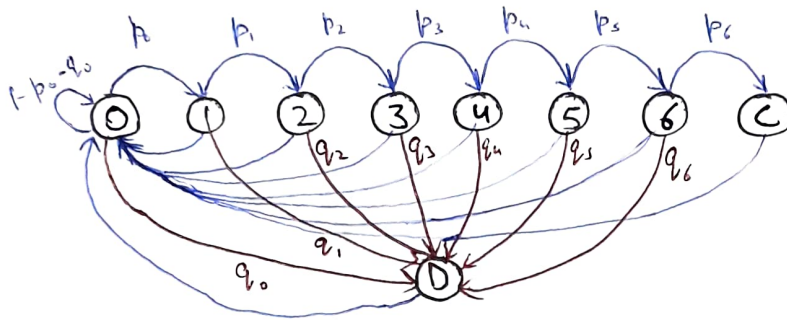


Prob 1States : 0, 1, 2, 3, 4, 5, 6, C, DState k : Missed min payment for the last k consecutive months

D : Bankruptcy

C : Account termination

From state i , transition to $i+1 \rightarrow$ with p_i D $\rightarrow q_i$ 0 $\rightarrow 1 - p_i - q_i$

$$p_{i,0} = 1 - p_i - q_i \quad \left\{ \begin{array}{l} \text{Probability of paying minimum amount and moving to state 0} \\ \text{while being in state } i \end{array} \right\} \quad i = 0, 1, \dots, 6$$

Assumption: When a customer account is terminated or the customer declares bankruptcy, the bank it is replaced with an active one, so that the number of accounts does not change. $\therefore p_{C,0} = p_{D,0} = 1$

TPM =

	0	1	2	3	4	5	6	C	D
0	0.937	0.033	0	0	0	0	0	0	0.03
1	0.931	0	0.048	0	0	0	0	0	0.021
2	0.873	0	0	0.09	0	0	0	0	0.037
3	0.783	0	0	0	0.165	0	0	0	0.052
4	0.713	0	0	0	0	0.212	0	0	0.075
5	0.573	0	0	0	0	0	0.287	0	0.135
6	0.489	0	0	0	0	0	0	0.329	0.182
C	1	0	0	0	0	0	0	0	0
D	1	0	0	0	0	0	0	0	0

Number of class = 1

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

∴ It is an irreducible Markov chain

$$\text{Period} = \gcd(1, 2, 3, \dots) = 1$$

∴ States are aperiodic and positive recurrent

∴ This is an irreducible ergodic Markov Chain

⇒ Stationary distribution exists

$$\pi_j = \sum_i \pi_i P_{ij}$$

$$\sum_i \pi_i = 1$$

$$\pi_1 = 0.033 \pi_0$$

$$\pi_2 = 0.048 \pi_1 = 1.584 \times 10^{-3} \pi_0$$

$$\pi_3 = 0.09 \pi_2 = 1.4256 \times 10^{-4} \pi_0$$

$$\pi_4 = 0.165 \pi_3 = 2.35224 \times 10^{-5} \pi_0$$

$$\pi_5 = 0.212 \pi_4 = 4.9867488 \times 10^{-6} \pi_0$$

$$\pi_6 = 0.287 \pi_5 = 1.431196906 \times 10^{-6} \pi_0$$

$$\pi_7 = 0.329 \pi_6 = 4.708637819 \times 10^{-7} \pi_0$$

$$\begin{aligned} \pi_8 &= 0.03 \pi_0 + 0.021 \pi_1 + 0.031 \pi_2 + 0.052 \pi_3 + 0.075 \pi_4 + 0.135 \pi_5 + 0.182 \pi_6 \\ &= 0.03076171899 \pi_0 \end{aligned}$$

$$\pi_0 + \pi_1 + \pi_2 + \dots + \pi_7 + \pi_8 = 1$$

$$\pi_0 (1 + 0.033 + 1.584 \times 10^{-3} + \dots + 0.030 \dots) = 1$$

$$\Rightarrow \pi_0 = \frac{1}{1.0655186} = 0.9385100507$$

$$\pi_1 = 3.097 \times 10^{-2}$$

$$\pi_2 = 1.486 \times 10^{-2}$$

$$\pi_3 = 1.331 \times 10^{-4}$$

$$\pi_4 = 2.207 \times 10^{-5}$$

$$\pi_5 = 4.68 \times 10^{-6}$$

$$\pi_6 = 1.343 \times 10^{-6}$$

$$\pi_7 = 4.419 \times 10^{-7}$$

$$\pi_8 = 0.02887$$

Prob 2

Total loss per customer per period = Loss due to bankruptcies + Loss due to account termination

$$\bullet \text{ Loss due to bankruptcies} = \sum_{k=0}^6 \pi_k q_k b_k$$

$$= 36.5530647676$$

$$\bullet \text{ Loss due to account termination} = \pi_6 p_6 b_6$$

$$= 1.89122004 \times 10^{-3}$$

$$\boxed{\text{Total} = \$36.55495599 \text{ per customer per period}}$$

$$\boxed{\text{Annual loss per customer} = \$438.6594719}$$

(35.55 x 12)
since period
= 1 month

$$\text{Total customers accounts} = 14 \times 10^6$$

$$\boxed{\text{Total annual loss} = \$6,141,232,607}$$

(≈ \$6.14 billion)

Prob 3

SPI pays 75% b_k

Policy P_m : Terminate account if customer misses m payments in a row.

$$\text{Bankruptcy loss} = \sum_{k=0}^{m-1} \pi_k q_k b_k$$

$$\text{Termination loss} = \pi_{m-1} \times P_{m-1} \times (0.25) b_{m-1}$$

$$\text{Retainer fee} = \$50,000$$

m	1	2	3	4	5	6	7
Bankruptcy losses	35.0190009251	36.378523	36.522374	36.543156	36.549566	36.552618	36.55361
Termination losses	9.63022525	0.7768711	0.087477	0.016966	0.0010185	0.00131195	0.0004728
Total	44.649	37.1553	36.60984	36.56014	36.5506	36.5539	36.5539
Final total (Add arkim fee)	44.6492	37.15538	36.611	36.56	36.5511	36.5511	36.5521

~~For all m = 1, 2, ..., 7~~

For m = 7, Total = 36.5539
which is less than 36.5545

~~Total loss with SPI services > Total loss without SPI services~~

~~VisaRupay should not take SPI services~~

VisaRupay should take SPI services and hand over accounts when m = 6

Prob 4

Similar analysis needs to be done

* Bankruptcy losses will be same

* Termination losses will be 100% instead of 25%.

Termination losses = [38.5209, 3.107, 0.3448, 0.0678, 0.0163, 0.0052, 0.00189]

Total: [73.5399, 39.486, 36.8722, 36.611, 36.5659, 36.5572, 36.5541]

m = 7

Prob 5

~~Cost~~ Cost per call = \$1

$$p_0 = 0.325 \rightarrow 0.25$$

\therefore ~~DTM~~ TPM will change

$\therefore \pi_i$'s will change

$$\pi_c = 0.25 \pi_0 = \$ 3.57799015 \times 10^{-7}$$

$$\cancel{\pi_0} \quad \pi_0 (1 + 0.03 + \dots + 0.03) = 1$$

$$\Rightarrow \pi_0 = \frac{1}{1.065115} = 0.9385101503$$

$$\begin{aligned} \text{Loss due to bankruptcies} &= \sum_{k=0}^6 \pi_k q_k b_k \\ &= 36.5530686456 \end{aligned}$$

$$\begin{aligned} \text{Loss due to termination} &= \pi_0 p_0 b_0 \\ &= 0.001437 \end{aligned}$$

$$\text{Total} = \$ 36.5545059491$$

$$\begin{aligned} \text{Calling cost} &= 1 \times \pi_c = 1.34319282 \times 10^{-6} \\ \text{per customer} \end{aligned}$$

$$\boxed{\text{Final total cost} = \$ 36.5545072923}$$

which is less than the cost in Part 2.

\therefore Policy of calling customer is cost-effective

Prob 6

$$\Phi_{K2}(\text{new}) = 1.5 * \Phi_{K2}$$

\therefore TPM will change

$$\pi_{D, \text{new}} = 1.5 * \pi_0 = 0.04614257849 \pi_0$$

$$\pi_0 = \frac{1}{1.08089955} = 0.925155349$$

Total loss: [61.27422, 54.55712, 54.09023, 54.05233,
(with SPI) 54.048233, 54.049134, 54.049855]

$$\begin{array}{l} \text{Total loss} \\ \text{(without SPI)} \end{array} \Bigg|_{m=7} = 54.05125$$

$$\text{Total loss (with SPI)} \Bigg|_{m=5} = 54.048231$$

VisRuPay should engage the services of SPI
and hand over accounts at $m=5$

Prob 7

$$\text{Bankruptcy loss} = \sum_{k=0}^{m-1} \pi_k b_k b_k$$

$$\text{Termination loss} = \pi_{m-1} * p_{m-1} * (0.4) b_{m-1}$$

$$\text{Retainer fee} = 0$$

$$\text{Total loss} = \begin{bmatrix} 50.427, 37.62151, 36.6623, 36.57038, \\ 36.556108, 36.55411, 36.5538 \end{bmatrix}$$

(with agency)

~~Agency share~~

$$\text{Min Cost with SPI} = \$ 36.5535$$

$$\text{Min Cost with Agency} = \$ 36.55411$$

Not better than SPI

But can be hired, as it does not exceed total cost

$$m = 6$$