Homework Assignment-I

Due date: Aug 30, 2019
Submit your assignment on A4 size paper and don't forget to staple all the pages before submission.

From Ross book (9th Edition):
Chapter 4: 8, 18, 21, 25, 34, 46, 52, 56

1.
A computer system can operate in two different modes. Every hour, it remains in the same mode or switches to a different mode according to the transition probability matrix

$$P = \begin{vmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{vmatrix}$$

a) Compute the 2-step transition probability matrix.
b) If the system is in Mode I at 5:30 pm, what is the probability that it will be in Mode I at 8:30 pm on the same day?

2.
A system has three possible states, 0, 1 and 2. Every hour it makes a transition to a different state, which is determined by a coin flip. For example, from state 0, it makes a transition to state 1 or state 2 with probabilities 0.5 and 0.5.

a) Find the transition probability matrix.
b) Find the three-step transition probability matrix.

3.
We observe the state of a system at discrete points in time. The system is observed only when it changes state. Define X_n as the state of the system after nth state change, so that $X_n = 0$, if the system is running; $X_n = 1$ if the system is under repair; and $X_n = 2$ if the system is idle. Assume that the matrix transition matrix is:

$$P = \begin{vmatrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

Compute the matrix P^n for all possible n.

4.
Each time a certain horse runs in a three-horse race, he has probability 1/2 of winning, 1/4 of coming in second, and 1/4 of coming in third, independent of the outcome of any previous race.

We have an independent trials process, but it can also be considered from the point of view of Markov chain theory. Find the TPM?

5.

We have two urns that, between them, contain four balls. At each step, one of the four balls is chosen at random and moved from the urn that it is in into the other urn. We choose, as states, the number of balls in the first urn. Find the TPM?

6.

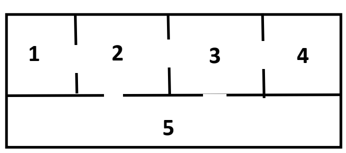
Consider a machine that works as follows. If it is up at the beginning of a day, it stays up at the beginning of the next day with probability p and fails with probability 1-p. It takes exactly 2 days for the repairs, at the end of which the machine is as good as new. Let Xn be the state of the machine at the beginning of day n, where the state is 0 if the machine has just failed, 1 if 1 days' worth of repair work is done on it, and 2 if it is up. Show that {Xn; n≥ 0} is a Markov chain, and deduce its transition probability matrix.

7.

Items arrive at a machine shop in a deterministic fashion at a rate of one per minute. Each item is tested before it is loaded onto the machine. An item is found to be nondefective with probability p and defective with probability 1 - p. If an item is found defective, it is discarded. Otherwise, it is loaded onto the machine. The machine takes exactly 1 minute to process the item, after which it is ready to process the next one. Let Xn be 0 if the machine is idle at the beginning of the nth minute and 1 if it is starting the processing of an item. Show that {Xn; n≥ 0} is a Markov chain, and deduce its transition probability matrix.

8**.**

Consider the maze shown below. Assume cell 1 and 4 are traps; the former contains food and the later contains electric shock. In all other rooms, the rat will stay 1 unit time and the select adjacent room randomly.



a) Model the status of the rat as a Markov chain; that is, state precisely what the state space is and what the transition probability matrix.
b) The mean time until rat enters a trap given it starts in cell i.

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