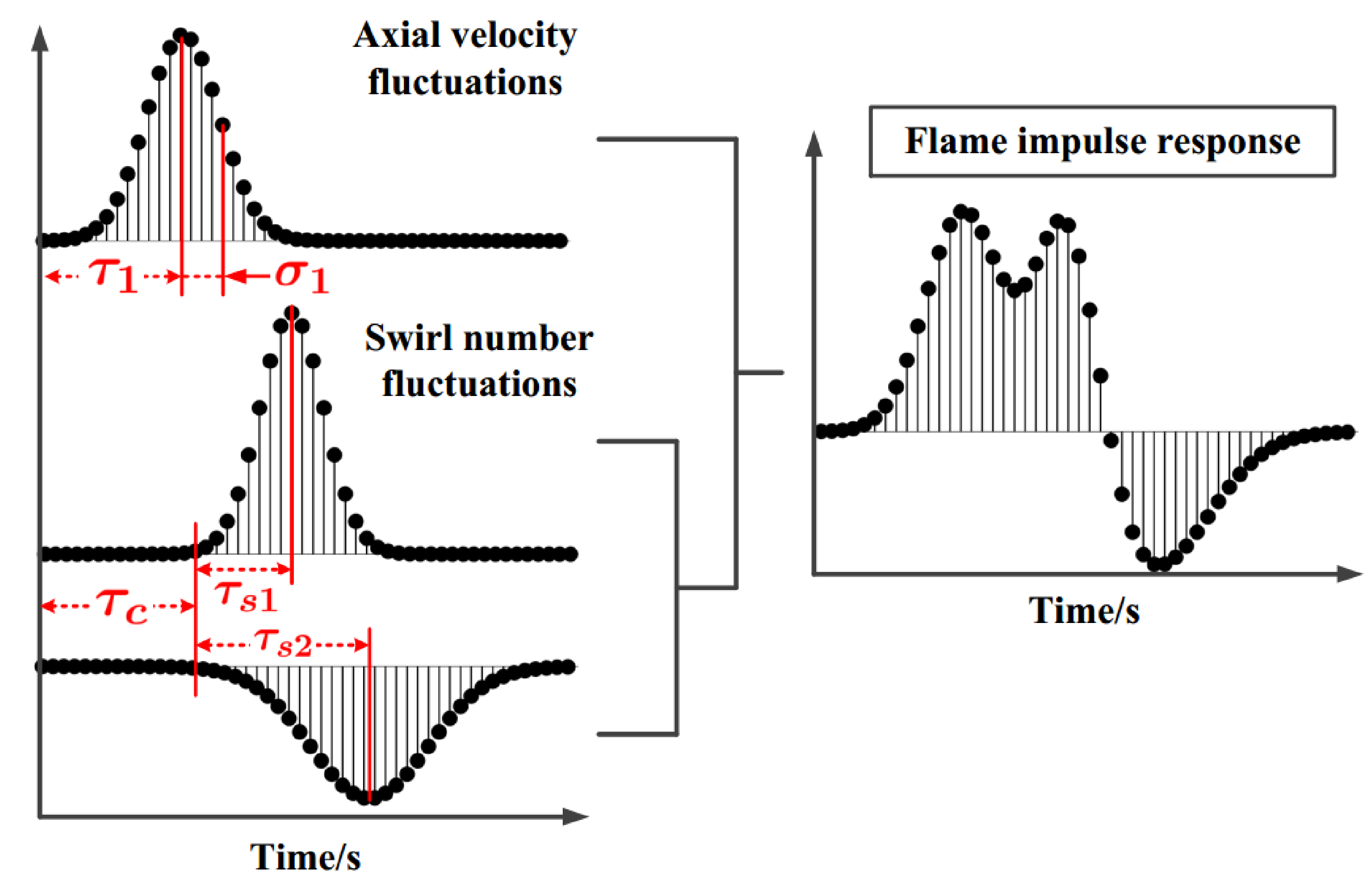

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	σ_1	$\sigma_1^0 = 0.7$	$0.9\sigma_1^0 \sim 1.1\sigma_1^0$
	τ_c	$\tau_c^0 = 3$	$2 \sim 4.8$
	τ_{s1}	$\tau_{s1}^0 = 1.8$	$0.9\tau_{s1}^0 \sim 1.1\tau_{s1}^0$
	τ_{s2}	$\tau_{s2}^0 = 3.3$	$0.9\tau_{s2}^0 \sim 1.1\tau_{s2}^0$
Acoustic BC	$\lvert R_{out}\rvert$	$\lvert R_{out}\rvert^0 = 0.9$	$0.6 \sim 1$

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Presentation overview

- Motivation
- Thermoacoustic problem settings
- Robust design tasks

Risk Analysis

Gaussian Process

→ Risk Diagram

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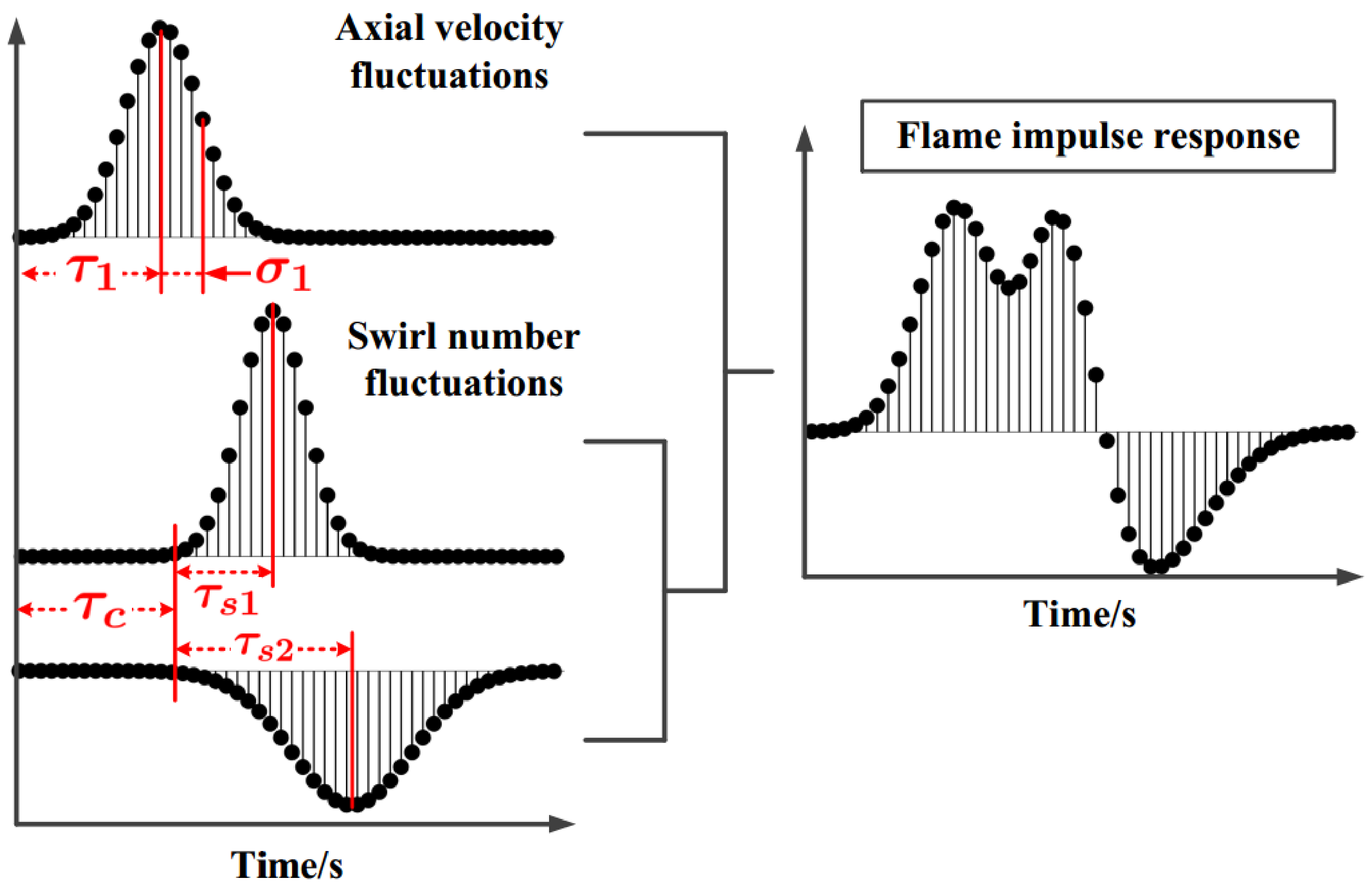
Risk analysis: setting the stage for the subsequent robust design analysis

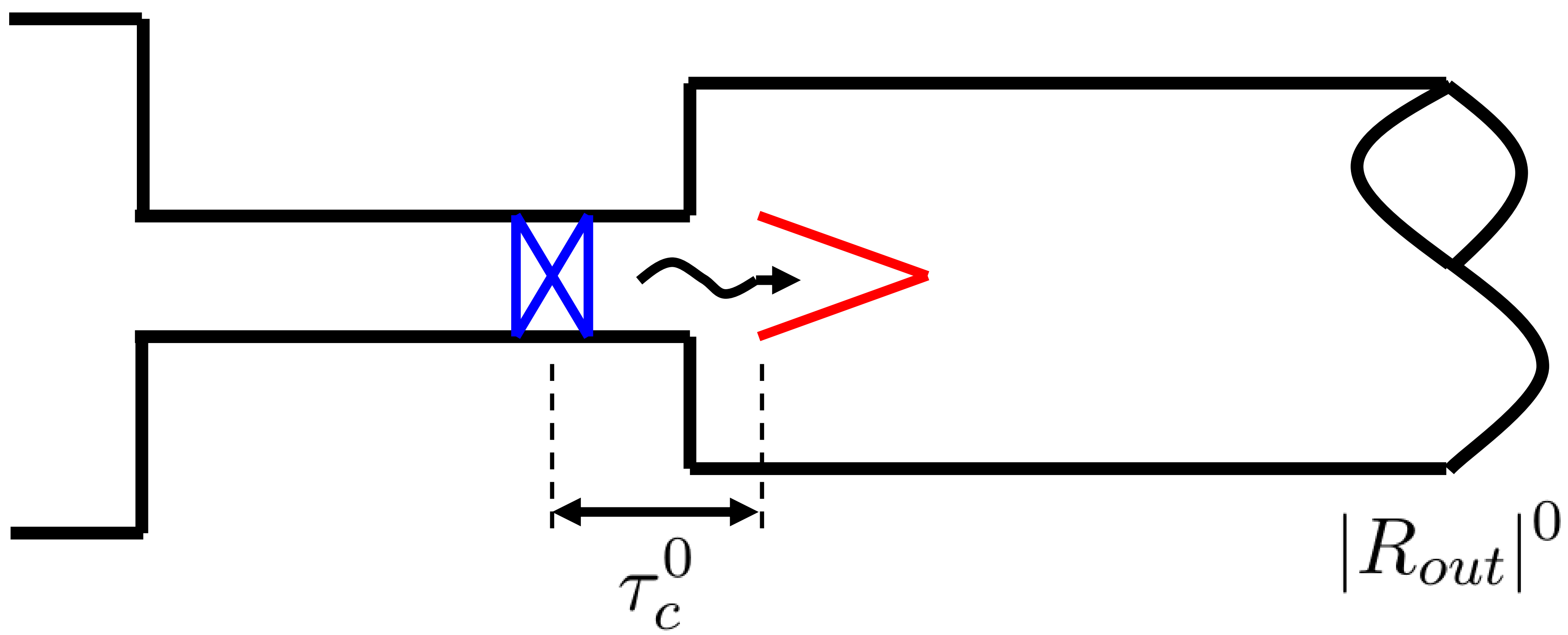
Flame model



Parameters		Nominal	Range
Flame (units: ms)	τ_1		$0.9\tau_1^0 \sim 1.1\tau_1^0$
	σ_1		$0.9\sigma_1^0 \sim 1.1\sigma_1^0$
	τ_c		
	τ_{s1}		$0.9\tau_{s1}^0 \sim 1.1\tau_{s1}^0$
	τ_{s2}		$0.9\tau_{s2}^0 \sim 1.1\tau_{s2}^0$
Acoustic BC	$\lvert R_{out}\rvert$		

"Q1: what is the risk factor of the system when uncertainties are presented in the flame parameter τ_1, σ_1, τ_{s1} and τ_{s2}?"

→ **Risk Analysis**



Known: $\tau_1, \sigma_1, \tau_{s1}, \tau_{s2} \sim \mathcal{U}$

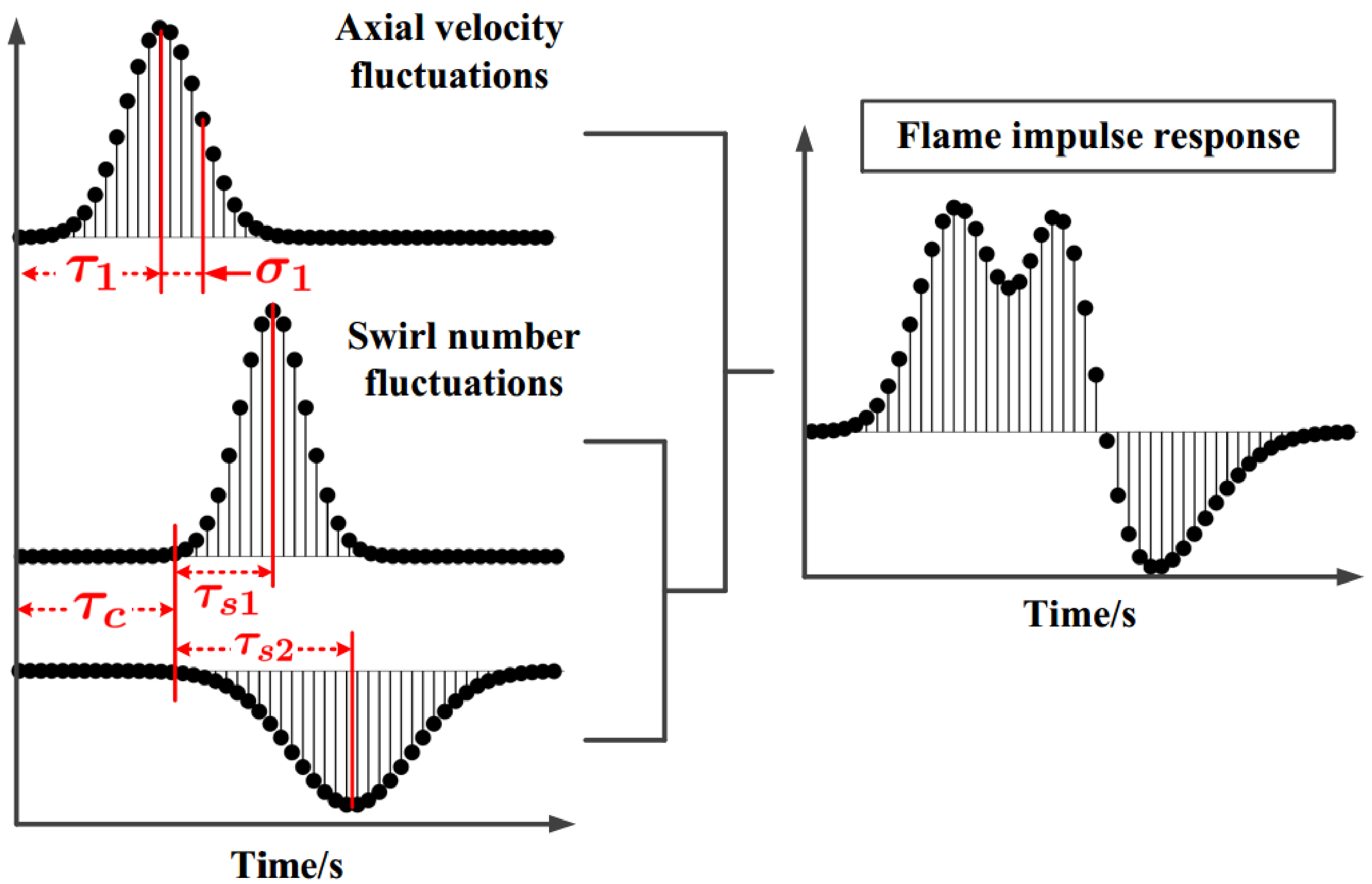
$\tau_c = \tau_c^*, \quad |R_{out}| = |R_{out}|^*$

Solve: $P_f^I = \int_0^\infty PDF(\alpha)d\alpha = ?$

$P_f^C = \int_0^\infty PDF(\alpha)d\alpha = ?$

Applying Monte Carlo directly on acoustic solvers is very expensive

Flame model



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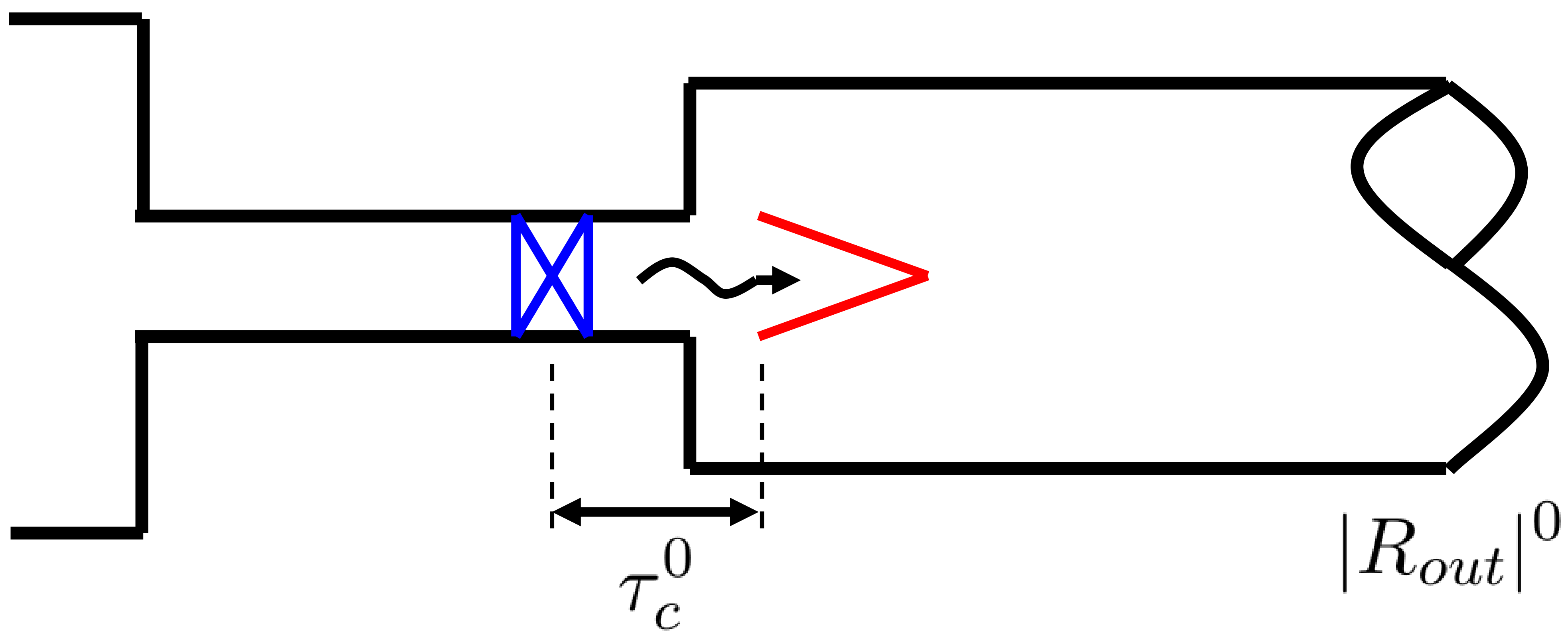
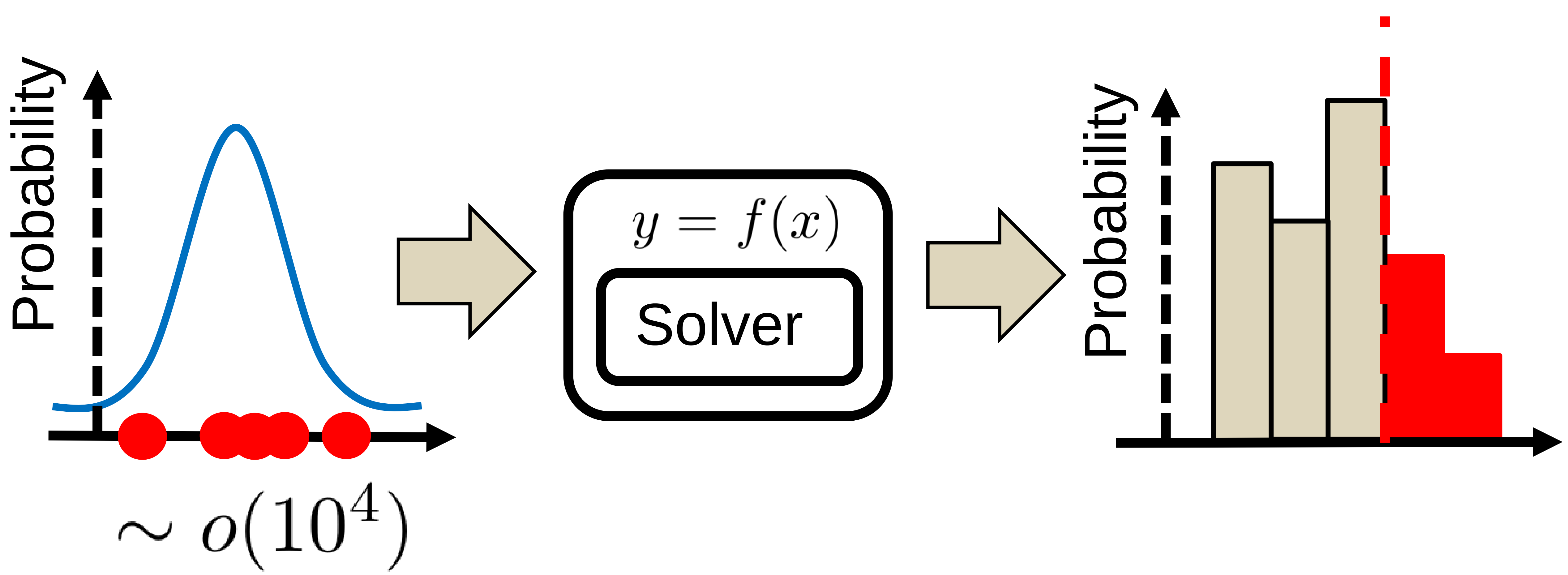
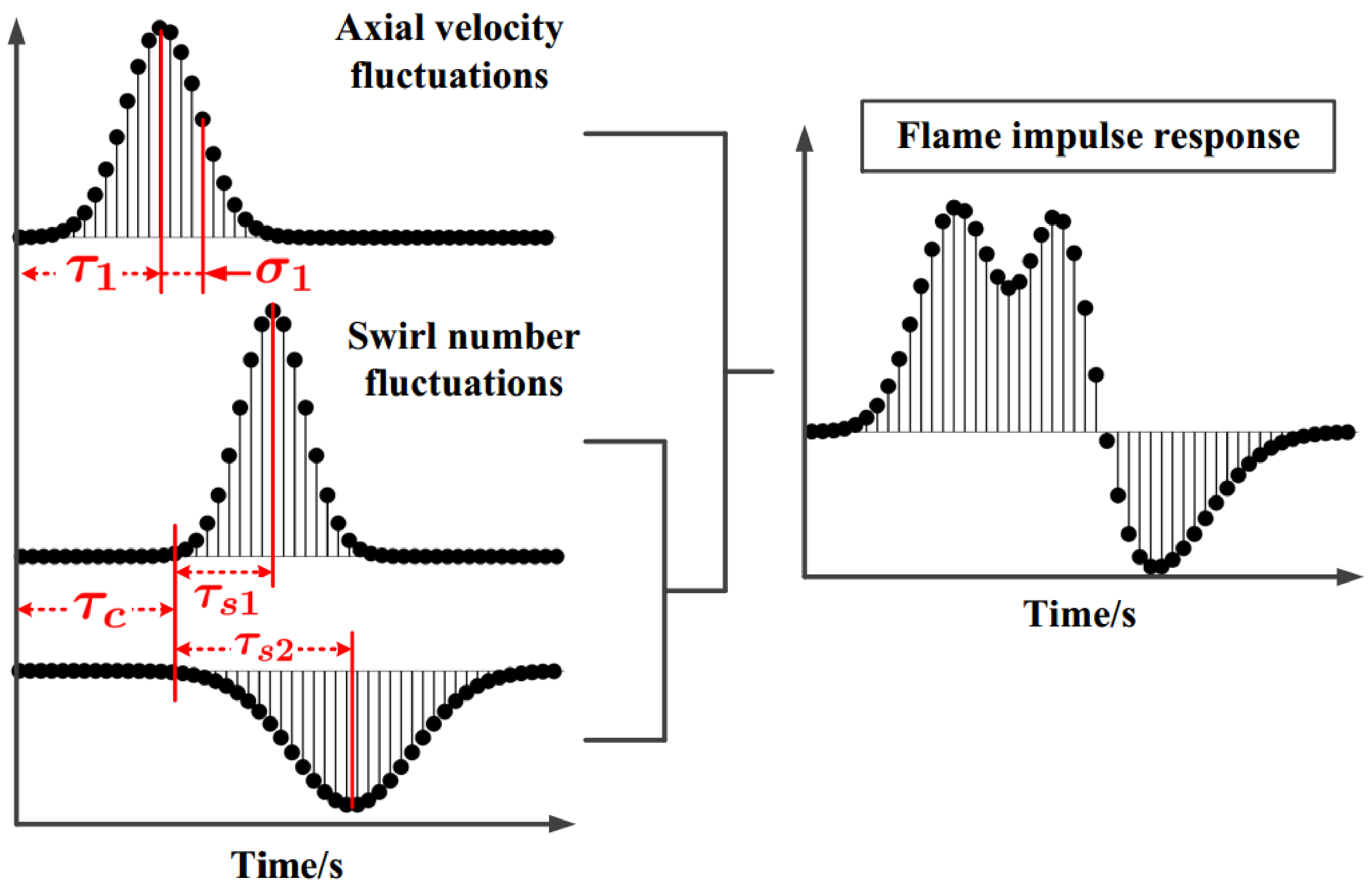




Surrogate modeling technique can significantly improve the efficiency of risk analysis

Flame model



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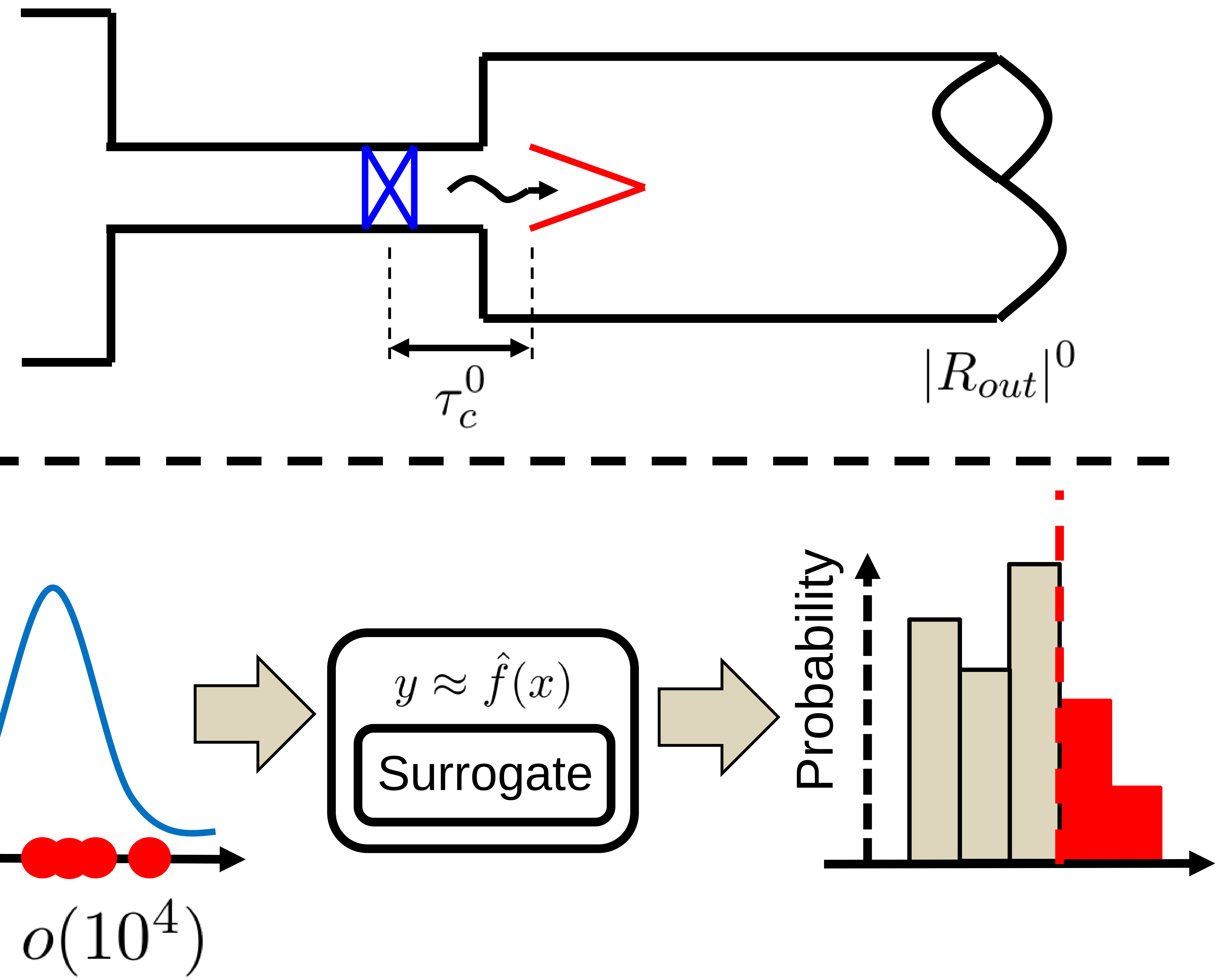
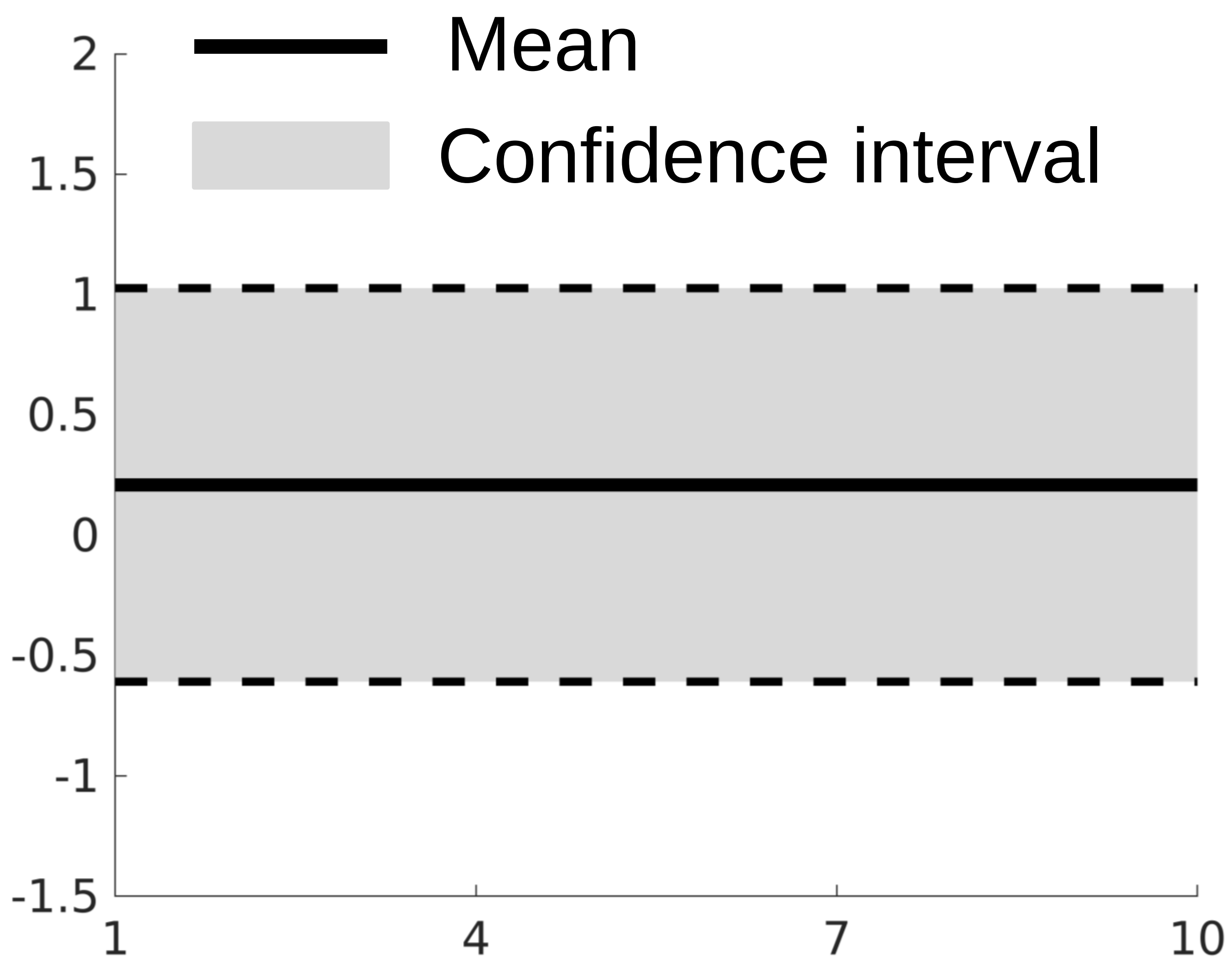
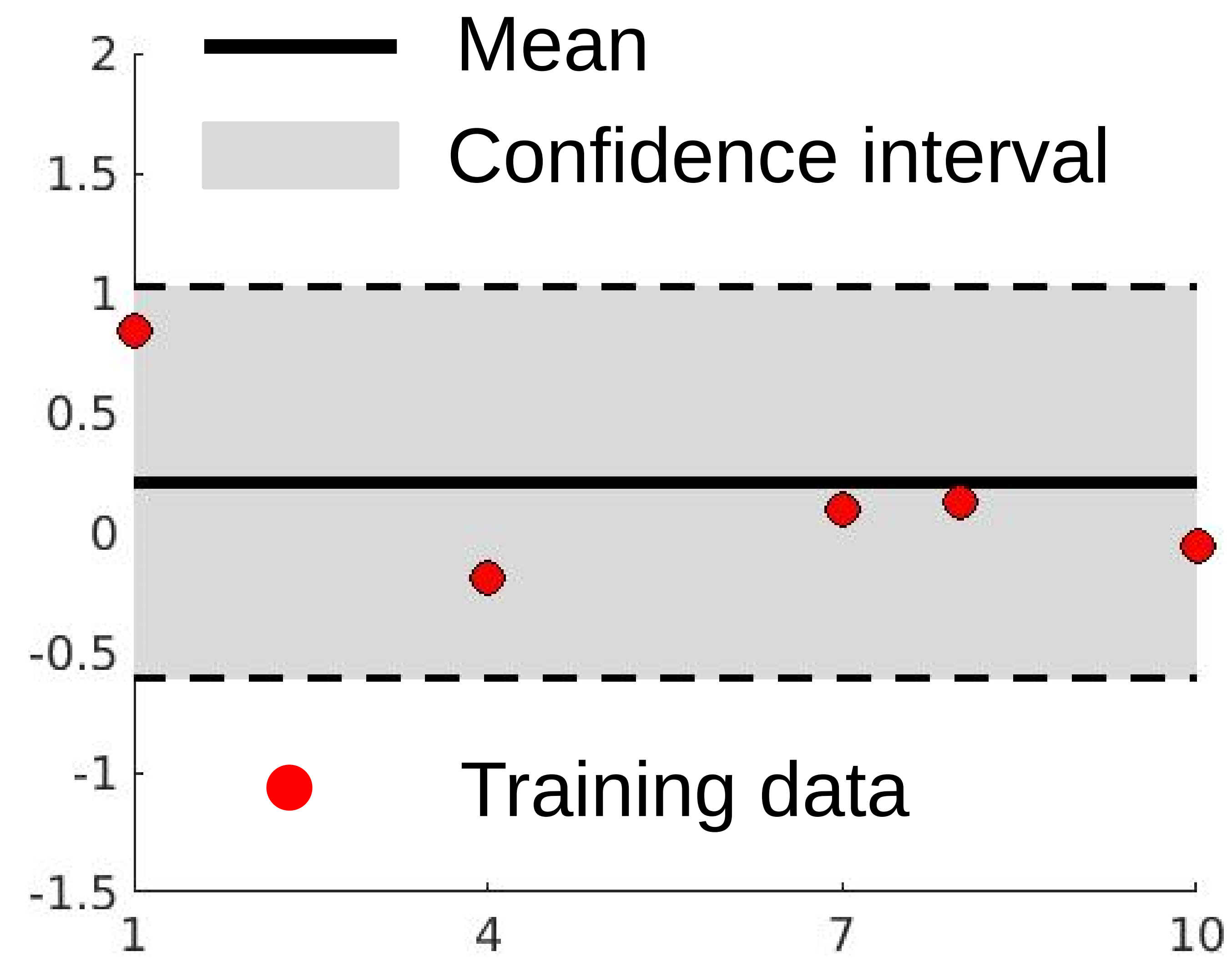
→ **Risk Analysis**



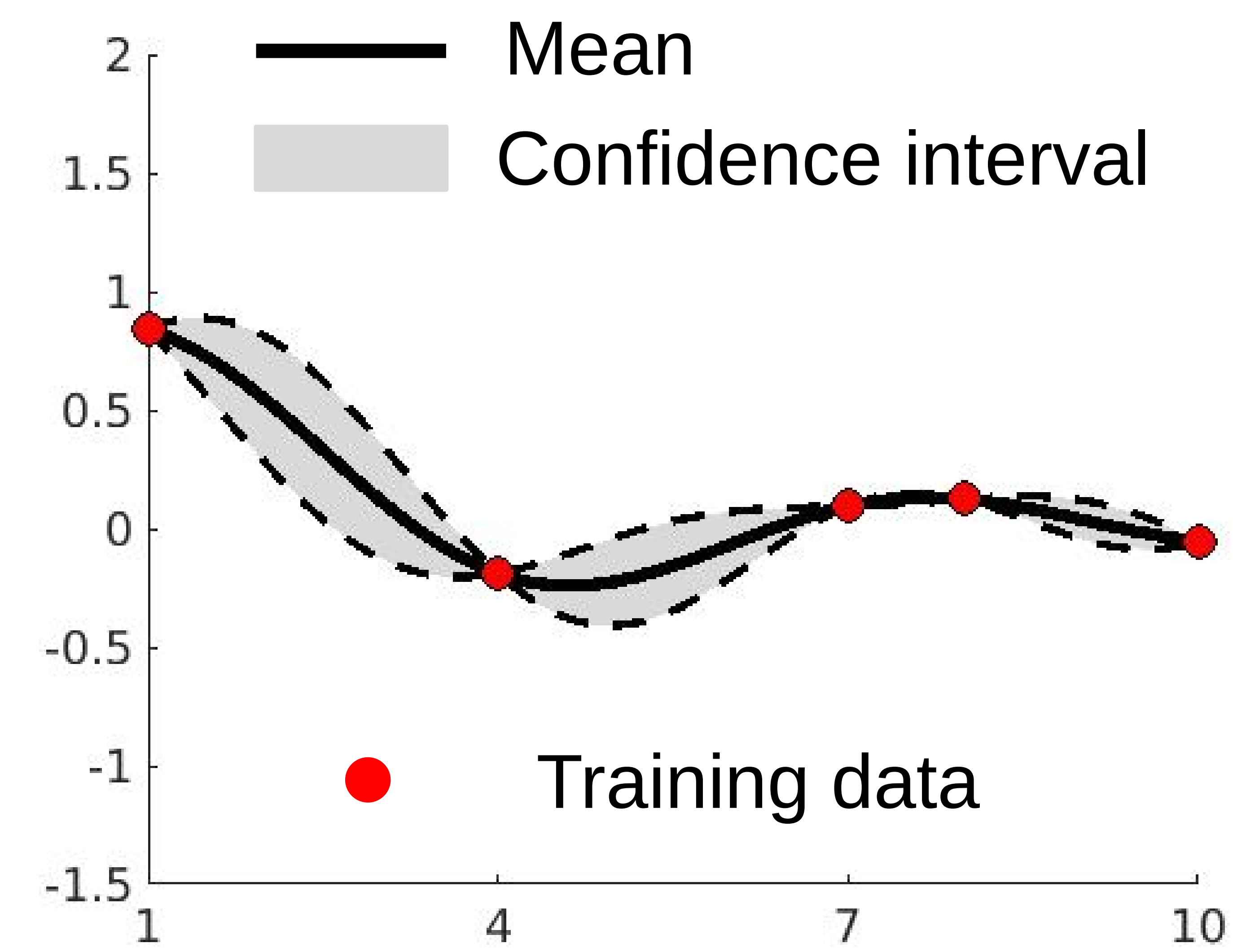
Gaussian Process is employed as the surrogate model in our study



Prior



Data



Posterior

$$f(x) \sim \mathcal{GP}(m(x), k(x,x'))$$

$$f^*(x) \sim \mathcal{GP}(m^*(x), k^*(x,x'))$$

$m(x)$: Polynomial

$k(x,x') = \sigma^2 \text{exp}(-\theta|x-x'|^2)$: Kernel

Gaussian Process is employed as the surrogate model in our study

Goal

$$\alpha^I \approx GP^I(\tau_1, \sigma_1, \tau_c, \tau_{s1}, \tau_{s2}, |R_{out}|)$$

$$\alpha^C \approx GP^C(\tau_1, \sigma_1, \tau_c, \tau_{s1}, \tau_{s2}, |R_{out}|)$$

Training

→ A total of 102 samples are used

→ Reuse for all design tasks

Gaussian Process models have delivered highly accurate risk analysis

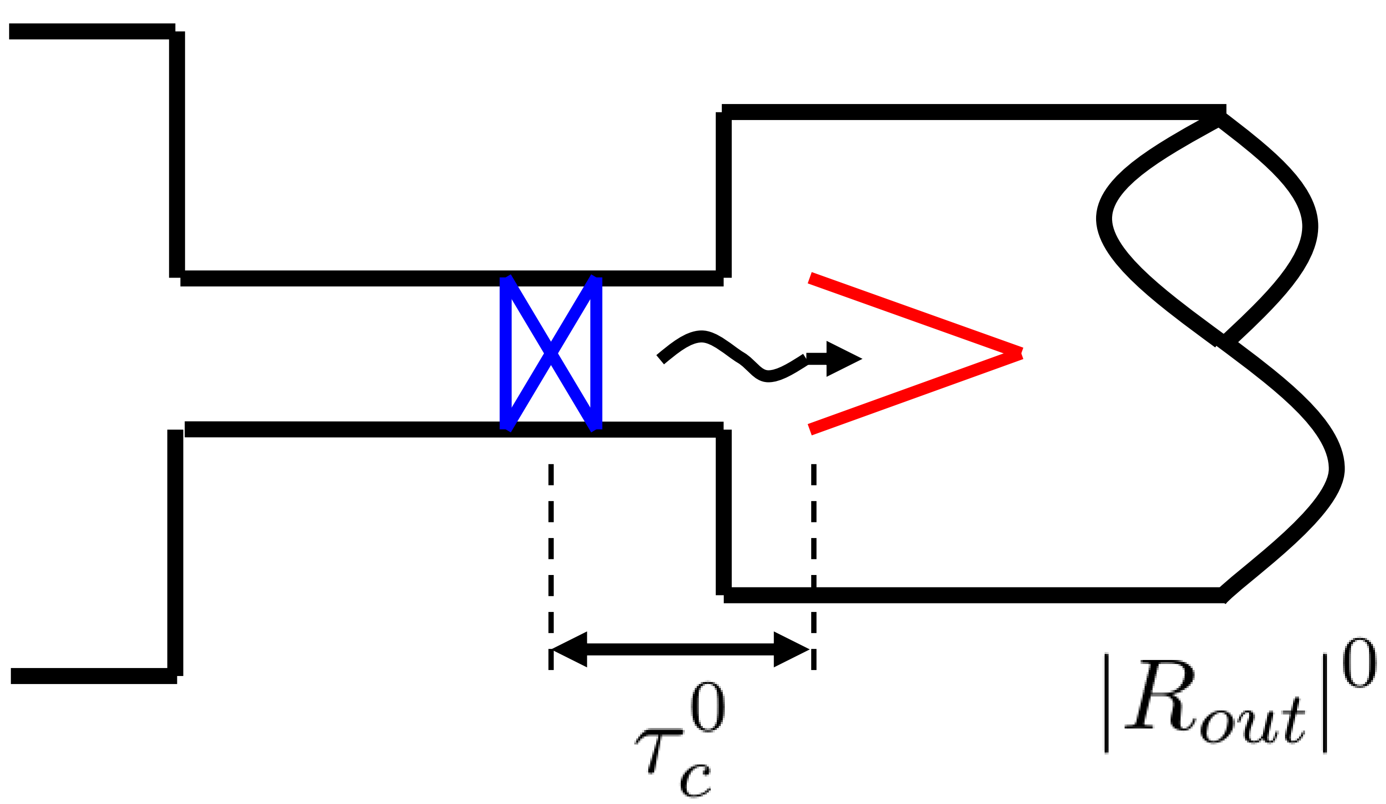
Q1: Risk Analysis

Known: $\tau_1, \sigma_1, \tau_{s1}, \tau_{s2} \sim \mathcal{U}$

$\tau_c = \tau_c^0, \ |R_{out}| = |R_{out}|^0$

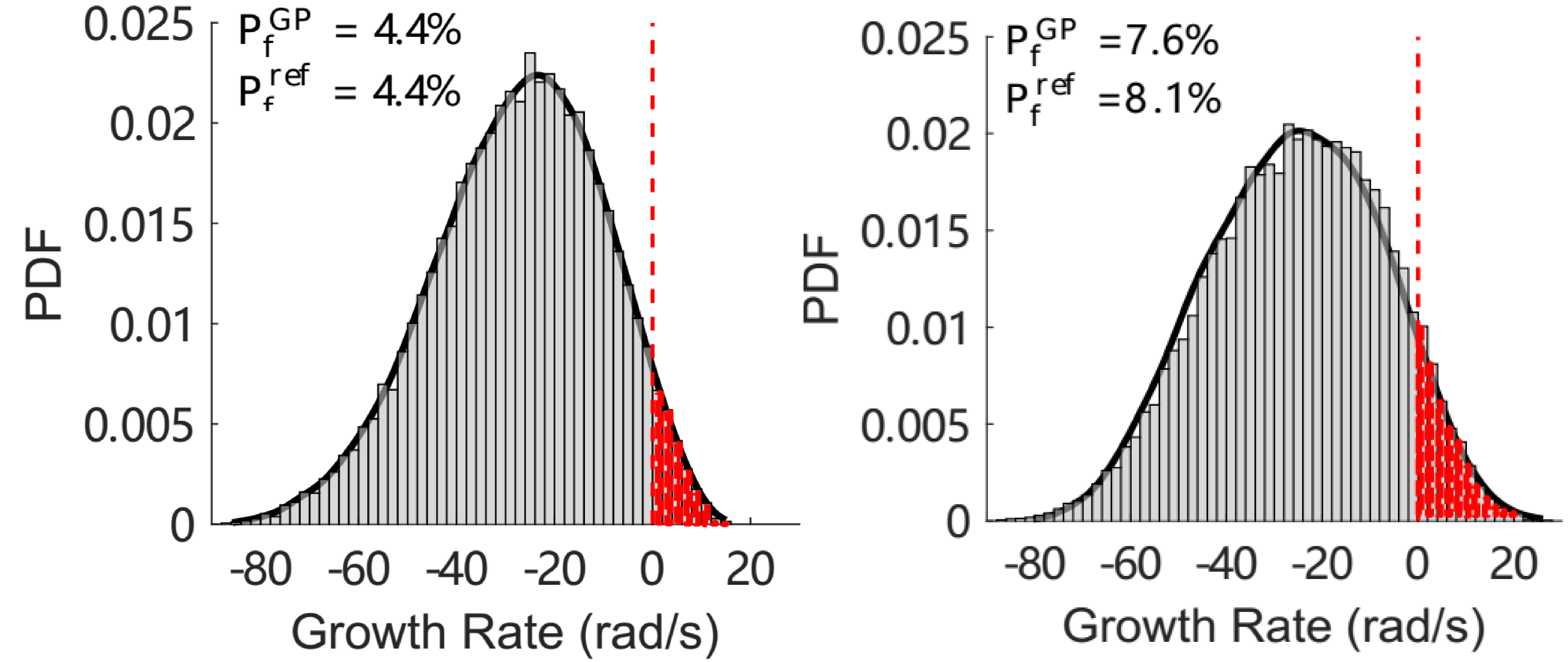
Solve: $P_f^I = \int_0^\infty PDF(\alpha) d\alpha = ?$

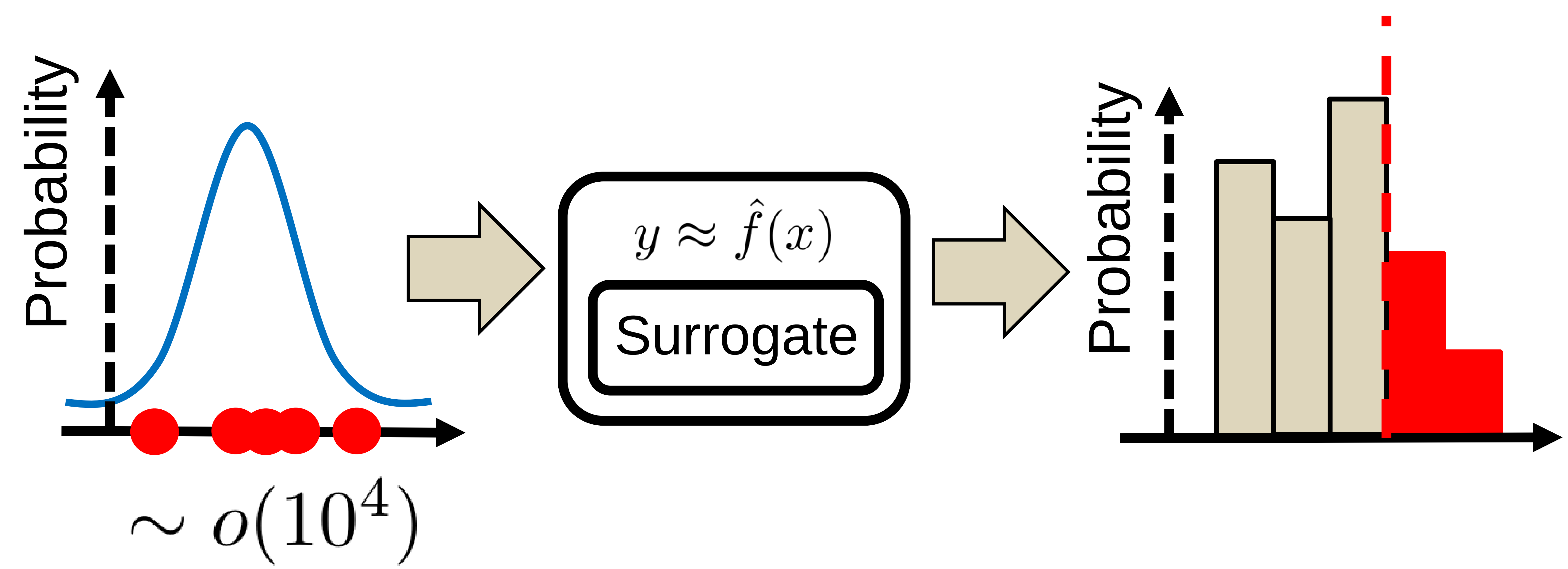
$P_f^C = \int_0^\infty PDF(\alpha) d\alpha = ?$



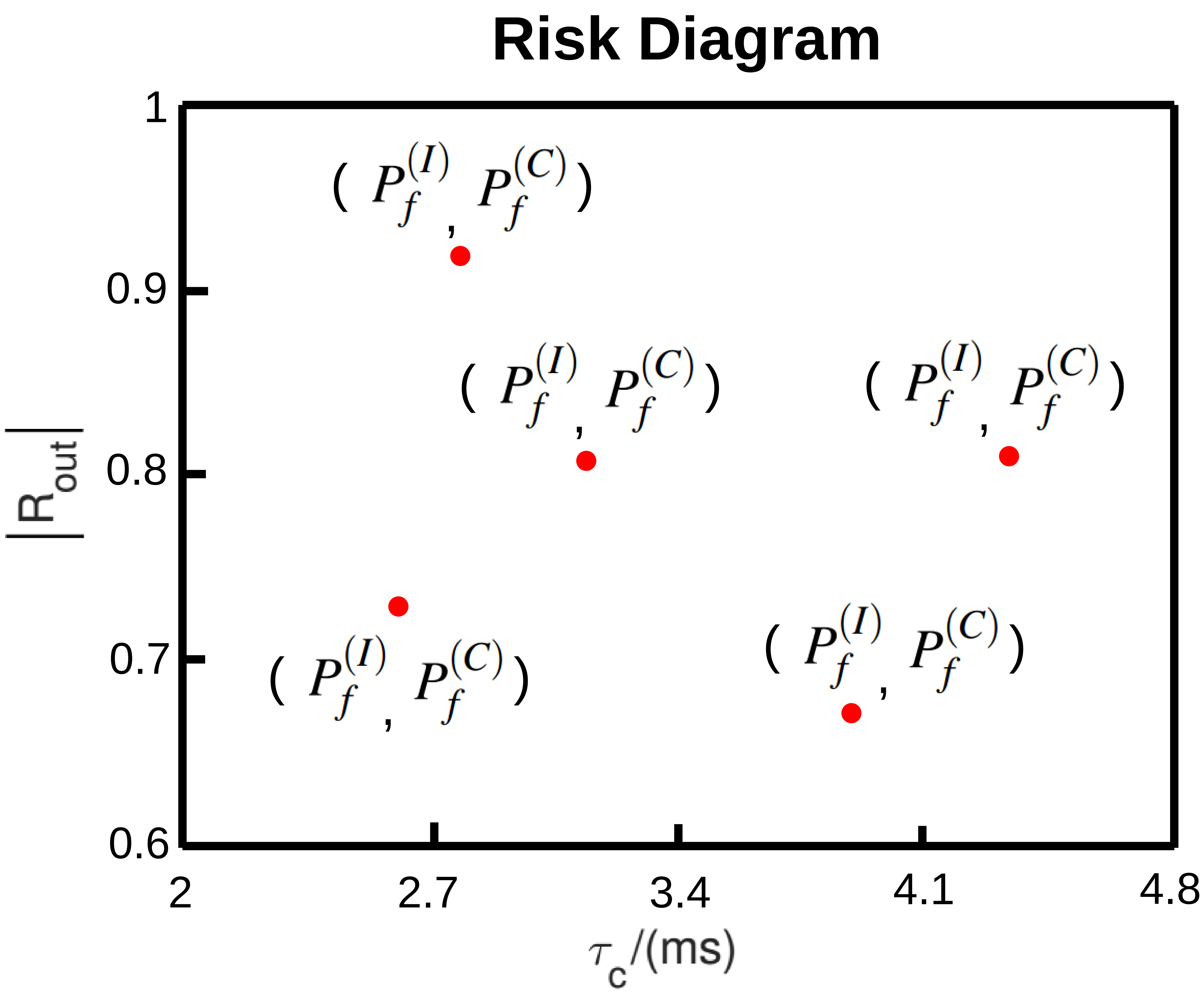
ITA Mode

Cavity Mode





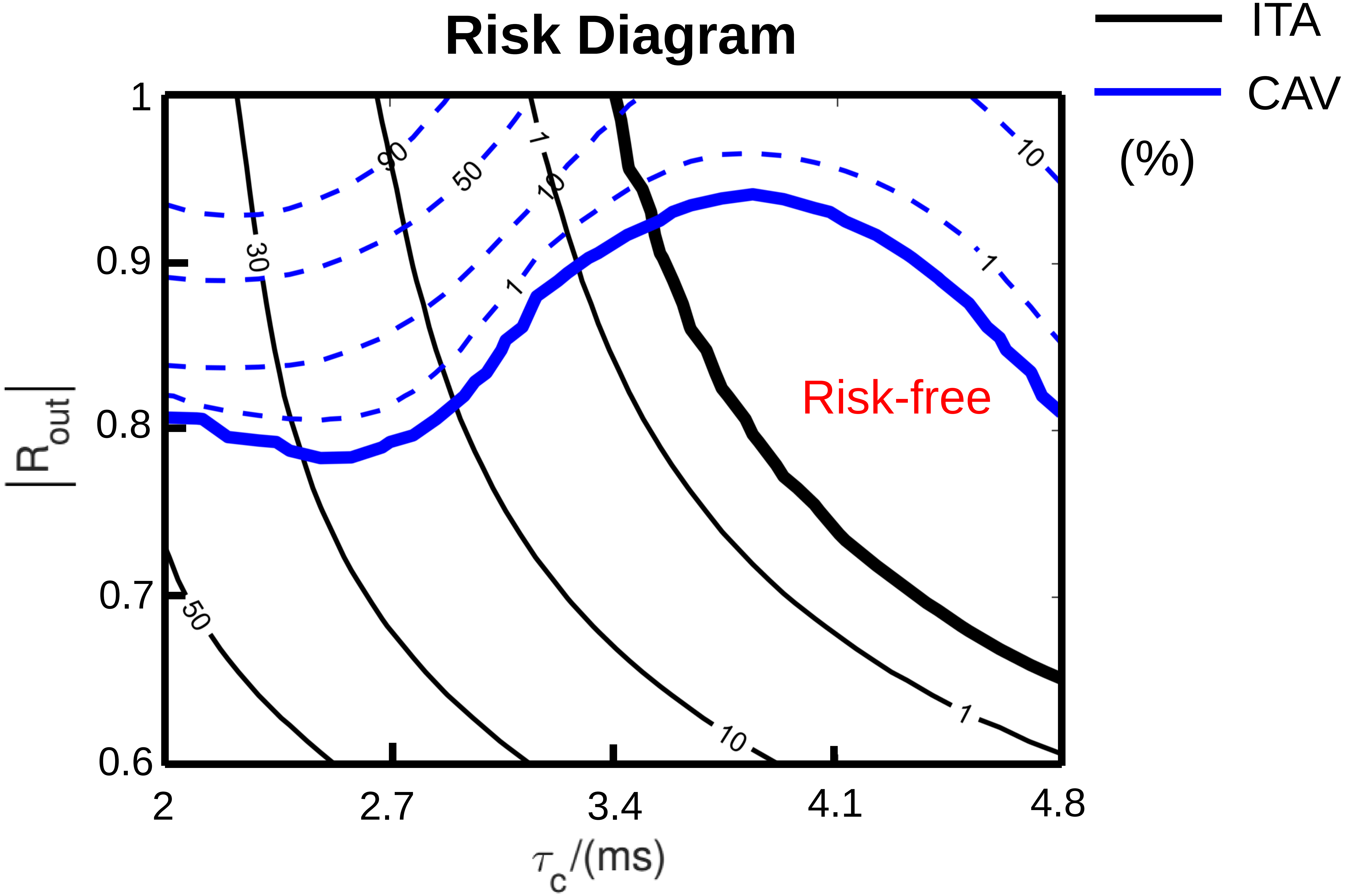
Risk diagram visualizes the risk pattern over the entire parameter space



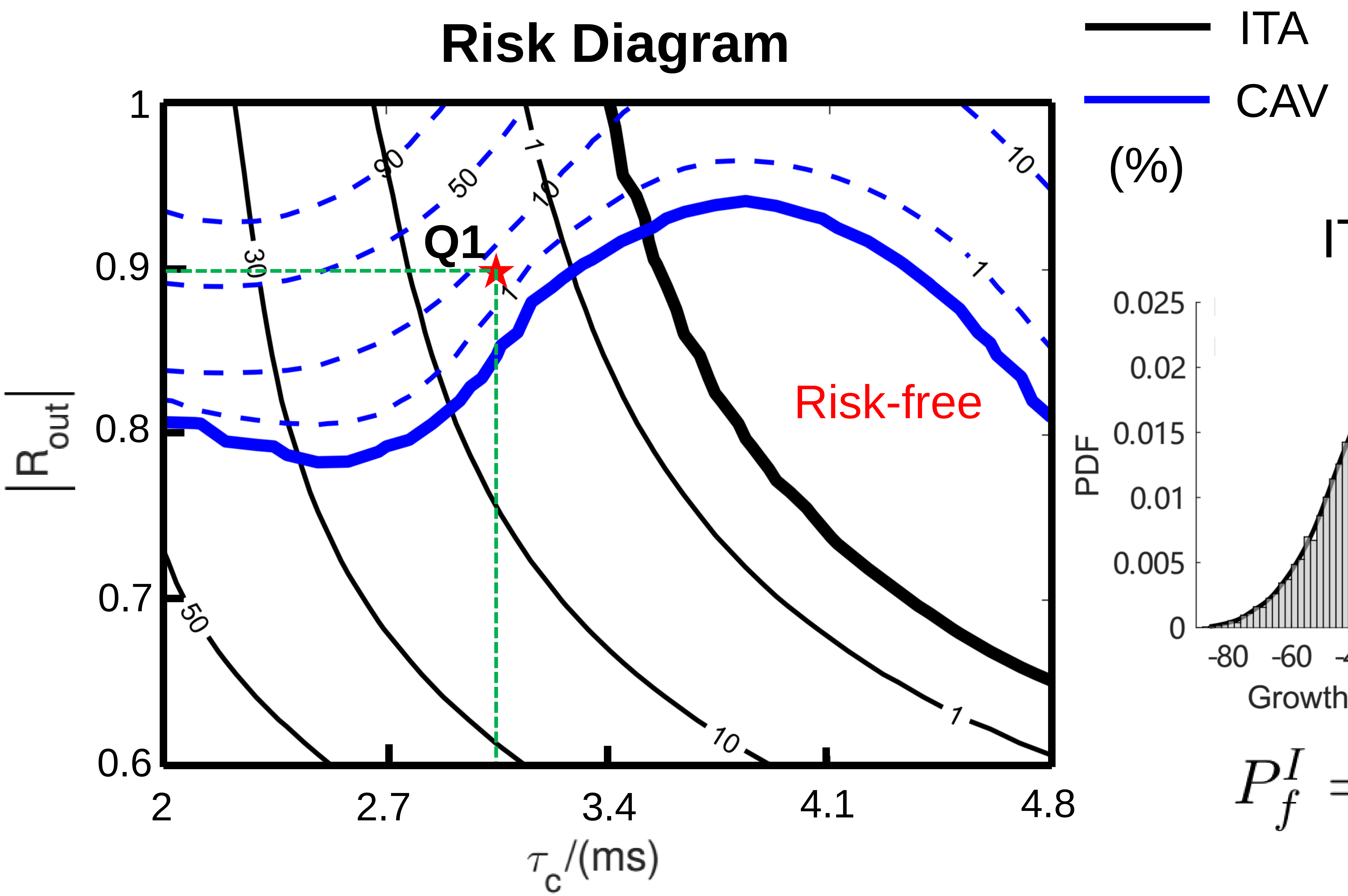
Parameters		Nominal	Range
	τ_1		$0.9\tau_1^0 \sim 1.1\tau_1^0$
Flame (units: ms)	σ_1		$0.9\sigma_1^0 \sim 1.1\sigma_1^0$
	τ_{s1}		$0.9\tau_{s1}^0 \sim 1.1\tau_{s1}^0$
	τ_{s2}		$0.9\tau_{s2}^0 \sim 1.1\tau_{s2}^0$
Acoustic BC			

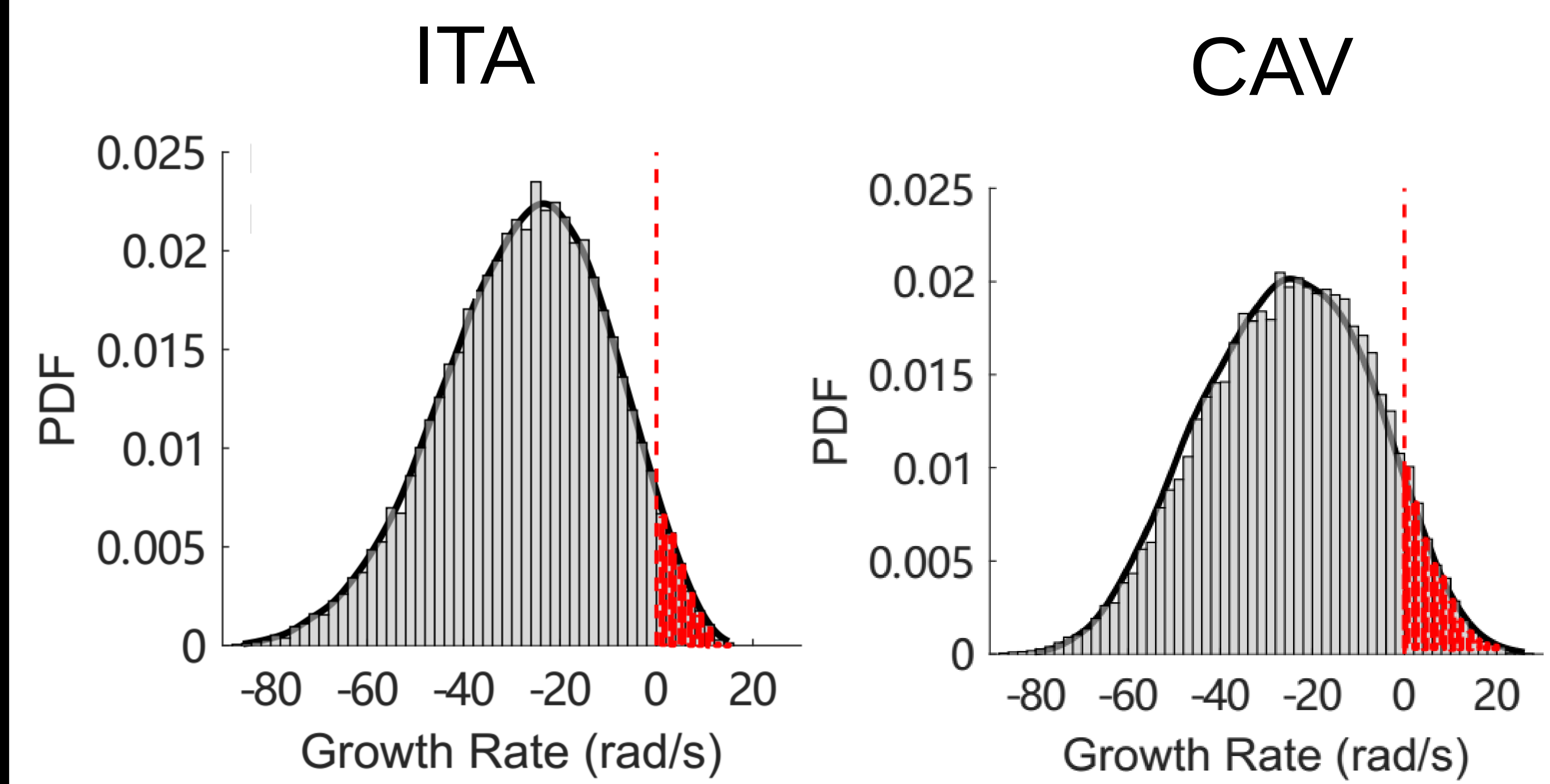
$$\tau_1, \sigma_1, \tau_{s1}, \tau_{s2} \sim \mathcal{U}$$

Risk diagram visualizes the risk pattern over the entire parameter space



Each point in risk diagram has two associated PDFs





$$P_f^I = 4.4\%$$

$$P_f^C = 7.8\%$$

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Risk Analysis
Gaussian Process

→ Risk Diagram

Ideal Control Design
Realistic Control Design
Tolerance Design

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Risk Diagram

Both modes have certain level of risk to be unstable

Q1: Risk Analysis

Known: $\tau_1, \sigma_1, \tau_{s1}, \tau_{s2} \sim \mathcal{U}$

$\tau_c = \tau_c^0, |R_{out}| = |R_{out}|^0$

Solve: $P_f^I = \int_0^\infty PDF(\alpha)d\alpha = ?$

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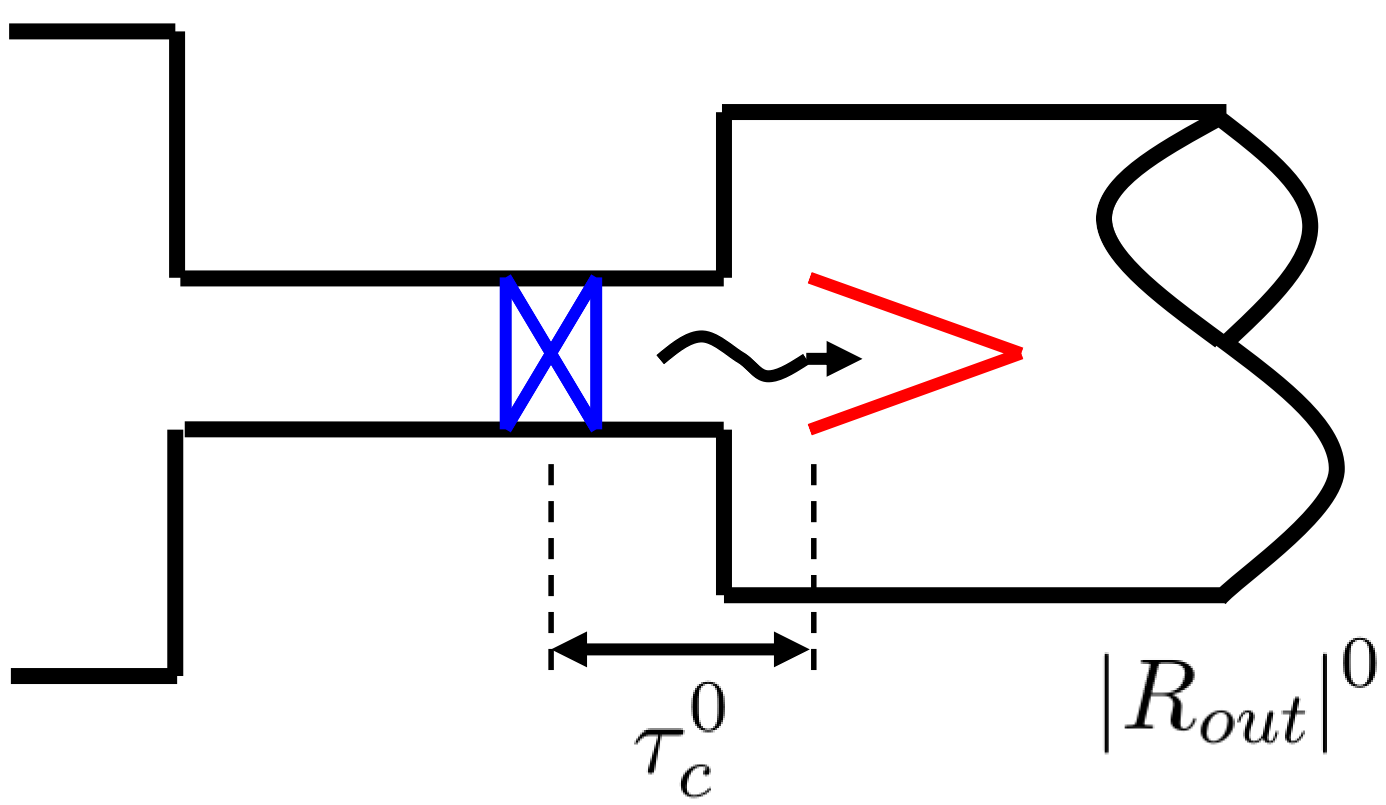
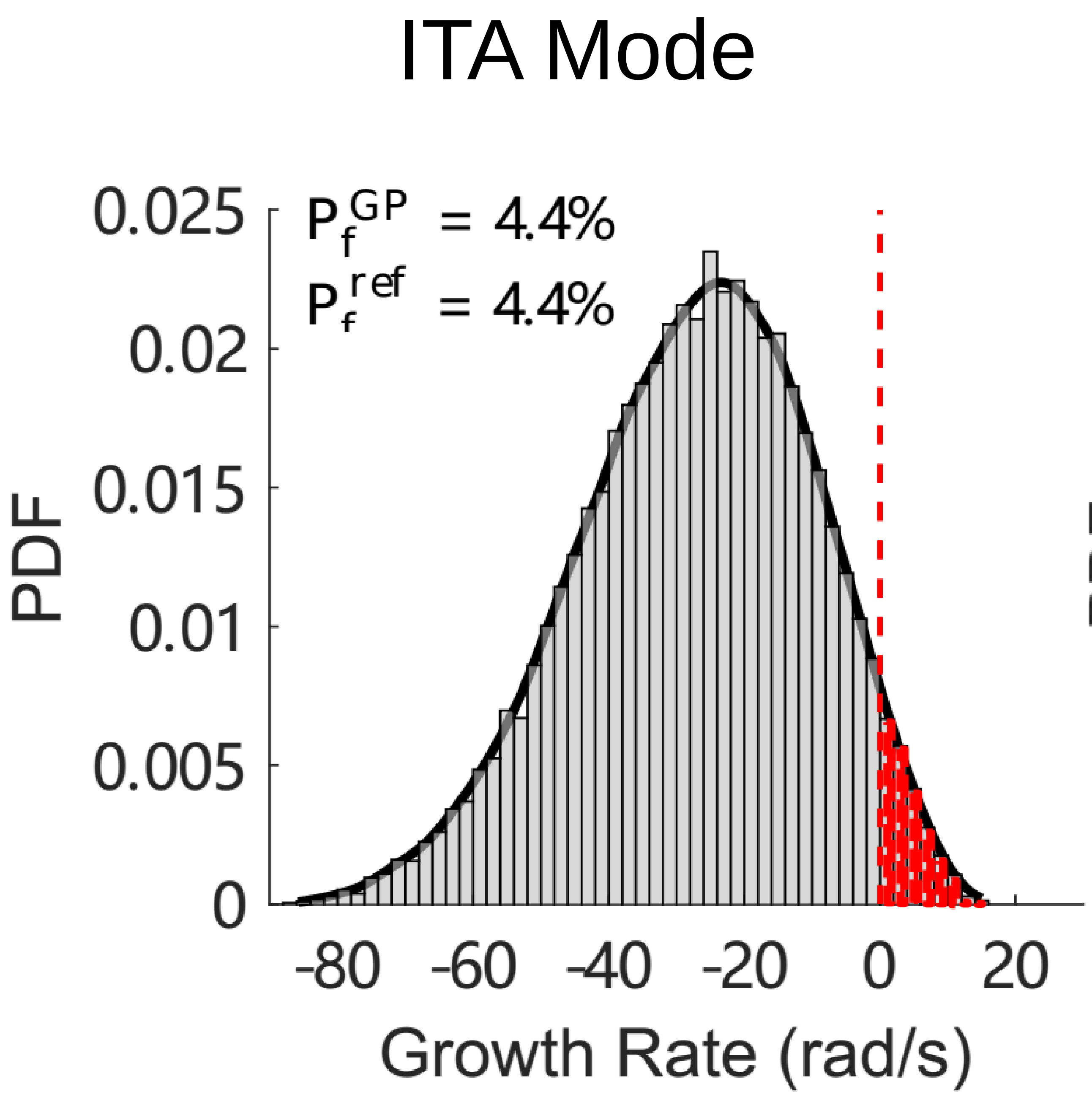
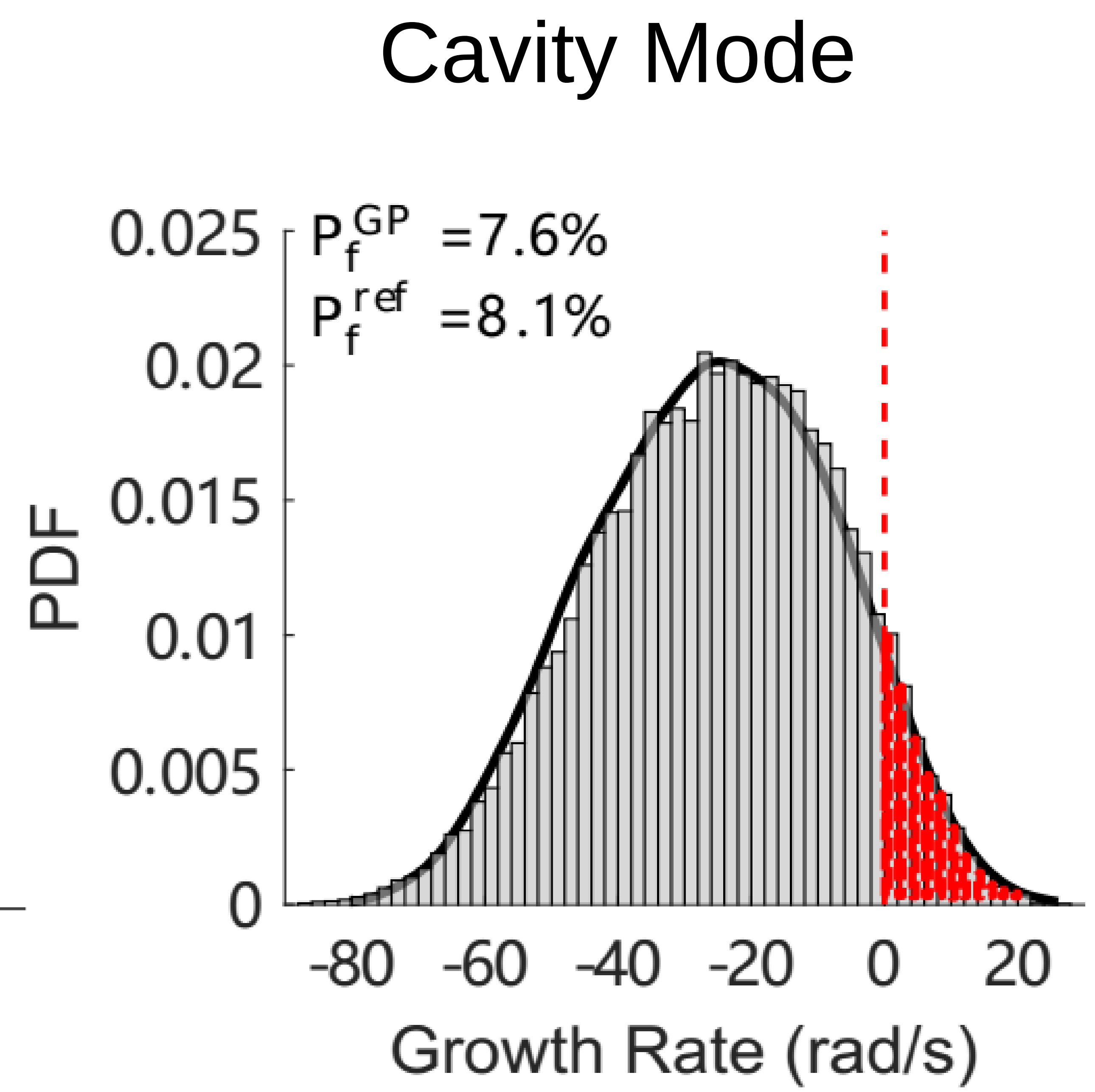
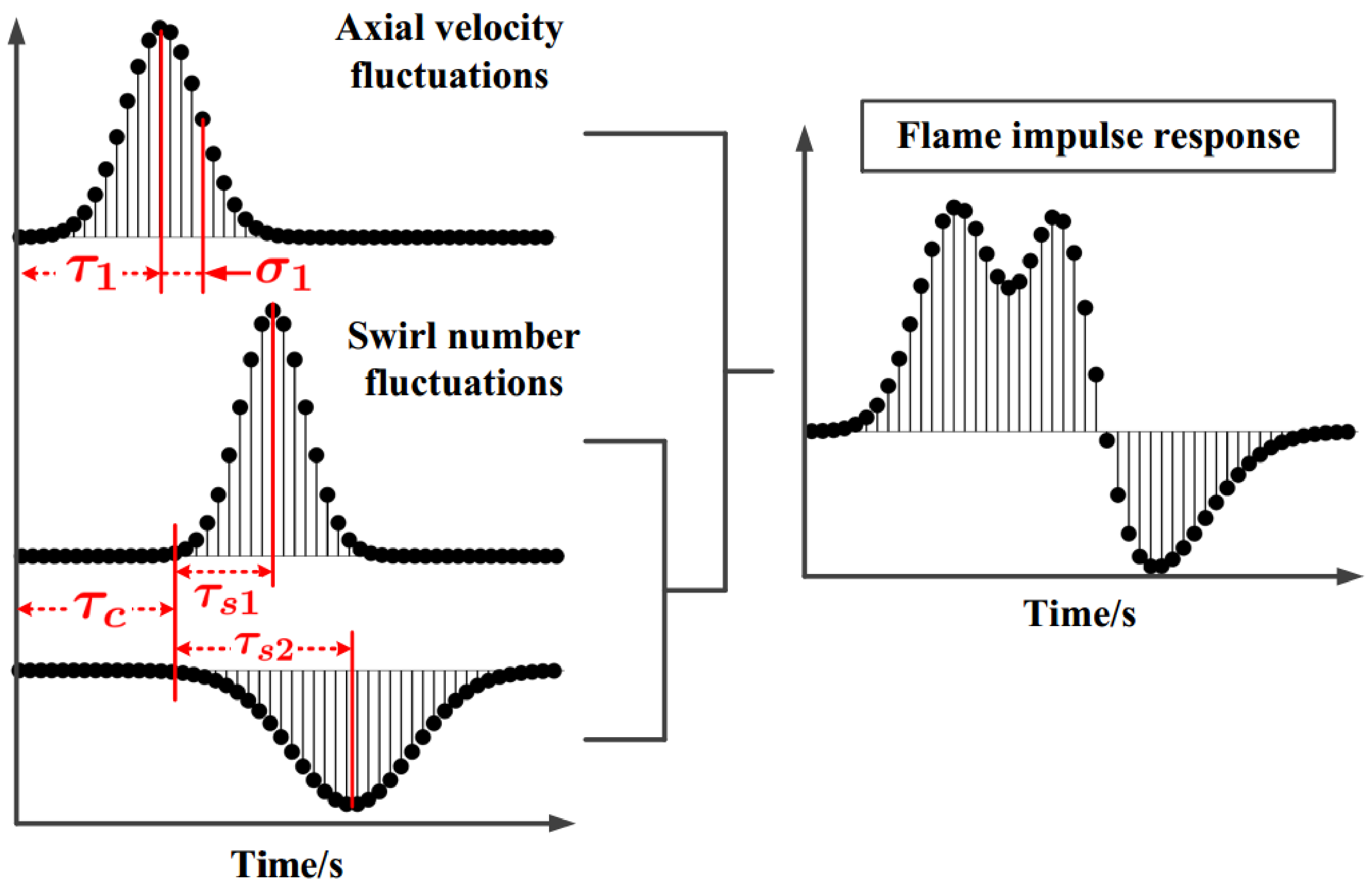






Ideal control design: A first step towards risk mitigation

Flame model

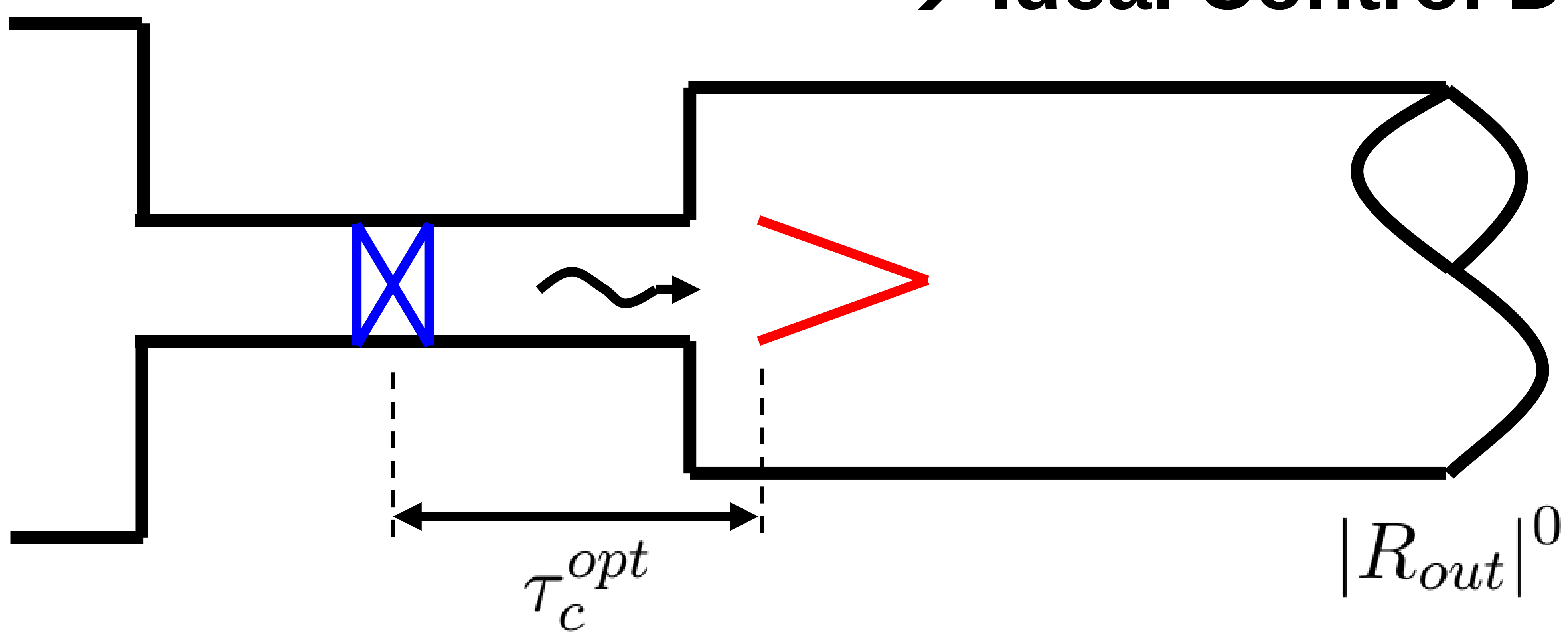


Known: $\tau_1, \sigma_1, \tau_{s1}, \tau_{s2} \sim \mathcal{U}$

$|R_{out}| = |R_{out}|^0$

"Q2: using τ_c as a control factor, what is the required minimum modification of τ_c to eliminate the risk of instability of both cavity and ITA mode simultaneously?"

→ Ideal Control Design



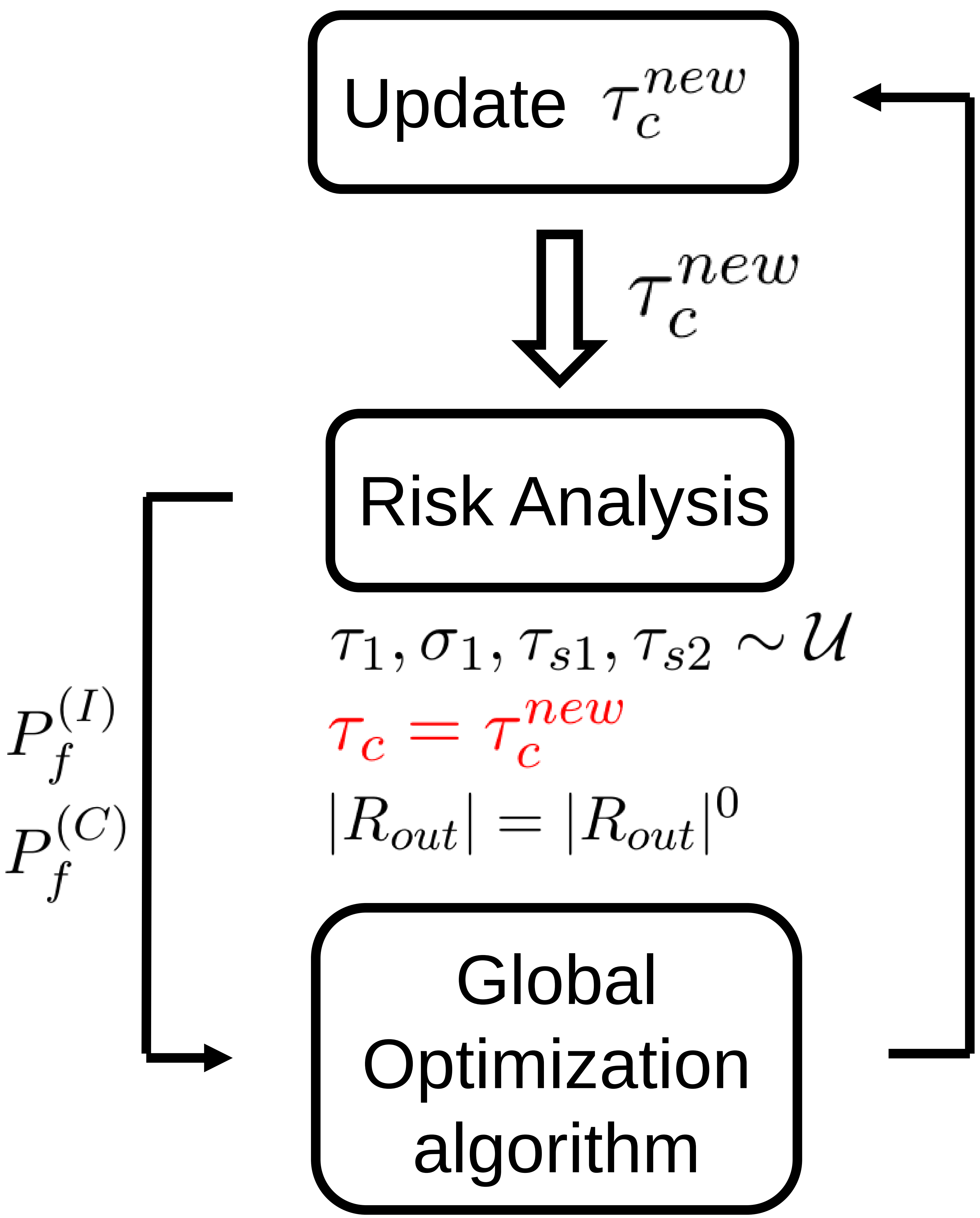
$$\min_{\tau_c} f(\tau_c) = (\tau_c - \tau_c^0)^2$$

$$\text{subject to}: \quad P_f^{(I)}(\tau_c) \leq 0.1\%$$

$$P_f^{(C)}(\tau_c) \leq 0.1\%$$

$$2\text{ms} \leq \tau_c \leq 4.8\text{ms}$$

Ideal control design: A first step towards risk mitigation



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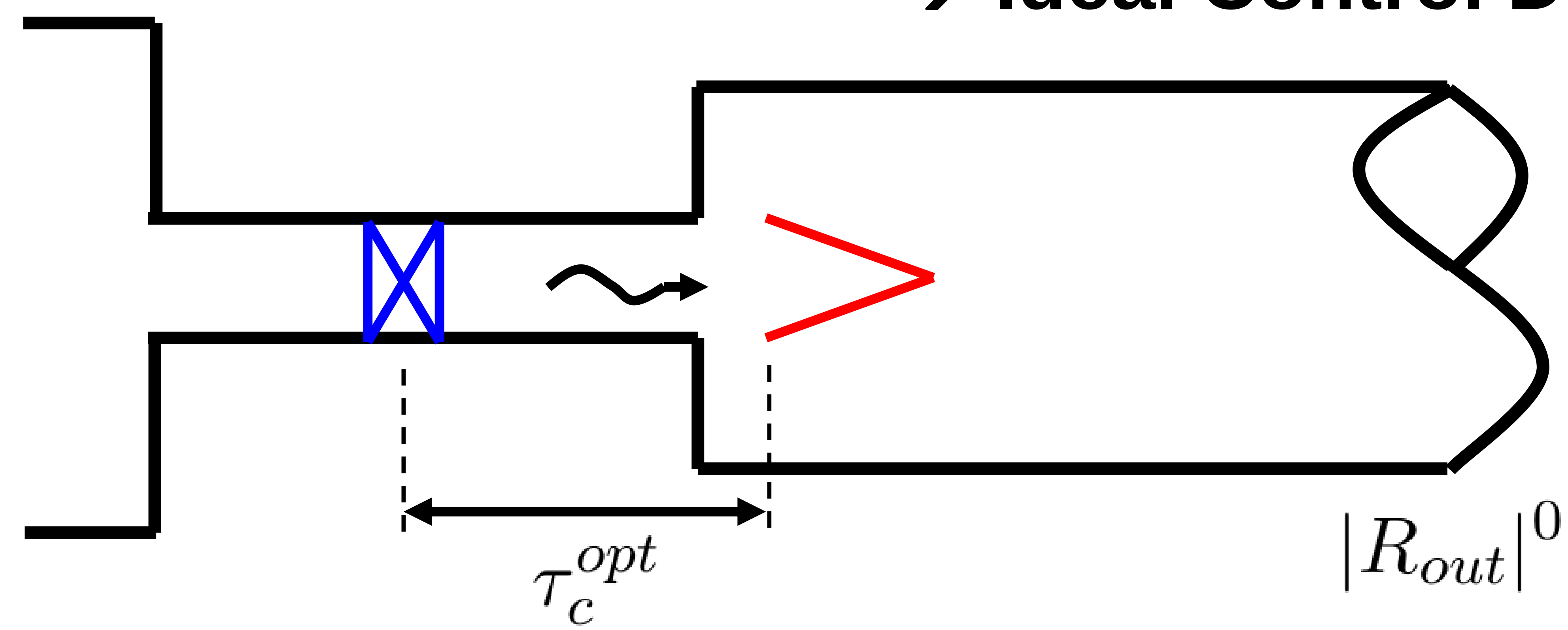

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$$\text{subject to :} \quad P_f^{(I)}(\tau_c) \leq 0.1\%$$

$$P_f^{(C)}(\tau_c) \leq 0.1\%$$

$$2\text{ms} \leq \tau_c \leq 4.8\text{ms}$$

Gaussian Process models have delivered highly accurate design

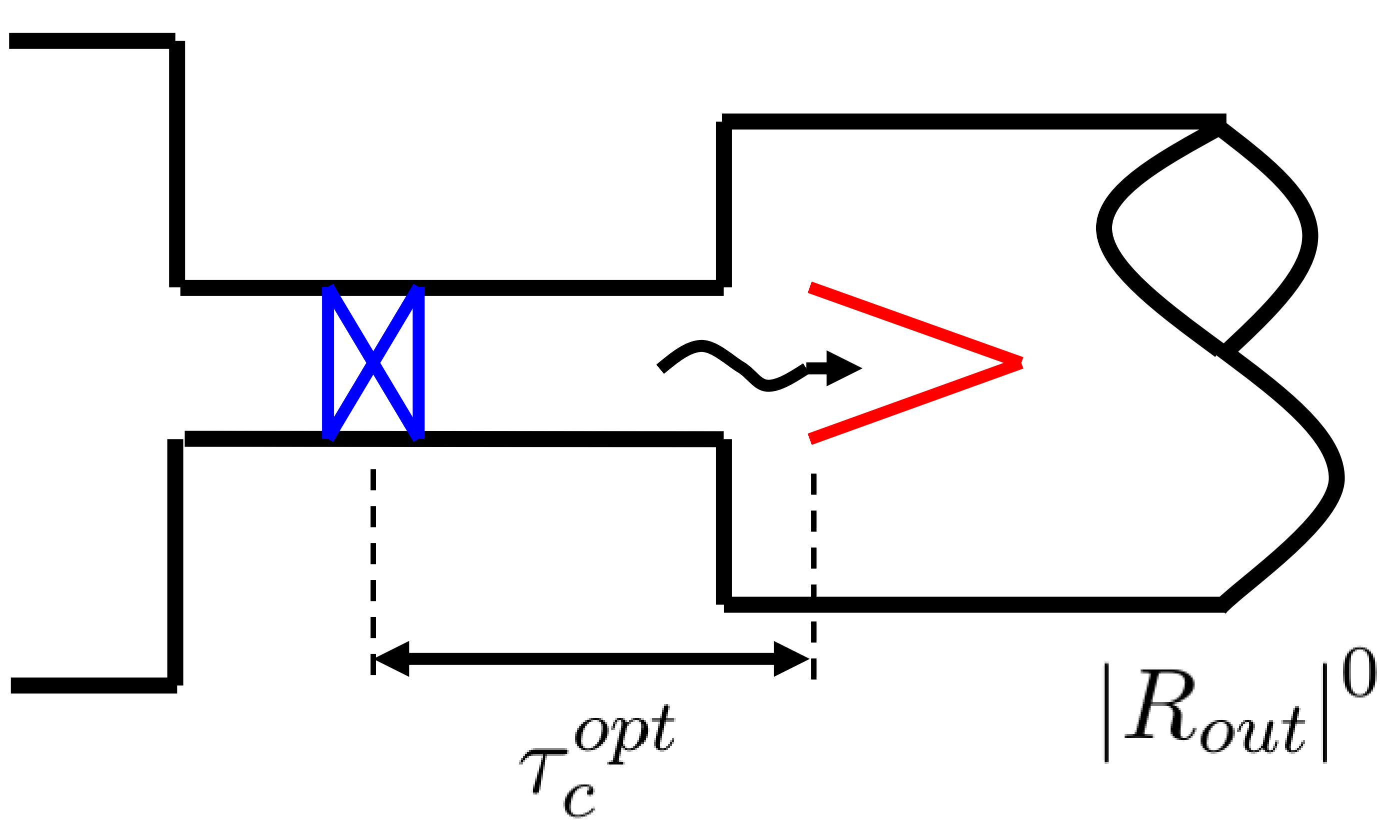
Q2: Ideal control design

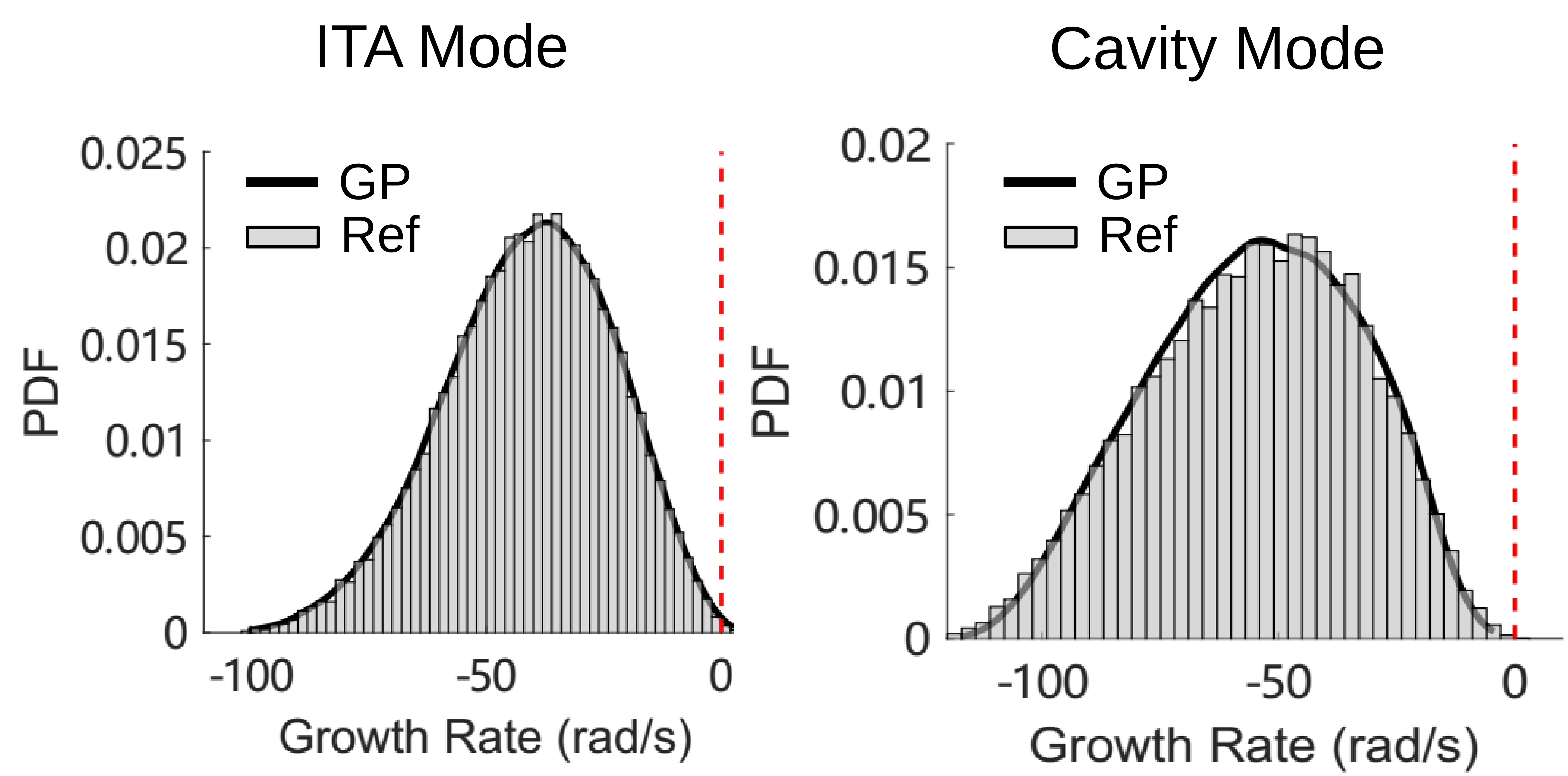
$$\min_{\tau_c} f(\tau_c) = (\tau_c - \tau_c^0)^2$$

$$\text{subject to :} \quad P_f^{(I)}(\tau_c) \leq 0.1\%$$

$$P_f^{(C)}(\tau_c) \leq 0.1\%$$

$$2\text{ms} \leq \tau_c \leq 4.8\text{ms}$$



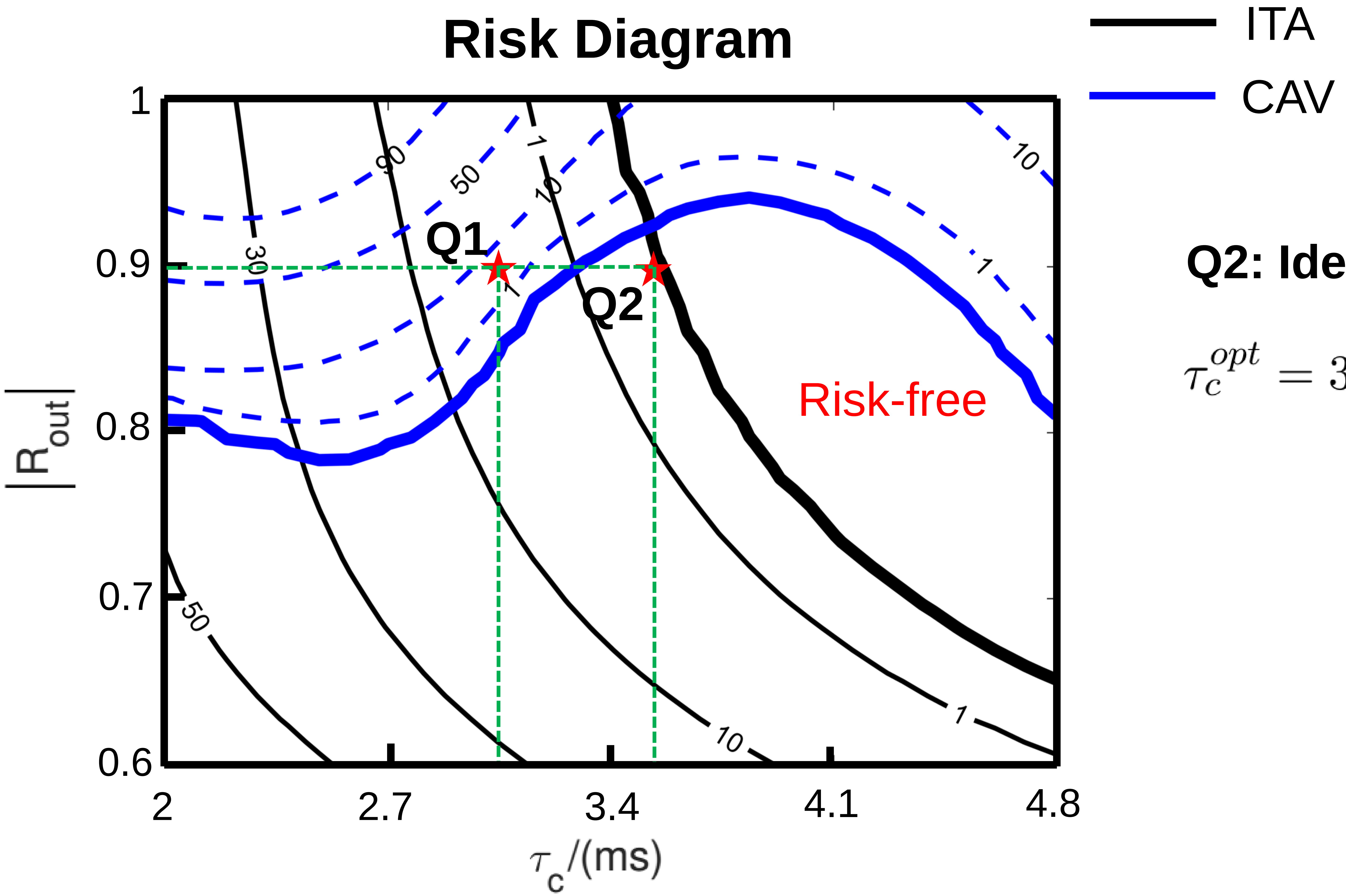


$$\tau_c^{opt} = 3.52\ ms \quad (\tau_c^0 = 3\ ms)$$

$$\tau_1, \sigma_1, \tau_{s1}, \tau_{s2} \sim \mathcal{U}$$

$$|R_{out}| = |R_{out}|^0$$

Risk diagram offers straightforward determination of the optimum design



Q2: Ideal control design

$$\tau_c^{opt} = 3.52\ ms$$

Presentation overview

- Motivation
- Thermoacoustic problem settings
- Robust design tasks

Risk Analysis

Gaussian Process

→ Risk Diagram

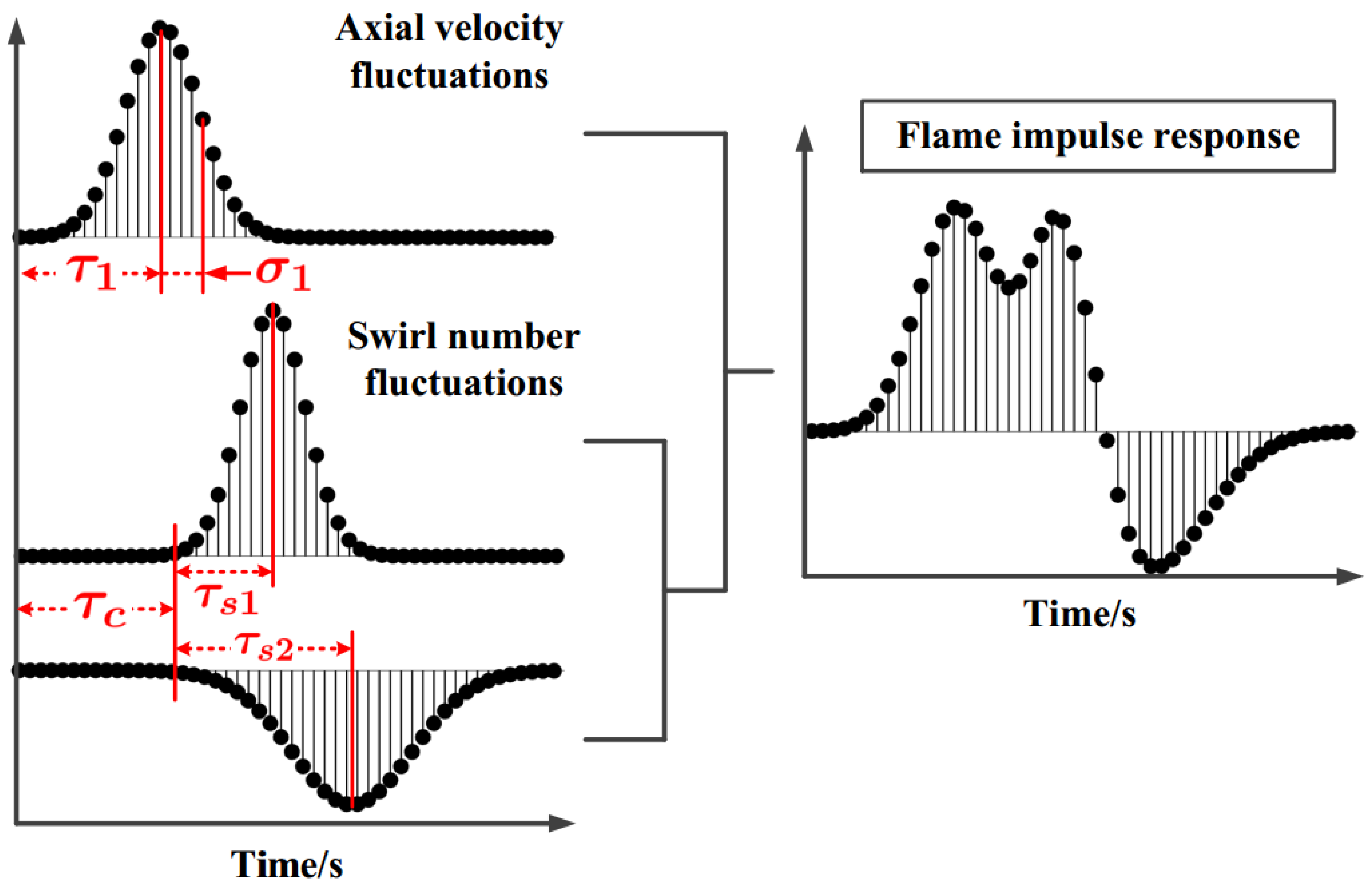
Ideal Control Design

Realistic Control Design

Tolerance Design

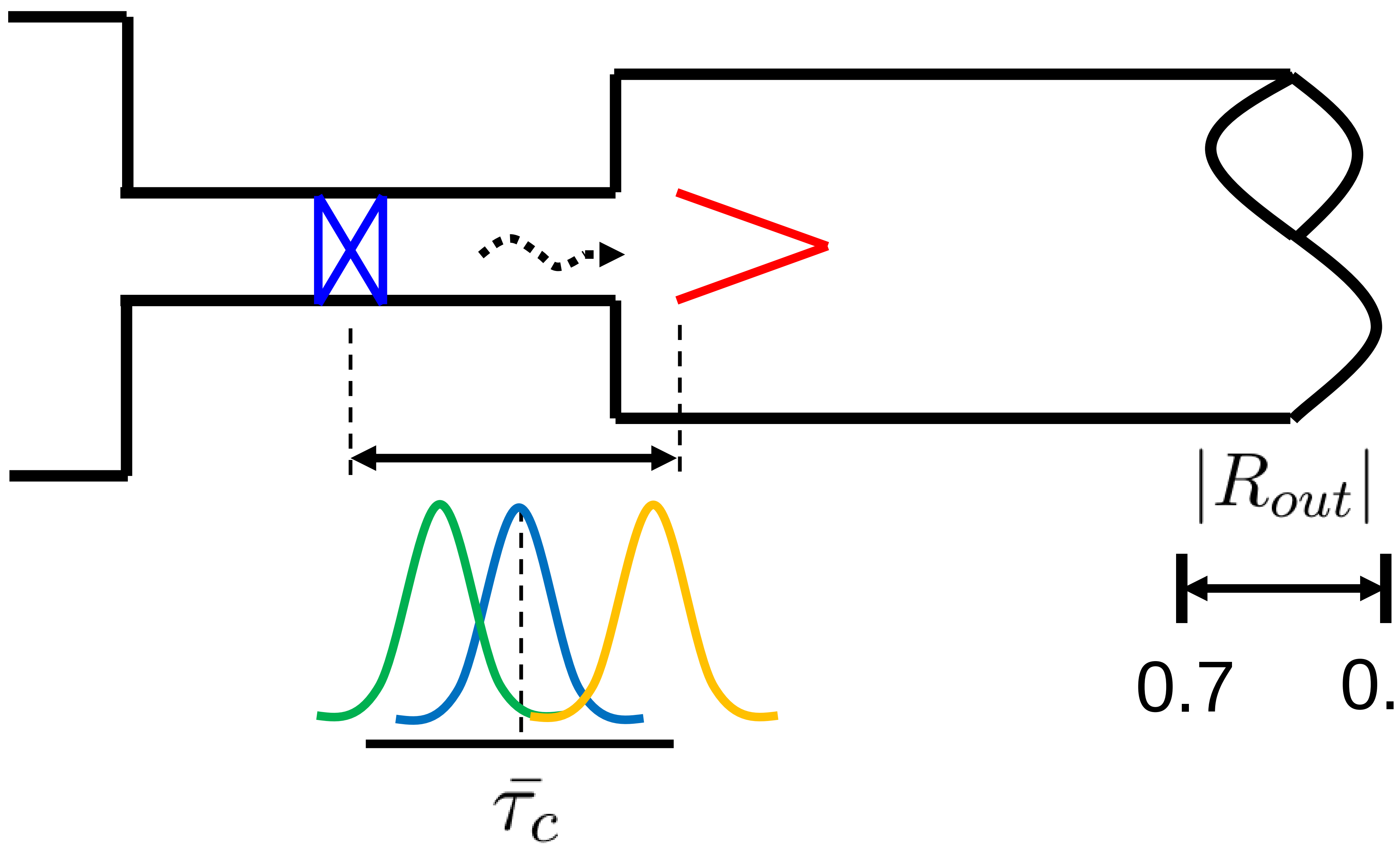
Realistic control design: further enhance the robustness of the design

Flame model



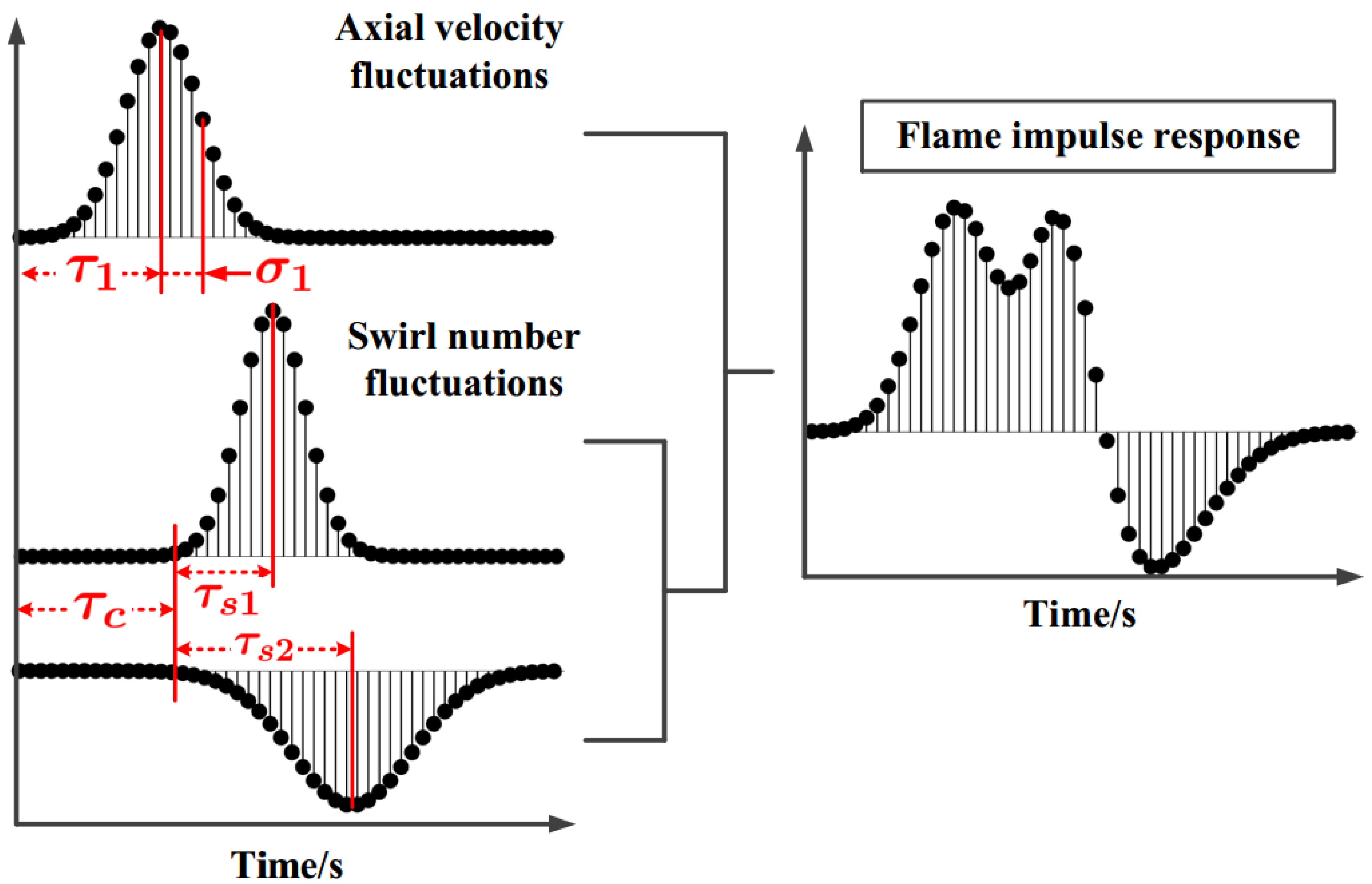
"Q3: in reality, we cannot perfectly control τ_c. Meanwhile, $|R_{out}|$ is also uncertain. Then how would these affect the decision made from Q2?"

→ Realistic Control Design



Realistic control design: further enhance the robustness of the design

Flame model



$$\min_{\bar{\tau}_c} f(\bar{\tau}_c) = (\bar{\tau}_c - \tau_c^0)^2$$

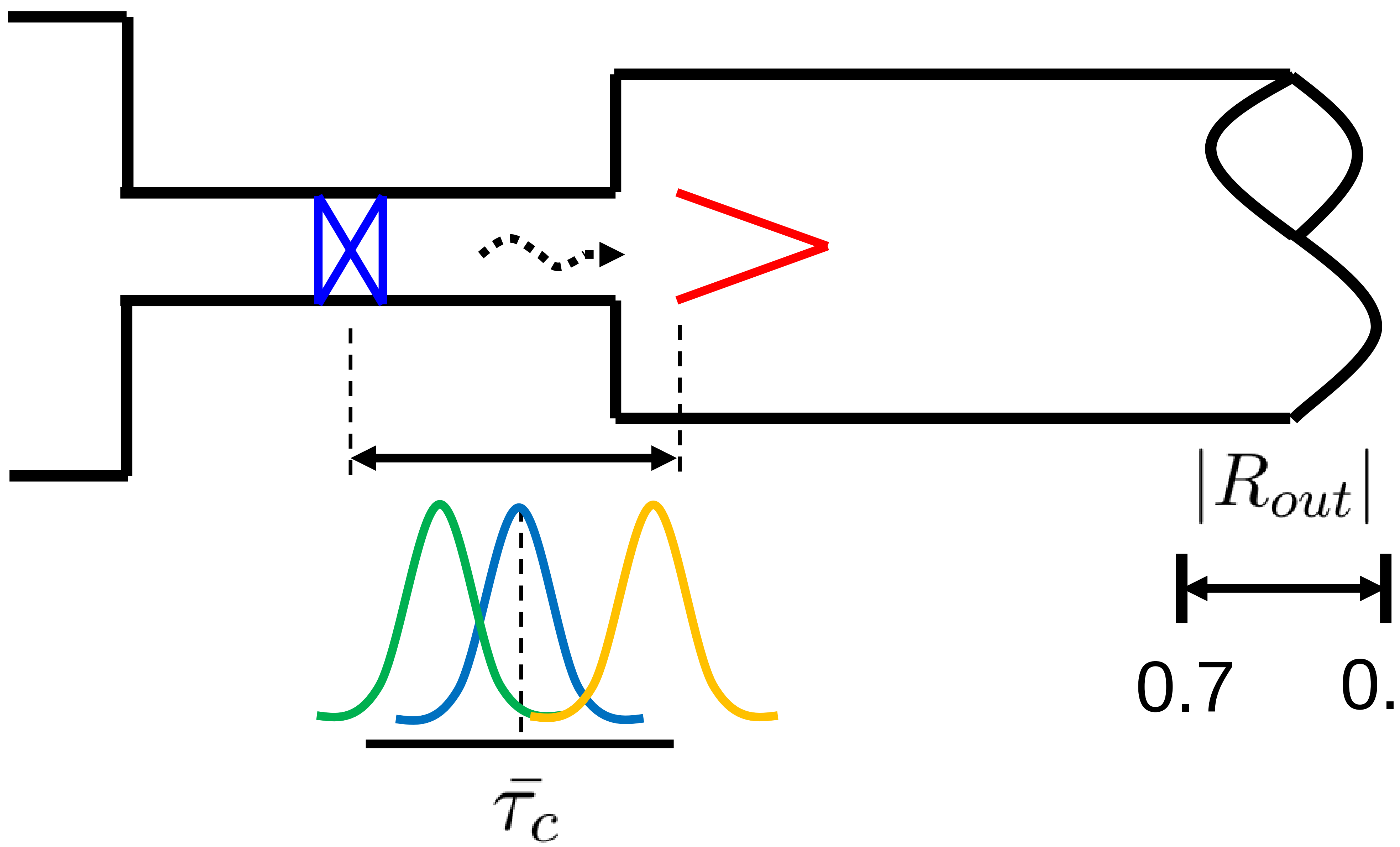
$$\text{subject to :} \quad P_f^{(I)}(\tau_c) \leq 0.1\%$$

$$P_f^{(C)}(\tau_c) \leq 0.1\%$$

$$\tau_c \sim \mathcal{N}(\bar{\tau}_c, (0.05\tau_c^0)^2)$$

"Q3: in reality, we cannot perfectly control τ_c. Meanwhile, $|R_{out}|$ is also uncertain. Then how would these affect the decision made from Q2?"

→ **Realistic Control Design**



Gaussian Process models have delivered highly accurate design

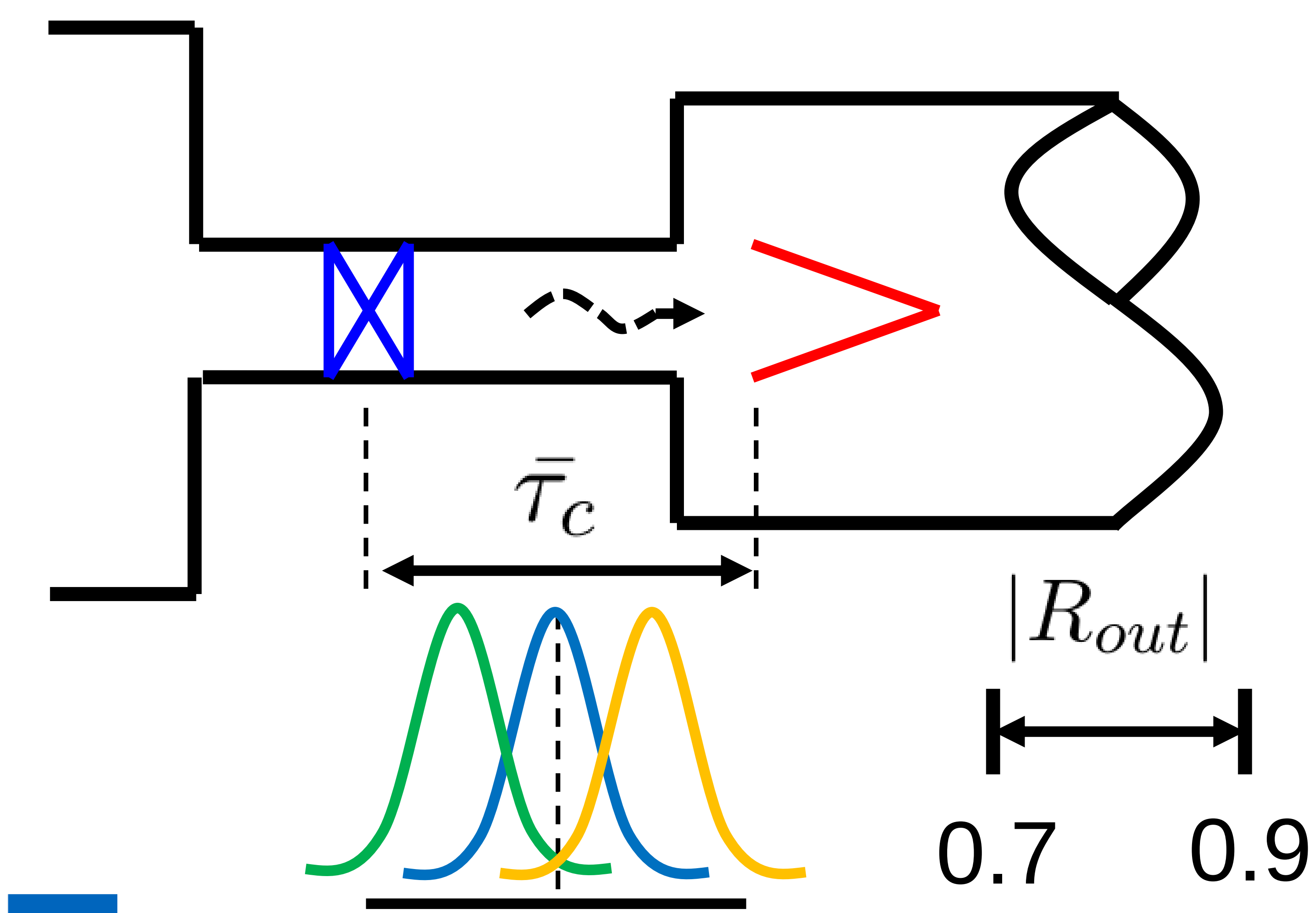
Q3: Realistic control design

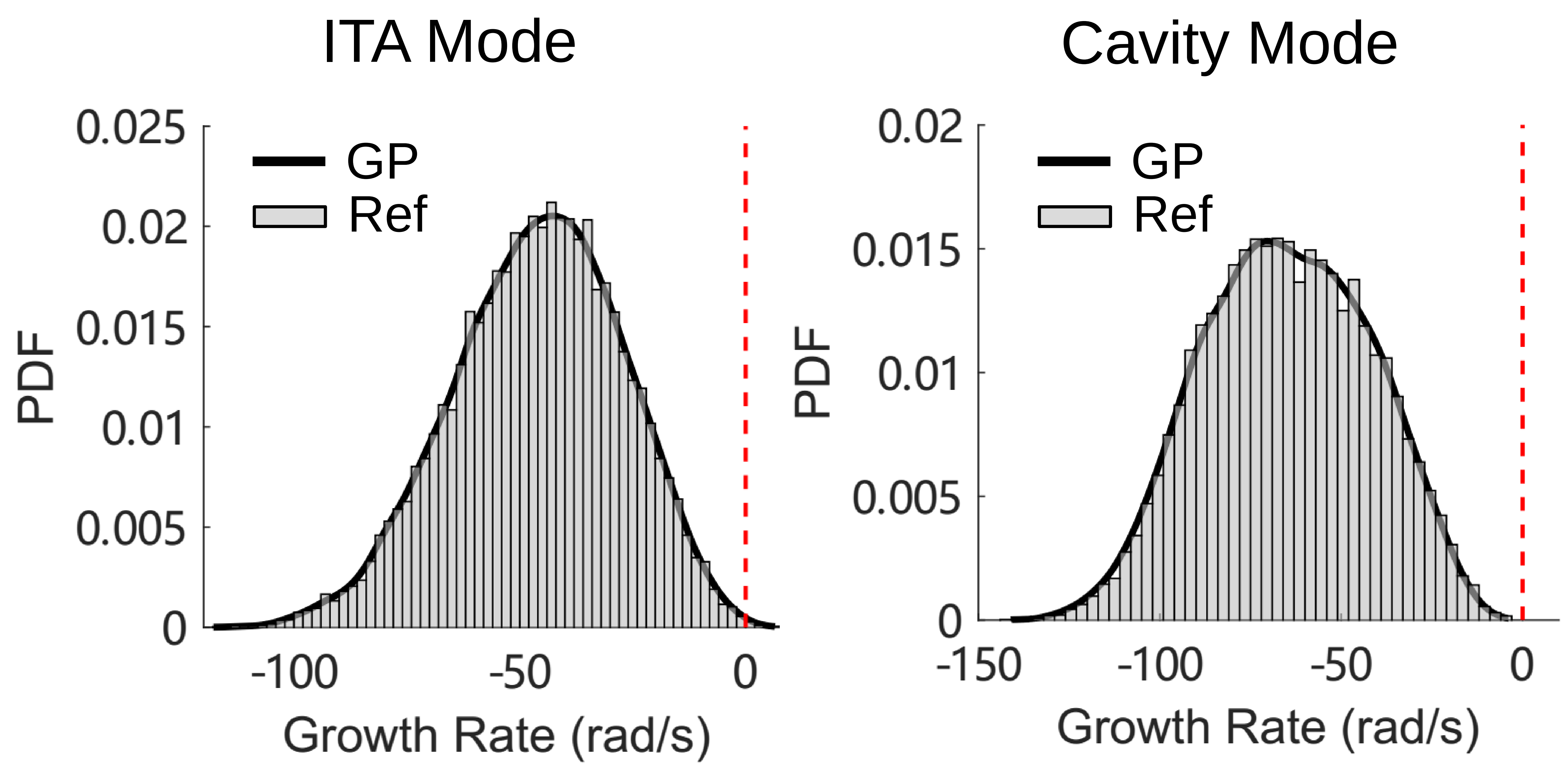
$$\min_{\bar{\tau}_c} f(\bar{\tau}_c) = (\bar{\tau}_c - \tau_c^0)^2$$

$$\text{subject to}: \quad P_f^{(I)}(\tau_c) \leq 0.1\%$$

$$P_f^{(C)}(\tau_c) \leq 0.1\%$$

$$\tau_c \sim \mathcal{N}(\bar{\tau}_c, (0.05\tau_c^0)^2)$$



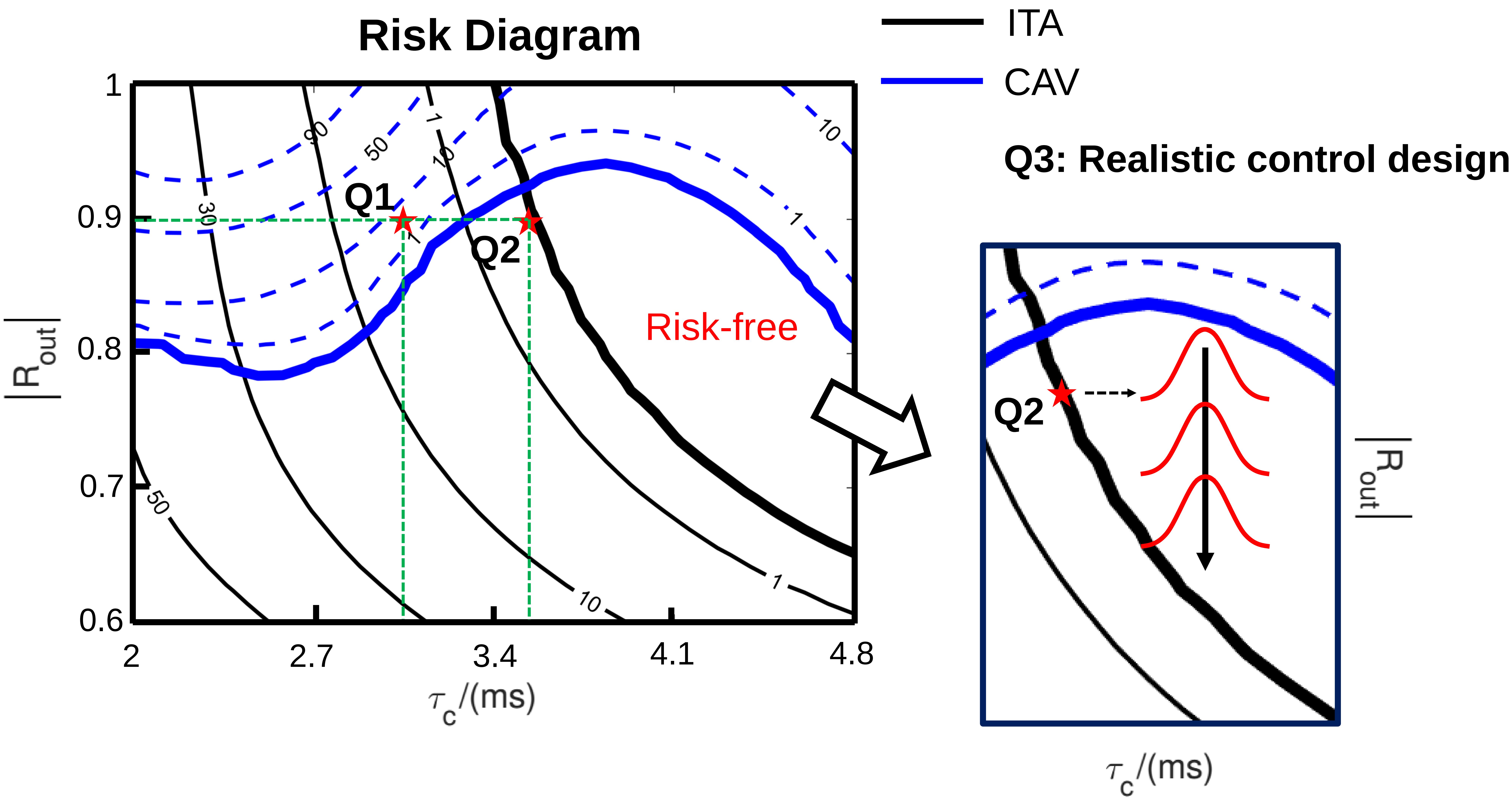


$$\bar{\tau}_c^{\,opt} = 4.06\ ms \qquad \tau_c \sim \mathcal{N}(\bar{\tau}_c^{\,opt}, (0.05\tau_c^0)^2)$$

$$(\tau_c^{opt} = 3.52\ ms) \qquad |R_{out}| \sim \mathcal{U}(0.7, 0.9)$$

$$\tau_1, \sigma_1, \tau_{s1}, \tau_{s2} \sim \mathcal{U}$$

Risk diagram indicates the direction for determining the optimum design



Presentation overview

- Motivation
- Thermoacoustic problem settings
- Robust design tasks

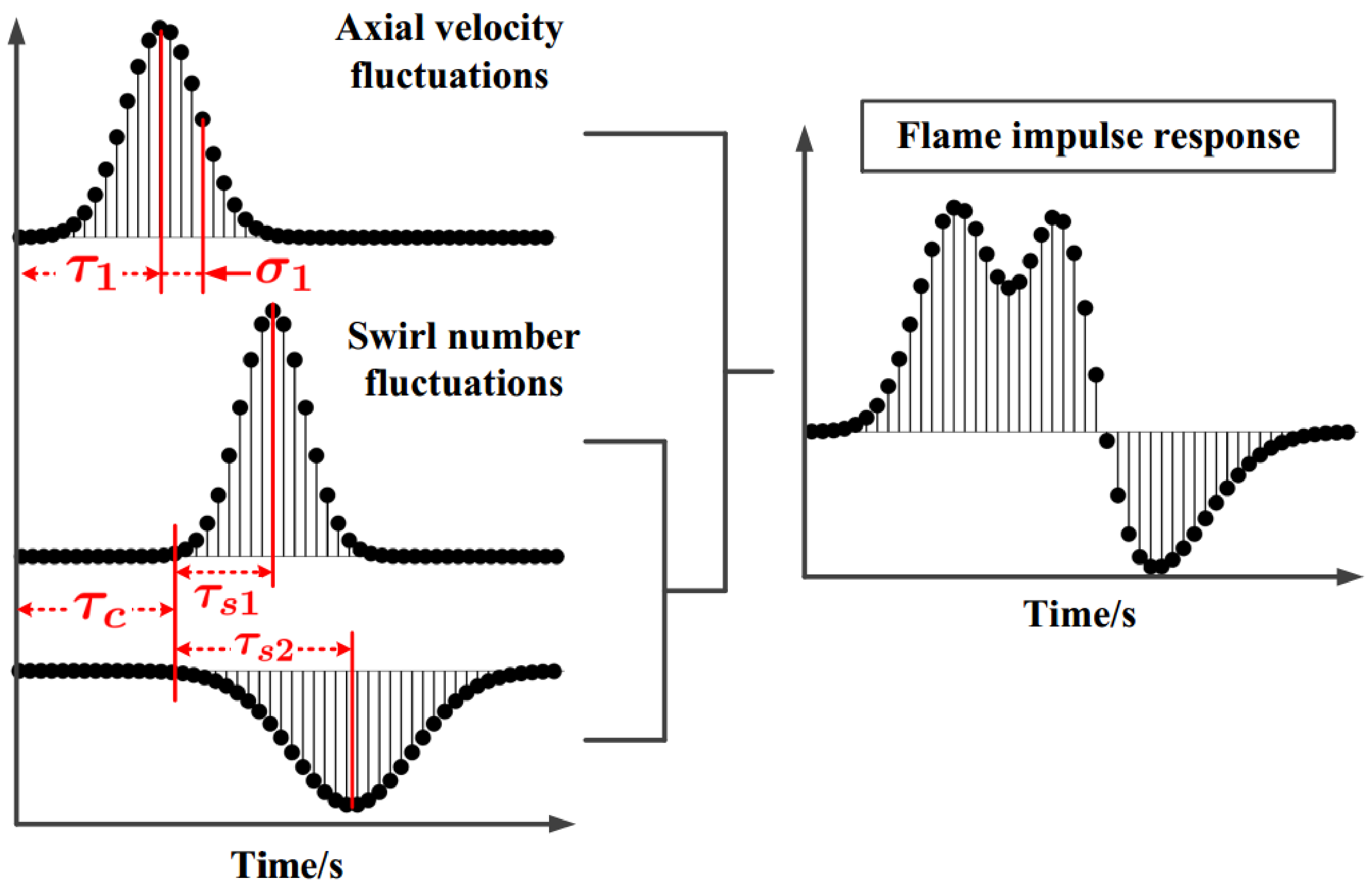
Risk Analysis
Gaussian Process

→ Risk Diagram

Ideal Control Design
Realistic Control Design
Tolerance Design

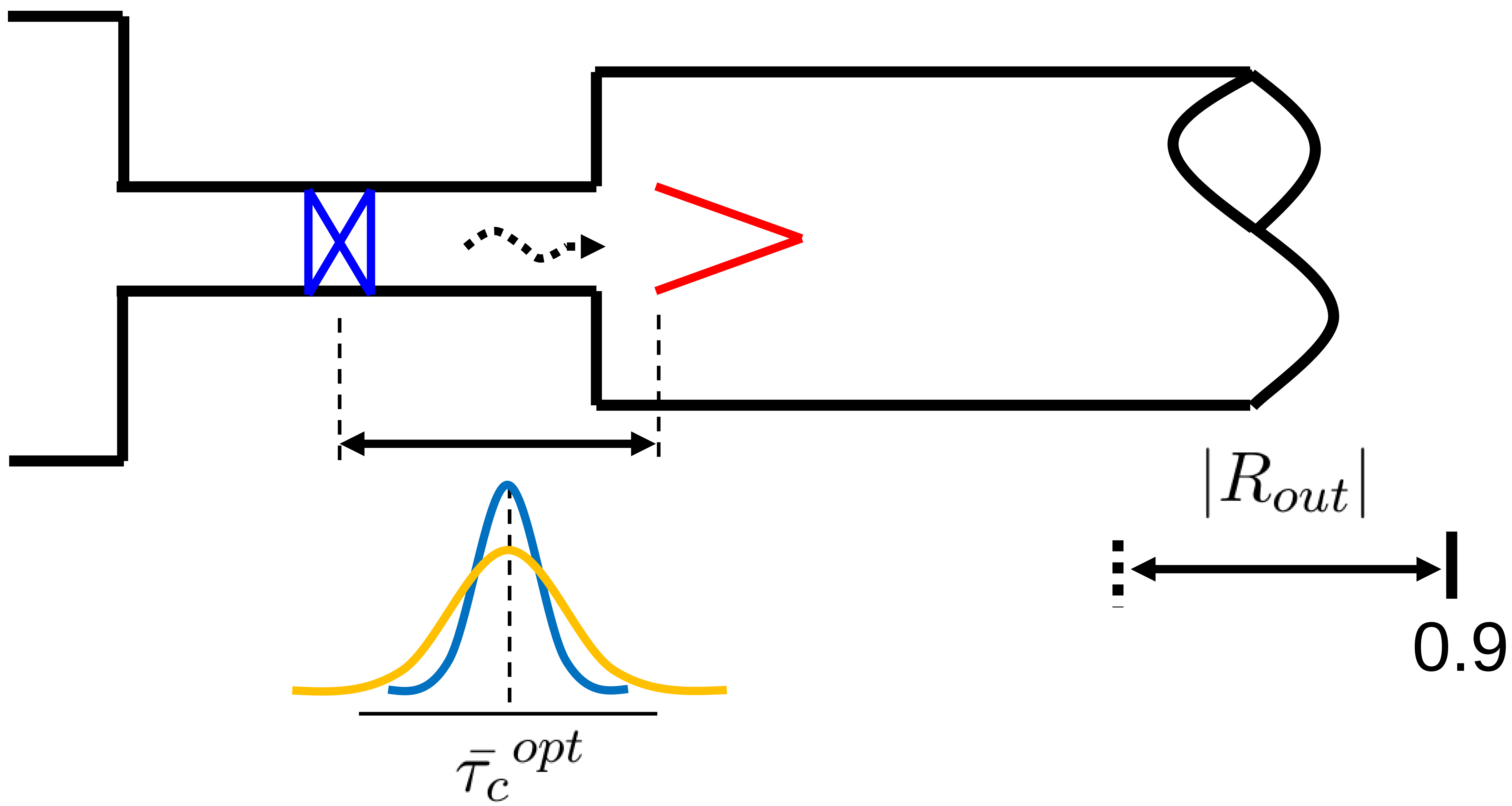
Tolerance design: A perspective of an inverse problem

Flame model



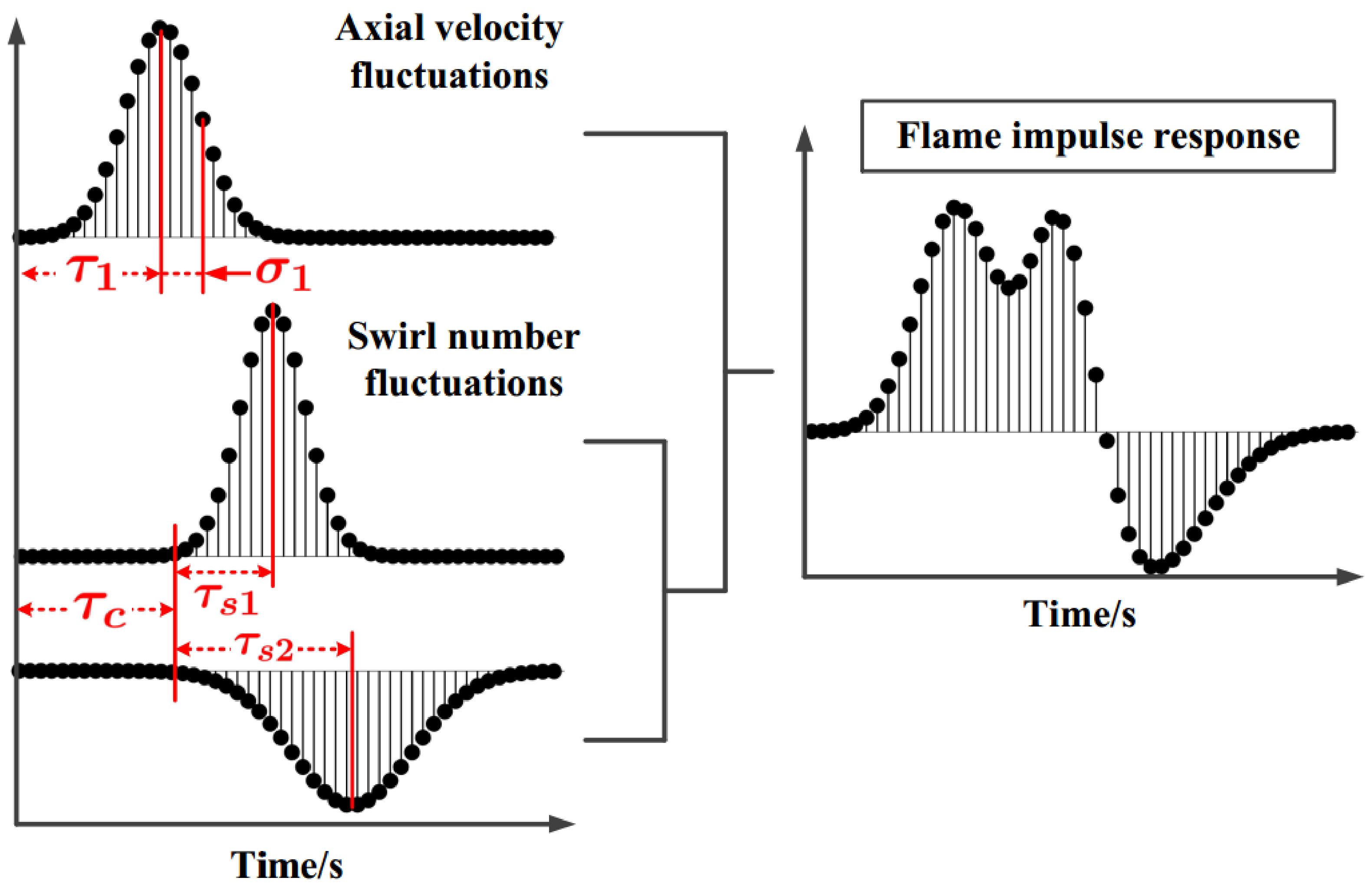
"Q4: given a certain threshold for risk factor, what are the maximum allowable variational ranges for τ_c and $|R_{out}|$?"

→ **Tolerance Design**



Tolerance design: A perspective of an inverse problem

Flame model

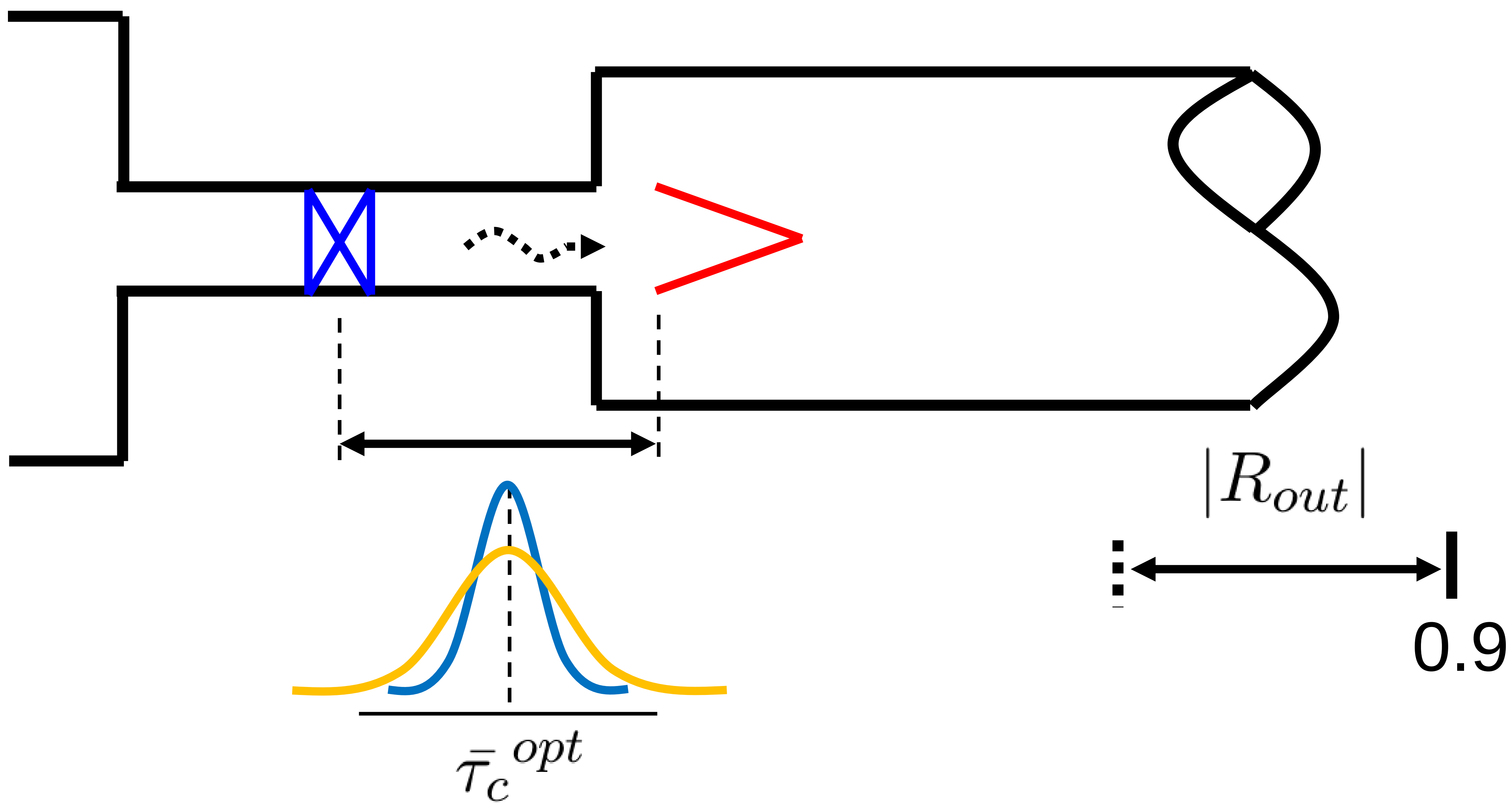


$$\max_{\sigma_{\tau_c}} f(\sigma_{\tau_c}) = \frac{\sigma_{\tau_c}}{\tau_c^0} \quad \min_{R_L} g(R_L) = \frac{R_L}{|R_{out}|^0}$$

$$\begin{aligned}
\text{subject to :} \quad & P_f^{(I)}(\tau_c, |R_{out}|) \leq 0.1\% \\
& P_f^{(C)}(\tau_c, |R_{out}|) \leq 0.1\% \\
& \tau_c \sim \mathcal{N}(\bar{\tau}_c, (\sigma_{\tau_c})^2) \\
& |R_{out}| \sim \mathcal{U}(R_L, 0.9)
\end{aligned}$$

"Q4: given a certain threshold for risk factor, what are the maximum allowable variational ranges for τ_c and $|R_{out}|$?"

→ Tolerance Design



Pareto front visualizes the trade-off between two objectives

Q4: Tolerance design

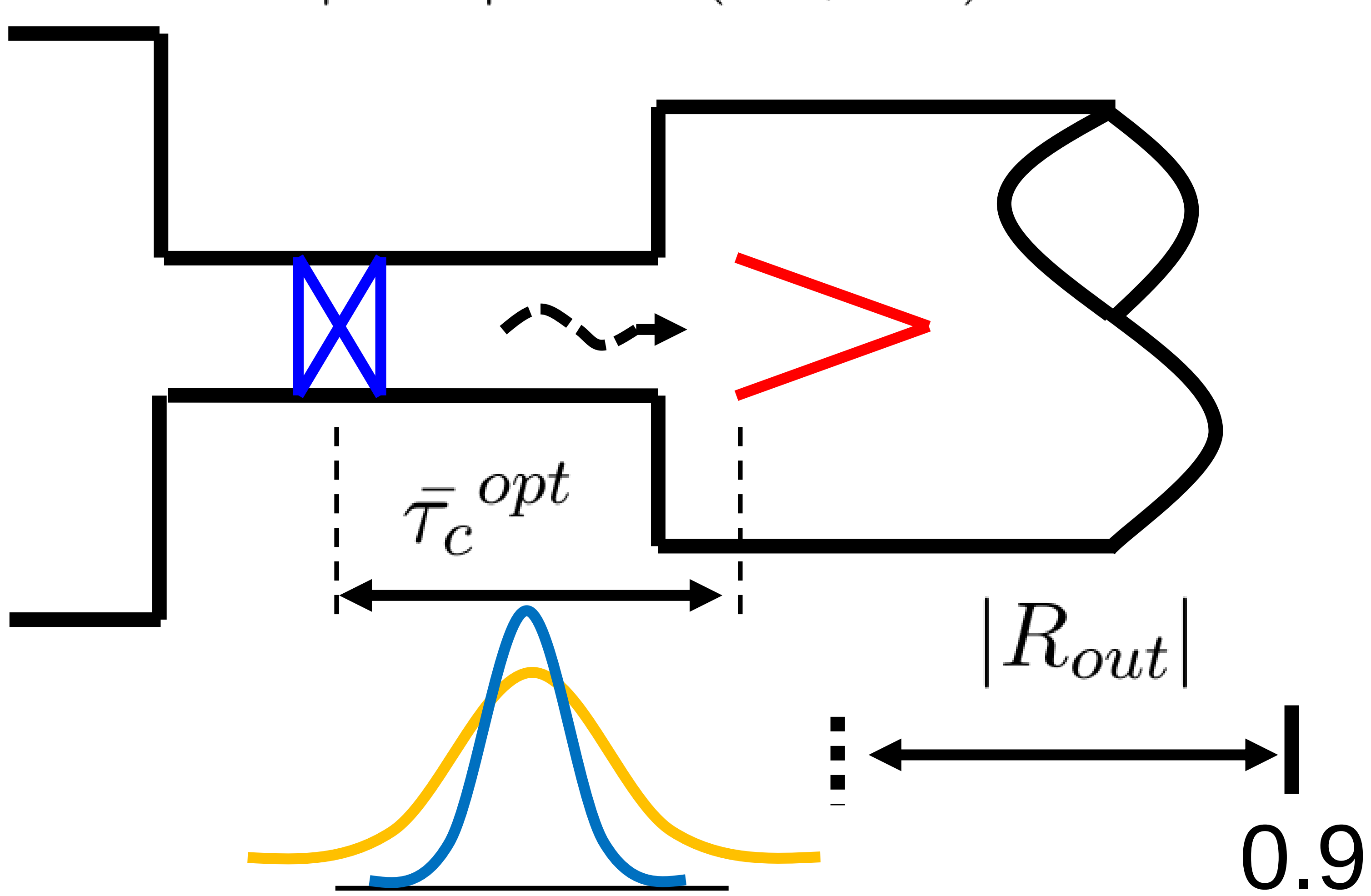
$$\max_{\sigma_{\tau_c}} f(\sigma_{\tau_c}) = \frac{\sigma_{\tau_c}}{\tau_c^0} \quad \min_{R_L} g(R_L) = \frac{R_L}{|R_{out}|^0}$$

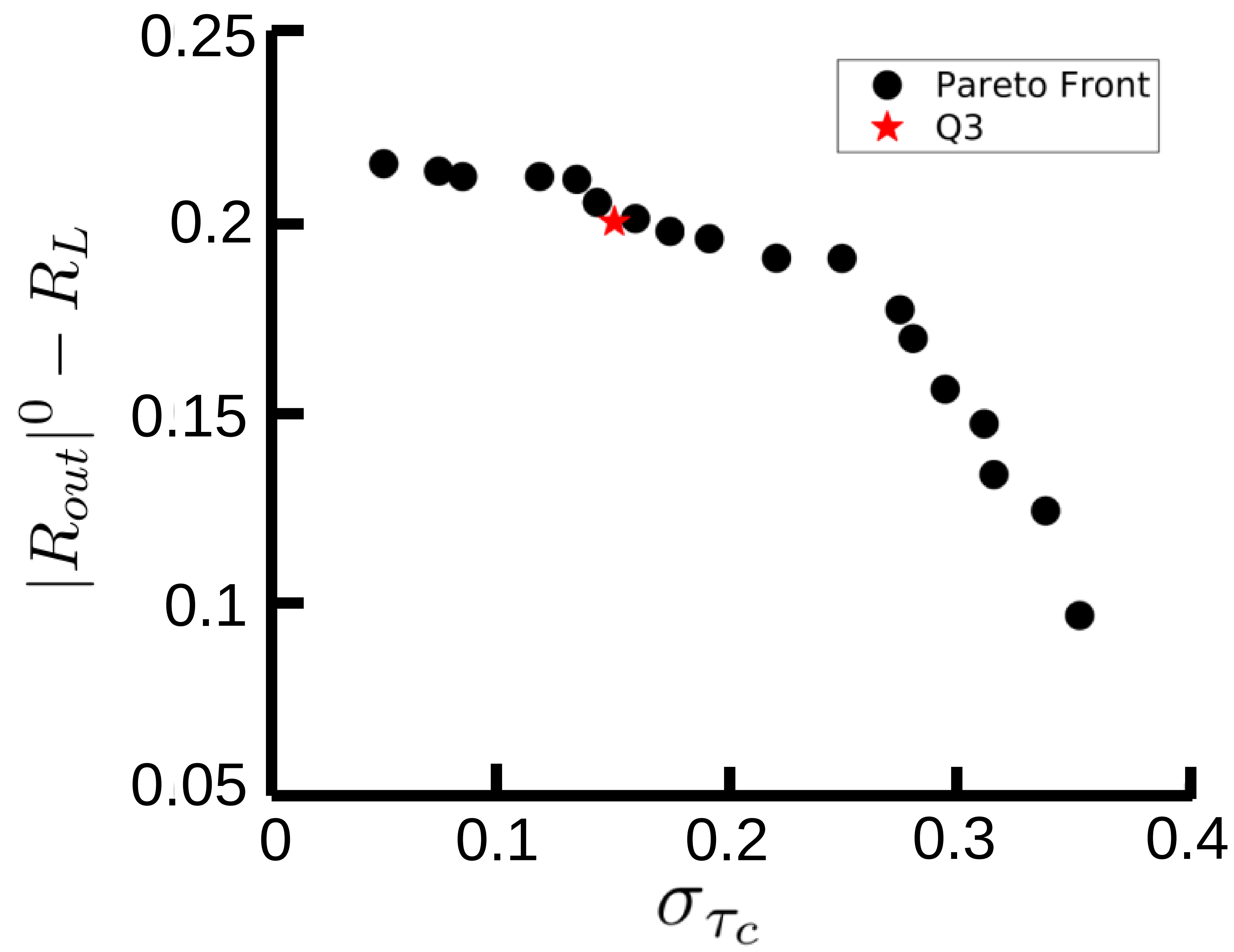
$$\text{subject to :} \quad P_f^{(I)}(\tau_c, |R_{out}|) \leq 0.1\%$$

$$P_f^{(C)}(\tau_c, |R_{out}|) \leq 0.1\%$$

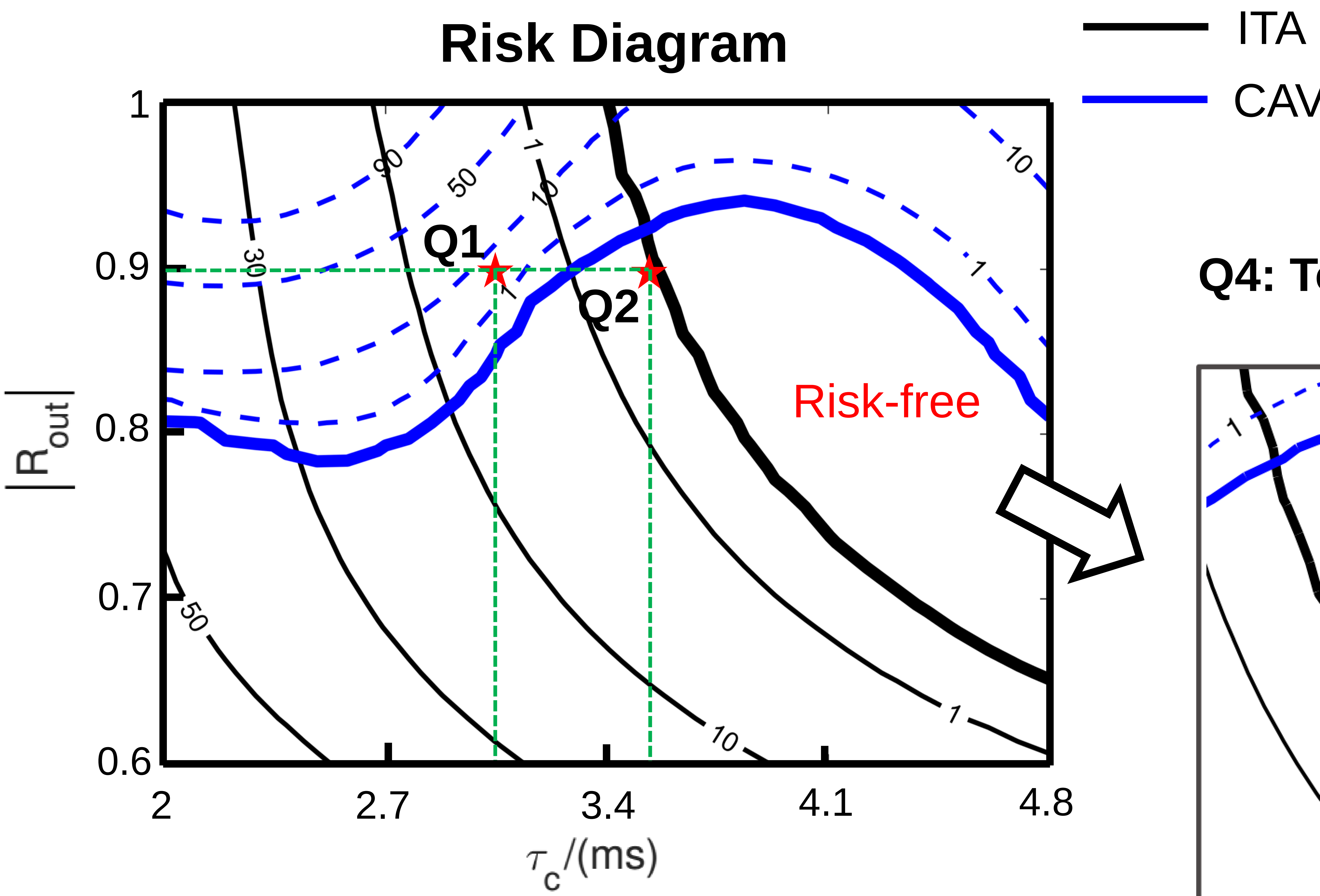
$$\tau_c \sim \mathcal{N}(\bar{\tau}_c, (\sigma_{\tau_c})^2)$$

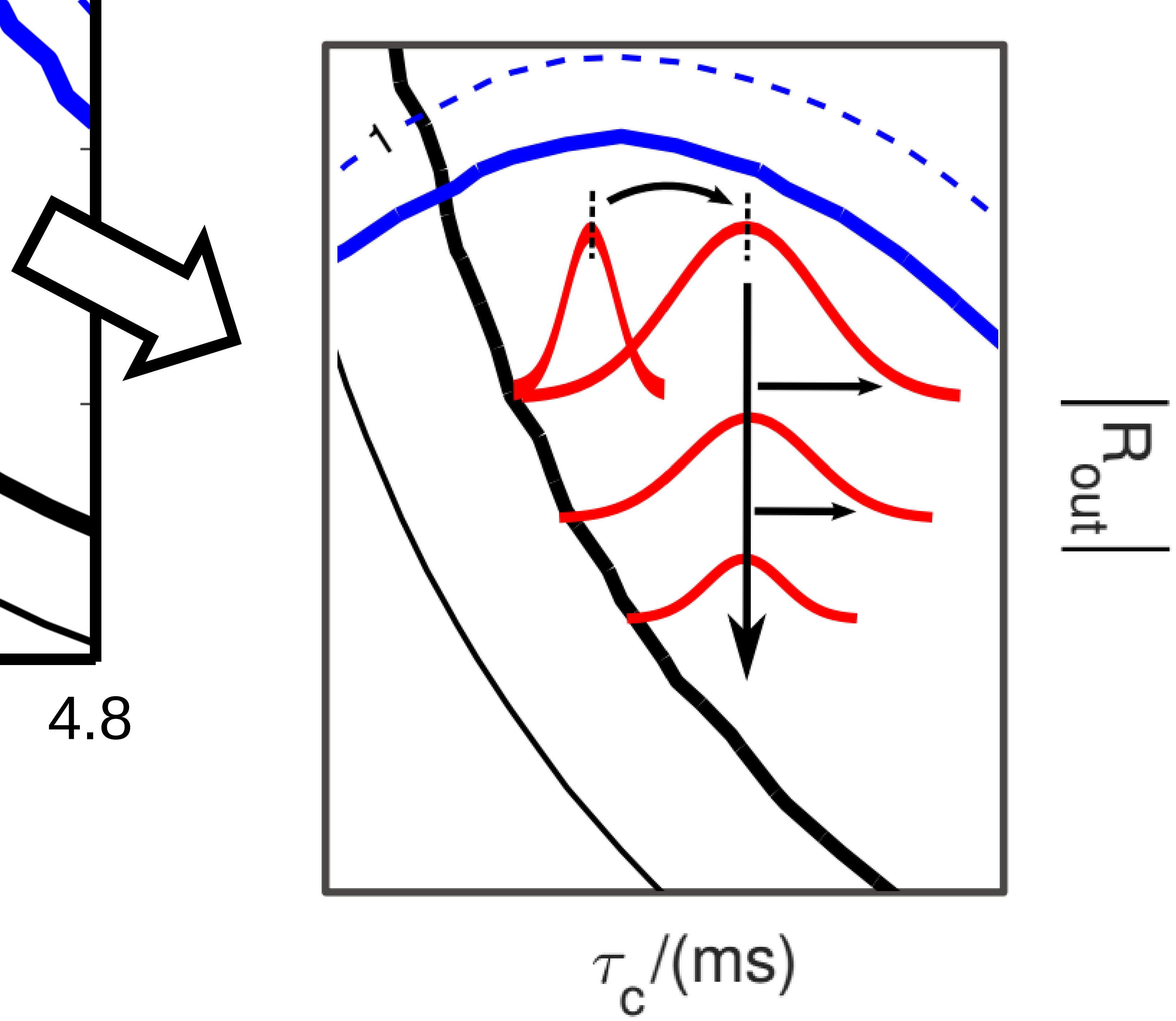
$$|R_{out}| \sim \mathcal{U}(R_L, 0.9)$$



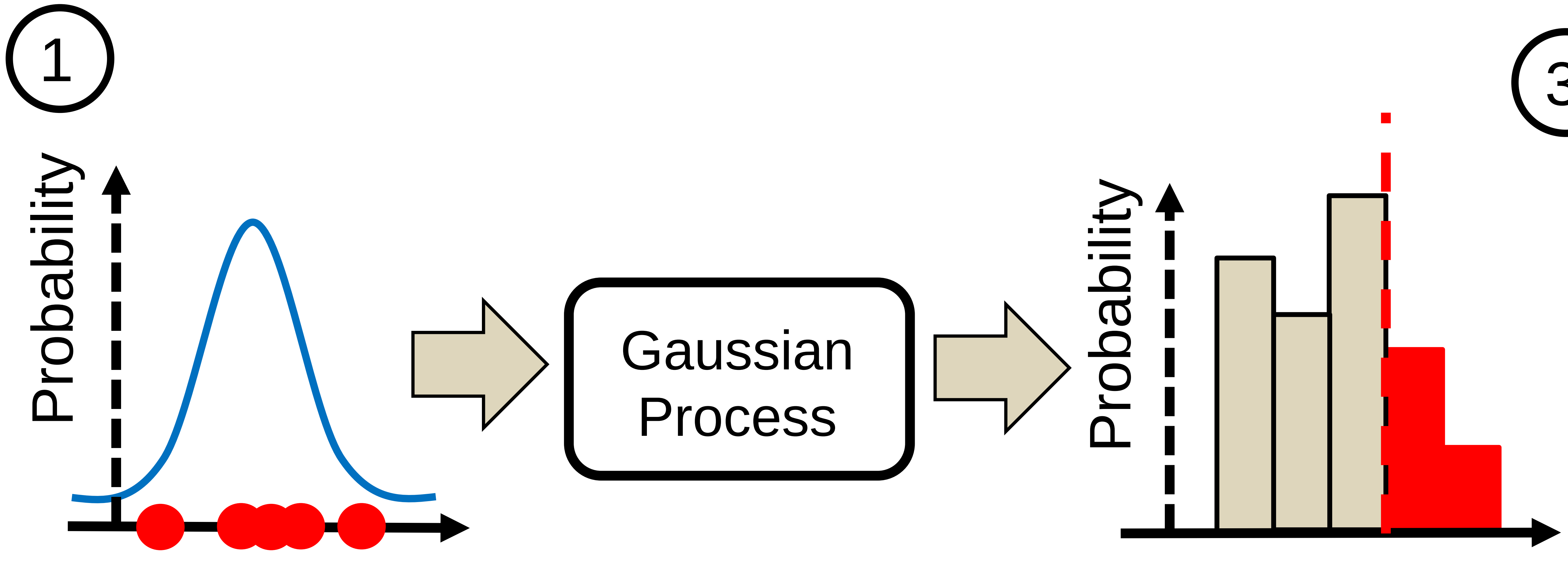


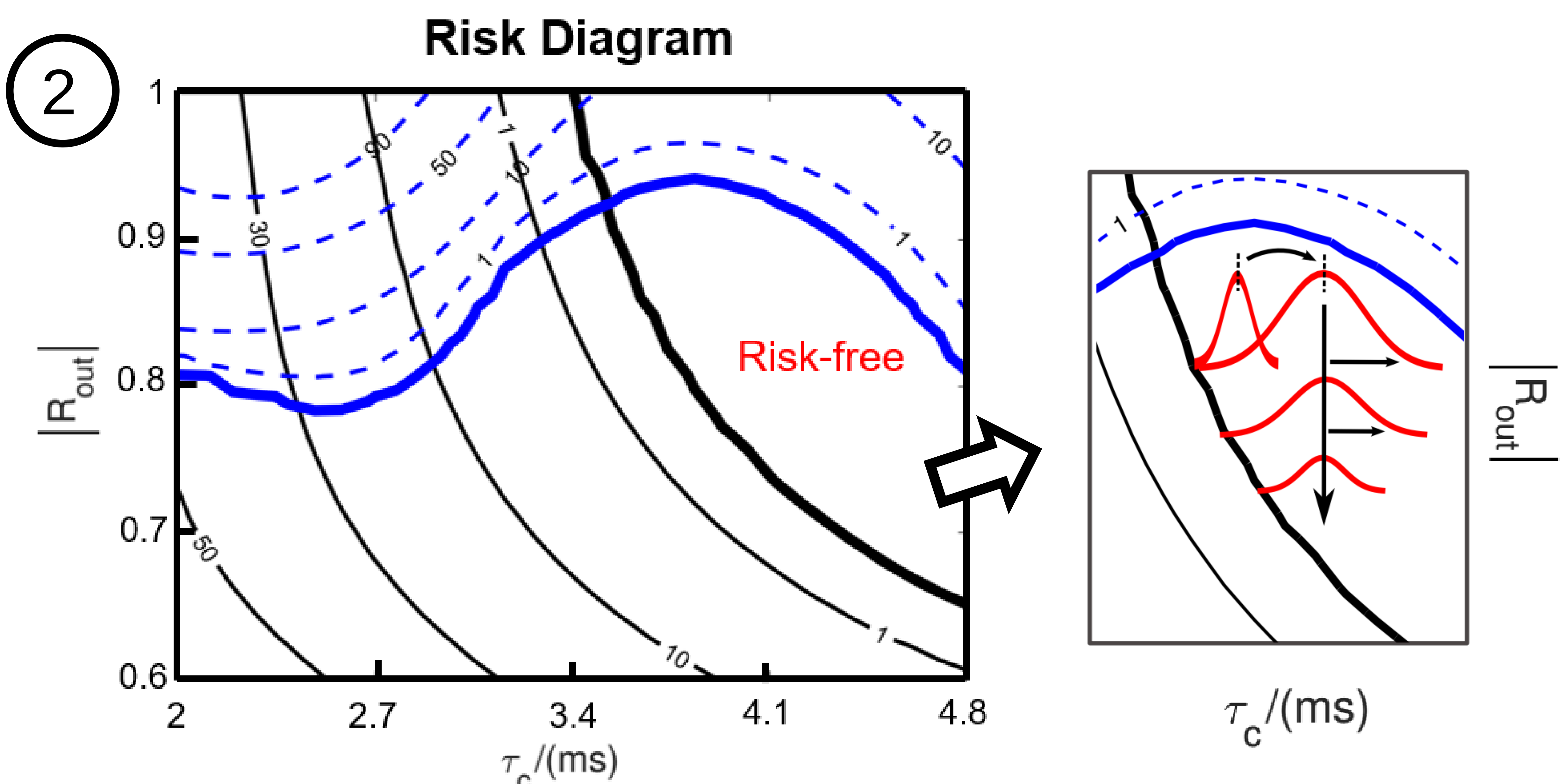
Risk diagram illustrates the trade-off between uncertainties of τ_c and R_{out}

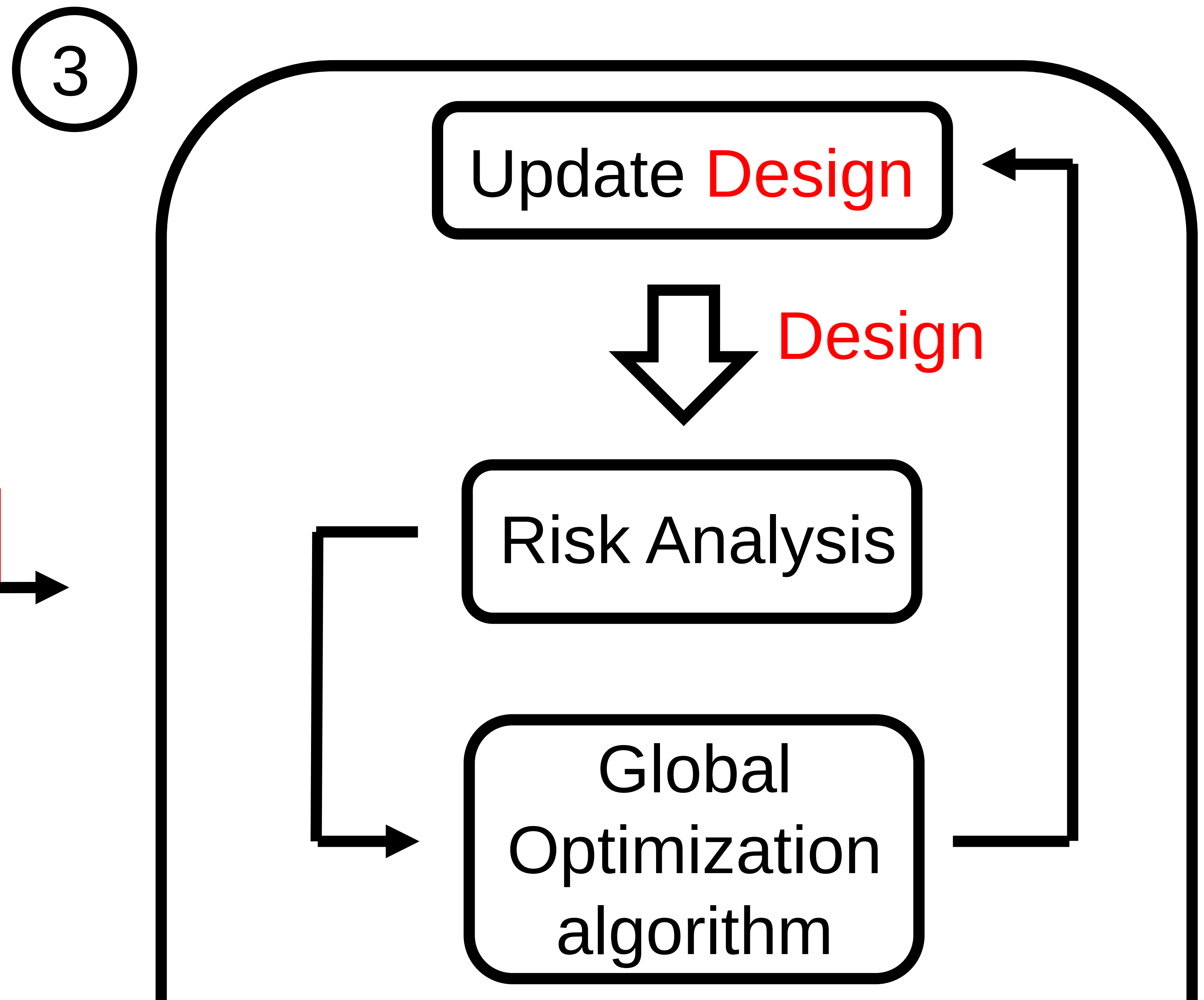




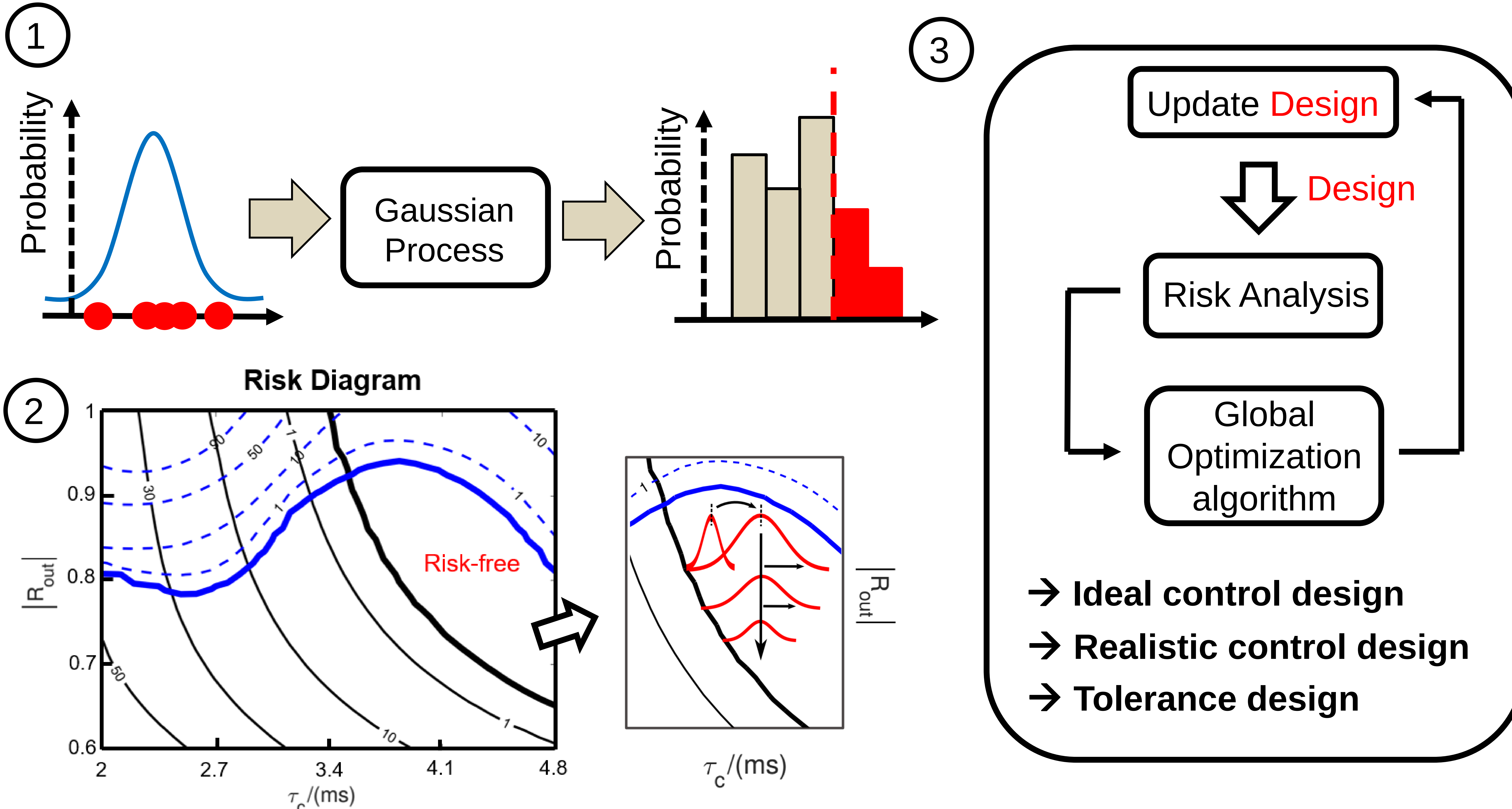
Conclusions







- → **Ideal control design**
- → **Realistic control design**
- → **Tolerance design**

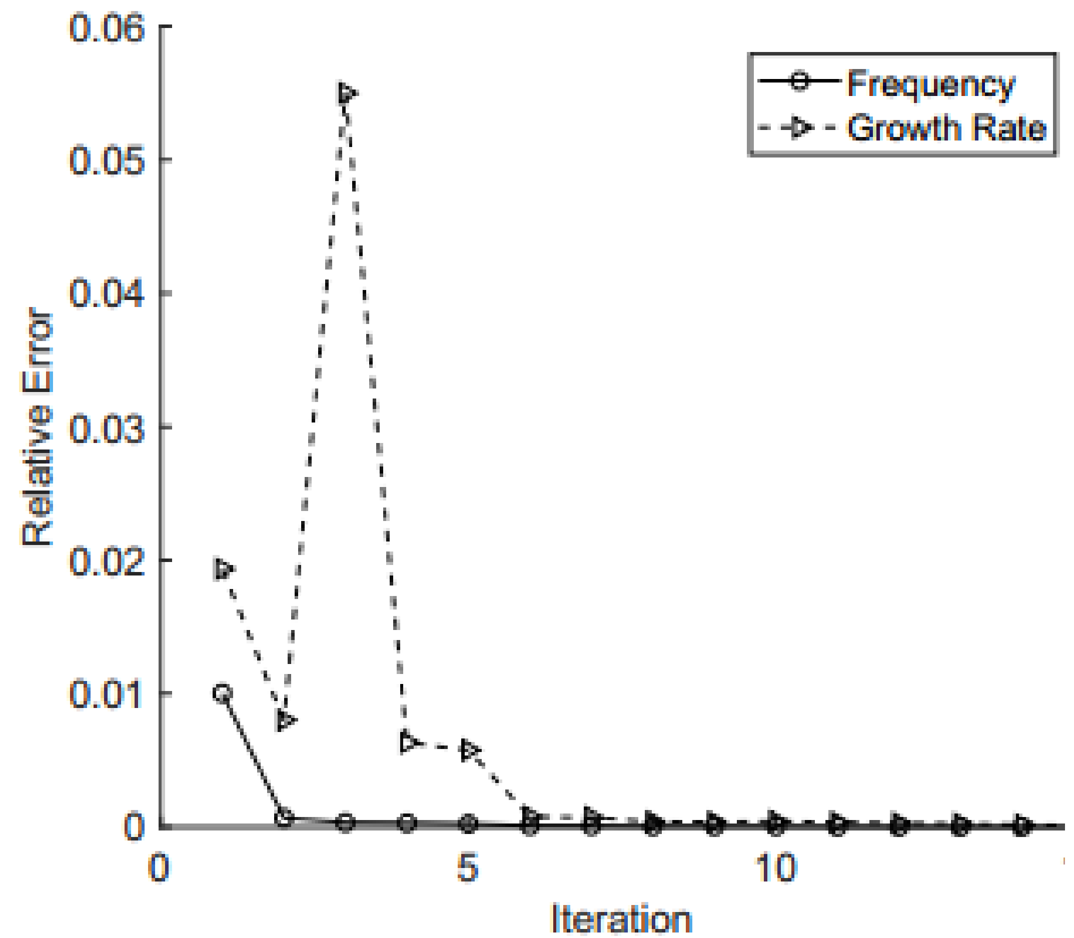


→ **Ideal control design**
→ **Realistic control design**
→ **Tolerance design**

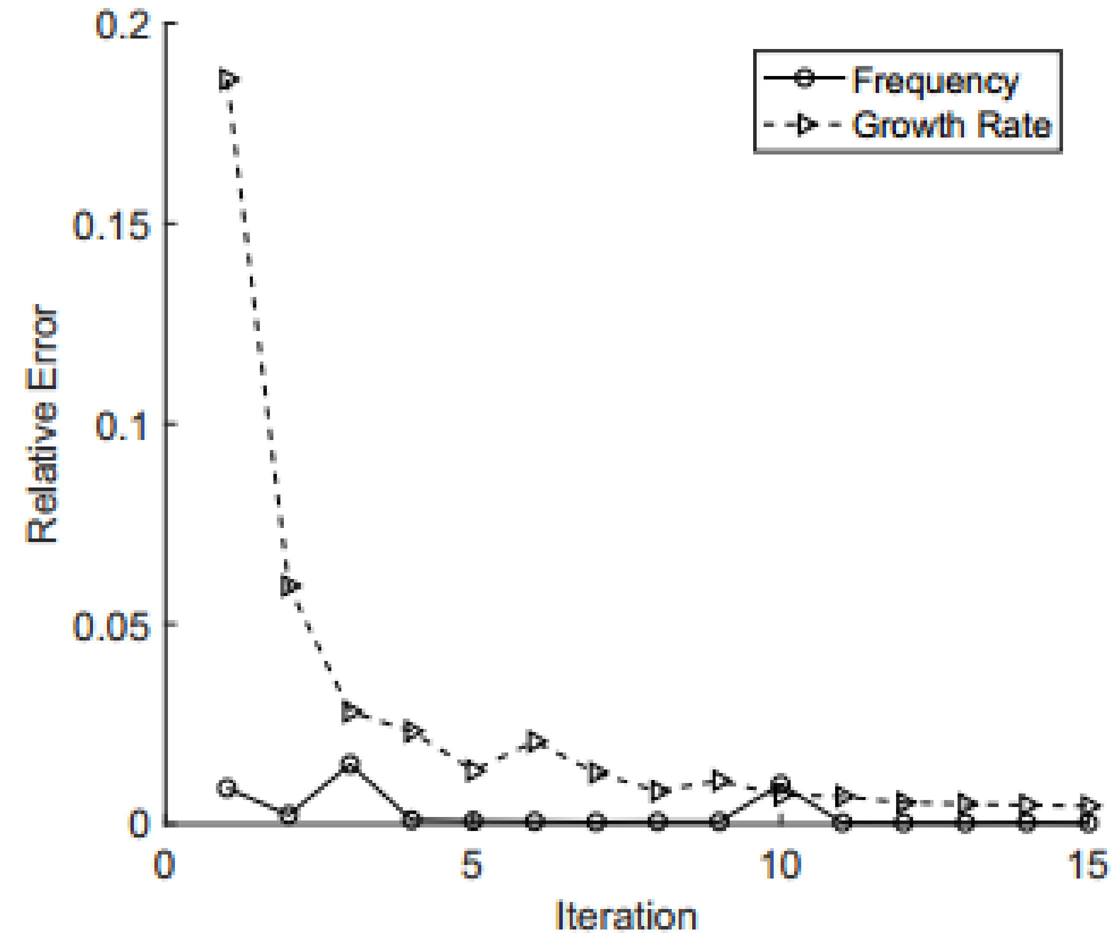
Back-up: GP model training process

Leave-one-out cross validation

$$GE = \frac{1}{N}\sum_{i=1}^{N}\left(f_i - \widehat{f}_i^{(-i)}\right)^2$$



(a) ITA mode



(b) cavity mode

N Total number of training samples

f_i the known response of the training sample $\boldsymbol{x}^i$

$\widehat{f}_i^{(-i)}$ the prediction at $\boldsymbol{x}^i$ using the GP model constructed upon all training samples except $(\boldsymbol{x}^i, f_i)$

Back-up: time costs

Laptop PC, CPU 2.30GHz

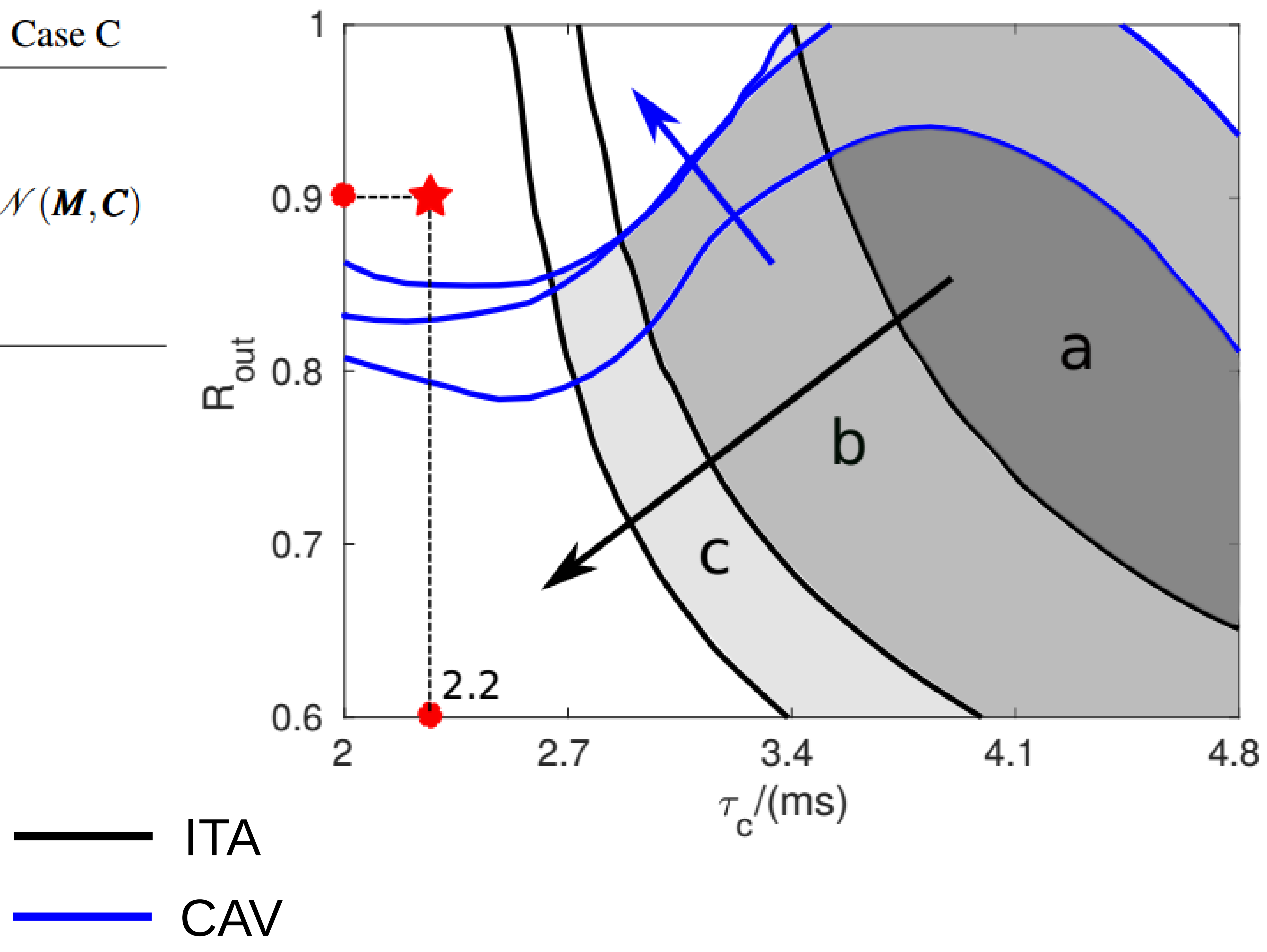
GP model training: 93s

Risk analysis (MC on acoustic solver): 271s

Risk analysis (MC on GP model): 1.3s

Back-up: Sensitivity analysis

Parameters	Case A	Case B	Case C
τ_1	$\mathscr{U}(0.9\tau_1^0, 1.1\tau_1^0)$	$\mathscr{N}(\tau_1^0, (0.03\tau_1^0)^2)$	
σ_1	$\mathscr{U}(0.9\sigma_1^0, 1.1\sigma_1^0)$	$\mathscr{N}(\sigma_1^0, (0.03\sigma_1^0)^2)$	$\mathscr{N}(\boldsymbol{M}, \boldsymbol{C})$
τ_{s1}	$\mathscr{U}(0.9\tau_{s1}^0, 1.1\tau_{s1}^0)$	$\mathscr{N}(\tau_{s1}^0, (0.03\tau_{s1}^0)^2)$	
τ_{s2}	$\mathscr{U}(0.9\tau_{s2}^0, 1.1\tau_{s2}^0)$	$\mathscr{N}(\tau_{s2}^0, (0.03\tau_{s2}^0)^2)$	



Back-up: take into account both parametric uncertainty and GP model uncertainty

Posterior

$$f^*(x) \sim \mathcal{GP}(m^*(x), k^*(x, x'))$$